Position funnel control for rigid revolute joint robotic manipulators with known inertia matrix

C. M. Hackl* and R.M. Kennel*

Abstract— **This paper presents funnel control for position control of rigid, revolute joint, n-degree-of-freedom (DOF) robotic manipulators with known inertia matrix. The multi-input multi-output (MIMO) funnel controller assures reference tracking with 'prescribed transient accuracy', i.e., for each joint, the absolute value of position *and* speed tracking error (difference between reference and actual value) is bounded by a prescribed, positive (possibly non-increasing) function of time (the 'funnel boundary'), respectively.**

I. INTRODUCTION

In this paper, funnel control — developed by Ilchmann et al. (see e.g. [1], [2], [3] and references therein) for systems with (asymptotically) stable zero-dynamics[1], known relative degree[2] and known sign of the high-frequency gain[3] — is applied for position control of rigid, revolute joint robotic manipulators. The presented result is a direct consequence and extension of 'funnel control for systems with relative degree two' (see [3]) and its application for position control of stiff one-mass and elastic two-mass systems (see [6]).

This far, for controller implementation, the inertia matrix of the robotic manipulator must be *known*, whereas centrifugal and Coriolis forces, friction, external disturbances (load torques) and gravitational effects may be (completely) *unknown* and, hence, are *not* compensated for. Due to 'Computer Aided Design' (CAD) of the mechanical parts of robotic manipulators, nowadays the structure of the inertia matrix and its parameters are usually available by the CAD software (see e.g. [7]).

Although funnel control may also work in the presence of saturated actuators (see e.g. [8], [3]) or for position control of elastic systems (see e.g. [9], [6]), in this paper, actuator saturation and elasticity in the joints (or the links) are neglected. The proposed MIMO funnel controller assures tracking with 'prescribed transient accuracy', i.e. position and speed tracking errors (differences between references and actual values) of each joint of the considered robotic manipulators evolve within a prescribed 'performance funnel'. Hence, for each joint, the control designer is *a priori* able to guarantee a desired minimum position *and* speed tracking accuracy by adequate design of the 'funnel boundary'. To illustrate the proposed MIMO funnel controller, it is implemented for position control of a planar (elbow-like) two degree-of-freedom (DOF) robotic manipulator and simulation results are shown. The following notation will be used:

$\mathbb{N}, \mathbb{R}, \mathbb{C}$	natural, real and complex numbers
$[a, b)$	interval from a to b (excluded)
$\mathbb{R}_{\geq 0} := [0, \infty)$	set of positive real numbers with zero
$\|\boldsymbol{v}\|$	$:= \sqrt{\boldsymbol{v}^\top \boldsymbol{v}}$, Euclidean norm of $\boldsymbol{v} = (v_1, \dots, v_n)^\top \in \mathbb{R}^n$, $n \in \mathbb{N}$
$\|\boldsymbol{M}\|$	$:= \max_{\|\boldsymbol{v}\|=1} \|\boldsymbol{M}\boldsymbol{v}\|$, induced matrix norm of $\boldsymbol{M} \in \mathbb{R}^{n\times m}$, $n, m \in \mathbb{N}$
$\det(\boldsymbol{M})$	determinant of matrix $\boldsymbol{M} \in \mathbb{R}^{n\times n}$
$\mathrm{diag}(a_1, \dots, a_n)$	diagonal matrix in $\mathbb{R}^{n\times n}$, $n \in \mathbb{N}$
$\boldsymbol{I}_n \in \mathbb{R}^{n\times n}$	$:= \mathrm{diag}(1, \dots, 1)$, the identity matrix
$\mathcal{C}^n(I; Y)$	space of n-times continuously differentiable functions mapping $I \to Y$
$\mathcal{L}^\infty_{(\mathrm{loc})}(I; Y)$	space of measurable, (locally) essentially bounded functions with norm:
$\|\boldsymbol{f}\|_\infty$	$:= \text{ess-sup}_{t\in I} \|\boldsymbol{f}(t)\|$
$\mathcal{W}^{k,\infty}(I; Y)$	space of bounded locally absolutely continuous functions with essentially bounded derivatives $\boldsymbol{f}^{(i)} \in \mathcal{L}^\infty(I; Y)$ for all $i \in \{1, \dots, k\}$ with norm:
$\|\boldsymbol{f}\|_{k,\infty}$	$:= \sum_{i=1}^{k} \|\boldsymbol{f}^{(i)}\|_\infty$

II. FUNNEL CONTROL (WITH DERIVATIVE FEEDBACK)

The idea of tracking with 'prescribed transient accuracy' for systems with asymptotically stable zero-dynamics, known relative degree and known sign of the high-frequency gain was initially presented in [10] utilizing a high-gain based switching controller which 'provides an arbitrarily good transient and steady-state response'. However, this controller invokes non-decreasing gains and switching, both is not desirable for mechatronic application.

Funnel control (see e.g. [1], [2], [3]) is also high-gain based but allows for gain decrease as well. Measurement noise is tolerated. From e.g. the root locus method, it is well known that systems with relative degree two might not be stabilizable by simple high-gain output feedback [3]. For systems with relative degree larger than or equal to two, a funnel controller must be used in conjunction with a compensator yielding a complex controller structure (due to backstepping) and gains occurring with $k(t)^7$ (huge noise amplification) [2]. Both is not reasonable for industrial application. In [11], another result in the spirit of funnel control has been introduced for systems with arbitrary-but-known relative degree. It does not rely on backstepping but requires partial-state feedback. In [12], it has been successfully applied for force/position control of unknown

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[1]For a detailed discussion of zero-dynamics, see [4, Sec. 4.3].

[2]The relative degree $r \geq 0$ of a system indicates which time derivative of the system's output is affected by the control action, see e.g. [4, p. 137].

[3]The 'high-frequency gain' describes the 'directional' effect of the control action on the r-th derivative of the system's output [5, p. 334].

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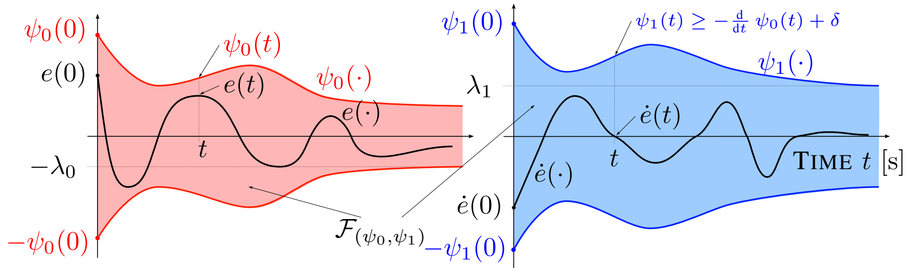

Fig. 1. Performance funnel $\mathcal{F}_{(\psi_0,\psi_1)}$ for error $e(\cdot)$ and derivative $\dot{e}(\cdot)$ with limiting functions $\psi_0(\cdot)$ and $\psi_1(\cdot)$.

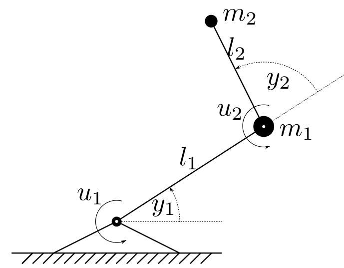

Fig. 2. Planar two DOF rigid revolute joint robotic manipulator.

robotic manipulators. However, the proposed controller is still complex (not 'simple'): it is recursively designed and relies on error transformations (complicating physical intuition). Moreover, the performance bounds are restricted to smooth, strictly decreasing functions.

In [3] and [6] it has been shown that funnel control may be extended to the relative-degree-two case retaining a 'simple' controller if derivative feedback is admissible (justified for position control).

