Parity Space Hybrid System Diagnosis under Model Uncertainty

Jorge Vento, Vicenç Puig and Ramon Sarrate

Abstract— **In this paper, diagnosis for hybrid systems using a parity space approach that considers model uncertainty is proposed. The hybrid diagnoser is composed of modules which carry out the mode recognition and diagnosis tasks both based on residuals generated using a model. Both tasks interact each other since the diagnosis module adapts himself according to the current mode of the hybrid system. Moreover, the methodology takes into account the parameter uncertainty using a passive robust strategy. An adaptive threshold for residual evaluation is generated and the parity space approach is used to design a set of residuals for each mode. The proposed fault diagnosis approach for hybrid systems is illustrated on a part of the Barcelona sewer network.**

I. INTRODUCTION

Most real systems are on-line controlled and supervised by means of automatic computer-based control systems. But, they are subject to faults that can appear in the plant components, sensors and actuators. Many of these systems present a behavior that changes with the operating mode, which can be modeled as a hybrid system. Thus, fault diagnosis using models, mostly developed for non-hybrid systems, should be extended to handle the hybrid system behavior.

Recently in the literature, model based techniques have been proposed to diagnose hybrid systems [1], [2], [3], [4]. The continuous behavior in each mode is described using differential equations. These techniques extend, in some way, existing model-based approaches for non-hybrid systems, being able to handle the continuous and discrete-event system behaviors. In hybrid systems, the diagnoser should be parameterized as a function of the current mode. Thus, the proposed diagnoser should be able to evaluate the behaviour of the hybrid system on-line, and to detect and isolate the mode and the faults. In [1], [2], the discrete-event behavior is modeled as a set of discrete modes, that can include nominal or faulty modes, and transitions between them are governed by events. Following the methodology proposed by [5], [6], a diagnoser combining the discrete and the continuous dynamics is built by means of a behaviour automaton. In [3], some guidelines are given on mode recognition and fault diagnosis based on a set of residuals. However, uncertainty is not taken into account in the model, nor a formal methodology to build a hybrid diagnoser is proposed.

The contribution of this paper is to present a fault diagnosis method for hybrid systems where the current operation mode is recognized by generating the set of residuals by means of the parity space approach and that takes into account the system parameter uncertainty in the residual evaluation. The robustness is enhanced using a passive strategy based on generating an adaptive threshold that considers model uncertainty in the residuals evaluation extending the results for the LTI case presented in [8].

The structure of this paper is the following. In Section 2, a hybrid model description that includes parameter uncertainty is presented. In Section 3, the hybrid system fault detection technique is introduced, while the fault isolation and mode recognition procedures are described in Section 4 and Section 5, respectively. In Section 6, an application case study based on the sewer network of the Barcelona city is used to assess the validity of the proposed approach. Finally, Section 7 summarizes the main paper conclusions.

II. PROBLEM SET-UP

A. Hybrid model

Let us consider that the model of the hybrid system to be diagnosed can be described by the following hybrid automaton $HA =< \mathcal{Q}, \mathcal{X}, \mathcal{U}, \mathcal{Y}, \mathcal{F}, \mathcal{G}, \mathcal{H}, \Sigma, \mathcal{T} >$, where:

- $\mathcal{Q} = \{q^i : i \in M\}$ is a set of discrete states and q^0 is the initial discrete state. The finite set $M = \{1, 2, \cdots, m\}$ indexes the nominal states.
- $\mathcal{X} \subseteq \Re^{nx}$ defines a discrete-time continuous state space, $\mathbf{x}(k) \in \mathcal{X}$ is the discrete-time state vector at sample k and $\mathbf{x}_0$ the initial state vector.
- $\mathcal{U} \subseteq \Re^{nu}$ defines a discrete-time continuous input space, $\mathbf{u}(k) \in \mathcal{U}$ is the discrete-time continuous input vector.
- $\mathcal{Y} \subseteq \Re^{ny}$ defines a discrete-time continuous output space, $\mathbf{y}(k) \in \mathcal{Y}$ is the discrete-time continuous output vector.
- $\mathcal{F}$ is a set of faults.
- $\mathcal{G} = \{g^i : i \in M\}$ defines a set of discrete-time state affine functions with parametric uncertainty for each mode $i \in M$:

$$\begin{aligned}\mathbf{x}(k+1) \quad &= \mathbf{A}^i(\widetilde{\boldsymbol{\theta}})\mathbf{x}(k) + \mathbf{B}^i(\widetilde{\boldsymbol{\theta}})\mathbf{u}(k) + \mathbf{F}_x^i(\widetilde{\boldsymbol{\theta}})\mathbf{f}(k) \\ &\quad + \mathbf{G}_x^i(\widetilde{\boldsymbol{\theta}})\end{aligned} \tag{1}$$

where $\mathbf{A}^i(\widetilde{\boldsymbol{\theta}}) \in \Re^{nx \times nx}$, $\mathbf{B}^i(\widetilde{\boldsymbol{\theta}}) \in \Re^{nx \times nu}$ and $\mathbf{G}_x^i(\widetilde{\boldsymbol{\theta}}) \in \Re^{nx \times 1}$ are the state matrices in mode i, and $\mathbf{f}(k) \in \Re^{nf}$ represents the system faults, with $\mathbf{F}_x^i(\widetilde{\boldsymbol{\theta}}) \in \Re^{nx \times nf}$

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being the fault distribution matrix in mode i. The model parameters $(\widetilde{\boldsymbol{\theta}})$ are considered time-invariant but bounded by an interval set, i.e., they belong to the set $\boldsymbol{\Theta} = \{\boldsymbol{\theta} \in \Re^{n\boldsymbol{\theta}} | \underline{\boldsymbol{\theta}} \leq \boldsymbol{\theta} \leq \overline{\boldsymbol{\theta}}\}$. This set represents the uncertainty on the exact knowledge of the real system parameters $(\widetilde{\boldsymbol{\theta}})$.

