

Model Predictive Control of Distributed Energy Resources

E. D. Mehleri, H. Sarimveis, L. G. Papageorgiou and N. C. Markatos

Abstract— Distributed energy resources (DER) are expected to play a critical role in the near future due to a number of advantages they have compared to conventional centralised power systems. DER systems typically involve many components that combine different technologies for producing and storing electrical energy and heat. Thus, operational optimisation and control of DER systems is a challenging task. In this work, the Model Predictive Control (MPC) rolling horizon approach is adopted to design a control strategy for DER systems. The proposed approach is not limited to single households but addresses DER systems in the neighbourhood level, taking into account the possibility of energy exchange among neighbouring households.

I. INTRODUCTION

IN recent year rapid changes are observed in the energy market due to the need to meet challenges and targets regarding energy savings and environmental protection. In addition, smart energy sensors and actuators allow end-users to make informed choices about the consumption of energy in their premises. In this dynamic environment, considerable attention has been given to the development of decentralised high-efficiency energy generation, known as distributed energy resource (DER) systems [1], which can employ a wide range of technologies including renewable energy resources [2]. Huang et al. [3] provided a comprehensive review of the current status of various DER technologies and discussed their operation and market environment. Driesen and Katiraei [4] reviewed different approaches for DER systems architecture and design. A number of recent papers proposed structural optimisation techniques for the optimal design of DER systems [5-6]. Operational optimisation and control of DER systems is a challenging task due to the integration of many different technologies in a single system. Among the different approaches, the Model Predictive Control (MPC) approach [7-8] has been proved satisfactory for intelligent control of DER systems and superior to standard control methods [9-10].

II. PROBLEM DESCRIPTION

We consider the development of a decision making

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strategy regarding the optimal operation of a DER system combined with a heating pipeline network and electricity transmission lines that satisfies the power and heat demands of a small neighbourhood consisted of a number of dwellings with estimated electricity and heating profiles. Every dwelling can satisfy its load by a Combined heat and power (CHP) plant, a Photovoltaic (PV) array, a storage tank and a back-up boiler. Surplus electricity can be delivered back to the grid, while the utility electricity can support the deficit electricity. Electricity can be transferred between the different dwellings in two different ways: Through the existing power transmission network or through a central storage tank, where energy is stored for later use.

The decision making strategy is built upon the MPC rolling horizon philosophy.

In the MPC optimisation problem, the following data are given: Dwellings, electricity and heat profiles for the prediction horizon; Electricity and gas tariff prices, prices of selling excess electricity; Solar irradiance for determining the power output of the PV array; Allocations and capacities of DER technologies; Pipeline heating network in the neighbourhood; Storage tank capacities; Initial values of all state variables.

The objective function to be minimised contains all the components that affect the operational cost of the DER system over the prediction horizon. The solution of the optimisation problem provides the optimal values of the decision variables over the future horizon. The key decision variables are: Electricity and heat production profiles per dwelling; Heat transfer amounts through pipeline network; Main flows of electricity and heat between grid and dwellings.

III. FORMULATION OF MPC PROBLEM

The MPC problem that is solved in each time period is formulated as a Mixed Integer Linear Programming (MILP) model.

A. System state

At the beginning of each time instant the control scheme measures the state of the system. The values of the states are used as initial conditions in the formulation of the MPC problem that corresponds to this particular time instant. The state consists of the energy levels of all storage elements (electricity $ES_{0,STO}$ and heat $H_{i,0,STO}, \forall i$) and the states of the CHP units, i.e. the states denoting if the CHP is in operation or not ($W_{i,0}, \forall i$) and the remaining time periods each CHP

unit must be in start-up mode ($ST_{i,0} \forall i$), in operation ($UT_{i,0} \forall i$), out of operation ($DT_{i,0} \forall i$).

