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Distributed LQG control of a water delivery canal with feedforward from measured consumptions

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***Abstract*— This work addresses the design of distributed LQG controllers for water delivery canals that include feedforward from local farmer water consumptions. The proposed architecture consists of a network of local control agents, each connected to one of the canal pools and sharing information with their neighbors in order to act in a coordinated way. In order to improve performance, the measurement of the outflow from each pool is used as a feedforward signal. Although the feedforward action is local, it propagates due to the coordination procedure. The paper presents the distributed LQG algorithm with feedforward and experimental results in a large scale pilot water delivery canal.**

***Index Terms*— Distributed Control, distributed LQG control, feedforward, water delivery canals.**

I. Introduction

A. Motivation

Water delivery canal systems are often spread over wide geographical areas, with actuators and local controllers in isolated spots [1], [2]. Furthermore, due to their physical characteristics, the use of pure decentralized control, *i. e.* structures in which local controllers act on the basis of pure local measurements, and without any exchange of information among them, may lead to poor performance or even instability. On the other way, completely centralized control architectures may not only be unfeasible due to the complexity of the transmission network involved, but highly unreliable as well, since communication links may be interrupted by hazardous causes.

These features provide a strong motivation for the employment of distributed control, in which a network of local control agents act in a coordinated way by communicating with their neighbors. If designed adequately, distributed control has a number of advantages:

- Simplicity of design of local controllers, conjugated with good overall performance;
- Increased reliability with respect to failures of either local control agents or communication links;
- Adequate management of local objectives, that may vary for canals crossing different administrative districts.

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The objectives associated with the management and control of water delivery canal systems are manifold and depend on the type of canal and the way it is operated. Usually, saving water is a major aim, with automatic control known to increase canal efficiency [3]. In some cases, the water is driven to the canal only during the periods of usage, while in other situations of interest a minimum level of water must always be ensured, for instance for ecological reasons. In any case, in irrigation canals, the key point is to ensure that the desired amount of water, during a specified period of time, is available to the farmers that require it, while simultaneously water level constraints are respected, *e. g.* to avoid over floods that represent water spillage and may damage certain types of canals or to ensure a minimum water level. If translated in terms of control system specifications, this means that the water levels must follow given references, with specified transients, and rejecting disturbances due to water extraction by users.

In the situation considered in this paper, the main concern is to keep the water level downstream of each pool close to specified values, rejecting disturbances induced by offtake consumptions. Hence, local upstream control is used and complemented with feedforward from accessible disturbances. Although other schemes could be considered that lead to a more efficient use of water [3], the concern here is to demonstrate how a LQG controller including feedforward terms can be modified to act in a distributed setting to regulate water level in a multiple pool canal.

B. Literature review

Centralized LQG control of hydrosystems has long been considered [8], [9], [10], [11]. Model predictive control (MPC) provides an approximation to LQG that has the advantage of allowing the incorporation of constraints in a easy way, but requires a higher computational load. Applications of MPC to water delivery canals include both centralized [12] and distributed examples [13]. In [14] a decentralized version of LQG for a multicanal system has been obtained in a way that has a tight connection with predictive control. For that sake, the overall system is decomposed in a number of interacting subsystems. A local control agent is then associated with each subsystem, in which the manipulated variables are computed by minimizing a receding horizon quadratic cost using the LQG algorithm. The coordination between the controllers of the local subsystems is achieved using the decomposition-coordination approach, based on dual optimization [15]. Accordingly, the manipulated variables are computed, at the beginning of each sampling interval, by

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iterating through two steps. First, the decomposition step is performed that consists, for each subsystem, of minimizing a Lagrangian formed by augmenting the local quadratic cost with a term embedding a constraint of continuity between different canal sections, multiplied by Lagrange multipliers. Then, the Lagrange multipliers are updated in the coordination step.

A different approach to obtain distributed versions of LQG consists in imposing a structure to the controller that matches the desired distributed architecture and to compute the gains using a gradient algorithm. In [16], pages 411-413, the distributed LQ regulator problem for a pair of coupled systems is solved by applying an algorithm for optimal control with an *a priori* imposed structure, described in [16], page 370. Estimates of the gradient based on the adjoint equation are suggested in [17] for deterministic plants and in [18] for stochastic plants.

