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# Design of an Interval Type-2 Fuzzy Logic Controller Based on Conventional PI Controller

Mortaza Aliasghary, İbrahim Eksin, Müjde Güzelkaya and Tufan Kumbasar

***Abstract*— Interval type-2 fuzzy logic controllers have attracted much research interests in recent years due to their ability to cope with uncertainty. In this paper, we propose a systematical methodology to construct an interval type-2 fuzzy logic controller. The main contribution of this methodology is to generate the rule base of an interval type-2 fuzzy logic controller based on an existing conventional PI controller. As a consequence a linear control law is transformed to a nonlinear structure. Thus, the designer can benefit from the nonlinear structure of the proposed controller and the extra degree of freedom of type-2 fuzzy sets. The simulation results have shown that the proposed controller can manage the uncertainties much better than linear conventional PI and type-1 fuzzy logic controllers.**

## I. Introduction

Fuzzy logic systems (FLSs) have been widely developed and utilized in many practical applications and engineering systems. However, the most important applications and studies about FLSs have been committed in the field of fuzzy logic control (FLC) [1]-[3]. The type-2 fuzzy sets (T2FSs) are the extension of the ordinary type-1 fuzzy sets which aim to model the uncertainty in the fuzzy logic systems. The T2FLS may be able to outperform its type-1 counterpart because of the additional degrees of freedom provided by the footprint of uncertainty (FOU) in their membership functions [4-6]. Nevertheless, the computations of type-2 fuzzy systems are more complex than type-1 fuzzy systems. Therefore, a special type of type-2 fuzzy logic sets called interval type-2 fuzzy sets is proposed in [7]. It has been shown that interval type-2 fuzzy sets are much more powerful tools to represent the inputs and/or outputs of fuzzy logic controller [8]. The benefits of interval type-2 fuzzy logic controllers (IT2-FLCs) are demonstrated in several control applications such as liquid-level process control [9]; autonomous mobile robots [10]; plants control [11]; bioreactor control [12]; pH control [13].

It is a known fact that Proportional-Integral (PI) and Proportional-Integral-Derivative (PID) controllers are the most popular ones used in industrial applications. Performance comparisons between conventional controllers and fuzzy logic controllers have been reported in the literature. In [14] Ying compared the performance of linear PID controllers with type-1 fuzzy logic controllers (T1-FLC) through different simulations and has shown that T1-FLC is advantageous only when the parameters of T1-FLC are properly designed. The main difficulty in T1-FLC design is to determine the parameters of the fuzzy logic controllers [15]. For this reason, the researchers proposed a general methodology to systematically construct a fuzzy logic controller based on the existence of a linear and nonlinear control law [16, 17].

Studies have been reported in the literature that the IT2-FLCs are generally more robust than T1-FLC [8]. Tan and Lai in [18] developed an interval type-2 fuzzy proportional controller with a variable gain. A practical approach for the design of PD and PI IT2-FLC has been proposed in [19]. Derivation and analysis of the Mamdani type of IT2F PI and PD controllers are studied in [20]. The analytical structure of a special class of IT2 fuzzy PI and PD controllers that uses the Karnik-Mendel algorithm for type-reduction was presented in [21].

In this paper, we propose a systematical methodology to construct an interval type-2 fuzzy logic controller. The methodology depends on a nonlinear mapping from an existing PI control law to IT2-FLC that captures the benefits of a PI controller in terms of simplicity and also can handle nonlinearity because of their type 2 fuzzy membership functions. The proposed nonlinear mapping could be done under certain circumstances that input type-2 membership functions are diamond-shaped and the closed-form inference engine given in [22] is used. When the footprint of uncertainty of the antecedent membership functions is taken to be zero, an identical mapping is accomplished between conventional PI controller and the proposed controller. If FOU is not equal to zero, then an additional degree of freedom is acquired and this provides an uncertainty cloud over the proposed controller. This provides the designer an additional tool to cope with the uncertainties and nonlinearities which may exist in the system to be controlled. The simulation studies have been performed to show that the proposed controller can manage the uncertainties much better than linear conventional PI and type-1 fuzzy logic controllers.

The paper is organized in five sections. In Section II the general structure of the proposed IT2-FLC is discussed in detail. In Section III, analytical derivations are done to show the mapping between the proposed control structure and the conventional PI controller. In Section IV, simulation studies are given to demonstrate the beneficial sides of FOU existing within the proposed controller under system parameter variations. Finally, discussions and conclusions are presented in Section V.