B. Objective function

The objective function of the model is to minimise the total operational cost for the residential energy system over the prediction horizon, which is partitioned into t time periods of equal duration pd . The revenue is the income from selling electricity to the grid. The cost includes the cost of purchasing electricity power from the grid, the cost of the consumed fuel, the operational and maintenance cost of the different units and the environmental cost. The objective function is then formulated as follows:

$$\min C_{TOTAL} = C_{OM}^{PV} + C_{OM}^B + C_{OM}^{CHP} + C_{OM}^{HST} + C_{OM}^{EST} + C_{PUR}^{GRID} + C_{CARBTAX} + C_{SAL}^{GRID} \quad (1)$$

The total cost for purchasing electricity is described by Equation (2), which is calculated by the cumulative amount of electricity purchased multiplied by the utility electricity rate.

$$C_{PUR}^{GRID} = \sum_i \sum_t pd \cdot P_{ELEC} \cdot E_{i,t}^{GRID} \quad (2)$$

The cost for carbon emission is expressed in Equation (3). It accounts for the carbon content of the purchased electricity and natural gas and is calculated by the cumulative carbon emission multiplied by the carbon tax rate.

$$C_{CARBTAX} = CT \cdot pd \cdot \left[\begin{aligned} & (CI_{ELEC} \cdot E_{i,t}^{GRID}) + (CI_{GAS}) \cdot \\ & \left(\frac{(E_{i,t,SELF}^{CHP} + E_{i,t,SAL}^{CHP} + E_{i,t,STO}^{CHP} + E_{i,t,TRA}^{CHP})}{n_e^{CHP}} \right) \\ & \left[\begin{aligned} & + \frac{H_{i,t}^B}{n_{th}^B} + Z_{i,t} \cdot F_{GAS,i} \end{aligned} \right] \end{aligned} \right] \quad (3)$$

The income from selling electricity back to the grid is described by Equation (4). It is calculated by the cumulative amount of electricity delivered to the grid by both CHP and PV multiplied by the electricity buy-back price.

$$C_{SAL}^{GRID} = \sum_i \sum_t pd \cdot P_{SAL}^{PV} \cdot E_{i,t,SAL}^{PV} + \sum_i \sum_t pd \cdot P_{SAL}^{CHP} \cdot E_{i,t,SAL}^{CHP} \quad (4)$$

The operational cost of the CHP plants is composed of the fuel cost, which is calculated based on cumulative fuel consumption for each period multiplied by the fuel price and the operational and maintenance cost.

$$C_{OM}^{CHP} = \sum_i \sum_t pd \cdot \left[\begin{aligned} & \left(\frac{(E_{i,t,SELF}^{CHP} + E_{i,t,SAL}^{CHP} + E_{i,t,STO}^{CHP} + E_{i,t,TRA}^{CHP})}{n_e^{CHP}} \right) \cdot (P_{GAS} / n_e^{CHP} + c_{OM}^{CHP}) \\ & + Z_{i,t} \cdot F_{GAS,i} \cdot P_{GAS} \end{aligned} \right] \quad (5)$$

Similarly, the operational cost of the back-up boilers is composed of the fuel cost, which is calculated by the

cumulative fuel consumption for each period multiplied by the fuel price and the operational and maintenance cost.

$$C_{OM}^B = \sum_i \sum_t pd \cdot (H_{i,t}^B \cdot (P_{GAS} / n_{th}^B + c_{OM}^B)) \quad (6)$$

The operational and maintenance cost of the heat storage tanks is calculated with cumulative heat energy multiplied by a unit maintenance cost coefficient:

$$C_{OM}^{HST} = \sum_i \sum_t pd \cdot c_{OM}^{HST} \cdot H_{i,t,STO} \quad (7)$$

The operational and maintenance cost of the electricity storage tank is calculated with cumulative electricity multiplied by a unit maintenance cost coefficient:

$$C_{OM}^{EST} = \sum_i \sum_t pd \cdot c_{OM}^{EST} \cdot ES_{i,t,STO} \quad (8)$$