A 'simple' funnel controller with derivative feedback for single-input single-output (SISO), relative degree two systems is given by (see e.g. [6, Theorem 3.3])

$$\left.\begin{array}{l} u(t) = k_0(t)^2\, e(t) + k_0(t)\, k_1(t)\, \dot{e}(t), \quad \text{with} \\ e(t) = y_{\text{ref}}(t) - y(t),\; y_{\text{ref}}(\cdot) \in \mathcal{W}^{2,\infty}(\mathbb{R}_{\geq 0};\mathbb{R}) \\ \text{and } k_i(t) := \dfrac{\varsigma_i(t)}{\psi_i(t) - |e^{(i)}(t)|} \quad \text{for } i \in \{0,1\} \end{array}\right\} \quad (1)$$

where $\varsigma_i(\cdot) \in \mathcal{W}^{1,\infty}(\mathbb{R}_{\geq 0}; [\underline{\varsigma}_i, \infty))$, $\underline{\varsigma}_i > 0$, $i \in \{0,1\}$ are scaling functions which allow for more degrees of freedom in controller design (e.g. a minimal gain may be fixed). Controller (1) utilizes two 'limiting functions' $\psi_0(\cdot)$ and $\psi_1(\cdot)$ for tracking error $e(\cdot)$ and for error derivative $\dot{e}(\cdot)$, respectively (see Fig. 1). In [6, Theorem 3.3] it has been shown that gains $k_0(\cdot)$, $k_1(\cdot)$, error $e(\cdot)$, derivative $\dot{e}(\cdot)$ and control input $u(\cdot)$ stay bounded.

Gain 'adaptation' in (1) is as follows: gain $k_0(\cdot)$ (or $k_1(\cdot)$) increases, if error $e(\cdot)$ (or $\dot{e}(\cdot)$) draws close to $\psi_0(\cdot)$ (or $\psi_1(\cdot)$) (more aggressive control) and decreases, if error $e(\cdot)$ (or $\dot{e}(\cdot)$) becomes small (more relaxed control).

The functions $\psi_0(\cdot)$ and $\psi_1(\cdot)$ are chosen from the set

$$\mathcal{B}_2 := \left\{ (\psi_0, \psi_1)\colon \mathbb{R}_{\geq 0} \to \mathbb{R}^2 \;\middle|\; \begin{array}{l} \text{(i) } \forall\, i \in \{0,1\}\; \exists\, c_i > 0\colon \psi_i(\cdot) \in \mathcal{W}^{1,\infty}(\mathbb{R}_{\geq 0}, [c_i,\infty)) \\ \text{(ii) } \exists\, \delta > 0 \text{ for a.a. } t \geq 0\colon \psi_1(t) \geq -\frac{\mathrm{d}}{\mathrm{d}t}\psi_0(t) + \delta \end{array} \right\}. \quad (2)$$

If $|e(0)| < \psi_0(0)$ and $|\dot{e}(0)| < \psi_1(0)$ (initial errors 'start' inside the funnel), for systems with stable zero-dynamics, relative degree two and known sign of the high-frequency gain, controller (1) assures tracking with 'prescribed transient accuracy': error $e(\cdot)$ and its derivative $\dot{e}(\cdot)$ evolve within the *prescribed* 'performance funnel' (see Fig. 1)

$$\mathcal{F}_{(\psi_0,\psi_1)} := \{\, (t, e_0, e_1) \in \mathbb{R}_{\geq 0} \times \mathbb{R} \times \mathbb{R} \;\mid\; |e_0| < \psi_0(t) \quad \text{and} \quad |e_1| < \psi_1(t) \,\} \quad (3)$$

(i.e. $(t, e(t), \dot{e}(t)) \in \mathcal{F}_{(\psi_0,\psi_1)}$ for all $t \geq 0$) with 'funnel boundary' $\partial\mathcal{F}_{(\psi_0,\psi_1)}(t) = (\psi_0(t), \psi_1(t))$ for all $t \geq 0$. Condition (i) in (2) implies that both limiting functions $\psi_0(\cdot)$ and $\psi_1(\cdot)$ are (absolutely) continuous and differentiable almost everywhere (with essentially bounded derivative). The asymptotic accuracies of the limiting functions (see Fig. 1) are given by

$$\lambda_0 := \liminf_{t\to\infty} \psi_0(t) \qquad \text{and} \qquad \lambda_1 := \liminf_{t\to\infty} \psi_1(t).$$

In most applications non-increasing performance funnels are desirable, however the boundary may increase as well. This might be reasonable if, due to large reference changes or sensor calibration/reset, error or its derivative will increase drastically leading to unacceptably large control actions. Condition (ii) in (2) is essential: only if an error derivative with $\operatorname{sign}(e(t))\dot{e}(t) < \frac{\mathrm{d}}{\mathrm{d}t}\psi_0(t)$ is admissible, then the error $e(t)$ 'may depart' from 'sub-boundary' $\psi_0(t)$ (see Fig. 1).

Example 2.1: Let $\Lambda_0 \geq \lambda_0 > 0$, $T_E > 0$ [s] and $\lambda_1 > 0$ [s], then a proper choice of a funnel boundary is given by

$$\begin{aligned} (\psi_0,\, \psi_1)\colon \mathbb{R}_{\geq 0} &\to (\lambda_0, \Lambda_0] \times (\lambda_1,\, (\Lambda_0 - \lambda_0)/T_E] \\ t &\mapsto \begin{pmatrix} (\Lambda_0 - \lambda_0)\exp(-t/T_E) + \lambda_0 \\ \frac{\Lambda_0 - \lambda_0}{T_E}\exp(-t/T_E) + \lambda_1 \end{pmatrix}. \end{aligned} \quad (4)$$

Boundary (4) is positive, non-increasing, bounded and smooth. Its asymptotic accuracies are given by $(\lambda_0,\, \lambda_1)$. The boundary 'starts' at $(\Lambda_0,\, \frac{\Lambda_0-\lambda_0}{T_E} + \lambda_1)$ and its derivative is essentially bounded by $\big((\Lambda_0 - \lambda_0)/T_E,\, (\Lambda_0 - \lambda_0)/T_E^2\big)$. By setting $\delta := \lambda_1$ and noting that $\psi_1(t) \geq -\dot{\psi}_0(t) + \delta$ for (almost) all $t \geq 0$ in (4), it is easy to see that boundary (4) is element of $\mathcal{B}_2$.

Controller (1) has been successfully applied for position control of stiff and elastic electrical drives (see [6]). It is a slight modified version of the originally introduced funnel controller $u(t) = k_0(t)^2\, e(t) + k_1(t)\, \dot{e}(t)$ in [3]. The modification in (1) allows for a better damped closed-loop system response (without overshoot, see [6]). A survey of feasible applications (with measurements) of funnel control for speed and position control of electrical drives and a comparison with 'standard' PI/PID controllers is given in [13].

III. Position Funnel Control of Rigid Revolute Joint Robotic Manipulators

In this section, it will be shown that (SISO) funnel controller (1) may be extended to the MIMO case and applied for position control of rigid, revolute joint robotic manipulators if the inertia matrix is known.