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## II. General Structure of the Proposed IT2-FLC

In the considered IT2-FLC, type-2 membership functions in the premise part and crisp numbers for the consequent part are used. The general rule structure of IT2-FLC is as follows [5]:

$$\text{Rule } l^{th}: \text{IF } x_1 \text{ is } \tilde{F}_1^j \text{ and } x_2 \text{ is } \tilde{F}_2^j \text{ Then } y^l \tag{1}$$

Where $x_1, x_2$ are the feedback error and the integral of error, respectively, $y^l (l = 1, \dots, M)$ is the consequent part, $M$ is the number of rules and $\tilde{F}_i^j$ denotes the type-2 membership functions for $j^{th}$ fuzzy set associated with the $i^{th}$ input. ($i = 1,2,\ j = 0,1$). The final output of the system can be written as

$$Y = \int_{f^1 \in [\underline{f}^1, \overline{f}^1]} \cdots \int_{f^M \in [\underline{f}^M, \overline{f}^M]} 1 / \frac{\sum_{l=1}^{M} f^l y^l}{\sum_{l=1}^{M} f^l} \tag{2}$$

where $\underline{f}^l$ and $\overline{f}^l$ are given by

$$\begin{aligned} \underline{f}^l(x) &= \underline{\mu}_{\tilde{F}_1^j}(x_1) * \underline{\mu}_{\tilde{F}_2^j}(x_2) \\ \overline{f}^l(x) &= \overline{\mu}_{\tilde{F}_1^j}(x_1) * \overline{\mu}_{\tilde{F}_2^j}(x_2) \end{aligned} \tag{3}$$

and $\overline{\mu}_{\tilde{F}_i^j}(x_i), \underline{\mu}_{\tilde{F}_i^j}(x_i)$ are the upper and lower membership functions, respectively. Here, the operator $*$ represents the t-norm, which is the product operator. The output of the IT2-FLC is achieved via the inference engine [22] as follows:

$$Y = \frac{\sum_{l=1}^{M} \underline{f}^l y^l}{\sum_{l=1}^{M} \underline{f}^l + \sum_{l=1}^{M} \overline{f}^l} + \frac{\sum_{l=1}^{M} \overline{f}^l y^l}{\sum_{l=1}^{M} \underline{f}^l + \sum_{l=1}^{M} \overline{f}^l} \tag{4}$$

In this paper, the diamond-shaped type-2 membership function [23] is used. Assume that two interval type-2 fuzzy sets cover the universe of discourse of the input variables. As it can be clearly seen in Fig. 1, the diamond-shaped type-2 membership function gets "0" or "1" values at both ends of the support and the kernel. In Fig.1, $\bar{F}_i^j$ represents the height of the $j^{th}$ fuzzy set associated with the $i^{th}$ input.

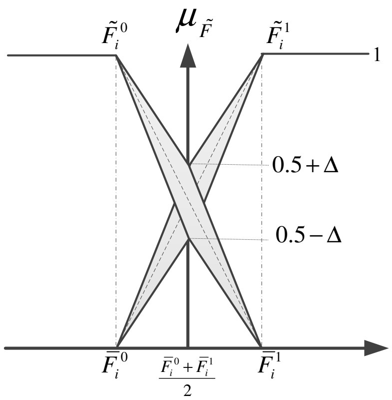



Fig. 1. The diamond-shaped type-2 membership function

Based on the proposed membership function, the upper membership function is given by

$$\overline{\mu}_{\tilde{F}_i^0} = \begin{cases} \dfrac{(\bar{F}_i^1 - x_i) + 2\Delta(x_i - \bar{F}_i^0)}{\bar{F}_i^1 - \bar{F}_i^0} & \bar{F}_i^0 \le x_i \le \bar{F}_i^0 + \bar{F}_i^1/2 \\ \dfrac{(\bar{F}_i^1 - x_i)(1 + 2\Delta)}{\bar{F}_i^1 - \bar{F}_i^0} & \bar{F}_i^0 + \bar{F}_i^1/2 \le x_i \le \bar{F}_i^1 \end{cases} \tag{5}$$

and the lower membership function is given by

$$\underline{\mu}_{\tilde{F}_i^0} = \begin{cases} \dfrac{(\bar{F}_i^1 - x_i) - 2\Delta(x_i - \bar{F}_i^0)}{\bar{F}_i^1 - \bar{F}_i^0} & \bar{F}_i^0 \le x_i \le \bar{F}_i^0 + \bar{F}_i^1/2 \\ \dfrac{(\bar{F}_i^1 - x_i)(1 - 2\Delta)}{\bar{F}_i^1 - \bar{F}_i^0} & \bar{F}_i^0 + \bar{F}_i^1/2 \le x_i \le \bar{F}_i^1 \end{cases} \tag{6}$$