C. Supply-demand relationships

A balance of supply and demand has to be achieved for both heat and electric power at each time instance. When referring to electricity, it can be met by purchasing electricity from the grid, electricity generated from the PV arrays and the CHP units, electricity from the power transmission network and electricity stored in the storage tank. Heat loads can be satisfied by the conventional boilers, the CHP units, heat stored in the storage tanks and by transferring heat among dwellings through the Heating Pipeline Network. The electric power balance is shown in Equation (9) and the heat balance is shown in Equation (10):

$$CLoad_{i,t}^{ELEC} = \quad (9)$$

$$E_{i,t}^{GRID} + E_{i,t,SELF}^{PV} + E_{i,t,SELF}^{CHP} + E_{i,t,OUT}^{EST} + E_{i,t,OUT}^{TL} \quad \forall i,t$$

$$CLoad_{i,t}^{HEAT} + H_{i,t,IN}^{HST} - H_{i,t,OUT}^{HST} - \sum_j (QH_{j,j,t} - QH_{i,j,t}) = H_{i,t}^B \quad (10)$$

$$+ (E_{i,t,SELF}^{CHP} + E_{i,t,SAL}^{CHP} + E_{i,t,STO}^{CHP} + E_{i,t,TRA}^{CHP}) \cdot HER \quad \forall i,t$$

Furthermore, the following constraints are set to prohibit the customer from buying and selling energy at the same time.

$$E_{i,t,SAL}^{CHP} + E_{i,t,SAL}^{PV} \leq M \cdot V_{i,t} \quad \forall i,t \quad (11)$$

$$E_{i,t}^{GRID} \leq CLoad_{i,t}^{ELEC} \cdot (1 - V_{i,t}) \quad \forall i,t \quad (12)$$

where M is an appropriate upper bound.

D. CHP unit

Eqs. (13-14) link the binary variables that describe the operation of the CHP units. Binary variable $Y_{i,t}$ takes the value of 1 only when CHP unit i starts up, while variable $X_{i,t}$ takes the value of 1 only when the unit shuts down.

$$W_{i,1} - W_{i,0} = Y_{i,1} - X_{i,1} \quad \forall i \quad (13)$$

$$W_{i,t} - W_{i,t-1} = Y_{i,t} - X_{i,t} \quad \forall i, t > 1$$

$$Y_{i,1} + X_{i,1} \leq 1 \quad \forall i \quad (14)$$

$$Y_{i,t} + X_{i,t} \leq 1 \quad \forall i, t > 1$$

The following set of equations forces the CHP unit i to be functional for at least UT_i time periods after startup:

$$\sum_{k=1}^{UT_i} W_{i,k} \geq UT_i \quad \forall i \quad (15)$$

$$\sum_{k=1}^{UT_i} W_{i,t+k} \geq UT_i \cdot Y_{i,t} \quad \forall i, t$$

The next set of equations ensures that a CHP unit is in operation, but still at startup mode for at least ST_i time periods after startup. Start-up mode means that the unit consumes gas but is not delivering electricity or heat:

$$\sum_{k=1}^{ST_i} Z_{i,k} \geq ST_i \quad \forall i \quad (16)$$

$$\sum_{k=1}^{ST_i} Z_{i,t+k} \geq ST_i \cdot Y_{i,t} \quad \forall i, t$$

Another set of equations forces the CHP units to be out of operation for at least DT_i time periods after shutdown:

$$\sum_{k=1}^{DT_i} (1 - W_{i,k}) \geq DT_i \quad \forall i \quad (17)$$

$$\sum_{k=1}^{DT_i} (1 - W_{i,t+k}) \geq DT_i \cdot X_{i,t} \quad \forall i, t$$

The performance characteristics of the CHP plant are described by Equation (18), which indicates that the CHP unit cannot generate more than its installed capacity when it is in operation and not in start-up mode. A lower bound on CHP electricity generation is also enforced in the same equation.

$$W_{i,t} \cdot E_{i,MIN}^{CHP} \leq E_{i,t,SELF}^{CHP} + E_{i,t,SAL}^{CHP} + E_{i,t,STO}^{CHP} + E_{i,t,TRA}^{CHP} \leq W_{i,t} \cdot E_{i,MAX}^{CHP} \quad \forall i, t \quad (18)$$

Eq. (19) indicates that no electricity is produced when the CHP unit is in start-up mode:

$$E_{i,t,SELF}^{CHP} + E_{i,t,SAL}^{CHP} + E_{i,t,STO}^{CHP} + E_{i,t,TRA}^{CHP} \leq (1 - Z_{i,t}) \cdot E_{i,MAX,t}^{CHP} \quad \forall i, t \quad (19)$$