- $\mathcal{H} = \{h^i : i \in M\}$ defines a set of discrete-time output affine functions with parametric uncertainty for each mode $i \in M$:

$$\mathbf{y}(k) = \mathbf{C}^i(\widetilde{\boldsymbol{\theta}})\mathbf{x}(k) + \mathbf{D}^i(\widetilde{\boldsymbol{\theta}})\mathbf{u}(k) + \mathbf{F}_y^i(\widetilde{\boldsymbol{\theta}})\mathbf{f}(k) + \mathbf{G}_y^i(\widetilde{\boldsymbol{\theta}}) \quad (2)$$

  where $\mathbf{C}^i(\widetilde{\boldsymbol{\theta}}) \in \Re^{ny \times nx}$, $\mathbf{D}^i(\widetilde{\boldsymbol{\theta}}) \in \Re^{ny \times nu}$ and $\mathbf{G}_y^i(\widetilde{\boldsymbol{\theta}}) \in \Re^{ny \times 1}$ are the output matrices in mode i, $\mathbf{F}_y^i(\widetilde{\boldsymbol{\theta}}) \in \Re^{ny \times nf}$ being the fault distribution matrix in mode i.
- $\Sigma = \Sigma_s \cup \Sigma_c \cup \Sigma_f$ is a set of events. Spontaneous mode switching events (Σ_s), input events (Σ_c) and fault events Σ_f are considered. Each spontaneous event $\sigma_s \subseteq \Sigma_s$ defines when the state vector intersects a jump surface $S_{\sigma_s} = \{\mathbf{x}(k) \in \mathcal{X} : s_{\sigma_s}(\mathbf{x}(k)) = \mathbf{0}\}$, with s_{σ_s} being a linear switching condition.
- Σ can be partitioned as $\Sigma_o \cup \Sigma_{uo}$ where Σ_o represents the set of observable events and Σ_{uo} represents the set of unobservable events. It is assumed that $\Sigma_f \subseteq \Sigma_{uo}$, $\Sigma_c \subseteq \Sigma_o$ and Σ_s can be contained in both partitions.
- $\mathcal{T} : \mathcal{Q} \times \Sigma \rightarrow \mathcal{Q}$ defines a discrete state transition function.

This hybrid automaton model results from an adaptation of [7], by introducing faults, events and uncertainty [8]. Other alternative descriptions can be found in the literature [1], [2], [3]

Alternatively, the model given by (1) and (2) can be expressed in input-output form using the shift q-operator (or delay operator), assuming zero initial conditions, as follows

$$\mathbf{y}(k) = \mathbf{M}^i(q^{-1}, \widetilde{\boldsymbol{\theta}})\mathbf{u}(k) + \mathbf{G}_f^i(q^{-1}, \widetilde{\boldsymbol{\theta}})\mathbf{f}(k) + \mathbf{Q}^i(q^{-1}, \widetilde{\boldsymbol{\theta}}) \quad (3)$$

where:

$$\begin{aligned}
\mathbf{M}^i(q^{-1}, \widetilde{\boldsymbol{\theta}}) &= \mathbf{C}^i(\widetilde{\boldsymbol{\theta}})(q\mathbf{I} - \mathbf{A}^i(\widetilde{\boldsymbol{\theta}}))^{-1}\mathbf{B}^i(\widetilde{\boldsymbol{\theta}}) + \mathbf{D}^i(\widetilde{\boldsymbol{\theta}}) \\
\mathbf{G}_f^i(q^{-1}, \widetilde{\boldsymbol{\theta}})) &= \mathbf{C}^i(\widetilde{\boldsymbol{\theta}})(q\mathbf{I} - \mathbf{A}^i(\widetilde{\boldsymbol{\theta}}))^{-1}\mathbf{F}_x^i(\widetilde{\boldsymbol{\theta}}) + \mathbf{F}_y^i(\widetilde{\boldsymbol{\theta}}) \\
\mathbf{Q}_y^i(q^{-1}, \widetilde{\boldsymbol{\theta}}) &= \mathbf{G}_y^i(\widetilde{\boldsymbol{\theta}})\frac{q}{q-1} \\
\mathbf{Q}^i(q^{-1}, \widetilde{\boldsymbol{\theta}}) &= \mathbf{Q}_x^i(q^{-1}, (\widetilde{\boldsymbol{\theta}})) + \mathbf{Q}_y^i(q^{-1}, \widetilde{\boldsymbol{\theta}}) \\
\mathbf{Q}_x^i(q^{-1}, \widetilde{\boldsymbol{\theta}}) &= \mathbf{C}^i(\widetilde{\boldsymbol{\theta}})(q\mathbf{I} - \mathbf{A}^i(\widetilde{\boldsymbol{\theta}}))^{-1}\mathbf{G}_x^i(\widetilde{\boldsymbol{\theta}})\frac{q}{q-1}
\end{aligned}$$

