Parameter Estimation by Solving Multivariate Polynomial System: A Synchronous Machine Example

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Abstract— **This paper describes a parameter estimation algorithm applicable for the model structures in the form of multivariate polynomial systems. An example of a synchronous machine nonlinear model is used throughout the paper to explain the contribution. The fundamental in the proposed algorithm is the idea of reformulating the least squares estimation problem having the polynomial model into a numerical linear algebra problem. Recorded signals are represented using the Lagrange interpolation at the Chebyshev points which allows accurate computation of signal derivatives and numerical integration as both are required in the algorithm. The paper explores sensitivity of the algorithm to the power of recorded signals (i.e. excitation intensity) and discusses impact of roundoff error.**

I. INTRODUCTION

ESTIMATING free parameters of a given nonlinear mathematical model structure from recorded signals is of the fundamental importance in many practical control problems. A model structure and characteristics of recorded signals (excitation) needs to be analysed to understand if the problem has global solution [1]. Algebraic methods were found very powerful in determining global identifiability [1] as well as in design of the estimation algorithms for various practical control tasks [2, 3, 4]. The contribution of this paper follows those ideas in applying the algebraic geometry [5], which are in general, a form of transformation of a nonlinear model represented with polynomial equations to a problem in linear algebra. The algorithm discussed in this paper is specially designed for parameter estimation of a synchronous machine connected to a power system represented by an infinite-power bus. The methodology presented through this example can be implemented in other estimation tasks and for more complex models. The model structure is formulated in such way that there are no unknown time-varying quantities and free parameters are constant for a specified data window. This is achieved by reformulating the equations so that only measured time-varying quantities enter the model equations. The model is globally identifiable as it can be rearranged as a linear regression [1].

An alternative approach in solving similar problems is simultaneous state and parameter estimation and it has been discussed before [6, 7]. It is based on discretization at the Chebyshev points, linearization of the model and application of Newton iterations. Integral formulation [7] has some numeric computation advantages if the model is of higher order. Algebraic approach presented in this paper avoids use of model linearization and Newton iterations and it is able to find global solution without worrying about initial point for Newton iterations.

In addition to measured signals, the algorithm presented in this paper requires signal derivatives. Care must be taken when computing derivatives from noisy signals. Practical signal differentiator must control the amount of noise through filtering [8, 9]. The differentiation technique presented in this contribution is based on spectral differentiation at the Chebyshev points as discussed in [6] and it is assumed that filtering is implemented in pre-processing stage.

The paper proposes rearrangement of the synchronous machine model into the overdetermined polynomial system. The least squares objective function is formulated to fit the model to data. Optimality condition leads to the nonlinear eiganvalue problem which is again the polynomial system but this time with the same number of equations and variables. The problem is transformed into the companion form and solved by using the standard linear eigenvalue technique. This contribution demonstrates how the specific estimation problem represented with polynomials can be solved using numerical linear algebra. In general, any *regular* polynomial system having as many equations as variables and with a finite number of zeros which are isolated can be transformed into a matrix eigenproblem [10].

The remainder of this paper is organized as follows: the rearrangement of the synchronous machine model into the overdetermined polynomial system structure is presented in Section 2; then in Section 3 we discuss the signal differentiation algorithm, formulation of the regular polynomial system resulting from the optimality condition of the least squares objective function and the solution method based on transformation of the system into a linear eigenvalue problem; the final section uses a simulation example to demonstrate accuracy of the method, but more importantly, discusses possible problems with numerical conditioning when the excitation signals are of low power.

II. THE MODEL STRUCTURE

A simple, so called "classical model", of a synchronous

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machine can be used in studying electromechanical response of a machine after a disturbance in power system [11]. The same model is usefully in the inverse setting: use measurements to estimate the parameters describing machine dynamic response, such as inertia constant and damping coefficient. The model for parameter estimation needs to be structured in such a way that the parameters we are estimating are constant in a specified time interval. In addition, we need to be able to record in the same time interval all time-varying quantities appearing in the model. In a sequel we reorganize the classical model to fulfil these requirements.