E. Back-up boilers

Constraint (20) poses upper and lower bounds on the heat generated by the back-up boilers:

$$H_{i,MIN}^B \leq H_{i,t}^B \leq H_{i,MAX}^B \quad \forall i, t \quad (20)$$

F. PV array

Eqs. (21-22) limit the electricity produced by the PV arrays by the capacity of the installed system and the amount of solar irradiation.

$$E_{i,t,SELF}^{PV} + E_{i,t,SAL}^{PV} + E_{i,t,STO}^{PV} + E_{i,t,TRA}^{PV} \leq A_i^{PV} \cdot I_t \cdot n_e^{PV} \quad \forall i, t \quad (21)$$

$$E_{i,t,SELF}^{PV} + E_{i,t,SAL}^{PV} + E_{i,t,STO}^{PV} + E_{i,t,TRA}^{PV} \leq A_i^{PV} \cdot Cp_{rat} \quad \forall i, t \quad (22)$$

G. Pipeline network

Eq. (23) defines upper bounds on heat transfer among dwellings:

$$QH_{i,j,t} \leq QH_{i,j,MAX} \quad \forall i, j, t \quad (23)$$

H. Heat storage tank

Additional constraints are needed to ensure the operation of the storage tank. Equation (24) states that for each storage tank the total amount of heat stored at the end of a period is equal to the heat stored at the end of the previous period plus the recovered heat that has been diverted towards storage during that period minus stored heat that is released to meet heat demand during that period.

$$H_{i,t,STO} = (1 - elc) \cdot H_{i,t-1,STO} + pd \cdot (H_{i,t,IN}^{HST} - H_{i,t,OUT}^{HST}) \quad \forall i, t \quad (24)$$

Eq. (25) ensures that heat stored in the storage tanks is within the tank capacities:

$$H_{i,t,STO} \leq H_{i,MAX}^{HST} \quad \forall i, t \quad (25)$$

The differences in heat stored in each storage tank between two consecutive time periods are constrained by the following inequalities:

$$-DRH_{i,STO} \leq H_{i,1,STO} - H_{i,0,STO} \leq URH_{i,STO} \quad \forall i \quad (26)$$

$$-DRH_{i,STO} \leq H_{i,t,STO} - H_{i,t-1,STO} \leq URH_{i,STO} \quad \forall i, t > 1$$

The next equation forces the heat stored in each storage tank at the end of the control horizon to be equal to the initial level:

$$H_{i,N,STO} = H_{i,0,STO} \quad \forall i \quad (27)$$

I. Electricity storage and electricity transfer

The electricity inventory balance is described by equation (28), similarly to the heat storage balance equation. The technology used for the electricity storage is a lead-acid battery with a charge loss ($cl=10\%$) and a discharge loss ($dl=15\%$). Equation (29) poses an upper bound on the level of electricity stored in the storage tank. Equation (30) describes the electric power transmissions between the buildings of the microgrid. The transmission line losses are taken as $trl=1\%$.

$$ES_{i,STO} = ES_{i-1,STO} + pd \cdot (1-cl) \cdot \sum_i (E_{i,t,STO}^{CHP} + E_{i,t,STO}^{PV}) - pd \cdot E_{i,t,OUT}^{EST} / (1-dl) \quad \forall t \quad (28)$$

$$ES_{i,STO} \leq ES_{MAX}^{EST} \quad \forall t \quad (29)$$

$$(1-trl) \cdot \sum_i (E_{i,h,TRA}^{CHP} + E_{i,h,TRA}^{PV}) = \sum_i E_{i,h,OUT}^{TL} / (1-trl) \quad \forall h \quad (30)$$

The differences in electricity stored in the central electricity storage tank between two consecutive time periods are constrained by the following inequalities:

$$-DRE_{STO} \leq ES_{1,STO} - ES_{0,STO} \leq URE_{STO} \quad (31)$$

$$-DRE_{STO} \leq ES_{i,STO} - ES_{i-1,STO} \leq URE_{STO} \quad \forall t > 1$$

The next equation forces the electricity stored in the storage tank at the end of the control horizon to be equal to the initial level:

$$ES_{N,STO} = ES_{0,STO} \quad (32)$$

The problem is solved using the GAMS software and in particular the CPLEX solver [11], which employs standard branch-and-bound procedures [12].