A. Model of robotic manipulator

In the following, let $n \in \mathbb{N}$ and consider a n-DOF robotic manipulator, given by the functional differential equation (see e.g. [14, p. 77], there without dynamic friction term)

$$\boldsymbol{M}(\boldsymbol{y}(t))\ddot{\boldsymbol{y}}(t) + \big(\boldsymbol{C}(\boldsymbol{y}(t),\dot{\boldsymbol{y}}(t)) + \boldsymbol{C}_V\big)\dot{\boldsymbol{y}}(t) + (\mathfrak{F}\dot{\boldsymbol{y}})(t) + \boldsymbol{g}(\boldsymbol{y}(t)) + \boldsymbol{d}(t) = \boldsymbol{u}(t), \quad \begin{pmatrix}\boldsymbol{y}(0)\\ \dot{\boldsymbol{y}}(0)\end{pmatrix} = \begin{pmatrix}\boldsymbol{y}^0\\ \boldsymbol{y}^1\end{pmatrix} \in \mathbb{R}^{2n}, \quad (5)$$

where $\boldsymbol{y}(t)$ in $[\mathrm{rad}]^n$ and $\dot{\boldsymbol{y}}(t)$ in $[\mathrm{rad/s}]^n$ represent position and speed (vector) at time $t \geq 0$, respectively. $\boldsymbol{M}(\cdot) \in \mathcal{C}(\mathbb{R}^n;\mathbb{R}^{n\times n})$ is the position dependent inertia matrix. Matrix $\boldsymbol{C}(\cdot,\cdot) \in \mathcal{C}(\mathbb{R}^n \times \mathbb{R}^n;\mathbb{R}^{n\times n})$ is the position and speed dependent centrifugal and Coriolis force matrix. $\boldsymbol{d}(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0};\mathbb{R}^n)$ represents an exogenous disturbance and $\boldsymbol{g}(\cdot) \in \mathcal{C}(\mathbb{R}^n;\mathbb{R}^n)$ is the position dependent gravity vector. The robot is actuated by joint torque vector $\boldsymbol{u}$ $[\mathrm{Nm}]^n$ (control input). Matrix $\boldsymbol{C}_V := \mathrm{diag}\{\nu_1, \ldots, \nu_n\} \in \mathbb{R}^{n\times n}$ with $\nu_1, \ldots, \nu_n \geq 0$ represents unbounded viscous friction effects and operator $\mathfrak{F}\colon \mathcal{C}(\mathbb{R}_{\geq 0};\mathbb{R}^n) \to \mathcal{L}^\infty(\mathbb{R}_{\geq 0};\mathbb{R}^n)$ allows to model bounded but nonlinear and dynamic friction effects (e.g. as covered by the LuGre friction model [15]). In [16] and [6], it has been shown (by experiments) that this separation into unbounded viscous part $\boldsymbol{C}_V\dot{\boldsymbol{y}}$ and bounded but nonlinear and dynamic part $\mathfrak{F}\dot{\boldsymbol{y}}$ is a reasonable assumption for speed or position control.

The following assumptions are imposed on model (5):

(A_1) the inertia matrix is uniformly bounded from above and below (see e.g. [17]), i.e. $\exists\, \overline{c}_{\boldsymbol{M}}, \underline{c}_{\boldsymbol{M}} > 0\ \forall \boldsymbol{y} \in \mathbb{R}^n:$

$$0 < \underline{c}_{\boldsymbol{M}} \boldsymbol{I}_n \leq \boldsymbol{M}(\boldsymbol{y}) = \boldsymbol{M}(\boldsymbol{y})^\top \leq \overline{c}_{\boldsymbol{M}} \boldsymbol{I}_n;$$

(A_2) the centrifugal and Coriolis force matrix is upper bounded (see e.g. [14, Sections 4.2]) as follows

$$\exists\, c_{\boldsymbol{C}} > 0\ \forall \boldsymbol{y},\boldsymbol{v},\boldsymbol{w} \in \mathbb{R}^n:\ \|\boldsymbol{C}(\boldsymbol{y},\boldsymbol{v})\boldsymbol{w}\| \leq c_{\boldsymbol{C}}\|\boldsymbol{v}\|\|\boldsymbol{w}\|;$$

(A_3) the gravity vector is uniformly bounded (see e.g. [14, Sections 4.3]), i.e. $\exists\, c_{\boldsymbol{g}} > 0\ \forall \boldsymbol{y} \in \mathbb{R}^n:\ \|\boldsymbol{g}(\boldsymbol{y})\| \leq c_{\boldsymbol{g}};$

(A_4) the friction operator is element of operator class[4] $\mathcal{T}$ and globally bounded, i.e. $\mathfrak{F} \in \mathcal{T}$ and

$$M_{\mathfrak{F}} := \sup\{\|(\mathfrak{F}\boldsymbol{\xi})(t)\| \mid t \geq 0,\ \boldsymbol{\xi}(\cdot) \in \mathcal{C}(\mathbb{R}_{\geq 0},\mathbb{R}^n)\} < \infty;$$

(A_5) the exogenous disturbance is bounded, i.e. $\boldsymbol{d}(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0};\mathbb{R}^n)$ and

(A_6) feedback of position $\boldsymbol{y}(\cdot)$ and speed $\dot{\boldsymbol{y}}(\cdot)$ is admissible.

Assumptions (A_1)-(A_3) are standard properties of rigid robotic manipulators having revolute joints exclusively (see e.g. [14, Sections 4.1-4.3]). Assumptions (A_4)-(A_6) are realistic for position control problems of mechatronic systems (see e.g. [6] and [18, p. 210-213 and 290-292]).

B. Position (MIMO) funnel controller

Funnel control in robotics has been introduced in [9]. Control objective is reference tracking of position reference $\boldsymbol{y}_{\mathrm{ref}}(\cdot) \in \mathcal{W}^{2,\infty}(\mathbb{R}_{\geq 0};\mathbb{R}^n)$. In [9], it could solely been shown that the *linear combination* of position and speed tracking error will evolve within a performance funnel. Hence, as severe drawback, either position error or speed error or both errors may leave the prescribed region. Therefore an extension of (SISO) funnel controller (1) with derivative feedback to robotic manipulators of form (5) is desirable to achieve tracking with prescribed transient accuracy for

$$\boldsymbol{e}(\cdot) = \boldsymbol{y}_{\mathrm{ref}}(\cdot) - \boldsymbol{y}(\cdot) \ \ \textit{and}\ \ \dot{\boldsymbol{e}}(\cdot) = \dot{\boldsymbol{y}}_{\mathrm{ref}}(\cdot) - \dot{\boldsymbol{y}}(\cdot),$$

separately. More precisely, for $(\boldsymbol{\psi}_0(\cdot),\boldsymbol{\psi}_1(\cdot)) \in \mathcal{B}_2^n$ (with $\mathcal{B}_2$ as in (2)), introduce the (MIMO) 'performance funnel'

$$\mathcal{F}_{(\boldsymbol{\psi}_0,\boldsymbol{\psi}_1)} := \big\{(t,\boldsymbol{\xi},\boldsymbol{\eta}) \in \mathbb{R}_{\geq 0} \times \mathbb{R}^n \times \mathbb{R}^n \;\big|\; \forall\, i \in \{1,\ldots,n\}:\ \ |\xi_i| < \psi_{0,i}(t) \wedge |\eta_i| < \psi_{1,i}(t)\big\} \quad (6)$$

and find a (MIMO) funnel controller which assures $(t,\boldsymbol{e}(t),\dot{\boldsymbol{e}}(t)) \in \mathcal{F}_{(\boldsymbol{\psi}_0,\boldsymbol{\psi}_1)}$ for all $t \geq 0$. If inertia matrix $\boldsymbol{M}(\cdot)$ is known, then the result directly follows from a straightforward extension of Theorem 3.3 in [6] to the MIMO case.