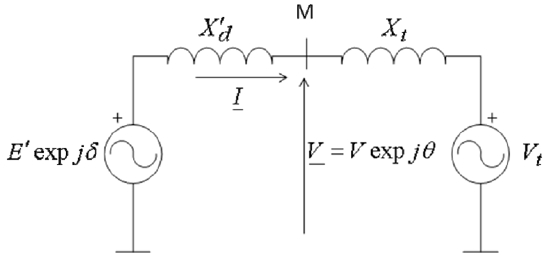

Fig. 1. The circuit representing a synchronous machine connected to a power system at the bus M where phasors of voltage $\underline{V}$ and current $\underline{I}$ are measured. The circuit comprises of the classical synchronous machine model [11] and the Thevenin's equivalent representing a power system.

The classical model of a synchronous machine connected to a power system represented with an infinite-power bus is shown in Fig. 1. Losses are neglected in this model. The infinite-power bus is a typical representation of a power system consisting of large number of machines with total power much larger then power of a connected machine. Thevenin's equivalent with constant parameters X_t and V_t represents the power system. This model is in the phasor (single-frequency) domain and assumes symmetrical three-phase system. Phase angle of the Thevenin's voltage V_t is set to be angle reference in the model. The classical machine model is derived from the constant flux linkage model by neglecting transient saliency, i.e. the transient reactances in d and q axis are equal: $X'_d = X'_q$ [11]. The phase angle δ of the internal transient voltage E' is used to represent rotor angle (the angle between the q-axis rotating with constant speed proportional to the nominal system frequency and the reference network voltage V_t). This is assumption valid only for short time period after dynamic response is initiated [11]. In addition X'_d and E' are assumed constant for the same time period [11]. Dynamics of the rotor angle δ is modelled using per unit version of the second Newton's low [11]:

$$\dot{\delta} = \omega, \qquad (1)$$

$$2H\dot{\omega} = -D\omega + \omega_0(P_m - P), \qquad (2)$$

where ω [rad/s] is the angular rotor speed relative to the synchronous speed $\omega_0 = 2\pi(50Hz)$, P_m is the per unit mechanical input power, P is the per unit active power delivered to the system, H [s] is the inertia constant and D is the damping coefficient in per unit.

In the estimation algorithm, we use sampled measurements recorded in a data window spanning the time interval immediately after disturbance inception when all mentioned assumptions are fulfilled. The machine connection point is the measurement point denoted by M in Fig. 1. Samples of positive sequence current $\underline{I}$ and voltage $\underline{V}$ phasors are recorded at the point M according to the IEEE Standard for Synchrophasors [12].

The complex power measured at the point M can be derived in terms of E' and V as,

$$\underline{S} = \underline{V}\,\underline{I}^* = \underline{V}\left(\frac{\underline{E}' - \underline{V}}{jX'_d}\right)^* = j\frac{VE'}{X'_d}\exp j(\theta - \delta) - j\frac{V^2}{X'_d}, \qquad (3)$$

where $(.)^*$ represents complex conjugate. The equations for active and reactive powers measured at M are obtained after separating (1) in real and imaginary parts:

$$P = \frac{E'V}{X'_d}\sin(\delta - \theta) \quad \text{and} \qquad (4)$$

$$Q = \frac{E'V}{X'_d}\cos(\delta - \theta) - \frac{V^2}{X'_d}. \qquad (5)$$

The first derivative of the active power equation (4),

$$\dot{P} = \frac{E'\dot{V}}{X'_d}\sin(\delta - \theta) + (\omega - \dot{\theta})\frac{E'V}{X'_d}\cos(\delta - \theta),$$

is transformed using expressions (4) and (5) in such a way that δ is eliminated,

$$\dot{P} = \frac{\dot{V}}{V}P + (\omega - \dot{\theta})(Q + \frac{V^2}{X'_d}),$$

and then rearranged to get a formula which gives unmeasured ω in function of measured values and X'_d:

$$\omega = \dot{\theta} + \left(Q + \frac{V^2}{X'_d}\right)^{-1}\left(\dot{P} - \frac{\dot{V}P}{V}\right). \qquad (6)$$

Replacing (6) and its first derivative in the motion equation (2), we obtain the desired structure of the model:

$$\left(A_0^T + A_1^T y_1 + A_2^T y_1^2\right)\begin{bmatrix}1\\ y_2\\ y_3\\ y_4\end{bmatrix} = 0, \qquad (7)$$

where

$$A_0 = -V^4\left[P, \quad -1, \quad \dot{\theta}, \quad \ddot{\theta}\right]^T,$$