IV. NUMERICAL STUDY

A. Description of the DER system

A case study is presented below with main focus on the Greek residential sector, comprising a network of 5 buildings in Athens (Greece). Following a rigorous design methodology [6] which excluded the PV technology, the capacity installed in the 5 dwellings is shown in Table I.

TABLE I
INSTALLED CAPACITY (HEAT STORAGE TANK/CHP)

Building	HEAT STORAGE TANK (KW)	CHP (KW power)
I1	2.9	0
I2	5.1	0
I3	11.2	15
I4	0.7	0
I5	1.4	0

Additionally, a heating pipeline network is implemented, which serves the transportation of heat from building I2 to buildings I1 and I5, from building I3 to I2 and from building I5 to I4. The design does not involve a central electricity storage tank. Electricity is exchanged among the dwellings only through the power transmission network.

B. Electricity and gas tariffs

The utility rate is 0.11 €/kWh [32]. The price of selling excess electricity to the grid is =0.08785 €/kWh for CHP units and 0.55 €/kWh for PV units. This price refers to the electricity buy-back price based on Greek governmental policies for PV systems up to 10 kWp and for residential CHP systems. Cost of natural gas is assumed 0.054 €/kWh.

The carbon tax of is 0.017 €/kg. The emission of CO2 for every kWh produced is 0.781 kg/kWh. The emission for every kWh natural gas is 0.184 kg/kWh. The electricity factor refers to end-use energy produced by the Greek electricity power mix, while the natural gas factor is based on lower heating value (LHV) and refers to kWh of input fuel.

C. Energy loads

In this study the electricity and heating load demands of a typical household in Athens (Greece) for one month (November), are used. Based on the average electricity profile of a typical household (110m²), the profiles of dwellings of different sizes have been extracted. In Figures 1 and 2 the electricity and heat load profiles for one day for building I3 are presented.

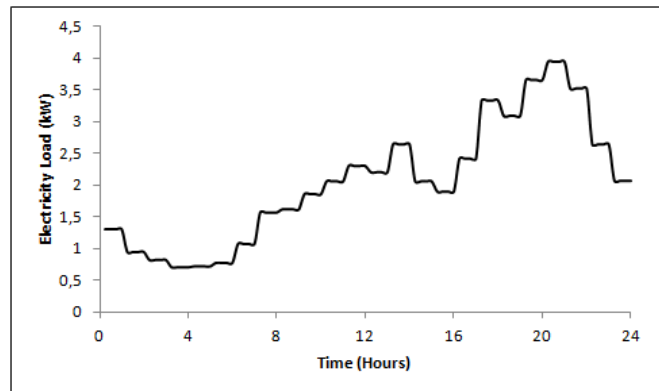


Figure 1. Electricity load of building I3 (kW)

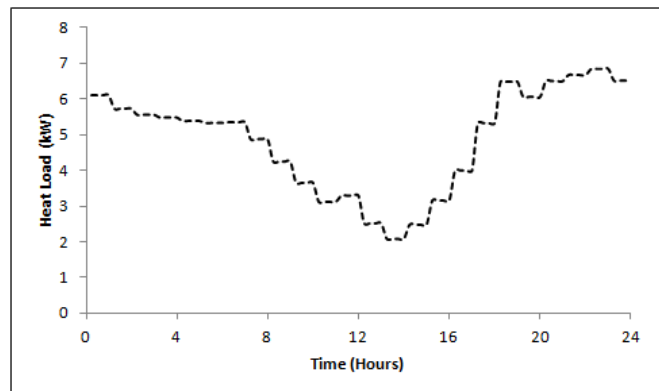


Figure 2. Heat load of building I3 (kW)

D. Results and Discussion

The aforementioned MPC problem was solved for a time period of 2 days using a prediction and control horizon of 48 15-min time intervals. The MILP model formulated at each solve has 4119 constraints and contains 5815 decision variables, 1200 of which are binary variables. The average time needed by CPLEX to solve the MILP problem to global optimality was 0.21 sec, while the longest run took 0.45 sec. All runs were performed in a regular PC running at 2.53 GHz. Taking into account that the action horizon is equal to 15 mins, the size of the system can be considerably scaled-

up before running into intractability problems. For very large-scale problems, distributed MPC algorithms need to be taken into account, which decompose the overall optimisation problem into several smaller-scale problems [13].