Theorem 3.1: Let $n \in \mathbb{N}$ and consider a n-th DOF rigid, revolute joint robotic manipulator of form (5) which satisfies assumptions (A_1)-(A_6). Then, for arbitrary position reference $\boldsymbol{y}_{\mathrm{ref}}(\cdot) \in \mathcal{W}^{2,\infty}(\mathbb{R}_{\geq 0},\mathbb{R}^n)$, funnel boundary $(\boldsymbol{\psi}_0(\cdot),\boldsymbol{\psi}_1(\cdot)) \in \mathcal{B}_2^n$, gain scaling $\varsigma_0(\cdot),\varsigma_1(\cdot) \in \mathcal{W}^{1,\infty}(\mathbb{R}_{\geq 0};[c,\infty)^n)$, $c > 0$ and initial value $(\boldsymbol{y}^0,\boldsymbol{y}^1) \in \mathbb{R}^{2n}$ satisfying $\forall\, i \in \{1,\ldots,n\}:$

$$|y_{\mathrm{ref,i}}(0) - y_i^0| < \psi_{0,i}(0) \ \text{ and } \ |\dot{y}_{\mathrm{ref,i}}(0) - y_i^1| < \psi_{1,i}(0), \quad (7)$$

the MIMO funnel controller

$$\boldsymbol{u}(t) = \boldsymbol{M}(\boldsymbol{y}(t))\Big(\boldsymbol{K}_0(t)^2\boldsymbol{e}(t) + \boldsymbol{K}_0(t)\boldsymbol{K}_1(t)\dot{\boldsymbol{e}}(t)\Big) \quad (8)$$

with gain matrices

$$\boldsymbol{K}_0(t) = \mathrm{diag}\big\{k_{0,1}(t), \ldots, k_{0,n}(t)\big\},$$
$$\boldsymbol{K}_1(t) = \mathrm{diag}\big\{k_{1,1}(t), \ldots, k_{1,n}(t)\big\}$$

where $\forall\, i \in \{1,\ldots,n\}$:

$$k_{0,i}(t) = \frac{\varsigma_{0,i}(t)}{\psi_{0,i}(t) - |e_i(t)|} \ \text{ and } \ k_{1,i}(t) = \frac{\varsigma_{1,i}(t)}{\psi_{1,i}(t) - |\dot{e}_i(t)|},$$

applied to (5) yields a closed-loop initial-value problem with the properties:

(i) there exists a solution[5] $(\boldsymbol{y},\dot{\boldsymbol{y}}) : [0,T) \to \mathbb{R}^{2n}$ which can be maximally extended and $T \in (0,\infty]$;

(ii) the solution $(\boldsymbol{y}(\cdot),\dot{\boldsymbol{y}}(\cdot))$ does not have finite escape

[4]Operator class $\mathcal{T}$ is defined in e.g. [6, Definition 2.1] or [1, Definition 1]. An explicit introduction of this operator class is not required for the further mathematical analysis and therefore omitted due to space limitations.

[5]Solutions of functional differential equations are considered in the sense of Carathéodory (see e.g. Section 5 in [1]): for $h \geq 0$, an open set $\mathcal{D} \subset \mathbb{R}^n$, a causal operator $\mathfrak{T}$ element of class $\mathcal{T}$ (see Definition 1 in [1]) and a Carathéodory function $\boldsymbol{f}\colon [-h,\infty) \times \mathcal{D} \times \mathbb{R}^m \to \mathbb{R}^n$ (see e.g. Footnote 4 in [1]), the initial trajectory problem of the functional differential equation

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(t,\boldsymbol{x}(t),(\mathfrak{T}\boldsymbol{x})(t)), \quad \boldsymbol{x}|_{[-h,0]} = \boldsymbol{x}^0 \in \mathcal{C}([-h,0])$$

with $\boldsymbol{x}^0(0) \in \mathcal{D}$ has a solution on $[-h,T)$, $T \in (0,\infty]$, i.e. an absolutely continuous function $\boldsymbol{x}(\cdot)\colon [-h,T) \to \mathbb{R}^n$ with $\boldsymbol{x}|_{[-h,0]} = \boldsymbol{x}^0$ which satisfies the above initial trajectory problem for almost all (a.a.) $t \in [0,T)$ and $\boldsymbol{x}(t) \in \mathcal{D}$ for all $t \in [0,T)$.

time, i.e. $T = \infty$;

(iii) the signals $\boldsymbol{e}(\cdot)$ and $\dot{\boldsymbol{e}}(\cdot)$ are uniformly bounded away from the boundary, i.e. $\forall\, i \in \{1, \ldots, n\}\ \exists \varepsilon_{0,i}, \varepsilon_{1,i} > 0\ \forall\, t \geq 0 : \psi_{0,i}(t) - |e_i(t)| \geq \varepsilon_{0,i}$ and $\psi_{1,i}(t) - |\dot{e}_i(t)| \geq \varepsilon_{1,i}$;

(iv) control action and gains are bounded, i.e. $\boldsymbol{u}(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0}; \mathbb{R}^n)$ and $\boldsymbol{K}_0(\cdot),\ \boldsymbol{K}_1(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0}; \mathbb{R}^{n \times n})$.

This far, exact knowledge of inertia matrix $\boldsymbol{M}(\cdot)$ is essential for the proof of Theorem 3.1. Only then the basic idea of the proof of Theorem 3.3 in [6] can be reused and allows to extend the (SISO) funnel controller (1) with derivative feedback to the MIMO case. Including inertia matrix $\boldsymbol{M}(\cdot)$ into the controller as in (8) yields a decoupled mechanical system (5), in the sense that actual error acceleration $\ddot{e}_i(t)$ of joint i is solely affected by

$$k_{0,i}(t)^2 e_i(t) + k_{0,i}(t)\, k_{1,i}(t) \dot{e}_i(t), \qquad i \in \{1, \ldots, n\},$$

which corresponds to SISO controller (1) (applied to each joint i). It was not possible to extend Theorem 3.3 in [6] to unknown or roughly known inertia matrices.

Remark 3.2 (Measurement noise): Usually, output (position) $\boldsymbol{y}(\cdot)$ in (5) is corrupted by measurement noise or sensor errors $\boldsymbol{n}(\cdot) \in \mathcal{W}^{2,\infty}(\mathbb{R}_{\geq 0}, \mathbb{R}^n)$ and output derivative (speed) $\dot{\boldsymbol{y}}(\cdot)$ is approximated by numerical differentiation (e.g. Euler method). Hence, the tracking error becomes $\boldsymbol{e}(\cdot) = \boldsymbol{y}_{\text{ref}}(\cdot) - (\boldsymbol{y}(\cdot) + \boldsymbol{n}(\cdot))$ and, for sufficiently accurate (high resolution) position sensors and sufficiently fast numeric differentiation, derivative feedback is justified and the error derivative is given by $\dot{\boldsymbol{e}}(\cdot) \approx \dot{\boldsymbol{y}}_{\text{ref}}(\cdot) - (\dot{\boldsymbol{y}}(\cdot) + \dot{\boldsymbol{n}}(\cdot))$. Then Theorem 3.1 ensures merely $|y_{\text{ref,i}}(t) - y_i(t)| < \psi_{0,i}(t) + \|n_i\|_\infty$ and $|\dot{y}_{\text{ref,i}}(t) - \dot{y}_i(t)| < \psi_{0,i}(t) + \|\dot{n}_i\|_\infty$ for all $t \geq 0$ and all $i \in \{1, \ldots, n\}$. To reduce noise sensitivity in application, simply choose sufficiently 'large' asymptotic accuracies $\lambda_{0,i} \gg \|n_i\|_\infty$ and $\lambda_{1,i} \gg \|\dot{n}_i\|_\infty$ for all $i \in \{1, \ldots, n\}$.