Another possibility to coordinate distributed controllers is to apply game theory concepts [19]. In this case, each control agent plays a game with their neighbors in which each player tries to optimize a local cost by assuming knowledge of the others control moves. In [20], by iterating the negotiation process, a Nash equilibrium is attained [22].

This last approach is used in this paper to develop a distributed version of LQ control. The controller agents play a game among themselves by trying, in successive iterations, to maximize their respective quadratic costs, while using for feedforward action the value of the manipulated variables of their neighbors that is computed in the previous round. Although convergence to a stabilizing controller is not in general ensured by the resulting Nash equilibrium, in the class of systems considered here stability is attained, together with a performance that is close to the optimum. Furthermore, this approach has the advantage of requiring a much lower computational load than algorithms relying on dual optimization and is the one followed in this work.

C. Paper contributions and structure

The paper contribution consists in the inclusion of feedforward with respect to accessible disturbances in a distributed LQG control algorithm and its experimental application to a water delivery canal, showing that the feedforward action propagates with the coordination scheme employed in order to improve the performance. A game approach is followed to obtain a distributed LQG architecture.

The paper is organized as follows: After the Introduction (this section) that motivates the problem, performs a concise literature review and describes the paper structure and contributions, the distributed control algorithm is presented in section II. The application to the canal, including experimental results is described in section III and finally, section IV draws conclusions.

II. Distributed LQG control with feedforward

The strategy to design a distributed LQG controller with feedforward consists in decomposing the plant in subsystems that interact, design controllers for each of them taken as accessible disturbances the interaction terms as well as other external disturbances that affect the plant and then design a coordination procedure.

A. Plant model

The plant to control is assumed to be decomposed in N local subsystems denoted $\mathcal{S}_i$, $i = 1, \ldots, N$ that are coupled only through their inputs and form a serial chain, *i. e.* system i interacts only with systems $i-1$ and $i+1$ whenever they exist (there are no systems 0 nor $N+1$). Each system $\mathcal{S}_i$ is modeled by the state space representation

$$x_i(k+1) = A_i x_i(k) + B_i u_i(k) + \Gamma_i \delta_i(k) \tag{1}$$

and output equation

$$y_i(k) = C_i x_i(k), \tag{2}$$

where $k \geq 0$ is a nonnegative integer that denotes discrete-time, $x_i \in \mathbb{R}^n$ is the state of subsystem $\mathcal{S}_i$, $u_i \in \mathbb{R}$ is the manipulated variable, $y_i \in \mathbb{R}$ is the measured output, $\delta_i \in \mathbb{R}^3$ is a vector accessible disturbance, and $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times 1}$, $C_i \in \mathbb{R}^{1\times n}$, and $\Gamma_i \in \mathbb{R}^{n\times 3}$ are matrices that define the parametrization of the model of subsystem $\mathcal{S}_i$. It is assumed that (A_i, B_i) is reachable and that (C_i, A_i) is detectable.

The vector of accessible disturbances is defined by

$$\delta(k) = \begin{bmatrix} u_{i-1}(k) \\ u_{i+1}(k) \\ d_i(k) \end{bmatrix}, \tag{3}$$

where d_i is an external signal that represents an accessible disturbance (*i. e.* a disturbance accessible for measure) that acts on the subsystem $\mathcal{S}_i$.

B. Local LQG control with feedforward

In this work the regulation problem is considered. As such, to each subsystem $\mathcal{S}_i$ associate a control law that minimizes the quadratic cost

$$J_i(u) = \frac{1}{2}\sum_{k=1}^{\infty}(x_i^T(k) Q_i x_i(k) + \rho_i u_i^2(k)), \tag{4}$$

where $\rho_i > 0$ is a positive weight in the control action and

$$Q_i = \begin{bmatrix} C_i^T C_i & 0 \\ 0 & 1 \end{bmatrix}, \tag{5}$$

when a series integrator is included and $Q_i = C_i^T C_i$ when it is not. Using the necessary conditions for optimality in the absence of set constraints given by [21] (pages 65-68), the control law is seen to be given by

$$u_{opt,i}(k) = -K_{LQ,i} x_i(k) + u_{ff,i}(k), \tag{6}$$

in which the state feedback gain is

$$K_{LQ,i} = -(\rho_i + B_i^T P_i B_i)^{-1} B_i^T P_i A_i, \tag{7}$$

and P_i satisfies the algebraic Riccati equation

$$P_i = A_i^T P_i \left[I + \frac{1}{\rho_i} B_i B_i^T P_i\right]^{-1} A_i + Q_i. \tag{8}$$