B. Overview of the proposed fault diagnosis approach

Model-based FDI relies on comparing the estimated behaviour of the system obtained from a non-faulty model with the real measured behaviour available through sensor measurements. The FDI algorithm design for hybrid systems takes into account which is the current operating mode i of the hybrid system to adapt the model used to generate the predicted output. Thus, a set of residuals adapted to the mode dynamic behaviour can be generated and evaluated as in the case of non-hybrid systems. The set of residuals for each mode including the uncertainty in parameters is given by :

$$\mathbf{r}^i(k, \boldsymbol{\theta}) = \mathbf{y}(k) - \widehat{\mathbf{y}}^i(k, \boldsymbol{\theta}) \quad (4)$$

where $\mathbf{y}(k)$ is the real behaviour and $\widehat{\mathbf{y}}^i(k, \boldsymbol{\theta})$ is estimated behaviour considering parameter uncertainty $\boldsymbol{\theta} \in [\underline{\boldsymbol{\theta}}, \overline{\boldsymbol{\theta}}]$. The predicted output can be obtained using any of the available model-based methods (observers or parity equations). The model-based fault diagnosis conceptual block diagram proposed for hybrid systems is shown in Fig. 1, which is composed by two modules: a fault detection/isolation module and a mode recognition module. Both modules use system inputs and outputs to recognize the system mode and to adapt the diagnosis module on-line.

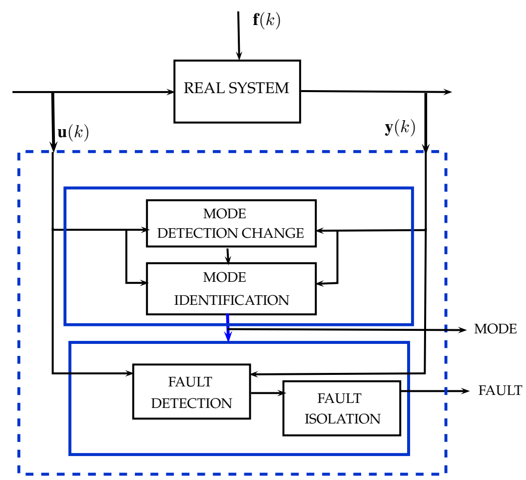

Fig. 1. Conceptual block diagram of hybrid system FDI module

III. FAULT DETECTION

The fault detection task is divided into two subtasks: residual generation and evaluation.

A. Residual generation

In this paper, the parity space approach [9] is used for residual generation. The residual expression generated for each mode is given by:

$$\mathbf{r}^i(k, \boldsymbol{\theta}) = \mathbf{W}^i(\boldsymbol{\theta})\bar{\mathbf{Y}}(k) - \mathbf{W}^i(\boldsymbol{\theta})\mathbf{T}_{u,p}^i(\boldsymbol{\theta})\bar{\mathbf{U}}(k) - \mathbf{W}^i(\boldsymbol{\theta})\mathbf{T}_{G,p}^i(\boldsymbol{\theta}) \quad (5)$$

where p is the residual order, $\mathbf{W}^i(\boldsymbol{\theta})$ is a matrix such that $\mathbf{W}^i(\boldsymbol{\theta})\mathbf{O}^i(\boldsymbol{\theta}) = 0$, and $\mathbf{T}_{u,p}^i(\boldsymbol{\theta})$, $\mathbf{O}^i(\boldsymbol{\theta})$ and $\mathbf{T}_{G,p}^i(\boldsymbol{\theta})$ matrices are given by:

$$\mathbf{T}_{u,p}^i(\boldsymbol{\theta}) = \begin{pmatrix} \mathbf{D}^i(\boldsymbol{\theta}) & \cdots & 0 & 0 \\ \mathbf{C}^i(\boldsymbol{\theta})\mathbf{B}^i(\boldsymbol{\theta}) & \cdots & 0 & 0 \\ \vdots & \ddots & & \vdots \\ \mathbf{C}^i(\boldsymbol{\theta})(\mathbf{A}^i(\boldsymbol{\theta}))^{p-1}\mathbf{B}^i(\boldsymbol{\theta}) & \cdots & & \mathbf{D}^i(\boldsymbol{\theta}) \end{pmatrix}$$

$$\mathbf{O}^i(\boldsymbol{\theta}) = \begin{pmatrix} \mathbf{C}^i(\boldsymbol{\theta}) \\ \mathbf{C}^i(\boldsymbol{\theta})\mathbf{A}^i(\boldsymbol{\theta}) \\ \vdots \\ \mathbf{C}^i(\boldsymbol{\theta})(\mathbf{A}^i(\boldsymbol{\theta}))^p \end{pmatrix}$$

$$\mathbf{T}^i_{G,q}(\boldsymbol{\theta}) = \begin{pmatrix} \mathbf{G}^i_y(\boldsymbol{\theta}) \\ \mathbf{C}^i(\boldsymbol{\theta})\mathbf{G}^i_x(\boldsymbol{\theta}) + \mathbf{G}^i_y(\boldsymbol{\theta}) \\ \vdots \\ \mathbf{C}^i(\boldsymbol{\theta})(\mathbf{A}^i(\boldsymbol{\theta}))^{p-1}\mathbf{G}^i_x(\boldsymbol{\theta}) + \cdots + \mathbf{G}^i_y(\boldsymbol{\theta}) \end{pmatrix}$$

and $\bar{\mathbf{Y}}(k) = \begin{bmatrix} \mathbf{y}(k-p) & \mathbf{y}(k-p+1) & \cdots & \mathbf{y}(k) \end{bmatrix}^t$, $\bar{\mathbf{U}}(k) = \begin{bmatrix} \mathbf{u}(k-p) & \mathbf{u}(k-p+1) & \cdots & \mathbf{u}(k) \end{bmatrix}^t$.