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$$A_1 = -V^2\Bigg[2PQ, \quad -2Q, \quad \dot{P} - \frac{\dot{V}}{V}P + 2\dot{\theta}Q,$$

$$\ddot{P} - 3\dot{P}\frac{\dot{V}}{V} + P\left(3\left(\frac{\dot{V}}{V}\right)^2 - \frac{\ddot{V}}{V}\right) + 2\ddot{\theta}Q\Bigg]^T,$$

$$A_2 = -Q\Bigg[PQ, \quad -Q, \quad \dot{P} - \frac{\dot{V}}{V}P + \dot{\theta}Q,$$

$$\ddot{P} - \dot{P}\left(\frac{\dot{V}}{V} + \frac{\dot{Q}}{Q}\right) - P\left(\frac{\ddot{V}}{V} - \left(\frac{\dot{V}}{V}\right)^2 - \frac{\dot{V}\dot{Q}}{VQ}\right) + \ddot{\theta}Q\Bigg]^T.$$

It should be noted that the entries in the vector functions A_0, A_1, A_2 are obtained from the measured quantities and their derivatives. Four unknown parameters are: $y_1 = X'_d$, $y_2 = P_m$, $y_3 = D/\omega_0$, $y_4 = 2H/\omega_0$.

III. ESTIMATION ALGORITHM

A. *Interpolation and Derivatives of the Recorded Signals*

The estimation algorithm should find parameters y_{1-4} by fitting the model (7) to the recorded data stored in the matrices (A_0, A_1, A_2). The model is valid for a short time interval $[0, T]$ after disturbance inception time ($t = 0$) and the measurements are obtained at the equispaced sampling times t_s within this data window: $0 \le t_s \le T$. To construct data matrices (A_0, A_1, A_2) we need the signals $z_1 = P$, $z_2 = Q$, $z_3 = V$, $z_4 = \theta$, which are derived from the synchronized current and voltage phasor measurements. Derivatives of those signals are also required. It is assumed that the filtering has been implemented by the phasor measurement instrument or in pre-processing stage and will not be discussed here. The instrument tracks only the fundamental frequency while all other frequencies and noise are sufficiently attenuated. Computation of derivatives after filtering stage is critical part of the estimation algorithm.

We propose for this task highly accurate and reliable method based on the Lagrange interpolation at the Chebyshev points [6, 7]. The equispaced signals are locally interpolated via the spline algorithm to find values at the $n+1$ Chebyshev points. The Chebyshev sampling points x_j are the extreme points of the Chebyshev polynomial of the first kind and degree n including the boundary points $x_0 = -1$ and $x_1 = 1$: $x_j = -\cos(j\pi/n)$, $j = 0, 1, \ldots, n$. The data window $[0, T]$ is mapped to the computation domain $[-1, 1]$ via $x = 2t/T - 1$. The Lagrange interpolation of measurements at the Chebyshev points within the computation interval $-1 \le x \le 1$ is [13]:

$$z_k(x) = \textstyle\sum_{j=0}^{n} z_k(x_j)\,\phi_j(x), \quad k = 1, \ldots, 4. \tag{8}$$

where $\phi_j(x) = \dfrac{c_j}{x - x_j} \Big/ \displaystyle\sum_{p=0}^{n} \dfrac{c_p}{x - x_p}$, and (9)

$c_j = (-1)^j/2$ for $j = 0, n$ and $c_j = (-1)^j$ otherwise.

The main advantage of the global interpolation at the Chebyshev points is their higher density at the interval boundaries. Interpolation at equispaced points is affected by Runge phenomenon where interpolation error in the proximity of boundaries increases exponentially with the increase of sampling rate. On the contrary, if we use the Chebyshev points, interpolation error will decrease exponentially with the sampling rate increase. This is very important feature for our application where short-duration signals containing transients are used for parameter estimation. We multiply signal interpolant (i.e. signal values tabulated at the Chebyshev points) by a matrix $\mathbf{D}$ to compute the interpolant of the signal derivative:

$$\begin{bmatrix} \dot{z}_k(x_0) \\ \dot{z}_k(x_1) \\ \vdots \\ \dot{z}_k(x_n) \end{bmatrix} = \mathbf{D} \begin{bmatrix} z_k(x_0) \\ z_k(x_1) \\ \vdots \\ z_k(x_n) \end{bmatrix}, \quad k = 1, \ldots, 4, \tag{10}$$

where the formulas for the entries of $\mathbf{D}$ are derivatives of $\phi_j(x_i)$ defined in (9):

$$\mathbf{D}_{ij} = \dot{\phi}_j(x_i) = \frac{c_j/c_i}{x_i - x_j}, \text{ and}$$

$$\mathbf{D}_{jj} = \dot{\phi}_j(x_j) = -\textstyle\sum_{i \ne j} \mathbf{D}_{ij}, \quad i = 0, 1, \ldots, n, \quad j = 0, 1, \ldots, n.$$

To compute second derivative we multiply a signal interpolant two times by a matrix $\mathbf{D}$.

B. *Matrix Polynomial Equation*

The objective function of the least-squares estimation problem with the model (7) is:

$$J = \mathbf{v}^T \left[\textstyle\int_{-1}^{1} \mathbf{A}(y_1)\,\mathbf{A}(y_1)^T\,dx\right] \mathbf{v}, \tag{11}$$

where $\mathbf{A}(y_1)^T = A_0^T + A_1^T y_1 + A_2^T y_1^2$ and

$$\mathbf{v} = [1 \quad y_2 \quad y_3 \quad y_4]^T.$$

The optimality condition $\partial J / \partial \mathbf{v} = 0$ for fixed y_1 results in the following system of four polynomial equations:

$$(\,\mathbf{C}_0 + \mathbf{C}_1 y_1 + \mathbf{C}_2 y_1^2 + \mathbf{C}_3 y_1^3 + \mathbf{C}_4 y_1^4\,)\,\mathbf{v} = 0, \tag{12}$$

where all matrices $\mathbf{C}_i$, $i = 1, \ldots, 4$, are symmetric and of the size 4×4. Their entries are calculated using the Clenshaw-Curtis numerical integration [14]:

$$\mathbf{C}_0 = \int_{-1}^{1} A_0\, A_0^T \; dx\,,$$

$$\mathbf{C}_1 = \int_{-1}^{1} \left(A_0\, A_1^T + A_1\, A_0^T \right) dx\,,$$

$$\mathbf{C}_2 = \int_{-1}^{1} \left(A_0\, A_2^T + A_1\, A_1^T + A_2\, A_0^T \right) dx\,,$$

$$\mathbf{C}_3 = \int_{-1}^{1} \left(A_1\, A_2^T + A_2\, A_1^T \right) dx\,, \text{ and}$$

$$\mathbf{C}_4 = \int_{1}^{1} A_2\, A_2^T \; dx\,.$$

The application of the Clenshaw-Curtis numerical integration is the natural choice since the entries in the matrices (A_0, A_1, A_2) are the interpolants at the Chebyshev points [14].

For the parameter $y_1 > 0$ the objective function (11) is globally nonnegative: $J \geq 0$ for $\forall\, \mathbf{v} \in \mathbf{R}^4$. In the nonlinear eigenvalue problem (12) there is a nonzero vector $\mathbf{v}$ such that $\mathbf{C}_0 + \mathbf{C}_1 y_1 + \mathbf{C}_2 y_1^2 + \mathbf{C}_3 y_1^3 + \mathbf{C}_4 y_1^4$ is singular. Roots of $\det(\mathbf{C}_0 + \mathbf{C}_1 y_1 + \mathbf{C}_2 y_1^2 + \mathbf{C}_3 y_1^3 + \mathbf{C}_4 y_1^4)$ correspond to the eigenvalues of the matrix (named $\mathbf{C} \in \mathbf{R}^{16\times16}$) in the following companion form:

$$-\underbrace{\begin{bmatrix} 0 & -\mathbf{I} & 0 & 0 \\ 0 & 0 & -\mathbf{I} & 0 \\ 0 & 0 & 0 & -\mathbf{I} \\ \mathbf{C}_4^{-1}\mathbf{C}_0 & \mathbf{C}_4^{-1}\mathbf{C}_1 & \mathbf{C}_4^{-1}\mathbf{C}_2 & \mathbf{C}_4^{-1}\mathbf{C}_3 \end{bmatrix}}_{\mathbf{C}} \begin{bmatrix} \mathbf{v} \\ y_1\mathbf{v} \\ y_1^2\mathbf{v} \\ y_1^3\mathbf{v} \end{bmatrix} = y_1 \begin{bmatrix} \mathbf{v} \\ y_1\mathbf{v} \\ y_1^2\mathbf{v} \\ y_1^3\mathbf{v} \end{bmatrix}. \qquad (13)$$