Proof: Step 1: Some preliminaries. It is easy to see, that (A_1) is equivalent to $\exists \underline{\gamma_0}, \overline{\gamma_0} > 0\ \forall\, \boldsymbol{y} \in \mathbb{R}^n$:

$$\underline{\gamma_0} \boldsymbol{I}_n \leq \boldsymbol{\Gamma}_0(\boldsymbol{y}) := \boldsymbol{M}^{-1}(\boldsymbol{y}) = \boldsymbol{\Gamma}_0(\boldsymbol{y})^\top \leq \overline{\gamma_0} \boldsymbol{I}_n. \tag{9}$$

For error $\boldsymbol{e}(t) = \boldsymbol{y}_{\text{ref}}(t) - \boldsymbol{y}(t)$, rewrite system (5) as

$$\frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} \boldsymbol{e}(t) \\ \dot{\boldsymbol{e}}(t) \end{pmatrix} = \begin{pmatrix} \dot{\boldsymbol{e}}(t) \\ \boldsymbol{\Gamma}_0(\boldsymbol{y}_{\text{ref}}(t) - \boldsymbol{e}(t)) \cdot \\ \Big[\big(\boldsymbol{C}(\boldsymbol{y}_{\text{ref}}(t) - \boldsymbol{e}(t),\, \dot{\boldsymbol{y}}_{\text{ref}}(t) - \dot{\boldsymbol{e}}(t)) + \boldsymbol{C}_V \big) \cdot \\ (\dot{\boldsymbol{y}}_{\text{ref}}(t) - \dot{\boldsymbol{e}}(t)) + (\mathfrak{F}(\dot{\boldsymbol{y}}_{\text{ref}} - \dot{\boldsymbol{e}}))(t) \\ + \boldsymbol{g}(\boldsymbol{y}_{\text{ref}}(t) - \boldsymbol{e}(t)) + \boldsymbol{d}(t) \Big] \end{pmatrix}$$
$$+ \begin{pmatrix} \boldsymbol{0} \\ \ddot{\boldsymbol{y}}_{\text{ref}}(t) \end{pmatrix} - \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{\Gamma}_0(\boldsymbol{y}_{\text{ref}}(t) - \boldsymbol{e}(t)) \end{pmatrix} \boldsymbol{u}(t), \ \begin{pmatrix} \boldsymbol{e}(0) \\ \dot{\boldsymbol{e}}(0) \end{pmatrix} = \begin{pmatrix} \boldsymbol{y}_{\text{ref}}(0) - \boldsymbol{y}^0 \\ \dot{\boldsymbol{y}}_{\text{ref}}(0) - \boldsymbol{y}^1 \end{pmatrix}. \tag{10}$$

Define the constants $\forall\, i \in \{1, \ldots, n\}$:

$$\begin{aligned} \underline{\varsigma}_{0,i} &:= \inf\nolimits_{t \geq 0} \varsigma_{0,i}(t) \quad \text{and} \quad \underline{\varsigma}_{1,i} := \inf\nolimits_{t \geq 0} \varsigma_{1,i}(t), \\ \lambda_{0,i} &:= \inf\nolimits_{t \geq 0} \psi_{0,i}(t) \quad \text{and} \quad \lambda_{1,i} := \inf\nolimits_{t \geq 0} \psi_{1,i}(t). \end{aligned} \tag{11}$$

Step 2: It is shown that Assertion (i) holds true, i.e. existence of a maximally extended solution. It suffices to consider system (5) in the form (10). For $\mathcal{F}_{(\boldsymbol{\psi}_0, \boldsymbol{\psi}_1)}$ as in (6) define the non-empty and open set

$$\mathcal{D} := \left\{ (\tau, \boldsymbol{\mu}, \boldsymbol{\xi}) \in \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \ \middle|\ (|\tau|, \boldsymbol{\mu}, \boldsymbol{\xi}) \in \mathcal{F}_{(\boldsymbol{\psi}_0, \boldsymbol{\psi}_1)} \right\}, \tag{12}$$

the function

$$\boldsymbol{f} : \mathbb{R}_{\geq 0} \times \mathcal{D} \times \mathbb{R}^n \to \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n,\ (t, (\tau, \boldsymbol{\mu}, \boldsymbol{\xi}), \boldsymbol{w}) \mapsto$$
$$\begin{pmatrix} 1 \\ \boldsymbol{\xi} \\ \boldsymbol{\Gamma}_0(\boldsymbol{y}_{\text{ref}}(t) - \boldsymbol{\mu}) \Big[\big(\boldsymbol{C}(\boldsymbol{y}_{\text{ref}}(t) - \boldsymbol{\mu},\, \dot{\boldsymbol{y}}_{\text{ref}}(t) - \boldsymbol{\xi}) + \boldsymbol{C}_V \big) \cdot \\ \cdot (\dot{\boldsymbol{y}}_{\text{ref}}(t) - \boldsymbol{\xi}) + \boldsymbol{w} + \boldsymbol{g}(\boldsymbol{y}_{\text{ref}}(t) - \boldsymbol{\mu}) + \boldsymbol{d}(t) \Big] + \ddot{\boldsymbol{y}}_{\text{ref}}(t) \\ - \operatorname{diag}\left\{ \frac{\varsigma_{0,1}(t)}{\psi_{0,1}(|\tau|) - |\mu_1|}, \ldots, \frac{\varsigma_{0,n}(t)}{\psi_{0,n}(|\tau|) - |\mu_n|} \right\}^2 \boldsymbol{\mu} \\ - \operatorname{diag}\Big\{ \frac{\varsigma_{0,1}(t) \varsigma_{1,1}(t)}{(\psi_{0,1}(|\tau|) - |\mu_1|)(\psi_{1,1}(|\tau|) - |\xi_1|)}, \ldots, \\ \frac{\varsigma_{0,n}(t) \varsigma_{1,n}(t)}{(\psi_{0,n}(|\tau|) - |\mu_n|)(\psi_{1,n}(|\tau|) - |\xi_n|)} \Big\} \boldsymbol{\xi} \end{pmatrix}$$

and the operator $\hat{\mathfrak{T}} : \mathcal{C}(\mathbb{R}_{\geq 0}; \mathbb{R} \times \mathbb{R}^{2n}) \to \mathcal{L}^\infty_{\text{loc}}(\mathbb{R}_{\geq 0}; \mathbb{R}^n)$, $(\hat{\mathfrak{T}}(\tau, \boldsymbol{\mu}, \boldsymbol{\xi}))(t) := (\mathfrak{F}\boldsymbol{\xi})(t)$. Then, for artifact $\tau \colon \mathbb{R} \to \mathbb{R},\ t \mapsto t$ and extended state variable $\hat{\boldsymbol{x}} := (\tau, (\boldsymbol{e}, \dot{\boldsymbol{e}}))$, the initial-value problem (10), (8) may be expressed in standard form

$$\frac{\mathrm{d}}{\mathrm{d}t} \hat{\boldsymbol{x}}(t) = \boldsymbol{f}(t, \hat{\boldsymbol{x}}(t), (\hat{\mathfrak{T}}\hat{\boldsymbol{x}})(t)), \ \ \hat{\boldsymbol{x}}(0) = \begin{pmatrix} 0 \\ \boldsymbol{y}_{\text{ref}}(0) - \boldsymbol{y}^0 \\ \dot{\boldsymbol{y}}_{\text{ref}}(0) - \boldsymbol{y}^1 \end{pmatrix}. \tag{13}$$

Choose a compact set $\mathfrak{C} \subset \mathcal{D} \times \mathbb{R}^n$ and observe that the following holds

$$\begin{aligned} &\exists\, m_{\mathfrak{C}}, M_{\mathfrak{C}} > 0\ \forall\, ((\tau, \boldsymbol{\mu}, \boldsymbol{\xi}), \boldsymbol{w}) \in \mathfrak{C} :\ \|((\tau, \boldsymbol{\mu}, \boldsymbol{\xi}), \boldsymbol{w})\| \leq M_{\mathfrak{C}} \\ &\wedge \min_{i \in \{1, \ldots, n\}} \{ \psi_{0,i}(|\tau|) - |\mu_i|,\ \psi_{1,i}(|\tau|) - |\xi_i| \} \geq m_{\mathfrak{C}}. \end{aligned} \tag{14}$$