Since uncertain parameters are included in the hybrid system model, the determination of $\mathbf{W}^i(\boldsymbol{\theta})$ is not a trivial task. In [10], a possible approach to address this issue is proposed. Here, a different approach based on the equivalence that exists between the parity space approach and input/output models is used [11]. Assume the system model in input/output form at a given operating point:

$$\mathbf{y}^i(q,\boldsymbol{\theta}) = \frac{b^i_p(\boldsymbol{\theta})q^p + b^i_{p-1}(\boldsymbol{\theta})q^{p-1} + \cdots + b^i_0(\boldsymbol{\theta})}{q^p + a^i_{p-1}(\boldsymbol{\theta})q^{p-1} + \cdots + a_0(\boldsymbol{\theta})}\mathbf{u}(q) \quad (6)$$

A way to construct the parity space residuals follows by defining

$$\mathbf{W}^i(\boldsymbol{\theta}) = \begin{bmatrix} a^i_0(\boldsymbol{\theta}) & \cdots & a^i_{p-1}(\boldsymbol{\theta}) & 1 \end{bmatrix} \quad (7)$$

since from the Cayley-Hamilton theorem, it can be proved that[1] $\mathbf{W}^i(\boldsymbol{\theta})obsv(\mathbf{A}(\boldsymbol{\theta}),\mathbf{C}(\boldsymbol{\theta})) = 0$ is satisfied by considering each output in (6) independently:

$$A^i(\boldsymbol{\theta})^p + a^i_{p-1}(\boldsymbol{\theta})A^i(\boldsymbol{\theta})^{p-1} + \cdots + a^i_0(\boldsymbol{\theta})A^i(\boldsymbol{\theta}) = 0$$

$$\Rightarrow \begin{bmatrix} a^i_0(\boldsymbol{\theta}) & \cdots & a^i_{p-1}(\boldsymbol{\theta}) & 1 \end{bmatrix} \begin{bmatrix} \mathbf{c}^i(\boldsymbol{\theta}) \\ \mathbf{c}^i(\boldsymbol{\theta})\mathbf{A}^i(\boldsymbol{\theta}) \\ \vdots \\ \mathbf{c}^i(\boldsymbol{\theta})\mathbf{A}^i(\boldsymbol{\theta})^p \end{bmatrix} = 0$$

where $\mathbf{A}^i(\boldsymbol{\theta}), \mathbf{c}^i(\boldsymbol{\theta})$ denote the system matrices of the state space of the transfer function given by (6). Moreover, it is satisfied that

$$\mathbf{W}^i(\boldsymbol{\theta})\mathbf{T}^i_{u,p}(\boldsymbol{\theta}) = \begin{bmatrix} b^i_0(\boldsymbol{\theta}) & \cdots & b^i_{p-1}(\boldsymbol{\theta}) & b^i_p(\boldsymbol{\theta}) \end{bmatrix}$$

and

$$\mathbf{W}^i(\boldsymbol{\theta})\mathbf{T}^i_{G,p}(\boldsymbol{\theta}) = \begin{bmatrix} g^i_0(\boldsymbol{\theta}) & \cdots & g^i_{p-1}(\boldsymbol{\theta}) & g^i_p(\boldsymbol{\theta}) \end{bmatrix}$$

Using this approach, the number of residuals is equal to the number of system outputs for each mode.

Alternatively, the residuals can be expressed using the input/output form [8] as follows:

[1] $obsv$ denotes the observability matrix

$$\mathbf{r}^{\circ i}(k,\boldsymbol{\theta}) = \mathbf{y}(k) - \mathbf{G}^i(q^{-1},\boldsymbol{\theta})\mathbf{u}(k) - \mathbf{H}^i(q^{-1},\boldsymbol{\theta})\mathbf{y}(k) - \mathbf{Q}^i_e(q^{-1},\boldsymbol{\theta}) \quad (8)$$

where $\mathbf{G}^i(q^{-1},\boldsymbol{\theta})$, $\mathbf{H}^i(q^{-1},\boldsymbol{\theta})$ and $\mathbf{Q}^i_e(q^{-1},\boldsymbol{\theta})$ correspond to the parameters of the input-output model in predictor form. Moreover, with the previous selection of $\mathbf{W}^i(\boldsymbol{\theta})$, an equivalence between input/ouput and parity space predictors can be established through the following relations:

$$\begin{aligned} \mathbf{H}^i(q^{-1},\boldsymbol{\theta}) &= \mathbf{I} - \mathbf{W}^i(\boldsymbol{\theta}) \begin{bmatrix} \mathbf{I}q^{-p} \\ \vdots \\ \mathbf{I} \end{bmatrix} \\ \mathbf{G}^i(q^{-1},\boldsymbol{\theta}) &= \mathbf{W}^i(\boldsymbol{\theta})\mathbf{T}^i_{\mathbf{u},p}(\boldsymbol{\theta}) \\ \mathbf{Q}^i_e(q^{-1},\boldsymbol{\theta}) &= \mathbf{W}^i(\boldsymbol{\theta})\mathbf{T}^i_{\mathbf{G},p}(\boldsymbol{\theta}) \end{aligned}$$