Key results are depicted in Table II and in Figures 3-7. Table I, shows that the selection of the DER approach leads to a 10% and 13% reduction of the Total and Environmental Cost, compared to a conventional system where all required electricity is purchased from the national grid, while heat is produced by conventional boilers.

TABLE II
BREAKDOWN OF THE TOTAL COST

	CONVENTIONAL SYSTEM	DER SYSTEM
Total Cost (€)	89.71	80.27
Operational Cost CHP (€)	-	110.08
Operational Cost of Boiler (€)	62.2	-
Operational Cost of Heat Storage Tank (€)	-	0.87
Environmental Cost (€)	6.2	5.4
Cost of Purchased Electricity (€)	21.31	0
Income from Selling Electricity to the Grid (€)	-	36.08

Figures 3-5 illustrate the operation of the CHP unit installed in building I3 and the distribution of the produced electricity among self-use, electricity delivered to other buildings and electricity sold to the grid.

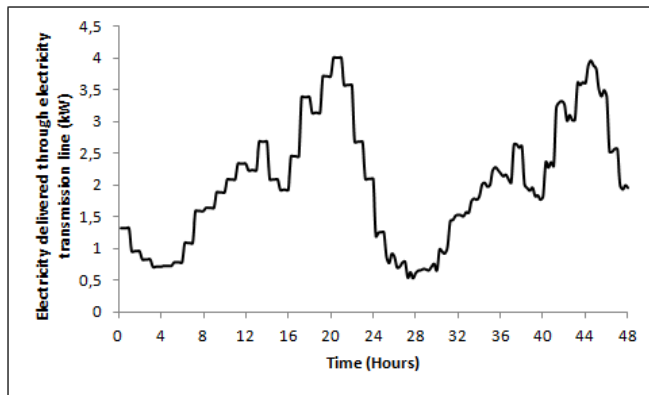


Fig. 3. Electricity transmitted through the electricity transmission line from the CHP unit (kW)

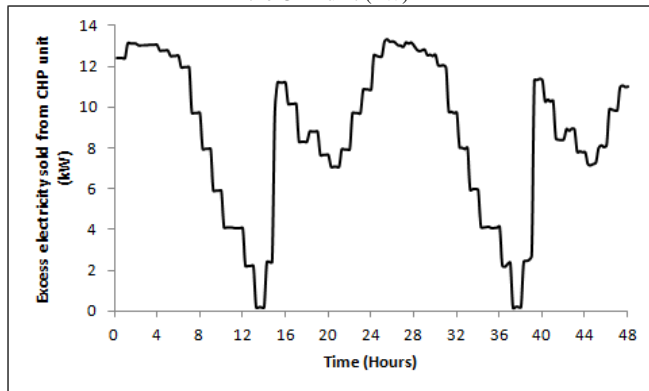


Fig. 4. Excess electricity from CHP unit sold to the grid (kW)

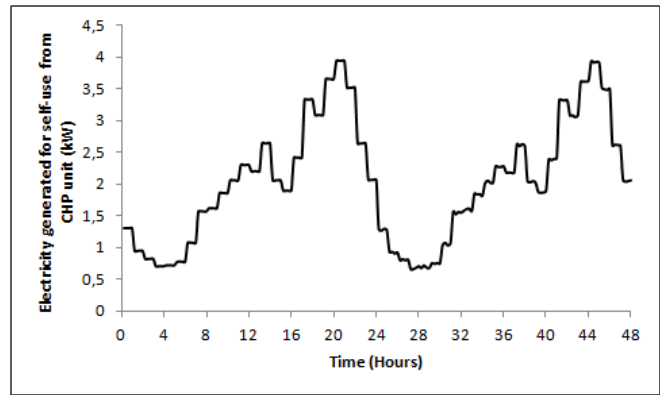


Fig. 5. Electricity generated for self-use from CHP unit (kW)

In Figure 6 the heat storage tank profiles per dwelling are presented, while Figure 7 depicts the main flows of heat between dwellings through the heating pipeline network.