The membership function $\tilde{F}_i^1$ can naturally be expressed as:

$$\underline{\mu}_{\tilde{F}_i^1} = 1 - \overline{\mu}_{\tilde{F}_i^0} \tag{7}$$

$$\overline{\mu}_{\tilde{F}_i^1} = 1 - \underline{\mu}_{\tilde{F}_i^0} \tag{8}$$

The location of the crisp numbers for the consequent part of IT2-FLC is the key feature of this study. Let us consider a linear PI controller described as

$$u = k_p e + k_i \int_{t_0}^{t} e\, dt = k_p x_1 + k_i x_2 \tag{9}$$

where $e$ and $\int_{t_0}^{t} e\, dt$ are the error and the integral of error, respectively and $u$ is the control signal. The general structure of the membership function of consequent part is as [16]

$$y^l = k_p \bar{F}_1^j + k_i \bar{F}_2^j \quad j \in \{0, 1\} \tag{10}$$

As it is seen from (10), the location of the crisp numbers for the consequent part of IT2-FLC directly incorporates the PI control law.

## III. Analytical Derivations for the Proposed IT2-FLC

Using (4) IT2-FLC output $Y$ for 2-input 1-output control system can be calculated as

$$Y = \frac{\sum_{l=1}^{4} \underline{f}^l y^l + \sum_{l=1}^{4} \overline{f}^l y^l}{\sum_{l=1}^{4} \underline{f}^l + \sum_{l=1}^{4} \overline{f}^l} = \frac{Y^N}{Y^D} \tag{11}$$

Since the number of the output sets is equal to $M = 2^2$, the membership function of consequent part can be given as

$$\begin{aligned} y^1 &= k_p \bar{F}_1^0 + k_i \bar{F}_2^0 \\ y^2 &= k_p \bar{F}_1^0 + k_i \bar{F}_2^1 \\ y^3 &= k_p \bar{F}_1^1 + k_i \bar{F}_2^0 \\ y^4 &= k_p \bar{F}_1^1 + k_i \bar{F}_2^1 \end{aligned} \tag{12}$$

Using the approach described by (10) and (12) the consequent part of rules is generated as in Table I. To simplify the derivation, we assume that $\frac{\bar{F}_i^0+\bar{F}_i^1}{2} \leq x_i \leq \bar{F}_i^1$ $i = 1,2$.

TABLE I
IT2-FLC RULE-BASE FOR A SYSTEM WITH 4 RULES

| $x_1$ \ $x_2$ | $\tilde{F}_2^{\,0}$ | $\tilde{F}_2^{\,1}$ |
|---|---|---|
| $\tilde{F}_1^{\,0}$ | $y^1$ | $y^2$ |
| $\tilde{F}_1^{\,1}$ | $y^3$ | $y^4$ |

By using (3) and (11) the following relations are obtained for $Y^D$ :

$$\begin{aligned} Y^D = {} & \underline{\mu}_{\tilde{F}_1^0}\underline{\mu}_{\tilde{F}_2^0} + \overline{\mu}_{\tilde{F}_1^0}\overline{\mu}_{\tilde{F}_2^0} + \underline{\mu}_{\tilde{F}_1^0}\underline{\mu}_{\tilde{F}_2^1} + \overline{\mu}_{\tilde{F}_1^0}\overline{\mu}_{\tilde{F}_2^1} \\ & + \underline{\mu}_{\tilde{F}_1^1}\underline{\mu}_{\tilde{F}_2^0} + \overline{\mu}_{\tilde{F}_1^1}\overline{\mu}_{\tilde{F}_2^0} + \underline{\mu}_{\tilde{F}_1^1}\underline{\mu}_{\tilde{F}_2^1} + \overline{\mu}_{\tilde{F}_1^1}\overline{\mu}_{\tilde{F}_2^1} \end{aligned} \tag{13}$$

Simplifying (13) we obtain

$$\begin{aligned} Y^D = {} & \left(\underline{\mu}_{\tilde{F}_1^0} + \underline{\mu}_{\tilde{F}_1^1}\right)\left(\underline{\mu}_{\tilde{F}_2^0} + \underline{\mu}_{\tilde{F}_2^1}\right) \\ & + \left(\overline{\mu}_{\tilde{F}_1^0} + \overline{\mu}_{\tilde{F}_1^1}\right)\left(\overline{\mu}_{\tilde{F}_2^0} + \overline{\mu}_{\tilde{F}_2^1}\right) \end{aligned} \tag{14}$$