The feedforward term is

$$u_{ff,i}(k) = (\rho_i + B_i^T P_i B_i)^{-1} B_i^T (g_i - P_i \Gamma_i \delta_i) \quad (9)$$

and the vector g_i satisfies the linear algebraic equation

$$M_i g_i = \Lambda(\delta_i), \quad (10)$$

where

$$M_i := I + A_i^T P_i \left[I + \frac{1}{\rho} B_i B_i^T P_i \right]^{-1} \frac{1}{\rho} B_i B_i^T - A_i^T \quad (11)$$

and

$$\Lambda_i(\delta_i) := -A_i^T P_i \left[I + \frac{1}{\rho} B_i B_i^T P_i \right]^{-1} \Gamma \delta_i. \quad (12)$$

In the situation in which the state x_i is not available for direct measure, it is replaced by an estimate generated locally with a Kalman filter, where the noise variances are selected according to the loop transfer recovery (LTR) technique [23].

C. Control agent coordination

If acting in isolation, neglecting subsystem interaction and in a pure decentralized way, local control agents of the type described lead to poor performance or, even, to instability. Therefore, their action is modified by a coordinate action procedure defined as follows:

Coordination algorithm
At the beginning of each sampling interval k set

$$j = 0 \quad (13)$$

and, for all i,

$$\bar{u}_{i,0}(k) = u_{opt,i}(k-1) \quad (14)$$

and execute in a recursive way the following cycle in the index variable j:

1) Set $j \leftarrow j + 1$;
2) For all control agents i compute $\bar{u}_{i,j}(k)$ using (6)-(12) in which

$$\delta_{i,j} = \begin{bmatrix} \bar{u}_{i-1,j-1}(k) \\ \bar{u}_{i+1,j-1}(k) \\ d_i(k) \end{bmatrix}. \quad (15)$$

3) If j is equal to a maximum pre-stipulated value $j_m ax$, then go to step 4. Otherwise, go to step 1.
4) For all i set

$$u_{opt,i}(k) = \bar{u}_{i,j_{max}}(k). \quad (16)$$

□

It is remarked that the definition of the disturbance $\delta_{i,j}$ by (15) means that two different feedforward terms are considered: Two terms are related with the coordination of the local control move with the control agents of the neighbor pools and one term is related with anticipation of the disturbance effect caused by the offtake.

Define

$$\bar{u}_{\cdot,j}(k) := \begin{bmatrix} \bar{u}_{1,j}(k) & \bar{u}_{2,j}(k) & \bar{u}_{3,j}(k) & \dots & \bar{u}_{N,j}(k) \end{bmatrix}^T$$

The above coordination algorithm is equivalent to propagate $\bar{u}_{\cdot,j}(k)$ using the following linear difference equation in j

$$\bar{u}_{\cdot,j+1}(k) = \Xi \bar{u}_{\cdot,j}(k) + \Upsilon, \quad (17)$$

where Γ_i^m is the m-th column of matrix Γ_i ($m = 1, 2, 3$),

$$\Xi = \begin{bmatrix} 0 & \beta_1\Gamma_1^2 & 0 & 0 & \dots & 0 \\ \beta_2\Gamma_2^1 & 0 & \beta_2\Gamma_2^2 & 0 & \dots & 0 \\ 0 & \beta_3\Gamma_3^1 & 0 & \beta_3\Gamma_3^2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \beta_N\Gamma_N^1 & 0 \end{bmatrix}, \quad (18)$$

$$\beta_i := -\left(\rho_i + B_i^T P_i B_i\right)^{-1} B_i^T P_i \quad (19)$$

and

$$\Upsilon := \begin{bmatrix} \gamma_1 & \gamma_2 & \dots & \gamma_N \end{bmatrix}^T, \quad (20)$$

with

$$\gamma_i := \beta_i \Gamma_i^3 d_i + \left(\rho_i + B_i^T P_i B_i\right)^{-1} B_i^T g_i. \quad (21)$$