B. Residual evaluation

The set of residuals generated for each mode are compared with a threshold value (zero in the ideal case). When the residual is larger than the threshold, it is concluded that the system is faulty. Otherwise, it is considered that the system is working properly. However, by considering the effect of the uncertain parameters $\boldsymbol{\theta}$ on the estimated output model response, an interval for $\widehat{y}^i(k)$ should be determined at every time instant instead of a single value. Thereby, $\widehat{y}^i(k,\boldsymbol{\theta})$ is bounded by the interval: $\left[\underline{\widehat{y}}^i(k,\boldsymbol{\theta}), \overline{\widehat{y}}^i(k,\boldsymbol{\theta})\right]$, where for each output such interval is computed as follows

$$\underline{\widehat{y}}^i_j(k,\boldsymbol{\theta}) = \min_{\theta\in\Theta}(\widehat{y}^i_j(k,\boldsymbol{\theta})) \quad \text{and} \quad \overline{\widehat{y}}^i_j(k,\boldsymbol{\theta}) = \max_{\theta\in\Theta}(\widehat{y}^i_j(k,\boldsymbol{\theta})) \quad (9)$$

with $j \in \{1,\cdots n_y\}$. In case that there is no fault, each system output fulfils:

$$y^i_j(k,\boldsymbol{\theta}) \in \left[\underline{\widehat{y}}^i_j(k,\boldsymbol{\theta}), \overline{\widehat{y}}^i_j(k,\boldsymbol{\theta})\right] \quad (10)$$

Alternatively, the previous fault detection test can be formulated using the residuals given by (4). A convenient way of considering the effect of parameter uncertainty in the evaluation of the residuals is using the nominal model $\widehat{y}^{\circ i}(k,\boldsymbol{\theta}^{\circ})$ obtained by considering $\boldsymbol{\theta} = \boldsymbol{\theta}^{\circ} \in \boldsymbol{\Theta}$. In the following, the notation $\widehat{y}^{\circ i}(k) \triangleq \widehat{y}^{\circ i}(k,\boldsymbol{\theta}^{\circ})$ will be assumed. Thus, the nominal residual can be evaluated as follows:

$$\mathbf{r}^{\circ i}(k) = \mathbf{y}(k) - \widehat{\mathbf{y}}^{\circ i}(k) \quad (11)$$

and the effect of parameter uncertainty will be bounded component wise by the following interval

$$\left[\underline{r}^{\circ i}_j(k), \overline{r}^{\circ i}_j(k)\right] \quad (12)$$

where:

$$\underline{r}^{\circ i}_j(k) = \underline{\widehat{y}}^i_j(k) - \widehat{y}^{\circ i}_j(k) \quad \text{and} \quad \overline{r}^{\circ i}_j(k) = \overline{\widehat{y}}^i_j(k) - \widehat{y}^{\circ i}_j(k) \quad (13)$$

being $\widehat{\underline{y}_j^i}(k)$ and $\overline{\widehat{y}}_j^i(k)$ the bounds of the j^{th} system output estimation computed obtained according to (9).

Once the residual (11) has been generated, it is evaluated component wise against the interval (12) to detect a fault

$$s_j^i(k) = \begin{cases} 1 & \text{if } r_j^{\circ i}(k) \notin \left[\underline{r}_j^{\circ i}(k), \overline{r}_j^{\circ i}(k)\right] \quad \text{(fault)} \\ 0 & \text{otherwise} \quad \text{(no fault)} \end{cases} \tag{14}$$

where the interval $\left[\underline{r}_j^{\circ i}(k), \overline{r}_j^{\circ i}(k)\right]$ can be seen as an adaptive threshold. Thus, the observed fault signature $\mathbf{s}^i(k) = [s_1^i(k), \cdots, s_{ny}^i(k)]$ is generated.

The threshold allows to establish the detectability limits. In particular, the next definition follows.

Definition 3.1: The l^{th} fault is detectable if there exists some residual that is sensitive to it, i.e., $r_j^{\circ i}(k) \notin \left[\underline{r}_j^{\circ i}(k), \overline{r}_j^{\circ i}(k)\right]$.

IV. FAULT ISOLATION

The isolation module is responsible of identifying the fault that has occurred in the system by checking the observed fault signature against the fault signatures stored in the theoretical fault signature matrix.