In this way, the polynomial system (12) is reduced to the linear eigenvalue problem. If the matrix $\mathbf{C}_4$ is singular or ill-conditioned we can formulate the generalized eigenvalue problem [15]. The eigenvalue problem is solved using MATLAB function "eig" which implements LAPACK routines [16]. In this problem, a maximum real and positive eigenvalue is the candidate solution for y_1 and all corresponding y_{2-4} are obtained from the eigenvector. In general, solution of the parameter estimation problem can be found by checking the values of the objective function (11) for all feasible candidate solutions. It is possible to refine the solution by using the Newton iterative method.

Although the model (7) is globally identifiable, accuracy of the estimation is greatly affected by measurement noise and roundoff errors when there is not sufficient intensity of excitation. The non-symmetrical matrix in the companion form (13) usually has poorly conditioned eigenvalues. Small perturbations in the matrix due to measurement noise and roundoff errors will lead to large perturbations in the eigenvalues. The sensitivity of the estimation algorithm to the intensity of excitation will be tested in the next section.

IV. SIMULATION EXAMPLE AND DISCUSSION

Accuracy of the proposed parameter estimation method is largely affected by roundoff errors. In addition the measurement noise will be amplified via differentiation and can influence the algorithm considerably. We discussed before possible methods to take care of the problem of noise amplification. These methods are either based on the use of the Adaptive Differentiation Filter [17] or by reformulating the model differential equation into an integral equation [7]. The effect of roundoff errors and noise is strongly linked to the intensity of excitation. For low power excitation the effect of roundoff errors and noise becomes intolerable.

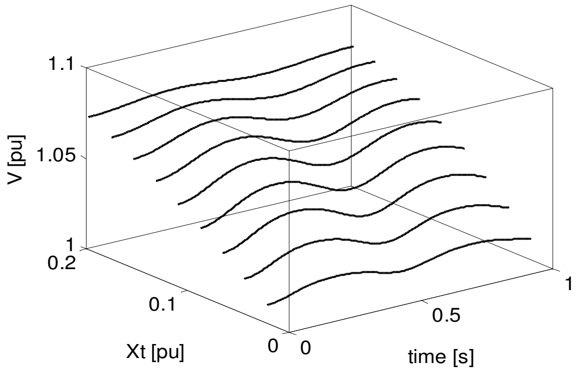

Fig. 2. The voltage magnitude signal recording in function of X_t.

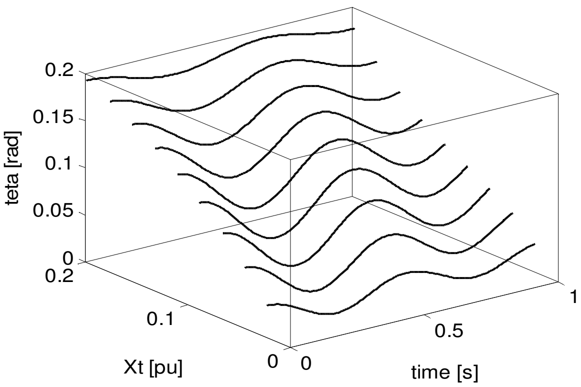

Fig. 3. The voltage phase angle signal recording in function of X_t.

To demonstrate the impact of the intensity of excitation, we devise a simple simulation experiment based on the circuit in Fig. 1 and the differential equations (1) and (2). The parameters of the pre-disturbance power system represented with the infinite-power busbar are: $\underline{V}_t = (1\; pu)\exp(j0\; rad)$ - Thevenin's equivalent voltage source and $X_t = 0.22\; pu$ - Thevenin's equivalent reactance. We simulate disturbances in the power system by changing X_t while $\underline{V}_t$ is kept constant. The series of 9 disturbances, from small to large, are simulated by changing X_t from the steady-state pre-disturbance value $0.22\; pu$ to the values $(0.9:0.1:0.1)\times 0.22\; pu$. The power of excitation will increase with decrease of X_t.