As such, the algorithm will converge if the spectral radius of Ξ satisfies the condition

$$\max |\lambda(\Xi)| < 1, \quad (22)$$

where $\lambda(\Xi)$ denotes the eigenvalues of Ξ. In a game theory framework the resulting equilibrium is a Nash equilibrium [22]. Although there are plants for which the Nash equilibrium does not ensure stability of the whole system, this is not the case for the application considered in this paper.

III. Control of a water delivery canal

The general distributed LQG controller with feedforward described in section II is now applied to a pilot water delivery canal. The canal used in the experimental tests is described in section III-A, section III-B presents some remarks on plant model identification, the controller structure is presented in III-C and the experimental results in III-D.

A. Canal description

The work reported in this paper was performed at the experimental canal of *Núcleo de Hidrulica and Controlo de Canais* (Universidade de Évora, Portugal), for which several studies previously conducted are available [4], [25], [26].

As shown in figure 1, the canal has four pools with a length of $35m$, separated by three undershoot gates, with the last pool ended by an overshoot gate. The maximum nominal design flow is $0.09\,\mathrm{m^3 s^{-1}}$. There are water offtakes downstream from each branch made by orifices in the channel walls, with additional pipes and valves and equipped with flow meters.

Water level sensors, placed inside an off-line stilling well connected to the bottom of the channel by a pipe, are installed downstream of each pool. The water level sensors allow to measure values between $0\,\mathrm{mm}$ and $900\,\mathrm{mm}$, a value that corresponds to the canal bank. The nomenclature is as follows (figure 1): For pool number i, $i = 1, \dots, 4$, the downstream level is denoted y_i and the opening of gate i is denoted u_i. Pool number i ends with gate number i.

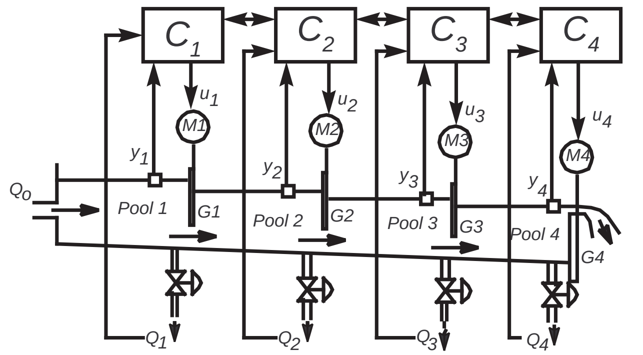

Fig. 1: Schematic view of the canal used in the experiments.

A motorized flow control valve connects a reservoir kept at a desired level to the canal inlet and delivers by gravity a specified discharge to the canal input. The flow of this discharge is controlled with a PI. The motors that drive each gate are controlled by a local PLC. In turn, local PLCs are connected to a central computer with a SCADA system. In order to allow fast controller prototyping, it is possible to connect via a wireless network to the central computer a computer running a MATLAB program that reads sensor signals and manipulates gate commands using the control algorithm.

B. Canal model identification

The dominant dynamics of water delivery canals is modeled by the Saint-Venant equations, a pair of nonlinear partial differential equations that embed mass and momentum conservation [3]. Although this representation is infinite dimensional, around a given operating point it can be approximated by finite dimensional linear state-space models (FDLSS). In order to obtain such models there are several possibilities. The first is to approximate numerically the Saint-Venant equations, either with a difference scheme such as the Preissman method [3] or orthogonal collocation. Another possibility consists in obtaining a transfer function directly by manipulating the Saint-Venant equations [4], [5]. Although this procedure has the advantage of providing a pencil of linear models for a wide range of operating conditions defined by water level and flow, it requires the estimation of physical parameters that may be difficult to obtain and, furthermore, it ignores the dynamics of elements that are very hard to capture from first principles modeling. To overcome this problems one may resort to system identification performed from plant data, eventually including static nonlinearity compensation to improve the linearity of the system, for instance, using as manipulated variables flows instead of gate positions [6], [7]. This last method has the drawback of representing the canal dynamics only around a given operating equilibrium point, but the advantage of capturing a tight approximation to the actual plant dynamics in a frequency range that is of significance for controller design.