The fault sensitivity of the residual can be determined using its internal form. For the parity space approach, this form is given by [9]:

$$\mathbf{r}^i(k) = \mathbf{W}^i(\boldsymbol{\theta})\mathbf{T}_{f,p}^i(\boldsymbol{\theta})\bar{\mathbf{F}}(k) \tag{15}$$

where $\mathbf{T}_{f,p}^i(\boldsymbol{\theta})$ is a matrix similar to $\mathbf{T}_{u,p}^i(\boldsymbol{\theta})$ replacing $\mathbf{D}^i(\theta)$ by $\mathbf{F}_y^i(\boldsymbol{\theta})$, $\mathbf{B}^i(\boldsymbol{\theta})$ by $\mathbf{F}_x^i(\boldsymbol{\theta})$ and $\bar{\mathbf{F}}(k)$ is a vector similar to $\bar{\mathbf{Y}}(k)$. According to [8], the residual fault sensitivity is given by

$$\mathbf{S}_f^i(q^{-1}) = \frac{\partial \mathbf{r}^i(k)}{\partial \mathbf{f}} \tag{16}$$

Thus, the residual internal form (15) allows to read directly the effect of the faults $\mathbf{f}$ in terms of the residual fault sensitivities. Thereby, using this residual expression, it is possible to see which faults affect the residuals and determine the fault signature matrix $\mathbf{FSM}^i$ corresponding to each mode i. The theoretical binary fault signature can be determined in an automatic way by means of the residual fault sensitivity (16). In particular, given the fault sensitivity of the j^{th} residual with respect to the l^{th} fault denoted as $s_f^i(j,l)$ (i.e, the element (j,l) of the sensitivity matrix $\mathbf{S}_f^i$), the element (j,l) of the fault signature matrix is determined as follows:

$$fsm^i(j,l) = \begin{cases} 1 & \text{if } s_f^i(j,l)(q^{-1}) \neq 0 \\ 0 & \text{if } s_f^i(j,l)(q^{-1}) = 0 \end{cases} \tag{17}$$

i.e., if the j^{th} residual in the mode i depends on the l^{th} fault then $fsm^i(j,l) = 1$, or 0 otherwise. The sensitivity using the parity space approach is given by:

$$\mathbf{S}_f^i(q^{-1}, \boldsymbol{\theta}) = \mathbf{W}^i(\boldsymbol{\theta})\mathbf{T}_{f,p}^i(\boldsymbol{\theta}) \begin{bmatrix} \mathbf{I}_{nf} q^{-p} \\ \vdots \\ \mathbf{I}_{nf} \end{bmatrix} \tag{18}$$

An important property for this module is fault isolability, which is defined next.

Definition 4.1: Two faults f_i and f_j are isolable if they are detectable and their columns in the fault signature matrix are different from the each other.

V. MODE RECOGNITION

The mode recognition task is implemented through the mode change detection and recognition modules (see Fig. 1).

1) Mode change detection: The aim of this module is to detect when a mode transition occurs in the hybrid system. The mode change detection from mode i to mode j is inferred when an inconsistency in the set of residuals of mode i is detected while at the same time the set of residuals corresponding to mode j are proved to be consistent.

Definition 5.1: Two modes q^i and q^j are said to be weakly non-discernible if and only if residuals $\mathbf{r}^{\circ i}(k)$ (generated considering the mode i model) and $\mathbf{r}^{\circ j}(k)$ (generated considering the mode j model) both belong to their intervals (i.e., $\mathbf{r}^{\circ i}(k) \in \left[\underline{\mathbf{r}}^{\circ i}(k), \overline{\mathbf{r}}^{\circ i}(k)\right]$, $\mathbf{r}^{\circ j}(k) \in \left[\underline{\mathbf{r}}^{\circ j}(k), \overline{\mathbf{r}}^{\circ i}(k)\right]$ holds) when they are computed using signals $(\mathbf{y}(k), \mathbf{u}(k))$ corresponding to mode q^i or mode q^j.

The notion of non-discernability was first introduced in [3], where necessary and sufficient conditions were provided for the parity space approach in the state space representation. Thus, the following property can be defined:

Definition 5.2: A mode change from mode q^i to mode q^j is detectable at time instant k if and only if the set of residuals of mode i are $\mathbf{r}^{\circ i}(k) \notin \left[\underline{\mathbf{r}}^{\circ i}(k), \overline{\mathbf{r}}^{\circ i}(k)\right]$ and the set of residuals of mode j are $\mathbf{r}^{\circ j}(k) \in \left[\underline{\mathbf{r}}^{\circ j}(k), \overline{\mathbf{r}}^{\circ j}(k)\right]$

This definition implies that a mode change from mode i to mode j is detectable if mode i and mode j are discernable.

2) Mode change isolation: Once a mode transition has been detected, the new mode should be identified. To identify it, the set of residuals of each possible successor mode are checked to verify which of them belong to their interval.

Definition 5.3: Two mode changes, $i \to j$ and $i \to l$ are isolable if the following conditions are satisfied at any time instant k:

1) Both mode changes are detectable
2) In the case of a mode change $i \to j$ the residuals satisfy: $\mathbf{r}^{\circ i}(k) \notin \left[\underline{\mathbf{r}}^{\circ i}(k), \overline{\mathbf{r}}^{\circ i}(k)\right]$, $\mathbf{r}^{\circ j}(k) \in \left[\underline{\mathbf{r}}^{\circ j}(k), \overline{\mathbf{r}}^{\circ j}(k)\right]$ and $\mathbf{r}^{\circ l}(k) \notin \left[\underline{\mathbf{r}}^{\circ l}(k), \overline{\mathbf{r}}^{\circ l}(k)\right]$
3) In the case of a mode change $i \to l$ the residuals satisfy: $\mathbf{r}^{\circ i}(k) \notin \left[\underline{\mathbf{r}}^{\circ i}(k), \overline{\mathbf{r}}^{\circ i}(k)\right]$, $\mathbf{r}^{\circ j}(k) \notin \left[\underline{\mathbf{r}}^{\circ j}(k), \overline{\mathbf{r}}^{\circ j}(k)\right]$ and $\mathbf{r}^{\circ l}(k) \in \left[\underline{\mathbf{r}}^{\circ l}(k), \overline{\mathbf{r}}^{\circ l}(k)\right]$.