Using the (7) and (8), (14) can be rewritten as

$$Y^D = 2\left[\left(\overline{\mu}_{\tilde{F}_1^0} - \underline{\mu}_{\tilde{F}_1^0}\right)\left(\overline{\mu}_{\tilde{F}_2^0} - \underline{\mu}_{\tilde{F}_2^0}\right) + 1\right] \tag{15}$$

and by using (5) and (6), (16) can be calculated as

$$\overline{\mu}_{\tilde{F}_i^0} - \underline{\mu}_{\tilde{F}_i^0} = \frac{4\Delta(\bar{F}_i^1 - x_i)}{\bar{F}_i^1 - \bar{F}_i^0} \tag{16}$$

Substituting (16) in (15), $Y^D$ is obtained as

$$Y^D = 2\left[\left(\frac{4\Delta(\bar{F}_1^1 - x_1)}{\bar{F}_1^1 - \bar{F}_1^0}\right)\left(\frac{4\Delta(\bar{F}_2^1 - x_2)}{\bar{F}_2^1 - \bar{F}_2^0}\right) + 1\right] \tag{17}$$

The nominator $Y^N$ of (11) can be written as

$$Y^N = \sum_{j=1}^{4}\left(\underline{f}^j + \overline{f}^j\right) y^j \tag{18}$$

Thus, $Y^N$ can be subsequently derived as

$$\begin{aligned} Y^N = {} & y^1\left(\underline{\mu}_{\tilde{F}_1^0}\underline{\mu}_{\tilde{F}_2^0} + \overline{\mu}_{\tilde{F}_1^0}\overline{\mu}_{\tilde{F}_2^0}\right) \\ & + y^2\left(\underline{\mu}_{\tilde{F}_1^0}\underline{\mu}_{\tilde{F}_2^1} + \overline{\mu}_{\tilde{F}_1^0}\overline{\mu}_{\tilde{F}_2^1}\right) \\ & + y^3\left(\underline{\mu}_{\tilde{F}_1^1}\underline{\mu}_{\tilde{F}_2^0} + \overline{\mu}_{\tilde{F}_1^1}\overline{\mu}_{\tilde{F}_2^0}\right) \\ & + y^4\left(\underline{\mu}_{\tilde{F}_1^1}\underline{\mu}_{\tilde{F}_2^1} + \overline{\mu}_{\tilde{F}_1^1}\overline{\mu}_{\tilde{F}_2^1}\right) \end{aligned} \tag{19}$$

Since the relations $\underline{\mu}_{\tilde{F}_i^1}(x_i) = 1 - \overline{\mu}_{\tilde{F}_i^0}(x_i)$ and $\overline{\mu}_{\tilde{F}_i^1}(x_i) = 1 - \underline{\mu}_{\tilde{F}_i^0}(x_i)$ are valid in the chosen membership structure, (19) can be rewritten as in (20).

$$\begin{aligned} Y^N = {} & (y^1 + y^4)\left(\underline{\mu}_{\tilde{F}_1^0}\underline{\mu}_{\tilde{F}_2^0} + \overline{\mu}_{\tilde{F}_1^0}\overline{\mu}_{\tilde{F}_2^0}\right) \\ & + (y^2 - y^4)\left(\underline{\mu}_{\tilde{F}_1^0} + \overline{\mu}_{\tilde{F}_1^0}\right) \\ & - (y^2 + y^3)\left(\underline{\mu}_{\tilde{F}_1^0}\overline{\mu}_{\tilde{F}_2^0} + \overline{\mu}_{\tilde{F}_1^0}\underline{\mu}_{\tilde{F}_2^0}\right) \\ & + (y^3 - y^4)\left(\underline{\mu}_{\tilde{F}_2^0} + \overline{\mu}_{\tilde{F}_2^0}\right) + 2y^4 \end{aligned} \tag{20}$$

From (12) it is obvious that $y^1 + y^4 = y^2 + y^3$ . Thus, we can simplify $Y^N$ as

$$\begin{aligned} Y^N = {} & (y^1 + y^4)\left(\underline{\mu}_{\tilde{F}_1^0}\underline{\mu}_{\tilde{F}_2^0} + \overline{\mu}_{\tilde{F}_1^0}\overline{\mu}_{\tilde{F}_2^0} - \underline{\mu}_{\tilde{F}_1^0}\overline{\mu}_{\tilde{F}_2^0} \right. \\ & \left. - \overline{\mu}_{\tilde{F}_1^0}\underline{\mu}_{\tilde{F}_2^0}\right) \\ & + (y^2 - y^4)\left(\underline{\mu}_{\tilde{F}_1^0} + \overline{\mu}_{\tilde{F}_1^0}\right) \\ & + (y^3 - y^4)\left(\underline{\mu}_{\tilde{F}_2^0} + \overline{\mu}_{\tilde{F}_2^0}\right) + 2y^4 \end{aligned} \tag{21}$$