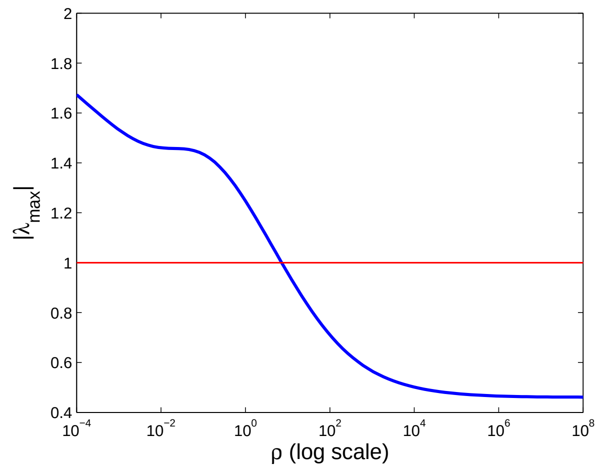

Fig. 2: The spectral radius of Ξ as a function of ρ.

According to the paper objectives, the dynamics of the canal considered is represented in this work by a FDLSS model, build by concatenating models for each of the pools where the adjacent pools affect interact only through their inputs. In turn, individual pool models are identified from plant data. In order to estimate the models, the plant is first driven to an equilibrium state. Around this equilibrium state the position of the gates is disturbed by a PRBS signal. The data collected is then filtered by a low pass filter and used for parameter estimation of an ARX model per pool, using with least squares. A state space of order 4 is finally obtained by converting the ARX models.

C. Canal control structure

As shown in figure 1 the control structure applied to the canal consists of four local control agents denoted $\mathcal{C}_1$ to $\mathcal{C}_4$. Each controller $\mathcal{C}_i$ receives as feedback the water level downstream measurement of pool i, y_i and uses as feedforward signal $d_i = Q_i$, the flow passing in offtale i. In addition, the controllers interchange information with their neighbors about the respective control decisions, in order to implement the coordination scheme described in section II-C. The sampling interval used by all the local controllers is 2 s.

In the distributed controller of the canal, the fulfilment of the convergence condition (22) depends on the choice of the weights ρ_i of the cost function (4). As shown in figure 2, in the case in which all the weights have a common value ρ, the spectral radius of Ξ is strictly smaller than 1 when ρ is bigger than a certain threshold.

In the experiment reported $\rho = 2 \times 10^6$, a value that, according to figure 2, leads to a value of the spectral radius that is less than 0,5 and, hence, to a fast convergence of the coordination procedure.

D. Experimental results

Figures 3 through 7 shows experimental results that have been obtained using in the pilot canal the distributed LQG

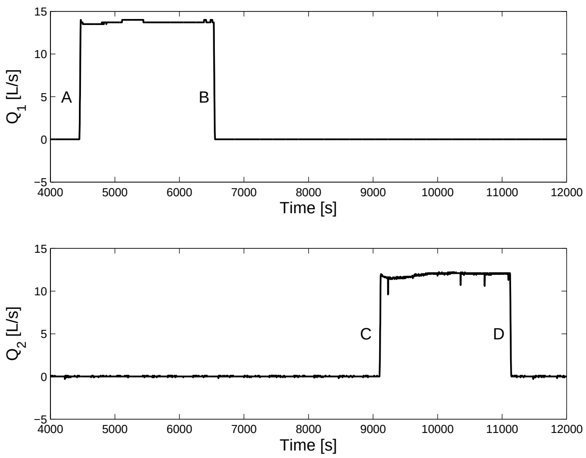

Fig. 3: Experimental results. Offtake flows of pool 1 (above) and pool 2 (below). These flows act as accessible disturbances.