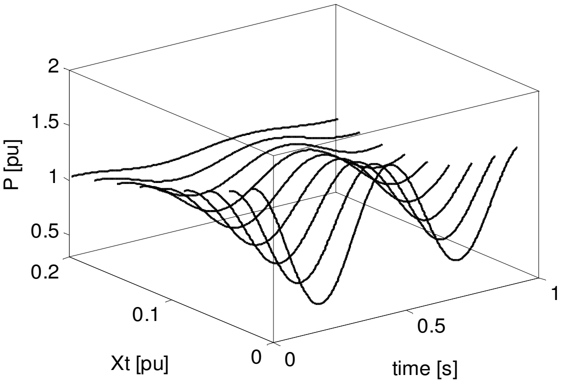

Fig.4. The active power signal recording in function of X_t.

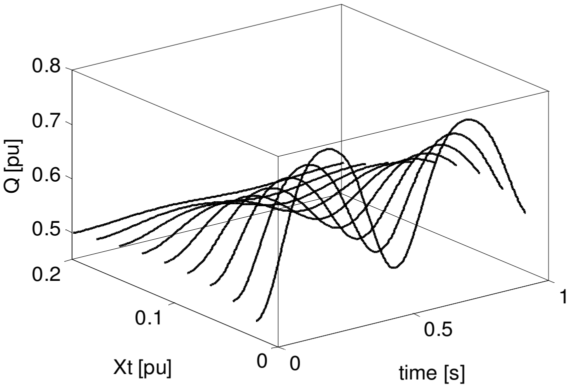

Fig.5. The reactive power signal recording in function of X_t.

The internal voltage source of the machine ($E' = 1.248\ pu$) is assumed constant in the first 1s after inception of a disturbance; as a consequence the parameter estimation algorithm is implemented for 1s data window. The voltage E' is eliminated when formulating the model structure (7). The parameters used in simulation and we want to estimate by fitting data to the structure (7) are: $X'_d = 0.3\ pu$, $P_m = 1\ pu$, $D = 5\ pu$ and $H = 4\ s$. The initial conditions of the differential equations (1) and (2) are:

$$\delta_0 = \sin^{-1}\frac{(X'_d + X_t)\,P_m}{E'\,V_t}\ rad \quad \text{and} \quad \dot{\delta}_0 = 0\ rad/s.$$

We reformulate the differential equations (1) and (2) by replacing the circuit expression for the electrical power,

$$2H\ddot{\delta} = -D\dot{\delta} + \omega_0 P_m - \omega_0 \frac{E'\,V_t}{X'_d + X_t}\sin\delta,$$

and solve them for a number of step changes of X_t that are simulating disturbances of various intensities. Measurements at the bus M in Fig. 1 are obtained by solving the circuit. We used the adaptive step size ODE solver to achieve the highest accuracy within the 1s data window and then by using a spline algorithm we find interpolants of the measurements (P, Q, V, θ) at the Chebushev points. The interpolants are differentiated using (10) and the data matrices (A_0, A_1, A_2) are composed. Since all entries of these matrices are Chebyshev interpolants, we use the Clenshaw-Curtis numerical integration to compute 4×4 matrices $\mathbf{C}_i$, $i = 1,\ldots,4$ and finally, construct the eigenvalue problem (13).

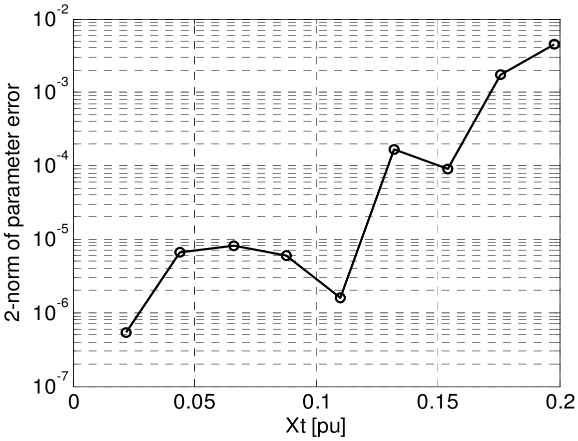

Fig.6. The estimation error in function of X_t.