Then, for $\boldsymbol{d}(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0};\, \mathbb{R}^n)$, $\boldsymbol{y}_{\text{ref}}(\cdot) \in \mathcal{W}^{2,\infty}(\mathbb{R}_{\geq 0};\, \mathbb{R}^n)$ and $\boldsymbol{\varsigma}_0(\cdot), \boldsymbol{\varsigma}_1(\cdot) \in \mathcal{W}^{1,\infty}(\mathbb{R}_{\geq 0}, \mathbb{R}^n_{>0})$, note that the function $\boldsymbol{f}(\cdot, \cdot, \cdot)$ has the following properties: (i) $\boldsymbol{f}(t, \cdot, \cdot)$ is continuous for each fixed $t \geq 0$, (ii) for each fixed $((\tau, \boldsymbol{\mu}, \boldsymbol{\xi}), \boldsymbol{w}) \in \mathcal{D} \times \mathbb{R}^n$ the function $\boldsymbol{f}(\cdot, (\tau, \boldsymbol{\mu}, \boldsymbol{\xi}), \boldsymbol{w})$ is measurable and (iii) for almost all $t \geq 0$ and for all $((\tau, \boldsymbol{\mu}, \boldsymbol{\xi}), \boldsymbol{w}) \in \mathfrak{C}$ the following holds

$$\begin{aligned} \|\boldsymbol{f}(t, (\tau, \boldsymbol{\mu}, \boldsymbol{\xi}), \boldsymbol{w})\| &\overset{(A_1)-(A_5),\ (14)}{\leq} 1 + M_{\mathfrak{C}} \\ + \overline{\gamma_0} \Big[c_{\boldsymbol{C}} (\|\dot{\boldsymbol{y}}_{\text{ref}}(t)\| &+ M_{\mathfrak{C}})^2 + \max\{\nu_1, \ldots, \nu_n\} (\|\dot{\boldsymbol{y}}_{\text{ref}}(t)\| \\ &+ M_{\mathfrak{C}}) + M_{\mathfrak{C}} + c_{\boldsymbol{g}} + \|\boldsymbol{d}(t)\| \Big] + \|\ddot{\boldsymbol{y}}_{\text{ref}}(t)\| \\ &+ \|\boldsymbol{\varsigma}_0(t)\| \big(\|\boldsymbol{\varsigma}_0(t)\| + \|\boldsymbol{\varsigma}_1(t)\| \big) M_{\mathfrak{C}} / m_{\mathfrak{C}}^2 =: l_{\mathfrak{C}}(t) \end{aligned}$$

where $l_{\mathfrak{C}}(\cdot) \in \mathcal{L}^\infty(\mathbb{R}_{\geq 0}; \mathbb{R}_{\geq 0}) \subset \mathcal{L}^1_{\text{loc}}(\mathbb{R}_{\geq 0}, \mathbb{R}_{\geq 0})$. Hence $\boldsymbol{f}(\cdot, \cdot, \cdot)$ is a Carathéodory function and invoking Theorem 5 in [1] yields existence of a solution $\hat{\boldsymbol{x}} : [0, T) \to \mathbb{R} \times \mathbb{R}^{2n}$ of the initial-value problem (13) with $\hat{\boldsymbol{x}}([0, T)) \in \mathcal{D}$, $T \in (0, \infty]$. Each solution can be extended to a maximal solution. Moreover $\boldsymbol{f}(\cdot, \cdot, \cdot)$ is essentially bounded and so, if $T < \infty$, then for any compact $\tilde{\mathfrak{C}} \subset \mathcal{D}$, there exists $\tilde{t} \in [0, T)$ such that $\hat{\boldsymbol{x}}(\tilde{t}) \notin \tilde{\mathfrak{C}}$. In the following, let $\hat{\boldsymbol{x}} := (\tau, \boldsymbol{e}, \dot{\boldsymbol{e}}) \colon [0, T) \to \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n$ be a fixed and maximally extended solution of the initial-value problem (13). Note that $(\boldsymbol{e}, \dot{\boldsymbol{e}}) : [0, T) \to \mathbb{R}^n \times \mathbb{R}^n$ solves the closed-loop initial-value problem (10), (8) for almost all $t \in [0, T)$. This shows Assertion (i).

Step 3: Some technical inequalities are introduced. In view of Step 1, $\boldsymbol{e}(\cdot)$ and $\dot{\boldsymbol{e}}(\cdot)$ are continuous on $[0, T)$

and evolve within the funnel $\mathcal{F}_{(\psi_0,\psi_1)}$ as in (6). Moreover, due to the properties of $\mathcal{B}_2$ in (2), it follows that $\forall\, i \in \{1,\dots,n\}$ $\forall\, t \in [0,T): |e_i(t)| < \psi_{0,i}(t) \le \|\psi_{0,i}\|_\infty$ and $|\dot{e}_i(t)| < \psi_{1,i}(t) \le \|\psi_{1,i}\|_\infty$. Hence

$$\forall\, t \in [0,T): \ \|\boldsymbol{e}(t)\| < \|\boldsymbol{\psi}_0\|_\infty \ \wedge\ \|\dot{\boldsymbol{e}}(t)\| < \|\boldsymbol{\psi}_1\|_\infty. \quad (15)$$

Define $\widehat{\boldsymbol{d}}\colon\ \mathbb{R}_{\ge 0} \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n;\ (t,\boldsymbol{\mu},\boldsymbol{\xi},\boldsymbol{w}) \mapsto \boldsymbol{\Gamma}_0(\boldsymbol{y}_{\text{ref}}(t)-\boldsymbol{\mu})\Big[\big(\boldsymbol{C}(\boldsymbol{y}_{\text{ref}}(t)-\boldsymbol{\mu},\, \dot{\boldsymbol{y}}_{\text{ref}}(t)-\boldsymbol{\xi})+\boldsymbol{C}_V\big)(\dot{\boldsymbol{y}}_{\text{ref}}(t)-\boldsymbol{\xi}) + \boldsymbol{w} + \boldsymbol{g}(\boldsymbol{y}_{\text{ref}}(t)-\boldsymbol{\mu}) + \boldsymbol{d}(t)\Big] + \ddot{\boldsymbol{y}}_{\text{ref}}(t) =: \widehat{\boldsymbol{d}}(t,\boldsymbol{\mu},\boldsymbol{\xi},\boldsymbol{w})$ and the constant

$$M := \overline{\gamma_0}\Big[c_{\boldsymbol{C}}\left(\|\dot{\boldsymbol{y}}_{\text{ref}}\|_\infty + \|\boldsymbol{\psi}_1\|_\infty\right)^2 + \max\{\nu_1,\dots,\nu_n\}\cdot \\ \cdot\left(\|\dot{\boldsymbol{y}}_{\text{ref}}\|_\infty + \|\boldsymbol{\psi}_1\|_\infty\right) + M_{\mathfrak{F}} + c_{\boldsymbol{g}} + \|\boldsymbol{d}\|_\infty\Big] + \|\ddot{\boldsymbol{y}}_{\text{ref}}\|_\infty. \quad (16)$$