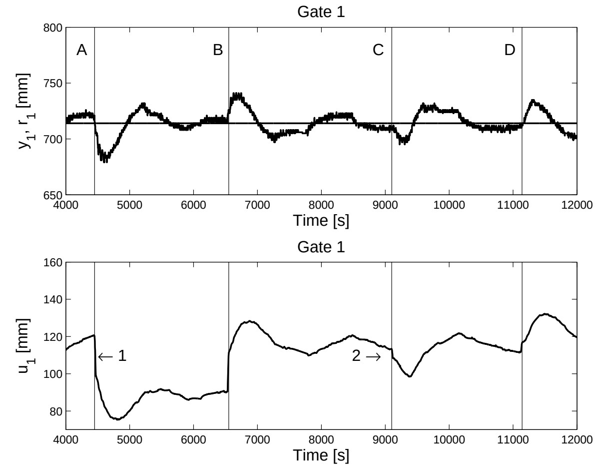

Fig. 4: Experimental results. Pool 1 downstream water level, y_1 and reference r_1, (above) and gate 1 position, u_1 (below).

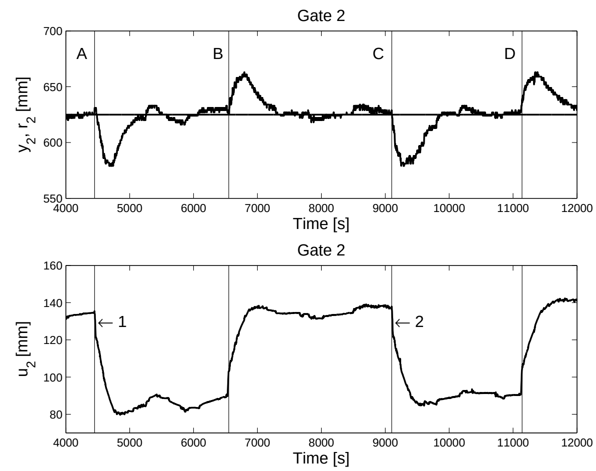

Fig. 5: Experimental results. Pool 2 downstream water level, y_2 and reference r_2, (above) and gate 2 position, u_2 (below).

controller with feedforward from offtake water consumptions. Figure 3 shows the offtake flows from pools 1 and 2, the other ones being kept at zero. Each of the figures from 4 till 7 shows, for one of the pools, the downstream water level and the corresponding reference, and the downstream gate position.

The experimental conditions are as follows. The flow entering the canal is $Q_0 = 35$l/s. The water level reference for all pools is kept constant during the whole experiment, with $r_1 = 714\,\mathrm{mm}$, $r_2 = 625\,\mathrm{mm}$, $r_3 = 567\,\mathrm{mm}$ and $r_4 = 400\,\mathrm{mm}$.

As shown in the upper graphic of figure 3, at the time indicated by the vertical lines in the plots labeled A, the flow of offtake 1 is changed from zero to $Q_1 = 13.7$ l/s, approximately. This flow is then cut to zero at the time corresponding to the vertical line labeled B. Similarly (see figure 3, below), at the time corresponding to the vertical line labeled C, the flow of offtake 2 is changed from zero to $Q_1 = 12$ l/s, approximately.

The changes in the offtake flows shown in figure 3 cause a disturbance to which local controllers react with both feedback and feedforward actions. When the offtake flow Q_1 increases, the water level of pool 1 will tend to decrease and the local controller will close gate 1 to compensate the flow of water lost through the offtake. This causes a reduction of the water flow passing through gate 1 from pool 1 to pool 2 and will in turn cause a tendency of the level of pool 2 to decrease. Similar reaction will then occur in gates 3 and 4.

While the feedback action depends on the existence of an error between the desired setpoint and the corresponding water level, the feedforward action depends on the value measured for the disturbance. When the disturbance grows, the feedforward action immediately forces the actuator to move to compensate its effect, even before a tracking error is seen by the controller. Therefore, feedforward is much faster than feedback when rejecting accessible disturbances. This is apparent in figure 4 since, when the offtake disturbance Q_1 grows from zero (vertical linea marked A), the local control agent $\mathcal{C}_1$ instantaneously partially closes gate 1 (see the graphic of u_1 in the part marked with the arrow labeled 1).

