On the stabilization of continuous fuzzy models using their discretized forms

A. Ellouze , J. Lauber, F. Delmotte, M. Chtourou and M. Ksantini

Abstract— **This paper proposes a new method to stabilize the continuous-time TS fuzzy models by using results from their discretized models. The proposed approach is structured as follows: first, a discrete model is obtained from the discretization of the continuous TS fuzzy model. Second the gains of a non-PDC controller ensuring the stabilization of the discrete model are determined. Third, we check if the discrete control law used continuously without any zero-order hold can stabilize the continuous TS model. It is obvious that two sets of LMIS conditions are obtained: one for the discrete model, and one for the continuous on. But these two sets of LMIS conditions are much easier to solve than a single set of BMIS conditions usually found when a non PDC control law is directly applied to a continuous model. Simulation examples illustrate the effectiveness of this approach.**

I. Introduction

During the last twenty years, Takagi–Sugeno (TS) fuzzy models [1] are becoming a useful tool to deal with control and observation of large class of nonlinear systems. In this context the models can be represented by a set of linear subsystems interconnected with non linear functions. Studies about TS models have been done in the continuous case as well as in the discrete one [4], [5], [7], [13], [14]. Usually, results for the discrete and the continuous cases are performed in parallel: the same approach is used in both cases, and leads to various sets of LMIs conditions. Sometimes there is no equivalence in the other case, because the method cannot be applied at all. In the discrete case, complex Lyapunov functions can be used leading to LMI problems, equivalent approach in the continuous case leads to at least BMI problems. Sometimes there is equivalence between the two cases, in the digital control framework for example. In this case a discrete control law is applied to a continuous model through a zero order hold. But eventually the closed loop is turned into a pure continuous model with the help of delays and Lyapunov- Krasovsky functional [15], [16].

The originality of this paper consists in the fact that, contrarily to what have been said previously, we want to fuse the two continuous and discrete cases to obtain new results about the continuous case. Indeed since very complex Lyapunov functions can be used only in the discrete case, we will first discretize a continuous model, to compute a complex control law associated to a certain type of Lyapunov function. Then we will directly check the stabilization of the closed loop of the continuous model by keeping the obtained discrete control law, turned into a continuous one without a zero order hold. This is much simpler than in other papers when the controller gains are not known. This way we can use more complex Lyapunov functions in the continuous case. Examples show that this is an interesting approach.

In this paper, the section 1 recalls the Takagi-Sugeno continuous fuzzy models and the stabilization conditions. Section 2 presents the discretization of fuzzy systems and conditions of stabilization. In Section 3, new stabilization conditions are proposed based on nonquadratic Lyapunov functions and non-PDC control laws. Simulation examples are provided in section 4. The conclusion is given in the last section.

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II. Takagi–Sugeno Fuzzy Model

A. Notations

For any positive scalar functions $h_i\left(z(\cdot)\right)\geq 0$, and matrices A_i with $i\in\{1,\dots,r\}$, we define the following notations:

$$A_{z(t)}=\sum_{i=1}^{r}h_i\left(z(t)\right)A_i \quad A_{z(t)z(t+1)}=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i\left(z(t)\right)h_j\left(z(t+1)\right)A_{ij}$$

and so on for multiple sums.

A $(*)$ indicates a transpose quantity. For example $A_{z(t)}^T P_{z(t)}(*)-P_{z(t-1)}<0$ stands for $A_{z(t)}^T P_{z(t)}A_{z(t)}-P_{z(t-1)}<0$ and $\begin{bmatrix}-P_{z(t-1)} & (*)\\ A_{z(t)} & -P_{z(t)}\end{bmatrix}<0$ for $\begin{bmatrix}-P_{z(t-1)} & A_{z(t)}^T\\ A_{z(t)} & -P_{z(t)}\end{bmatrix}<0$.