Invoking Assumptions (A_2), (A_3), (A_4) and (A_5) yields

$$\text{for a.a. } t \in [0,T): \ \left\|\widehat{\boldsymbol{d}}\big(t,\boldsymbol{e}(t),\dot{\boldsymbol{e}}(t),(\mathfrak{F}(\dot{\boldsymbol{y}}_{\text{ref}}-\dot{\boldsymbol{e}}))(t)\big)\right\| \overset{(9)}{\le} \\ \overline{\gamma_0}\Big[c_{\boldsymbol{C}}\,\|\dot{\boldsymbol{y}}_{\text{ref}}(t)-\dot{\boldsymbol{e}}(t)\|^2 + M_{\mathfrak{F}} + c_{\boldsymbol{g}} + \|\boldsymbol{d}\|_\infty \\ +\max\{\nu_1,\dots,\nu_n\}\,\|\dot{\boldsymbol{y}}_{\text{ref}}(t)-\dot{\boldsymbol{e}}(t)\|\Big] + \|\ddot{\boldsymbol{y}}_{\text{ref}}\|_\infty \overset{(15),(16)}{\le} M,$$

hence we have $\forall\, i \in \{1,\dots,n\}$ for a.a. $t \in [0,T)$:

$$|\widehat{d}_i\big(t,\boldsymbol{e}(t),\dot{\boldsymbol{e}}(t),(\mathfrak{F}(\dot{\boldsymbol{y}}_{\text{ref}}-\dot{\boldsymbol{e}}))(t)\big)| \le M. \quad (17)$$

Inserting (8) into (10) and invoking (17) yields

$$\forall\, i \in \{1,\dots,n\}\ \forall\, t \in [0,T): \\ -M - k_{0,i}(t)^2\, e_i(t) - k_{0,i}(t)\, k_{1,i}(t)\, \dot{e}_i(t) \le \ddot{e}_i(t) \\ \le M - k_{0,i}(t)^2\, e_i(t) - k_{0,i}(t)\, k_{1,i}(t)\, \dot{e}_i(t). \quad (18)$$

Step 4: For all $i \in \{1,\dots,n\}$ it is shown that $|e_i(\cdot)|$ is uniformly bounded away from the boundary $\psi_{0,i}(\cdot)$; more precisely for positive

$$\varepsilon_{0,i} \le \min\Bigg\{\frac{\lambda_{0,i}}{4},\ \frac{\psi_{0,i}(0)-|e_i(0)|}{2},\ \frac{\frac{1}{2}\delta_i \underline{\varsigma}_{0,i}^2\lambda_{0,i}}{\beta+\sqrt{\beta^2+2\delta_i^2\underline{\varsigma}_{0,i}^2\lambda_{0,i}M}}, \\ \frac{\frac{1}{2}\delta_i\underline{\varsigma}_{0,i}\lambda_{0,i}}{2\|\varsigma_{1,i}\|_\infty\|\psi_{1,i}\|_\infty+\sqrt{4\|\varsigma_{1,i}\|_\infty^2\|\psi_{1,i}\|_\infty^2+2\delta_i^2\lambda_{0,i}(M+\|\dot{\psi}_{1,i}\|_\infty)}}\Bigg\}, \quad (19)$$

with $\beta := 2\underline{\varsigma}_{0,i}\|\varsigma_{1,i}\|_\infty\|\psi_{1,i}\|_\infty+\delta_i\left(\|\psi_{1,i}\|_\infty + \|\dot{\psi}_{0,i}\|_\infty\right)^2$, $\lambda_{0,i}$, $\lambda_{1,i}$, $\underline{\varsigma}_{0,i}$ *and* $\underline{\varsigma}_{1,i}$ *as in* (11), $\delta_i = \delta$ *as in* (2) *and* M *as in* (16), *it holds that* $\psi_{0,i}(t) - |e_i(t)| \ge \varepsilon_{0,i}$ *for all* $i \in \{1,\dots,n\}$ *and all* $t \in [0,T)$. Choose $i \in \{1,\dots,n\}$ arbitrarily and note that for $\varepsilon_{0,i}$ as in (19) the Steps 3a-e in the proof of Theorem 3.3 in [6] go through without changes (setting $\gamma = 1$ and $\varepsilon_{0,i} = \varepsilon_0$). Hence the claim of Step 4 holds true.

Step 5: For all $i \in \{1,\dots,n\}$ it is shown that $|\dot{e}_i(\cdot)|$ is uniformly bounded away from the boundary $\psi_{1,i}(\cdot)$; more precisely for positive

$$\varepsilon_{1,i} \le \min\Bigg\{\lambda_{1,i}/2,\ \psi_{1,i}(0) - |\dot{e}_i(0)|, \\ \frac{\frac{1}{2}\underline{\varsigma}_{0,i}\underline{\varsigma}_{1,i}\lambda_{1,i}\varepsilon_{0,i}^2}{\|\psi_{0,i}\|_\infty(M+\|\dot{\psi}_{1,i}\|_\infty)\varepsilon_{0,i}^2+\|\varsigma_{0,i}\|_\infty^2\|\psi_{0,i}\|_\infty^2}\Bigg\}, \quad (20)$$

with M as in (16) *and $\varepsilon_{0,i}$ as in* (19)*, it holds that $\psi_{1,i}(t) - |\dot{e}_i(t)| \ge \varepsilon_{1,i}$ for all $i \in \{1,\dots,n\}$ and all $t \in [0,T)$.* Again choose $i \in \{1,\dots,n\}$ arbitrarily and observe that identical arguments as in Step 4 of the proof of Theorem 3.3 in [6] (setting $\gamma = 1$ and $\varepsilon_{1,i} = \varepsilon_1$) show the claim of Step 5.

Step 6: It is shown that Assertions (ii)-(iv) hold true. For M as in (16), $\varepsilon_{0,i}$ as in (19) and $\varepsilon_{1,i}$ as in (20), $i \in \{1,\dots,n\}$ define

$$\widetilde{\mathfrak{C}} := \Big\{(t,\boldsymbol{\mu},\boldsymbol{\xi}) \in [0,T] \times \mathbb{R}^{2n} \ \Big|\ \forall\, i \in \{0,\dots,n\}: \\ |\mu_i| \le \psi_{0,i}(t) - \varepsilon_{0,i}\ \wedge\ |\xi_i| \le \psi_{1,i}(t) - \varepsilon_{1,i}\Big\}.$$

Let $\mathcal{D}$ be as in Step 2. If $T < \infty$ then $\widetilde{\mathfrak{C}} \subset \mathcal{D}$ and contains the whole graph of the solution $t \mapsto (\boldsymbol{e}(t),\dot{\boldsymbol{e}}(t))$, which contradicts maximality of the solution. Hence $T = \infty$. Assertion (iii) follows from Step 4 and Step 5. Moreover, Step 4 and Step 5 with boundedness of $\boldsymbol{\varsigma}_0(\cdot)$ and $\boldsymbol{\varsigma}_1(\cdot)$ on $\mathbb{R}_{\ge 0}$ imply that $\boldsymbol{K}_0(\cdot)$ and $\boldsymbol{K}_1(\cdot)$ are uniformly bounded on $\mathbb{R}_{\ge 0}$, respectively. Then, from (15), (A_1) and (8), it follows that $\boldsymbol{u}(\cdot)$ is uniformly bounded on $\mathbb{R}_{\ge 0}$, hence Assertion (iv) is shown. This completes the proof. ∎

C. Simulation

For illustration, the MIMO funnel controller (8) with derivative feedback is applied to the planar two DOF rigid, revolute joint robotic manipulator depicted in Fig. 2. The planar (elbow-like) robot has two revolute joints actuated by u_1 and u_2 [Nm], respectively. The links are assumed massless and have length l_1 and l_2 [m]. Point masses m_1 and m_2 [kg] are attached to their distal ends, respectively. Control objective is position control of joint angles y_1 and y_2 [rad] with prescribed transient accuracy. The mathematical model of this robot is given by (see e.g. [18, p. 259ff.])