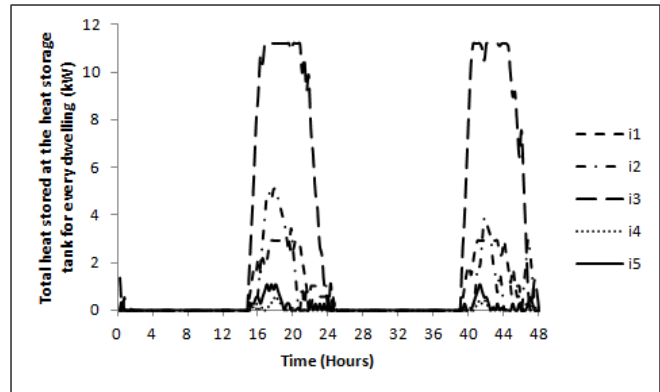


Fig. 6. Heat stored in the storage tank for every dwelling (kW)

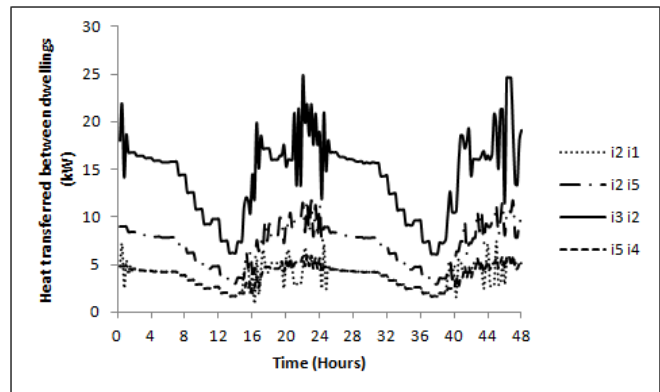


Fig. 7. Heat transferred between dwellings through the heating pipeline network (kW)

NOMENCLATURE

Parameters

CI_{ELEC}	Carbon intensity of electricity, kg CO_2 /kWh electricity
CI_{GAS}	Carbon intensity of natural gas, kg CO_2 /kWh natural gas
cl	Charge loss of the electrical storage tank
c_{OM}^B	Operational and maintenance cost of boiler, €/kWh
c_{OM}^{CHP}	Operational and maintenance cost of CHP unit, €/kWh
c_{OM}^{EST}	Operational and maintenance cost of electrical storage tank, €/kWh
c_{OM}^{HST}	Operational and maintenance cost of heat storage tank, €/kWh
CT	Carbon tax of CO_2 , €/kg
dl	Discharge loss of the electrical storage tank
$DRH_{i,STO}$	Upper bound on the decrease of the heat stored in the storage tank for dwelling i between two consecutive periods
DRE_{STO}	Upper bound on the decrease of the electricity stored in the electricity tank between two consecutive periods
DT_i	Number of consecutive time periods the CHP unit i must be shut-down after stopping
elc	Energy loss coefficient, %
$E_{i,MAX}^{CHP}$	Upper bound on energy generated by the CHP unit i , kW
$E_{i,MIN}^{CHP}$	Lower bound on energy generated by the CHP unit i , kW
ES_{MAX}^{EST}	Upper bound on electricity stored in the electrical storage tank, kW
$F_{GAS,i}$	Natural gas consumed by CHP unit i during start-up, KW
HER	Heat to electricity ratio
$H_{i,MIN}^B$	Lower bound on heat generated by the boiler for every dwelling i , kW
$H_{i,MAX}^B$	Upper bound on heat generated by the boiler for every dwelling i , kW
$H_{i,MAX}^{HST}$	Upper bound on heat stored in the heat storage tank for every dwelling i , kW
n_{th}^B	Thermal efficiency of the boiler, %
n_e^{CHP}	Electrical efficiency of the CHP unit, %
n_e^{PV}	Electrical efficiency of the PV panel, %
It_h	Irradiance at period h , kW/ m^2
pd	Period duration, hours