Using these notations, a continuous T-S fuzzy system can be described as a polytopic form with linear models blended together by non linear functions [1]:

$$\dot{x}(t)=A_{z(t)}^c x(t)+B_{z(t)}^c u(t) \qquad (1)$$

Where $x(t)\in R^n$ is the state vector, $u(t)\in R^m$ is the input vector, $z(t)$ is the vector of premises, r is the number of rules of the fuzzy model and A_i^c and B_i^c are matrices of appropriate dimensions, defined in the continuous case. The nonlinear weights h_i satisfy the convex sum property:

$$h_i\left(z(t)\right)\geq 0,\ i=1,2,...,r,\ \sum_{i=1}^{r}h_i\left(z(t)\right)=1 \qquad (2)$$

B. Quadratic stabilization conditions

This section recalls some results obtained with the classical PDC law for continuous TS fuzzy model [3]. The PDC (Parallel Distributed Compensation) controller that stabilizes the Takagi-Sugeno fuzzy system is given by:

$$u(t) = -K^c_{z(t)} x(t) \tag{3}$$

From equation (3), we obtain the Takagi-Sugeno closed loop for continuous fuzzy system as follows:

$$\dot{x}(t) = \left(A^c_{z(t)} - B^c_{z(t)} K^c_{z(t)}\right) x(t) \tag{4}$$

The synthesis of the controller (3) consists in finding the gains K^c_i that ensure the closed loop stabilization of continuous fuzzy system (4).

A quadratic Lyapunov function is mainly used:

$$V\left(x(t)\right) = x^T(t) P x(t),\ P = P^T > 0 \tag{5}$$

Theorem 1 [4]**:** The equilibrium of the continuous fuzzy control system (4) is globally asymptotically stable if a common matrix $X = X^T > 0$ and matrices M_i satisfying these LMIs that can be effectively solved via LMI control toolbox [13]:

$$\begin{cases} \Upsilon_{ii} < 0 & i = 1,2,...,r \\ \dfrac{2}{r-1}\Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} < 0 & i,j = 1,2,...,r, \quad i \neq j \end{cases} \tag{6}$$

with:

$$\Upsilon_{ij} = A^c_i X + X A^{cT}_i - B^c_i M_j - B^{cT}_i M^T_j,\ X = P^{-1},\ K^c_i = M^{-1}_i X$$

III. Discretization Of T-S Models And Condition Of Stabilization In The Discrete Case

Several discretization methods are presented in the literature as Lie series, Taylor series, the Euler approximation... The Euler approximate discrete time model is adopted in this paper which is a preliminary paper that we hope will open a new path. Of course better results from more accurate discretization methods can be expected.

The discretization of the TS fuzzy model with a sampling period T_e is given by:

$$x(k+1) = \sum_{i=1}^{r} h_i(z(k))\left(A^d_i x(k) + B^d_i u(k)\right) \tag{7}$$

where h_i are the same nonlinearities as in section1 and

$$A^d_i = I + T_e \cdot A^c_i,\ B^d_i = T_e \cdot B^c_i$$

(A^d_i and B^d_i are defined in the discrete case)

To stabilize the discretized model, the nonquadratic Lyapunov function is considered as [4]:

$$V(x(k)) = x^T(k) G_{z(k)} P_{z(k)} G^{-1}_{z(k)} x(k),\ P_z = \sum_{i=1}^{r} h_i P_i \quad P_i = P^T_i \tag{8}$$

and the non-PDC control law given by:

$$u(k) = -K^d_{z(k)} G^{-1}_{z(k)} x(k) \tag{9}$$

The following theorem presents the stabilization conditions that allow to find appropriate gains K^d_i and G_i to guarantee the closed loop stabilization of system (7).

Theorem 2 [7]: The equilibrium of the closed loop for the discrete TS fuzzy system (7) is globally asymptotically stable if there exist a common symmetric and positive definite matrices P_i, K^d_i and G_i satisfying these LMIs:

$$\begin{cases} \Upsilon^k_{ii} < 0 & i,k = 1,2,...,r \\ \dfrac{2}{r-1}\Upsilon^k_{ii} + \Upsilon^k_{ij} + \Upsilon^k_{ji} < 0 & i,j,k = 1,2,...,r, \quad i \neq j \end{cases} \tag{10}$$

with $\Upsilon^k_{ij} = \begin{bmatrix} -P_i & (*) \\ A^d_i G_j - B^d_i K^d_j & -G_k - G^T_k + P_k \end{bmatrix}$

The next section presents new stabilization conditions for the continuous TS fuzzy systems through the discrete controller.