A. Hybrid diagnoser

A diagnoser for hybrid systems based on the conceptual scheme of Fig. 1 combine all previous modules in order to track the mode change sequence and to detect and isolate the potential system faults. Algorithm 1 briefly describes the reasoning carried out by the diagnoser based on the residuals.

Algorithm 1 Hybrid Diagnoser

1: $i \leftarrow 0$
2: **repeat**
3: Evaluate $\mathbf{r}^{\circ i}(k)$ according to (5)
4: **until** $\mathbf{r}^{\circ i}(k) \notin \left[\underline{\mathbf{r}}^{\circ i}(k), \overline{\mathbf{r}}^{\circ i}(k)\right]$
5: **for all** j such that $q^j \in \{q : \exists\sigma \in \Sigma, q = \mathcal{T}(q^i, \sigma)\}$ **do**
6: Evaluate $\mathbf{r}^{\circ j}(k)$ according to (5)
7: **if** $\mathbf{r}^{\circ j}(k) \in \left[\underline{\mathbf{r}}^{\circ j}(k), \overline{\mathbf{r}}^{\circ j}(k)\right]$ **then**
8: **print** `Transition from mode` i `to` j
9: $i \leftarrow j$
10: **goto line** 2
11: **end if**
12: **end for**
13: **for all** faults in the system, $f \in \mathcal{F}$ **do**
14: **if** $\mathbf{s}^{\circ i}(k) = \mathbf{fsm}^i(\bullet, f)$ **then**
15: **print** `Fault` f `occurred`
16: **STOP**
17: **end if**
18: **end for**
19: **print** `Unknown event`

The algorithm checks whether $\mathbf{r}^{\circ i}(k) \in \left[\underline{\mathbf{r}}^{\circ i}(k), \overline{\mathbf{r}}^{\circ i}(k)\right]$ holds or not for current mode. Else, two hypothesis should be verified: a mode change or a fault occurrence. In case of a mode change, the set of residuals of a successor mode will fulfill: $\mathbf{r}^{\circ j}(k) \in \left[\underline{\mathbf{r}}^{\circ j}(k), \overline{\mathbf{r}}^{\circ j}(k)\right]$. In the case of a fault, the set of binary residuals in the current mode are compared with the theoretical fault signature to isolate the fault. It is assumed that a mode change and a fault do not occur at the same time. The main reason of such assumption is because residuals are used for both detecting mode changes and faults.

VI. APPLICATION CASE STUDY

A. Description

The application case study is based on a part of the Barcelona sewer network. Sewer networks present several elements exhibiting numerous operating modes depending of the sewer flows, i.e., behave as a hybrid system. Fig. 2 shows the model of the considered part of the Barcelona network using the virtual tank modeling approach [12].

The elements that appear in the Fig. 2 are: two virtual tanks (T_0, and T_1), one real tank (T_2), three limnimeters to measure the sewer levels (L_{39}, L_{41} and L_{47}), two rain gauges to measure the input rain intensity in the virtual tanks (P_{19} and P_{16}), and two redirection gates placed downstream T_0 and T_1, which allow to change the flow direction. In this particular case study, fixed position gates have been assumed.

Some phenomena like overflows in sewers and tanks (dash lines illustrate this overflow situation in Fig. 2) can appear and change their behaviour. A hybrid model is used in order to describe such behaviour and to design a hybrid diagnoser to detect and isolate faults. The Diagnoser reasons according to Algorithm 1, and it is built based on the methodology presented in [6].

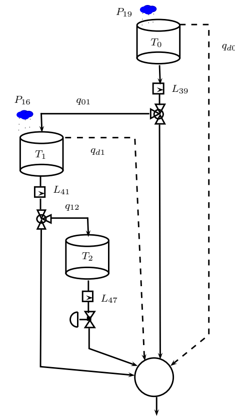

Fig. 2. Barcelona test catchment

B. Modeling

The hybrid automata HA describing the sewer network is illustrated in Fig. 3. There are 4 discrete states (i.e., $w = 4$) corresponding to the overflow or no overflow conditions of the virtual tanks. In the figure such conditions are represented by *O* and *WO*, respectively. For example, state 1 means that no tank is in overflow situation, state 2 means that only T_0 is in overflow, and so on. The initial state corresponds to $q^0 = q^1$. Transitions are bound to spontaneous mode switching events (e.g., no input events are considered) which are represented in the figure as inequalities. Such events are unobservable since state variables (e.g., tank volumes) are not measured.