P_{ELEC}	Grid electricity price, €/kWh
P_{GAS}	Natural gas price, €/KWh
P_{SAL}^{CHP}	Price of selling excess electricity from CHP, €/kWh
P_{SAL}^{PV}	Price of selling excess electricity from PV, €/kWh
$QH_{i,j,MAX}$	Upper bound on heat transferred from dwelling i to j , kW
ST_i	Number of consecutive time periods the CHP unit i must be in operation but still in start-up mode after start-up
trl	Electricity transmission line losses, %
$URH_{i,STO}$	Upper bound on the increase of the heat stored in the storage tank for dwelling i between two consecutive periods
URE_{STO}	Upper bound on the increase of the electricity stored in the electricity tank between two consecutive periods
UT_i	Number of consecutive time periods the CHP unit i must be in operation after start-up

Variables

C_{TOTAL}	Total cost, € during control horizon
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Positive variables

A_i^{PV}	Surface of the PV panels in every dwelling i , m^2
$C_{CARBTAX}$	Total environmental cost, €
C_{PUR}^{GRID}	Total cost for purchased electricity, €
C_{SAL}^{GRID}	Income from selling electricity to the grid, €
C_{OM}^B	Operational cost of boiler, €
C_{OM}^{CHP}	Operational cost of CHP, €
C_{OM}^{EST}	Operational cost of electricity storage tank, €
C_{OM}^{HST}	Operational cost of heat storage tank, €
$E_{i,t}^{GRID}$	Purchased electricity from the grid for every dwelling i at period t , kW
$E_{i,t,SAL}^{CHP}$	Excess electricity sold from CHP unit for every dwelling i at period t , kW
$E_{i,t,SELF}^{CHP}$	Electricity generated for self-use from CHP unit for every dwelling i at period t , kW
$E_{i,t,STO}^{CHP}$	Electricity stored from CHP unit for every dwelling i at period t , kW
$E_{i,t,TRA}^{CHP}$	Electricity delivered from CHP unit through electricity transmission lines

	for every dwelling i at period t , kW
$E_{i,t,SAL}^{PV}$	Excess electricity sold from PV for every dwelling i at period t , kW
$E_{i,t,SELF}^{PV}$	Electricity generated for self-use from PV for every dwelling i at period t , kW
$E_{i,t,STO}^{PV}$	Electricity stored from PV for every dwelling i at period t , kW
$E_{i,t,TRA}^{PV}$	Electricity delivered from PV through electricity transmission lines for every dwelling i at period t , kW
$E_{i,t,OUT}^{EST}$	Electricity released by the electricity storage tank to meet the electricity load for every dwelling i at period t , kW
$E_{i,t,OUT}^{TL}$	Electricity transmitted through electricity transmission lines to meet the electricity load for every dwelling i at period t , kW
$ES_{i,t,STO}$	Electricity stored in the electricity storage tank at period t , kWh
$H_{i,t}^B$	Generated heat by the boiler for every dwelling i at period t , kW
$H_{i,t,IN}^{HST}$	Excess heat stored at the storage tank for every dwelling i at period t , kW
$H_{i,t,OUT}^{HST}$	Heat released by the storage tank for every dwelling i at period t to meet the heating load, kW
$H_{i,t,STO}$	Total heat stored at the storage tank for every dwelling i at period t , kWh
$QH_{i,j,t}$	Heat transferred from dwelling i to j at period t , kW

Binary variables

$V_{i,t}$	1 if dwelling i may sell excess electricity to the grid at period h ; 0 if it may buy from the grid
$W_{i,t}$	1 if the CHP unit in dwelling i is in operation during period h ; 0 otherwise
$X_{i,t}$	1 if the CHP unit in dwelling i shuts down in period h ; 0 otherwise
$Y_{i,t}$	1 if the CHP unit in dwelling i starts up in period h ; 0 otherwise
$Z_{i,t}$	1 if the CHP unit in dwelling i is in operation but still in start-up mode during period h ; 0 otherwise

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