IV. New Stabilization Conditions

An interesting novel approach is presented in this paper. The idea of this approach is based on these steps:

- First, we use the method of discretization described in section III to obtain a discrete model from the continuous TS fuzzy model (1).
- Second, by applying the theorem 2 on the discretized model obtained previously, we determine the gains of a non-PDC controller K^d_i and G_i ensuring the stabilization of the discrete model.
- Third, given the continuous TS fuzzy model (1), by keeping the gains K^d_i and G_i determined previously, a new continuous control law will be applied to model (1) which is given by:

$$u(t) = -K^d_{z(t)} G^{-1}_{z(t)} x(t) \tag{11}$$

Considering (11), the closed loop TS fuzzy continuous model will be presented by:

$$\dot{x}(t) = \left(A^c_{z(t)} - B^c_{z(t)} K^d_{z(t)} G^{-1}_{z(t)}\right) x(t) \tag{12}$$

- Finally, based on lyapunov function, we check if the control law (11), used continuously without any zero-order hold, can stabilize the continuous TS fuzzy model (12).

Remark 1: If we used, directly, a continuous non PDC controller in time for the model (1), we found a set of BMIs conditions that can be difficult to solve since the BMI solver cannot guarantee that any existing solution will be definitely found. The advantage of the proposed approach is, so, that it enables to obtain two sets of LMIS conditions: one for the discrete model, and one for the continuous on. These two sets of LMIS conditions are much easier to solve than a single set of BMIS and allow to guarantee the feasibility of solution if it exists. These LMIS conditions can be effectively solved via LMI control toolbox [12].

To check the stabilization of the closed loop (12), we consider the following non quadratic Lyapunov function:

$$V\left(x(t)\right) = x^T(t) P_z x(t) \tag{13}$$

The proposed stabilization conditions depend on the upper bound of the time-derivative of the membership functions. From the properties of the membership functions in (2), it is well known that [5]:

$$\sum_{j=1}^{r}\dot{h}_j\left(z(t)\right)=0\ ,\ \forall z(t) \tag{14}$$

From equations (2) and (14), we can introduce a symmetric matrix M_3 such that:

$$\mathbf{M}_3=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i\dot{h}_j x^T(t)M_3x(t)=0 \tag{15}$$

The same technique in [8,9] is applied to obtain new conditions. By introducing slack matrix variables into the LMIs, we consider the following null product that will serve stabilization conditions:

$$2\left[x^T(t)M_1+\dot{x}^T(t)M_2\right]\left[\dot{x}(t)-(A_z^c-B_z^cK_z^dG_z^{-1})x(t)\right]=0 \tag{16}$$

with M_1 and M_2 arbitrary matrices.

According to these preliminaries, the next theorem presents sufficient news conditions to guarantee stabilization of the continuous model(4).

Theorem 3: Consider the TS fuzzy system (4) and the gain matrices K_i^d and G_i given by theorem 2. The equilibrium of model (12) is globally asymptotically stable if there exist a scalar ρ ($0<\rho<1$) such that $h_i+\rho\dot{h}_i>0$ for all i and x, symmetric positive definite matrix P_i, symmetric matrix M_3 and arbitrary matrices M_1 and M_2 satisfying these LMIs:

$$\begin{cases}\Upsilon_{iii}<0, & i=1,2,...,r\\ \Upsilon_{iij}+\Upsilon_{iji}+\Upsilon_{jii}<0, & i,j=1,2,...,r,\quad i\neq j\\ \Upsilon_{ijk}+\Upsilon_{ikj}+\Upsilon_{jik}+\Upsilon_{jki}+\Upsilon_{kij}+\Upsilon_{kji}<0, & \\ \quad i=1,2,...,r-2,\quad j=i+1,...,r-1,\quad k=j+1,...,r & \\ P_i+M_3<0, & i=1,2,...,r\end{cases} \tag{17}$$

with

$$\Upsilon_{ijk}=\begin{bmatrix}-G_k^TM_1A_i^cG_j+G_k^TM_1B_i^cK_j^d+(*) & (*)\\ -\frac{1}{\rho}G_k^TP_jG_i-\frac{1}{\rho}G_k^TM_3G_i & \\ P_iG_j-M_2A_i^cG_j+M_1^TG_j+M_2B_iK_j^d & M_2+M_2^T\end{bmatrix} \tag{18}$$

Proof:

Choose the Lyapunov function (13). Consider its time derivative and add the null terms (15) and (16), it follows that:

$$\begin{aligned}\dot{V}(x(t))&=2x^T(t)P_z\dot{x}(t)+x^T(t)(\dot{P}_z+\mathbf{M}_3)x(t)\\ &+2\left[x^T(t)M_1+\dot{x}^T(t)M_2\right]\left[\dot{x}(t)-(A_z^c-B_z^cK_z^dG_z^{-1})x(t)\right]\end{aligned}$$