In order to simplify (21), the following equations are derived separately from the previous equations.

$$\underline{\mu}_{\tilde{F}_i^0} + \overline{\mu}_{\tilde{F}_i^0} = \frac{2(\bar{F}_i^1 - x_i)}{\bar{F}_i^1 - \bar{F}_i^0} \tag{22}$$

$$\begin{aligned} & \underline{\mu}_{\tilde{F}_1^0}\underline{\mu}_{\tilde{F}_2^0} + \overline{\mu}_{\tilde{F}_1^0}\overline{\mu}_{\tilde{F}_2^0} - \underline{\mu}_{\tilde{F}_1^0}\overline{\mu}_{\tilde{F}_2^0} - \overline{\mu}_{\tilde{F}_1^0}\underline{\mu}_{\tilde{F}_2^0} \\ & \qquad = \frac{2(\bar{F}_1^1 - x_1)(\bar{F}_2^1 - x_2)(8\Delta^2)}{(\bar{F}_1^1 - \bar{F}_1^0)(\bar{F}_2^1 - \bar{F}_2^0)} \end{aligned} \tag{23}$$

Substituting (12), (22) and (23) into (21), $Y^N$ is obtained as

$$\begin{aligned} Y^N = {} & \left(k_p(\bar{F}_1^0 + \bar{F}_1^1) + k_i(\bar{F}_2^0 + \bar{F}_2^1)\right) \\ & \times\left(\frac{2(\bar{F}_1^1 - x_1)(\bar{F}_2^1 - x_2)(8\Delta^2)}{(\bar{F}_1^1 - \bar{F}_1^0)(\bar{F}_2^1 - \bar{F}_2^0)}\right) \\ & + 2\left(k_p x_1 + k_i x_2\right) \end{aligned} \tag{24}$$

Finally, the output of IT2-FLC can be found as follows

$$Y = \frac{P(x_i, \bar{F}_i^j, K, \Delta) + 2\left(k_p x_1 + k_i x_2\right)}{2\left[Q(x_i, \bar{F}_i^j, \Delta) + 1\right]} \tag{25}$$

where

$$\begin{aligned} P\left(x_i, \bar{F}_i^j, K, \Delta\right) = {} & \left(k_p(\bar{F}_1^0 + \bar{F}_1^1) + k_i(\bar{F}_2^0 + \bar{F}_2^1)\right) \\ & \times\left(\frac{2(\bar{F}_1^1 - x_1)(\bar{F}_2^1 - x_2)(8\Delta^2)}{(\bar{F}_1^1 - \bar{F}_1^0)(\bar{F}_2^1 - \bar{F}_2^0)}\right) \end{aligned} \tag{26}$$

$$Q(x_i, \bar{F}_i^j, \Delta) = \left(\frac{4\Delta(\bar{F}_1^1 - x_1)}{\bar{F}_1^1 - \bar{F}_1^0}\right)\left(\frac{4\Delta(\bar{F}_2^1 - x_2)}{\bar{F}_2^1 - \bar{F}_2^0}\right) \tag{27}$$

Here $x_1$ and $x_2$ are the inputs $e$ and $\int_{t_0}^{t} e\,dt$, respectively. Note that if $\Delta = 0$, it is clear that $Q\left(x_i, \bar{F}_i^j, \Delta\right) = 0$ and $P\left(x_i, \bar{F}_i^j, K, \Delta\right) = 0$. So, IT2-FLC reduces to a T1-FLC and the fuzzy controller has an identical output to the PI control law as following