The drawback is that feedforward is unable to reject unaccessible disturbances and is significantly less robust than feedback, and hence requires a much better plant model to be designed successfully. This motivated the inclusion of a parameter that multiples u_{ff} and provides a tuning knob to compensate for errors in the plant static gain.

The above features of feedforward control are well known [24]. A new fact to remark is the way the coordination procedure propagates the feedforward action to the other local control agents. Although the control agent that manipulates gate 2 does not directly receive the feedforward signal from the accessible disturbance affecting pool 1, the negotiation between $\mathcal{C}_1$ and $\mathcal{C}_2$ leads also to an instantaneous action of $\mathcal{C}_2$ that is apparent when looking at the part of the graph of u_2 in figure 5 marked with the arrow labeled 1.

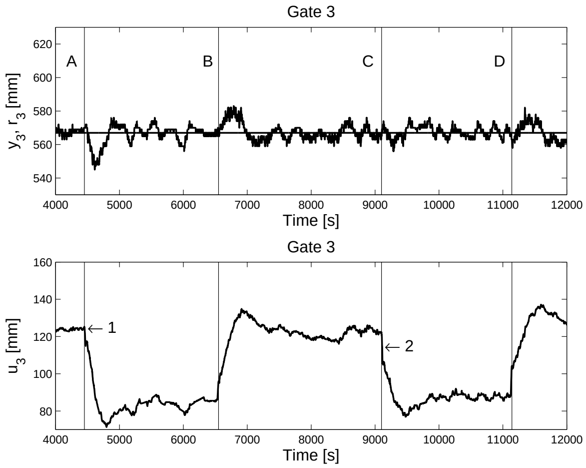

Fig. 6: Experimental results. Pool 3 downstream water level, y_3 and reference r_3, (above) and gate 3 position, u_3 (below).

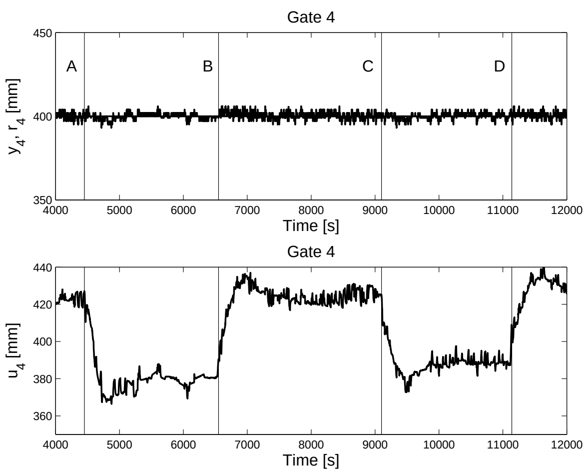

Fig. 7: Experimental results. Pool 4 downstream water level, y_4 and reference r_4, (above) and gate 4 position, u_4 (below).

Reciprocally, when the disturbance Q_2 grows, not only $\mathcal{C}_2$ forces the gate 2 to close instantaneously (arrow 2 in figure 5), but also gate 1 is closed (arrow 2 in figure 4).

These situations repeat throughout the test in several other situations, with converse situations taking place when the disturbing offtake flows are reduced to zero.

IV. Conclusions

A distributed LQG controller with feedforward from accessible disturbances induced by water consumption offtakes has been described and applied to a water delivery canal. The propagation of feedforward action throughout the local controllers, associated to the coordination procedure, leads to a faster response to disturbances with respect to feedback action alone and has been demonstrated experimentally.

Experiments not reported here show that the proposed LQG controller closely approximates the performance of the optimal multivariable LQG.

References

[1] I. Mareels, E. Weyer, S. K. Ooi, M. Cantoni, Y. Li and G. Nair (2005). Systems engineering for irrigation systems: Successes and challenges. *Prep. 16th IFAC World Congress*, Prague, Czech Republic.

[2] M. Cantoni, E. Weyer, Y. Li, S. K. Ooi, I. Mareels and M. Ryan. Control of large-scale irrigation networks. Proc. IEEE, vol. 95, No. 1, pp. 75-91, 2007.

[3] X. Litrico and V. Fromion (2009). *Modeling and Control of Hydrosystems*. Springer.