So, $\dot{V}(x(t))<0$ if the condition holds:

$$\begin{bmatrix}-M_1A_z^c+M_1B_z^cK_z^dG_z^{-1}+(*)+\dot{P}_z+\mathbf{M}_3 & (*)\\ P_z-M_2A_z^c+M_1^T+M_2B_z^cK_z^dG_z^{-1} & M_2+M_2^T\end{bmatrix}<0 \tag{19}$$

We multiply the last condition on the left and right by congruence with the matrix $diag(G_z^{-1}\ I)$ and we hold:

$$\begin{bmatrix}-G_z^TM_1A_z^cG_z+G_z^TM_1B_z^cK_z^d+(*) & (*)\\ +G_z^T\dot{P}_zG_z+G_z^T\mathbf{M}_3G_z & \\ P_zG_z-M_2A_z^cG_z+M_1^TG_z+M_2B_z^cK_z^d & M_2+M_2^T\end{bmatrix}<0 \tag{20}$$

which leads to:

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}h_ih_k\left(h_j\Psi_{ijk}+\dot{h}_j\Phi_{ijk}\right)<0 \tag{21}$$

with:

$$\Psi_{ijk}=\begin{bmatrix}-G_k^TM_1A_i^cG_j+G_k^TM_1B_i^cK_j^d+(*) & (*)\\ P_iG_j-M_2A_i^cG_j+M_1^TG_j+M_2B_i^cK_j^d & M_2+M_2^T\end{bmatrix}$$

$$\Phi_{ijk}=\begin{bmatrix}G_k^TP_jG_i+G_k^TM_3G_i & 0\\ 0 & 0\end{bmatrix}$$

Assuming that there exist a scalar ρ ($0<\rho<1$) such that $h_i+\rho\dot{h}_i>0$ for all i and x, then (21) becomes:

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}h_ih_k\left(h_j\Psi_{ijk}+\frac{h_j+\rho\dot{h}_j-h_j}{\rho}\Phi_{ijk}\right)<0 \tag{22}$$

which is equivalent to

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}h_ih_k\left(h_j\left(\Psi_{ijk}-\frac{1}{\rho}\Phi_{ijk}\right)+\frac{h_j+\rho\dot{h}_j}{\rho}\Phi_{ijk}\right)<0 \tag{23}$$

So sufficient conditions to satisfy (23) are:

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}h_ih_kh_j\left(\Psi_{ijk}-\frac{1}{\rho}\Phi_{ijk}\right)<0 \tag{24}$$

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}h_ih_k\left(\frac{h_j+\rho\dot{h}_j}{\rho}\right)\Phi_{ijk}<0 \tag{25}$$

Equation (25) can be reduced to: $P_j+M_3<0$; $j=1,2,...,r$.

As shown in [10], (24) with Υ_{ijk} defined in (18) can be written as:

$$\begin{aligned}&\sum_{i=1}^{r}h_i^3\Upsilon_{iii}+\sum_{i=1}^{r}\sum_{i\neq j}^{r}h_i^2h_j(\Upsilon_{iij}+\Upsilon_{iji}+\Upsilon_{jii})\\ &+\sum_{i=1}^{r-2}\sum_{j=i+1}^{r-1}\sum_{k=j+1}^{r}h_ih_jh_k(\Upsilon_{ijk}+\Upsilon_{ikj}+\Upsilon_{jik}+\Upsilon_{jki}+\Upsilon_{kij}+\Upsilon_{kji})<0\end{aligned}$$

which gives the sufficient conditions in (17).

As shown in [6], the value of ρ is given, a posteriori, such that $h_i+\rho\dot{h}_i>0$ for all i and x.Thus the proof is completed and the TS continuous fuzzy system (12) is asymptotically stable by new stabilization conditions which are in the form of LMIS and can be solved via LMI control toolbox [12].

Remark 2: Compared with others works in the literature [6], the authors used a non PDC controller and a non quadratic lyapunov function. The approach presented in their paper is more complicated. In fact, to avoid the problem of BMIS conditions, they used a Lyapunov function depending on gain

$G^{-1}_{z(t)}$ which is the form of: $V(x(t)) = x^T(t)G^{-1}_{z(t)}x(t)$. Since, if they used a Lyapunov function such (13), they will fall into BMI problem. On the other side, the advantage of our proposed approach is to lead, directly, to LMIS conditions using very complex control law and Lyapunov function (13). May be the novel approach doesn't improve the conservatism of the results obtained in the literature but we have been able to reach a new approach which is simpler than other ones when the controller gains are not known.