$$Y = k_p x_1 + k_i x_2 = k_p e + k_i \int_{t_0}^{t} e\,dt \tag{28}$$

TABLE II
THE PERFORMANCE VALUES FOR THE SYSTEM WITH PARAMETER UNCERTAINTY

| | $k=1$, $\tau=10$ | | $k=1.5$, $\tau=10$ | | $k=0.5$, $\tau=10$ | | $k=1$, $\tau=8$ | | $k=1$, $\tau=12$ | | $L=4$ | | $L=1.5$ | | $k=1.4$, $\tau=11, L=3$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $ITAE$ | $T_s$ | $ITAE$ | $T_s$ | $ITAE$ | $T_s$ | $ITAE$ | $T_s$ | $ITAE$ | $T_s$ | $ITAE$ | $T_s$ | $ITAE$ | $T_s$ | $ITAE$ | $T_s$ |
| PI T1-FLC | 43.82 | 22.33 | 516.5 | 87.33 | 30.75 | 17.83 | 85.39 | 30.91 | 34.12 | 17.25 | 8784 | 429 | 11.31 | 9.31 | 1690 | 156.1 |
| IT2-FLC Δ= 0.1 | 42.64 | 22.25 | 435.7 | 77.15 | 30.09 | 17.85 | 80.59 | 30.8 | 33.48 | 17.37 | 6826 | 331.5 | 10.6 | 9.22 | 1301 | 131.3 |
| IT2-FLC Δ= 0.2 | 39.97 | 21.42 | 286.6 | 57.20 | 28.52 | 17.90 | 70.04 | 26.37 | 32.18 | 17.84 | 3731 | 209.3 | 8.73 | 8.83 | 733.5 | 88.5 |
| IT2-FLC Δ= 0.3 | 37.44 | 17.95 | 181.6 | 42 | 28.39 | 18.12 | 59.73 | 26.60 | 31.57 | 18.82 | 2012 | 144.5 | 6.83 | 7.85 | 420.3 | 64 |
| IT2-FLC Δ= 0.4 | 36.12 | 18.83 | 124 | 37.26 | 29.89 | 18.50 | 52.21 | 26.36 | 32.35 | 20.32 | 1276 | 112.81 | 7.085 | 4.80 | 271.7 | 51.7 |

## IV. SIMULATION STUDIES

In this section, the simulation results of the proposed controller are presented. In order to compare the performance of the proposed controller with the conventional PI controller and T1-FLC, the following uncertain first-order plus time delay (FOPTD) system is considered:

$$G(s) = \frac{Ke^{-Ls}}{\tau s + 1} \qquad (29)$$

The nominal parameters of the FOPTD model are $K = 1$, $\tau = 10$ (s) and $L = 2.5$ (s). The uncertainty intervals of the parameters are $K = [0.5,1.5]$, $\tau = [8,12]$ and $L = [1.5,4]$. The PI parameters are found according to the Ziegler-Nichols method, as $k_p = 3.6$ and $k_i = 0.432$. Two diamond shaped membership functions are used for both IT2-FLC and T1-FLC as shown in Fig. 1. By substituting $k_p$ and $k_i$ into (12), the consequents of the rules will be formed. In order to make a comparison of the IT2-FLC with the PI (and naturally T1-FLC) two performance measures are considered which are Integral Time Absolute Error (ITAE) and Settling Time ($T_s$). The unit step reference signal is applied to the closed-loop system.

Fig. 2 shows the step responses of the nominal plant in (29) when Δ= 0. It is clear from (28) and Fig. 2 that the results of IT2-FLC are identical to T1-FLC and PI controller. By increasing the value of Δ in IT2-FLC, the linear PI controller (T1-FLC) will be transformed to a nonlinear PI controller as shown in (25). A step response on the nominal plant with various values of Δ is shown in Fig. 3 and the performance measures are given in Table II.

In order to investigate the robustness of the proposed IT2-FLC, the nominal plant parameters are varied. The ITAE performance index and settling time for the new model parameters are shown in Table II. Fig. 4 shows the step responses and the control signals of the system by varying the static gain $K$ of the plant. From Fig. 4, it is clear the proposed controller is more capable in eliminating the oscillation in comparison to the conventional PI (T1-FLC). Secondly, the parameter $\tau$ is changed within the interval of uncertainty and the results are shown in Fig. 5. As it is tabulated in Table II, increasing of the value of Δ does not necessarily improve the performance of the controller.