[4] X. Litrico, V. Fromion, J.-P. Baume, C. Arranja and M. Rijo (2005). Experimental validation of a methodology to control irrigation canals based on Saint-Venant equations, *Control Engineering Practice*, 13(11):1425-1437.

[5] S. K. Ooia, M.P.M. Krutzenb and E. Weyer (2005). On physical and data driven modelling of irrigation channels. *Control Engineering Practise*, 13:461471.

[6] Weyer, E. (2001). System identification of an open water channel. *Control Engineering Practise*, 9:12891299.

[7] K. Eurén and E. Weyer (2007). System identification of open water channels with undershot and overshot gates. *Control Engineering Practise*, 15:813824.

[8] L. W. Mays (1997). *Optimal Control of Hydrosystems*. Marcel Dekker.

[9] E. Weyer (2003). LQ control of an irrigation channel. *Proc. 42nd IEEE CDC*, 750-755.

[10] X. Litrico and D. Georges (2001). Robust LQG control of single input multiple-outputs dam-river systems. *Int. J. of Systems Science*, 32 (6): 795-805.

[11] E. Bautista, A. J. Clemmens and R. J. Strand (2006). Salt River Project Canal Automation Pilot Project: Simulation Tests. *J. of Irrigation and Drainage Engineering*, 132(2): 143-152.

[12] O. Begovich, V. Ruiz, G. Besançon, F. Georges and C. Aldana (2007). Predictive control with constraints of a multi-pool irrigation canal prototype. *Latin American Applied Research*, 37(3):177185.

[13] J. M. Igreja, J. M. Lemos, F. M. Cadete, L. M. Rato and M. Rijo (2012). Control of a water delivery canal with cooperative distributed MPC. *ACC 2012*, Montreal, Canada.

[14] El Fawal, D. Georges and G. Bornard (1998). Optimal control of complex irrigation systems via decomposition-coordination and the use of augmented Lagrangian. *Proc. 1998 IEEE Int. Conf. Systems, Man and Cybernetics*, San Diego, CA.

[15] S. Boyd and L. Vandenberghe. *Convex Optimization*, Cambridge University Press, 2004.

[16] F. L. Lewis and V. L. Syrmos. *Optimal Control*, 2nd ed. John Wiley & Sons, Inc. 1995.

[17] K. Martensson and A, Rantzer. Sub-Optimality Bound on a Gradient Method for Iterative Distributed Control Synthesis. *Proc. 19th Int. Symp. on Mathematical Theory of Networks and Systems MTNS 2010*, Budapest, Hungary, pp. 1999-2003, 2010.

[18] A. Rantzer and K. Martensson. Gradient methods for iterative distributed control. *Proc. 48th IEEE Conf. on Decision and Control*, Shanghai, China, pp. 549-554, 2009

[19] J. M. Maestre, D. Muñoz de la Peña, E. F. Camacho and T. Alamo. Distributed model predictive control based on agent negotiation. *J. Proc. Control*, vol. 21, pp. 685-697, 2011.

[20] Zhang, Y. and S. Y. Li (2007). Networked model predictive control based on neighbourhood optimization for serially connected large-scale processes. *J. Process Control*, 17(1): 37-50.

[21] G. C. Goodwin, M. A. Seron and J. A. de Doná (2005). *Constrained control and estimation*, Springer.

[22] J. N. Webb (2006). *Game theory – Decisions, interaction and evolution*, Springer.

[23] Z. Zhang and J. S. Freudenberg (1993). Discrete-time loop transfer recovery for systems with nonminimum phase zeros and time delays. *Automatica*, 29(2): 351-363.

[24] G. Stephanopoulos (1984). *Chemical process control: An introduction to theory and practice*. Prentice-Hall International.

[25] I. Guenova Welz, X. Litrico, V. Fromion, M. Rijo and P. O. Malaterre (2005). Stability and performance analysis of classical decentralized control of irrigation canals. *Prep. 16th IFAC World Congress*, Prague, Czech Republic.

[26] J. M. Lemos, F. Machado, N. Nogueira, L. Rato and M. Rijo (2009). Adaptive and non-adaptive model predictive control of an irrigation channel. *Networks and Heterogeneous Media*, 4(2):303-324.