V. Simulation Examples

To illustrate the effectiveness of the proposed approach, it will be applied to two examples.

Example 1:

Consider a continuous TS fuzzy model with three rules described by the following matrices:

$$A_1^c = \begin{bmatrix} 0.4336 & 3.4969 & -1.2608 \\ -1.2417 & -0.1268 & -1.7609 \\ 7.0123 & 4.1182 & 0.9172 \end{bmatrix},\ B_1^c = \begin{bmatrix} -1.8533 \\ -1.6675 \\ -0.1836 \end{bmatrix}$$

$$A_2^c = \begin{bmatrix} 1.7982 & 6.2225 & -0.5677 \\ -2.4373 & 2.1246 & -1.7632 \\ 4.8669 & 5.9119 & 0.5389 \end{bmatrix},\ B_2^c = \begin{bmatrix} -2.4991 \\ -0.0891 \\ -1.0981 \end{bmatrix}$$

$$A_3^c = \begin{bmatrix} 0.2450 & 4.7009 & -2.2986 \\ -1.0859 & 0.1313 & -3.0472 \\ 6.6137 & 3.4311 & 2.4282 \end{bmatrix},\ B_3^c = \begin{bmatrix} -1.7080 \\ -0.9472 \\ -1.4954 \end{bmatrix}$$

The stabilization of this model by solving the LMI conditions given by theorem 1, leads to unfeasible solutions. Whereas, using the discretized model based on the Euler approximation (7) with $T_e = 0.01$ s and solving conditions in theorem 2, this system has feasible solutions.

Whereas, using the discretized model based on the Euler approximation (7) with $T_e = 0.01$ s and solving conditions in theorem 2, this system has feasible solutions and the controller gains matrices are given by:

$$K_1^d = \begin{bmatrix} -2.0282 & -1.0470 & -0.0903 \end{bmatrix}$$

$$K_2^d = \begin{bmatrix} -2.2865 & -0.7839 & -0.3577 \end{bmatrix}$$

$$K_3^d = \begin{bmatrix} -3.7375 & -1.7905 & -1.7104 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} 0.3292 & 0.1155 & -0.3531 \\ 0.1331 & 0.1659 & 0.0336 \\ -0.3652 & 0.0279 & 0.6908 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 0.3628 & 0.1417 & -0.3418 \\ 0.1227 & 0.1609 & 0.0347 \\ -0.3273 & 0.0460 & 0.6911 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} 0.3887 & 0.1510 & -0.3465 \\ 0.1459 & 0.1710 & 0.0337 \\ -0.3328 & 0.0393 & 0.6701 \end{bmatrix}$$

By choosing $\rho = 0.2$ and using these values of gains matrices, the stabilization of this continuous model has been guaranteed, and we obtain the following matrices satisfying theorem 3:

$$P_1 = \begin{bmatrix} 25.9891 & -25.7170 & 14.0613 \\ -25.7170 & 29.4479 & -14.4243 \\ 14.0613 & -14.4243 & 9.7277 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 21.5867 & -22.9398 & 11.9588 \\ -22.9398 & 34.2722 & -12.6771 \\ 11.9588 & -12.6771 & 8.5710 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 25.2560 & -26.3256 & 13.9525 \\ -26.3256 & 34.5528 & -14.8826 \\ 13.9525 & -14.8826 & 9.9607 \end{bmatrix}$$

$$M_1 = \begin{bmatrix} -29.4918 & 12.2679 & 5.2836 \\ 30.7485 & -20.2625 & -2.3825 \\ -16.5706 & 5.9882 & 1.7429 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} -0.5143 & 0.6078 & 0.3072 \\ 0.5316 & -0.9654 & -0.0353 \\ -0.0564 & -0.3291 & -0.6680 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} -29.3312 & 30.0103 & -16.2904 \\ 30.0103 & -41.4499 & 17.0608 \\ -16.2904 & 17.0608 & -11.3331 \end{bmatrix}$$

For the simulation, the following premise is used: $z(t) = -0.5\left(1+\sin\left(x_3(t)\right)\right)$ with $-\frac{\pi}{2} \le x_3(t) \le \frac{\pi}{2}$

The membership functions are defined as indicated in [11]. For $\rho = 0.2$, the conditions $h_i + \rho\dot{h}_i > 0$ for all i and x are checked with the initial conditions given by $x(0) = \begin{bmatrix} 0.5 & 0.5 & 0.5 \end{bmatrix}$.