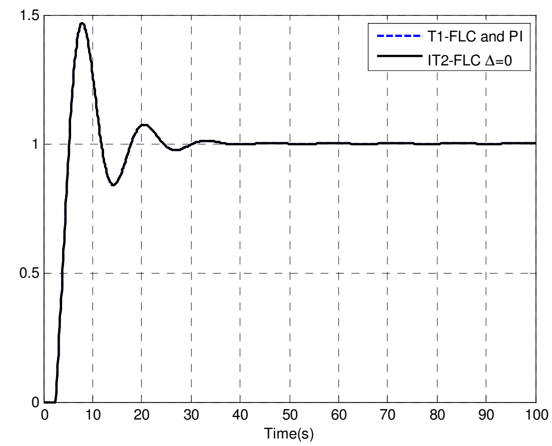



Fig. 2. Illustration of the step responses on the nominal plant with Δ= 0

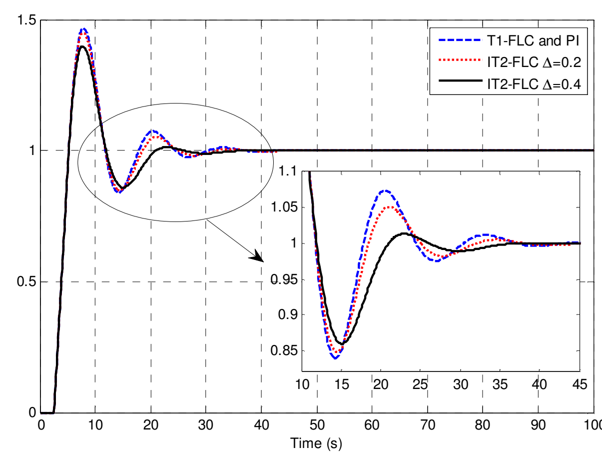
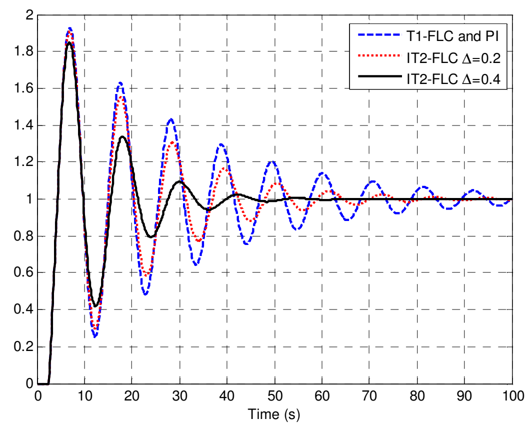
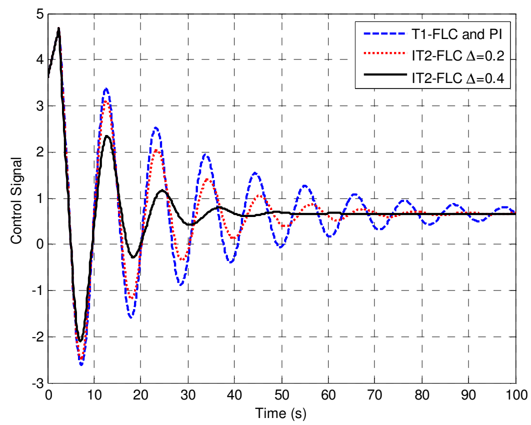



Fig. 3. Illustration of the step responses with different values of Δ




(a)




(b)

Fig. 4. Illustration of the (a) the step responses (b) the control signals on the plant by varying $k = 1.5$

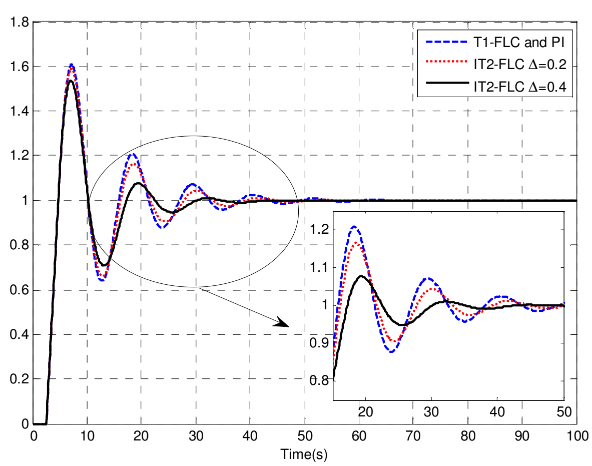
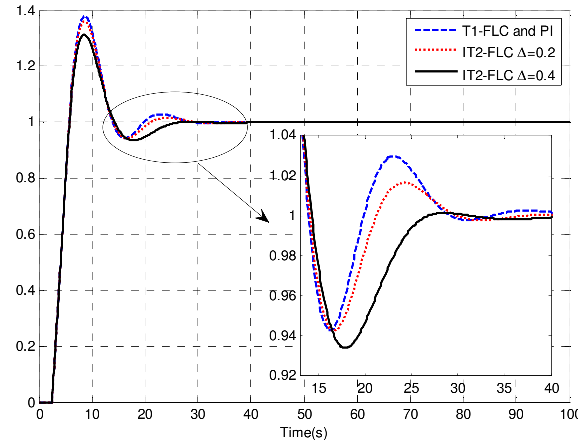
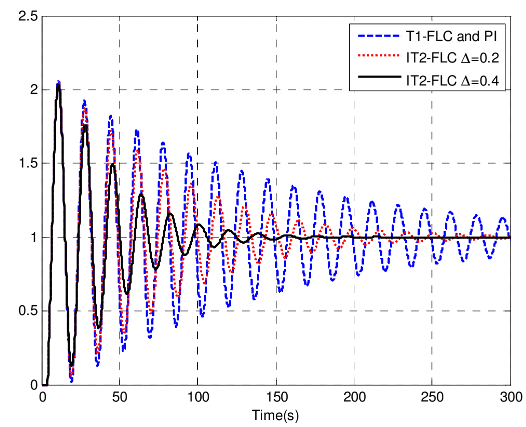