$$\boldsymbol{M}(\boldsymbol{y}(t))\,\ddot{\boldsymbol{y}}(t) + \boldsymbol{C}(\boldsymbol{y}(t),\dot{\boldsymbol{y}}(t))\,\dot{\boldsymbol{y}}(t) + \boldsymbol{g}(\boldsymbol{y}(t)) = \boldsymbol{u}(t) \quad (21)$$

with initial value $(\boldsymbol{y}(0),\dot{\boldsymbol{y}}(0)) = (\boldsymbol{0},\boldsymbol{0})$ and inertia matrix

$$\boldsymbol{M}\colon \mathbb{R}^2 \to \mathbb{R}^{2\times 2},\ \boldsymbol{y} \mapsto \boldsymbol{M}(\boldsymbol{y}) := \\ \begin{bmatrix} m_1 l_1^2 + m_2\,(l_1^2 + l_2^2 + 2 l_1 l_2 \cos(y_2)), & m_2(l_2^2 + l_1 l_2 \cos(y_2)) \\ m_2(l_2^2 + l_1 l_2 \cos(y_2)), & m_2 l_2^2 \end{bmatrix}, \quad (22)$$

centrifugal and Coriolis force matrix

$$\boldsymbol{C}\colon \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^{2\times 2}, \quad (\boldsymbol{y},\boldsymbol{v}) \mapsto \boldsymbol{C}(\boldsymbol{y},\boldsymbol{v}) := \\ \begin{bmatrix} -2 m_2 l_1 l_2 \sin(y_2) v_1, & -m_2 l_1 l_2 \sin(y_2) v_2 \\ -m_2 l_1 l_2 \sin(y_2) v_1, & 0 \end{bmatrix}$$

and gravity vector

$$\boldsymbol{g}\colon \mathbb{R}^2 \to \mathbb{R}^2, \quad \boldsymbol{y} \mapsto \boldsymbol{g}(\boldsymbol{y}) := \\ g\begin{pmatrix} m_1 l_1 \cos(y_1) + m_2(l_1\cos(y_1) + l_2\cos(y_1+y_2)) \\ m_2 l_2 \cos(y_1+y_2) \end{pmatrix}$$

where $g = 9.81\ \left[\mathrm{kg\,m^2}\right]$ is the (rounded) gravity constant. It is easy to see that the planar robot (21) satisfies Assumptions (A_1)-(A_6). The details are omitted. For simplicity, friction, gears, disturbances and measurement errors (e.g. noise) are neglected. The simulation is performed in Matlab/Simulink using solver `ode45 (Dormand-Prince)` with variable

robot (21)	$m_1 = m_2 = 1$ [kg], $l_1 = l_2 = 1$ [m], $g = 9.81$ $[\mathrm{m/s^2}]$, $(\boldsymbol{y}(0), \dot{\boldsymbol{y}}(0)) = (\mathbf{0}, \mathbf{0})$
reference	$(y_{\mathrm{ref},1}(\cdot),\, y_{\mathrm{ref},2}(\cdot))$ as in top of Fig. 3
initial error	$\boldsymbol{e}(0) = (\pi/2, -\pi/4)^\top$ $[\mathrm{rad}]^2$
(MIMO) funnel controller (8)	$\boldsymbol{M}(\boldsymbol{y})$ as in (22), $(\psi_{0,i}(\cdot),\, \psi_{0,i}(\cdot))$, $i \in \{1, 2\}$ as in (4) with $(\Lambda_{0,1},\, \Lambda_{0,2}) = 5\,\boldsymbol{e}(0)$ $[\mathrm{rad}]^2$, $(\lambda_{0,1},\, \lambda_{0,2}) = (\pi/18,\, \pi/18)$ $[\mathrm{rad}]^2$, $(T_{E,1},\, T_{E,2}) = (1.32,\, 1.63)$ $[\mathrm{s}]^2$, $(\lambda_{1,1},\, \lambda_{1,2}) = (\pi,\, \pi)$ $[\mathrm{rad/s}]^2$, $\varsigma_0(\cdot) = (\psi_{0,1}(\cdot),\, \psi_{0,2}(\cdot))$, $\varsigma_1(\cdot) = 10\,(\psi_{1,1}(\cdot),\, \psi_{1,2}(\cdot))$

TABLE I

ROBOT AND CONTROLLER PARAMETERS FOR SIMULATION.

step size (max. 10^{-3} [s]). Robot and controller parameters are collected in Tab. I. The simulation results for closed-loop system (21), (8) are shown in Fig. 3. The funnel controller (8) achieves tracking with prescribed transient accuracy for joint positions $y_1(\cdot)$ & $y_2(\cdot)$ and joint velocities $\dot{y}_1(\cdot)$ & $\dot{y}_2(\cdot)$, respectively. Both joint position errors $e_1(\cdot)$ & $e_2(\cdot)$ and both joint velocity errors $\dot{e}_1(\cdot)$ & $\dot{e}_2(\cdot)$ evolve within the performance funnel. Since gears are not considered, the simulated motor torques $u_1(\cdot)$ & $u_2(\cdot)$ are seemingly large. In 'real world' gears (with ratios $\gg 10$) reduce the torque on motor side.

IV. Future Work

Future work will focus on i) providing measurement results, ii) making knowledge of the inertia matrix obsolete and iii) including elasticity in the mathematical analysis (i.e. extending the results to nowadays rigid link, elastic joint robotic manipulators).

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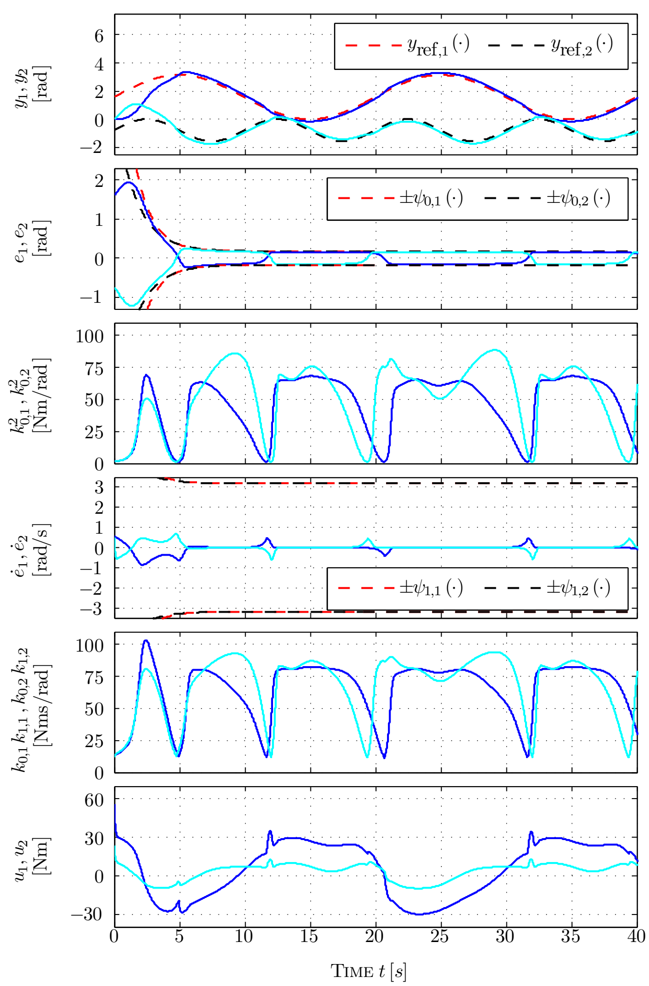

Fig. 3. Simulation results for closed-loop system (21), (8): joint 1 and joint 2 (from top to bottom: position $y_1(\cdot)$, $y_2(\cdot)$; position error $e_1(\cdot)$, $e_2(\cdot)$; proportional gain $k_{0,1}(\cdot)^2$, $k_{0,2}(\cdot)^2$; speed error $\dot{e}_1(\cdot)$, $\dot{e}_2(\cdot)$; derivative gain $k_{0,1}(\cdot)k_{1,1}(\cdot)$, $k_{0,2}(\cdot)k_{1,2}(\cdot)$ and torque $u_1(\cdot)$, $u_2(\cdot)$).

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