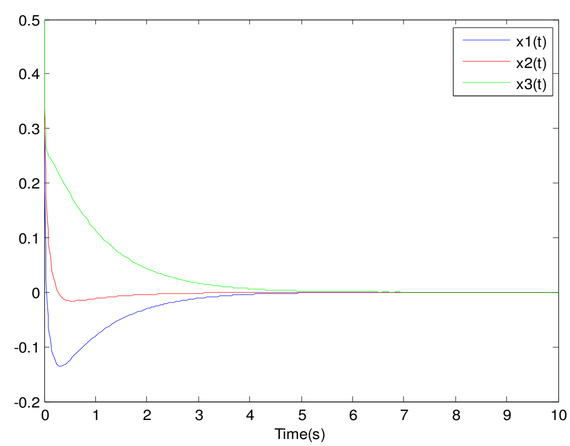

Fig. 1 Evolutions of the state variable $x_1(t), x_2(t), x_3(t)$

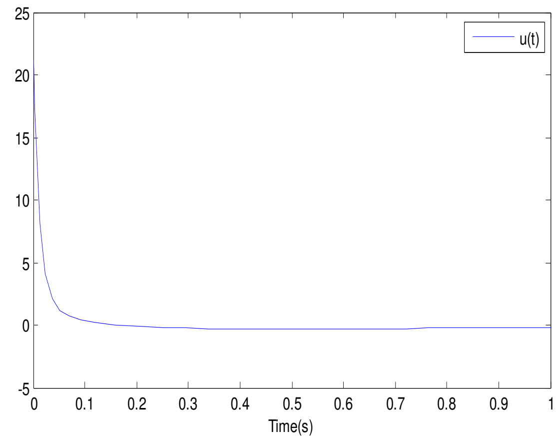

Fig. 2 Evolutions of the control signal $u(t)$

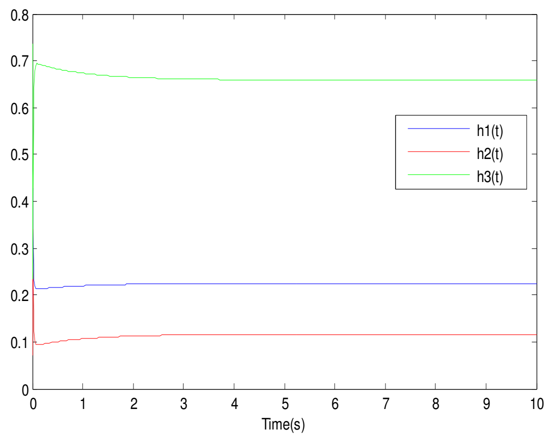

Fig. 3 Evolutions of the membership functions $h_i + \rho \dot{h}_i$

Figs. 1-3 illustrate respectively, the closed-loop results for the continuous system (12), the control law (11) and the membership functions $h_i + \rho \dot{h}_i$.

As can be seen, the evolutions of the state response and the control signal showing, respectively, in Fig.1 and Fig.2, lead to interesting results regarding fast convergence in the stabilization of this continuous TS fuzzy system. Thus, it can be concluded that the state response of the closed loop continuous system is asymptotically stabilized by the gains of a non-PDC controller ensuring the stabilization of its discrete model. The evolution of the membership functions $h_i + \rho \dot{h}_i$ is depicted in Fig.3, from which we can see that $h_i + \rho \dot{h}_i > 0$ for all i and x.