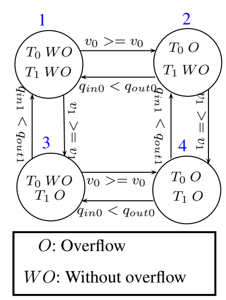

Fig. 3. Hybrid automata for the sewer network

For instance, the predictor used for residual generation corresponding to mode 3 is

$$\hat{\mathbf{y}}^{\circ 3}(k) = \begin{bmatrix} \theta_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \theta_2 \end{bmatrix} \mathbf{y}(k) + \begin{bmatrix} \theta_3 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{u}(k) + \begin{bmatrix} 0 \\ \theta_4 \\ \theta_5 \end{bmatrix} \tag{19}$$

It has been obtained using the state space model of the network presented in Fig. 3 using the matrix $\mathbf{W}(\boldsymbol{\theta})$ defined as explained in Section III. The uncertain parameters have been estimated using the algorithm proposed by [13] leading to the following intervals: $\theta_1 \in [0.7083, 0.8657]$, $\theta_2 \in [0.8460, 1.0340]$, $\theta_3 \in [1.0162, 1.2420] \cdot 10^4$, $\theta_4 \in [3.3942, 4.1485]$ and $\theta_5 \in [0.1196, 0.1462]$.

The fault set $\mathcal{F}$ includes faults in the output sensors (f_{L39}, f_{L41} and f_{L47}) as well as faults in the input sensors (f_{P19} and f_{P16}). Applying (18), the theoretical fault signature matrix is obtained selecting $\mathbf{F}_y^i = \begin{bmatrix} \mathbf{0} & \mathbf{I} \end{bmatrix}$ and $\mathbf{F}_x^i = \begin{bmatrix} -\mathbf{B}^i & \mathbf{0} \end{bmatrix}$ to represent output and input sensor faults respectively.

C. Results

A simulation scenario that illustrates the system state tracking and fault diagnosis is presented in this section. Fig. 4 shows in solid line the simulated system state evolution, while the dashed line is the state sequence estimated by the diagnoser. A delay is present since the residuals have a first order dynamic behavior.

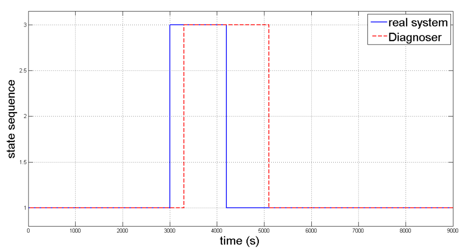

Fig. 4. Mode change detection using interval models

The state sequence is $\{q^1, q^3, q^1\}$. Initially, neither virtual tank is in overflow. Next, T_1 is in overflow whereas later T_1 leaves the overflow condition. Fig. 5 illustrates the residual evolution during the simulation. Notice that, for instance, when a transition from mode $q^1 \rightarrow q^3$ occurs then $\mathbf{r}^{\circ 1}(k) \notin \left[\underline{\mathbf{r}}^{\circ 1}(k), \overline{\mathbf{r}}^{\circ 1}(k)\right]$ and $\mathbf{r}^{\circ 3}(k) \in \left[\underline{\mathbf{r}}^{\circ 3}(k), \overline{\mathbf{r}}^{\circ 3}(k)\right]$ holds. Remark that all modes are discernable according to the criterion explained in Section V-.1.

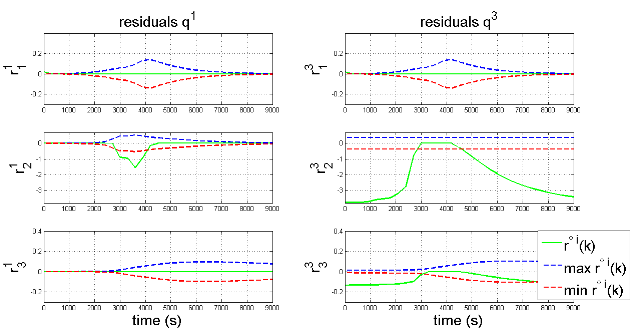

Fig. 5. Mode change detection using interval models

Finally, an additive fault in sensor L_{39} appears at time $3600s$. When the fault appears the residuals of mode q^3 are activated and the diagnoser stops. Remark in Fig. 6 that, when the fault appears in mode q^3, then $\mathbf{r}^{\circ 3}(k) \notin \left[\underline{\mathbf{r}}^{\circ 3}(k), \overline{\mathbf{r}}^{\circ 3}(k)\right]$ holds. In fact, the observed fault signature is $\mathbf{s}^i(k) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^t$, which corresponds to the theoretical fault signature obtained applying (18).

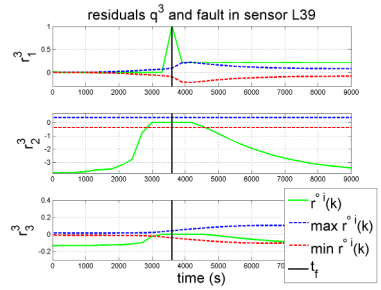

Fig. 6. Fault detection using interval models

VII. CONCLUSIONS

In this paper, a methodology and architecture to design a diagnoser in the framework of hybrid systems considering parameter uncertainty has been proposed. The methodology is robust since it considers modeling errors in the detection process. The parity space equations are used to evaluate the residuals on-line eliminating the dependence of the state variables, and the uncertainty is determined based on the equivalence that there exists between input/output models and parity equations. The performance of the proposed approach has been successfully tested in a part of the Barcelona sewer network

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