Fig. 5. (a)

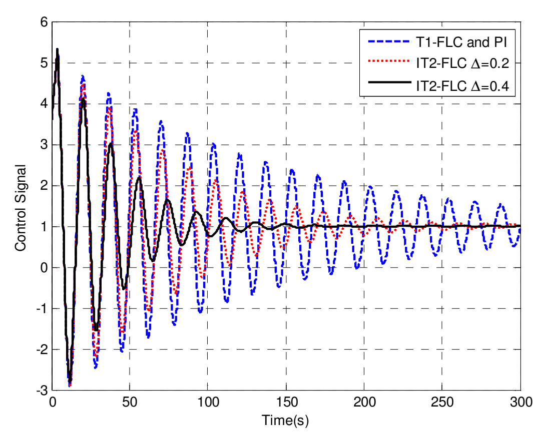



(b)

Fig. 5. Illustration of the step responses on the plant by varying $\tau$ (a) $\tau = 8$ (b) $\tau = 12$




(a)



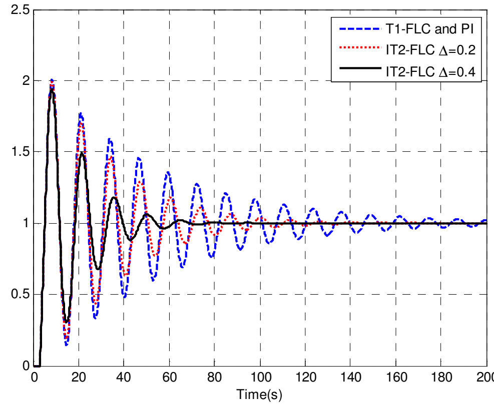

(b)

Fig. 6. Illustration of (a) the step responses (b) the control signals on the plant by varying the time delay $L = 4$

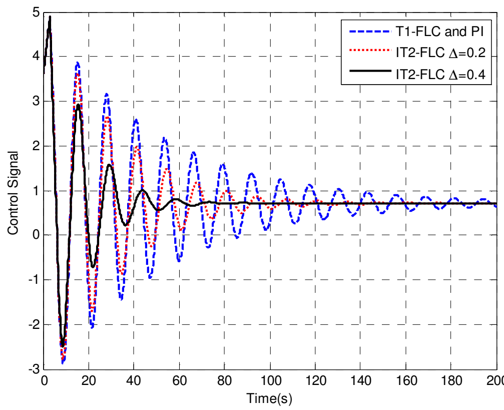
The system response is illustrated in Fig. 6 for the case of varying on the time delay of the process to 4. It is easily seen from Fig. 6 that the proposed controller eliminates the oscillations much better than PI controller. Finally, all process parameters are changed as $k = 1.4,\ \tau = 11,\ L = 3$ and the corresponding system responses are shown in Fig. 7. The performance values of the proposed IT2-FLC compared to the linear PI controller can be found in Table II.

## V. Conclusion

In this paper, a systematical methodology to construct an interval type-2 fuzzy logic controller is proposed. The methodology depends on a nonlinear mapping from an existing PI control law to IT2-FLC that captures the benefits of a PI controller in terms of simplicity and also can handle nonlinearity because of their type 2 fuzzy membership functions. The proposed control structure is achieved under




(a)




(b)

Fig. 7. Illustration of the (a) step responses (b) control signals on the plant with the parameters $k = 1.4,\ \tau = 11,\ L = 3$

circumstances that input type-2 membership functions are diamond-shaped and a certain closed-form inference engine is used. If the FOU of the IT2-FLC is zero then the obtained control law is identical to the conventional PI controller. If FOU is not equal to zero, then an additional degree of freedom is acquired and this provides the designer an additional tool to cope with the uncertainties. Results show that the proposed controller can manage the uncertainties much better than linear conventional PI and type-1 fuzzy logic controllers. It can also be concluded that increasing the value of FOU (Δ) does not necessarily improve the performance of the controller.

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