Example 2:

Consider a continuous TS fuzzy model with four rules described by the following matrices:

$$A_1^c = \begin{bmatrix} -0.2148 & -2.5854 \\ 3.4708 & 0.7516 \end{bmatrix},\ A_2^c = \begin{bmatrix} -0.5332 & -2.6571 \\ 4.2280 & -0.7629 \end{bmatrix},$$

$$A_3^c = \begin{bmatrix} 0.4848 & -2.6106 \\ 4.7066 & -0.2436 \end{bmatrix},\ A_4^c = \begin{bmatrix} -0.8060 & -1.8184 \\ 4.1234 & -0.4564 \end{bmatrix}$$

$$B_1^c = \begin{bmatrix} 1.1449 \\ 1.9387 \end{bmatrix},\ B_2^c = \begin{bmatrix} 0.4442 \\ 2.3260 \end{bmatrix},$$

$$B_3^c = \begin{bmatrix} -0.3627 \\ 2.4011 \end{bmatrix},\ B_4^c = \begin{bmatrix} -1.0255 \\ 0.8189 \end{bmatrix}$$

This model doesn't have a solution by solving LMIs condition (theorem 1). Its discretized model is obtained using the Euler approximation (7) with $T_e = 0.05$ s. By solving conditions in theorem 2, this system has feasible solutions.

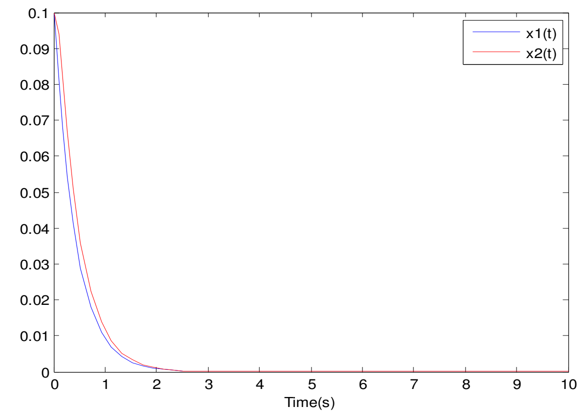

Fig. 4 Evolution of the state variable $x_1(t), x_2(t)$

For the simulation, we choose the following premises:

$$z_1(t) = x_1^2(t) \text{ with } -1 \le x_1(t) \le 1$$

$$z_2(t) = -\frac{1}{2}\left(1 + \sin\left(x_2(t)\right)\right) \text{ with } -\frac{\pi}{2} \le x_2(t) \le \frac{\pi}{2}$$

The membership functions are defined as indicated in [11].

As an example, for $\rho = 0.8$ the conditions $h_i + \rho \dot{h}_i > 0$ are verified a posteriori with the initial conditions given by $x(0) = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}$.

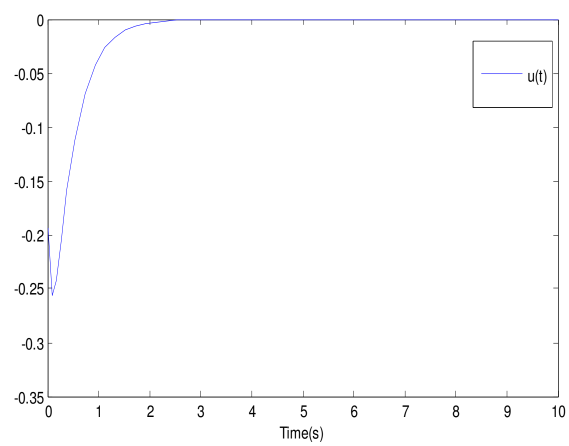

Fig. 5 Evolutions of the control signal $u(t)$

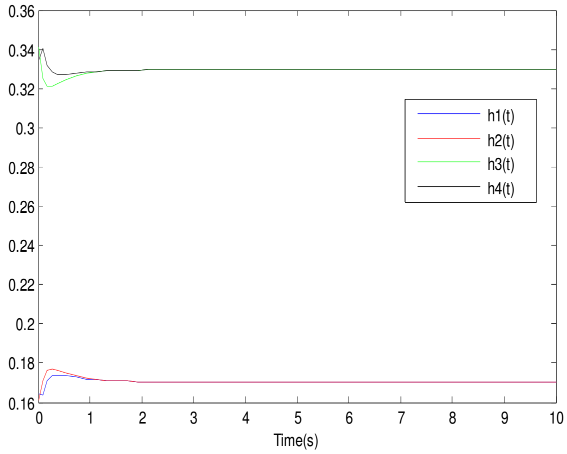

Fig. 6 Evolutions of the membership functions $h_i + \rho\dot{h}_i$

VI. CONCLUSION

In this paper we propose to simplify the stabilization of a continuous TS fuzzy model. Usually trying to use a non quadratic lyapunov function leads to BMIS. However in this paper we obtain LMI. The trick is to first discretize the model. Indeed it is well known for a long time that using non quadratic lyapunov functions in the discrete case is not a problem and may lead to LMIs quite easily. Then we use the control law computed in the discrete case directly for the continuous model. Of course the discrete control law is applied continuously. This way we can use very complex control law in the continuous case while avoiding the problem of getting BMIS sets of conditions. At the end, we have one set of LMIS conditions for the discrete case, and another one when checking the stability of the closed loop for the continuous case. In the future we will try more complex control law and we will start to study the effect of the discretization step.