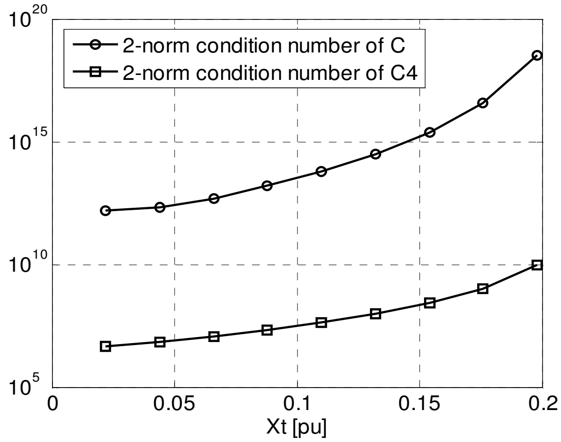

Fig.7. The condition numbers of C and C4 in function of X_t.

The measured signals when X_t is varied are shown in Figures 2 to 5. It can be observed that the intensity of excitation increases when X_t moves away from the steady-state value. The change of X_t simulates real disturbances in power system. Small change can be due to load variation or discrete control actions as for example change of transformer tap-changer position. Large change is due to a fault in transmission system when one or more lines are disconnected. The estimation accuracy is computed as the second norm of the 4×1 vector representing difference between 4 estimated and true parameter values y_{1-4}. It can be seen in Fig. 6 how the accuracy depends on the excitation intensity. The worst accuracy is for X_t close to the steady-state value ($0.9\times0.22\,pu$) and the best for a large disturbance simulated by the large change of X_t ($0.1 \times 0.22\,pu$). The reason for this sensitivity of the algorithm to the intensity of excitation is the roundoff error. In Fig. 7 the condition number of the companion matrix $\mathbf{C}$ of the eigenvalue problem (13) in function of X_t is presented. For small disturbances (X_t close to the pre-disturbance value) the matrix $\mathbf{C}$ has very large condition number and it becomes ill-conditioned which impacts the estimation accuracy. The matrix $\mathbf{C}_4$ also has the condition number increasing to the very large values. The obvious improvement of the algorithm, especially for the estimation using low power signals, would be to avoid inversion of $\mathbf{C}_4$ and solve the generalised eigenvalue problem.

V. CONCLUSIONS

In this paper we proposed the nonlinear parameter estimation technique applied in modelling a synchronous machine connected to a power system. Structure of the model is devised in such a way that all time-varying quantities which are not measured are eliminated. The model is valid for short time period after a disturbance when the machine can be represented with constant internal voltage source behind the reactance. This time period is used to record samples of voltage and current phasor measurements at the machine terminals. These measurements are used to calculate active and reactive power signals, which together with voltage magnitude and phase angle measurements, are forming the set of input signals. All signals are converted into the interpolants at the Chebyshev points and the differentiation of those interpolants has been used to approximate signal derivatives required in the model. It should be noted that we can also formulate the model as an integral equation by successive model integration. Such a form will require numerical integration of signals which has advantage to differentiation in respect to noise filtering. The estimation problem is formulated as the least-square problem and the optimality condition leads to the system of polynomial equations written in the matrix polynomial form (12). The interpolant form of measurements is exploited when calculating coefficients of the polynomial equations. They are calculated with highly accurate Clenshaw-Curtis numerical integration.

Making the parameter estimation to work for globally identifiable machine model when the intensity of excitation is low is essentially the challenge in numerical computation. The approach we have followed is based on transformation of the system with matrix polynomial (12) into an equivalent linear eigenvalue problem (13) which can be solved with standard numerical tools. The simulation experiment was devised to test the method and the results obtained were reported in the paper. We noticed that the problem is not well-conditioned for low power excitation. In this regard, it is worth exploring possibility of using the Chebyshev polynomial bases instead of the monomial basis in the matrix polynomial in (12). Instead of companion matrix formulation of the eigenvalue problem (13), we can use the three-term recurrence for the Chebyshev polynomials to construct the better conditioned colleague matrix formulation [18]. It is expected that the parameter estimation based on the solution of the generalized eigenvalue problem with the colleague matrix structure will be much better conditioned problem. This will be discussed in the next paper.

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