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REFERENCES

[1] G. T. Takagi, M. Sugeno, *Fuzzy identification of systems and its applications to modeling and control*, IEEE Transactions on SMC, vol.15, n°1, p.116-132, 1985.

[2] S. Boyd, L.El Ghaoui, E. Feron, V. Balakrishnan, *Linear Matrix Inequalities in systems and control theory*, SIAM, Philadelphia, 1994.

[3] H.O. Wang, K. Tanaka, M. Griffin, *An approach to fuzzy control of non linear systems :stability and design issues*, IEEE Transactions on Fuzzy Systems, p.14-23, 1996.

[4] H.D. Tuan, P. Apkarian, T. Narikiyo, Y. Yamamoto, *Parameterized linear matrix inequality techniques in Fuzzy control system design,* IEEE Transaction on Fuzzy Systems, vol.19, n°2, p. 324-332, 2001.

[5] K. Tanaka, T. Hori, H.O. Wang, *A multiple Lyapunov function approach to stabilization of fuzzy control systems,* IEEE Transaction on Fuzzy Systems , vol.11, n°4, p.582–589, 2003.

[6] X.H. Chang, G.H. Yang, *Relaxed stabilization conditions for continuous-time Takagi–Sugeno fuzzy control systems*, Information Sciences, p.3273–3287, 2010.

[7] T.M. Guerra, L. Vermeiren, *LMI-based relaxed nonquadratic stabilization conditions for nonlinear systems in the Takagi–Sugeno's form,* Automatica, vol. 40, n°5, p. 823–829, 2004.

[8] F.O. Souza, R.M. Palhares, P.Y. Ekel, *Asymptotic stability analysis in uncertain multi-delayed state neural networks via Lyapunov–Krasovskii theory*, Math. Comput. Model., 2007.

[9] L.A. Mozelli, R.M. Palhares, G S.C. AvellarL, *A systematic approach to improve multiple Lyapunov function stability and stabilization conditions for fuzzy systems*, Information Sciences 179, p. 1149–1162, 2009.

[10] C.H. Fang, Y.S. Liu, S.W. Kau, L. Hong, C.H. Lee, *A NewLMI-Based Approach to Relaxed Quadratic Stabilization of T-S Fuzzy Control Systems*, IEEE Transaction on Fuzzy Systems, vol.14 , n°3, June 2006.

[11] K. Tanaka, H.O. Wang, *Fuzzy control systems design and analysis. A linear matrix inequality approach*, John Wiley & Sons, New York, 2001.

[12] P. Gahinet, A. Nemirovski, A.J. Laub, M. Chilali, *LMI Control Toolbox*, The MathWorks Inc., Natick, 1995.

[13] K. Tanaka, H.O. Wang, *Fuzzy control systems design and analysis.A linear matrix inequality approach.* John Wiley and Sons, New York. 2001.

[14] B.C. Ding, H.X. Sun, P. Yang, *Further studies on LMI-based relaxed stabilization conditions for nonlinear systems in Takagi-Sugeno's form, Automatica* Vol. 42, p. 503-508, 2006.

[15] Y. Cao, P.M. Frank, M.L. Hadjili, *stability analysis and synthesis of nonlinear time-delay systems via linear Takagi-Sugeno fuzzy models,* Fuzzy Sets and Systems, vol.124, P. 213-229, 2001.

[16] Y. Cao, P.M. Frank, M.L. Hadjili, Improved delay-dependent robust stabilization conditions of uncertain T-S fuzzy systems with time-varying delay, Fuzzy Sets and Systems, vol.159, P. 2713-2729, 2008.

[17] T-M. Guerra, H. Kerkeni, J. Lauber, L. Vermeiren, An efficient Lyapunov function for discrete TS models: observer design. IEEE Transactions on Fuzzy Systems, 20 (1), pp. 187-192, 2012.