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Lead-time identification for inventory control of the supply chain

C.A Garcia and A.Ibeas and R. Vilanova and J. Herrera

***Abstract*— In this paper, an Internal Model Control (IMC) scheme is incorporated in production inventory control systems in a complete supply chain. The IMC control scheme is enhanced with a novel method for the online identification of lead times based on a multimodel scheme. In this way, all benefits of the IMC scheme can be exploited. Simulation examples illustrate the effectiveness of the approach.**

I. INTRODUCTION

A common Supply Chain includes the necessary entities to provide goods from the factory to the customer. In this way, all entities must perform inventory replenishment policies, the aim of them is maintaining enough stock to satisfy customer demand. Many undesirable effects may appear when an inventory replenishment policy is implemented such as the instability and the well known *bullwhip effect* [1]. One of the main causes of all these phenomena is attributed to the lead time that consists of a time period ordering delay and a time period of physical production or distribution delay, specially when it is not properly known.

Recently, Control Engineering based approaches have become an alternative to design effective inventory replenishment policies. Thus, [2] presents a proportional-integral (PI) and cascade control as inventory replenishment policies, being the design of this controller focused on the mitigation of the bullwhip effect. Moreover, in order to counteract the lead time effects in the inventory control, [3] introduces the Internal Model Control (IMC) which is a robust control approach as a novel decision replenishment policy. The IMC scheme allows to compensate the delay effects. However, the changes and uncertainty in the delay through time, which is a typical situation in supply chains are not considered. Moreover these works considered a single echelon while a real supply chain is composed of many echelons.

In this work, we advocate on a decentralized control approach based on an IMC delay compensation scheme for the multivariable supply chain. Its main advantage is that in this scheme there are three controllers to tackle the nominal stability, the relationInventory level vs Inventory target and the relation Inventory level vs Demand separately. However, Internal model control scheme has a drawback: the system's delay has to be known beforehand to perform its perfect compensation. This situation is not feasible when the delay changes during the process which is a common situation in supply chains. An alternative to overcome this problem is to include a lead time identification method in the supply chain operation.

It is difficult to find works that deal with lead time identification in entire supply chains since most of the works normally deal with a single echelon. In [4] a recursive prediction error method (RPEM) is proposed to identify the lead time online in a unique echelon (SISO system). Then one parameter of the Control System is adjusted according the identified lead time. An important disadvantage of the aforementioned work is its theoretical complexity.

In this work, a delay identification algorithm is proposed for the complete supply chain being able to identify the delays among the different echelons describing the supply chain. The identified values of the delays are then used to adjust the delay compensation in a IMC based decentralized compensation scheme.

The proposed identification scheme consist of a battery of different models operating in parallel [5]. Each model includes the same rational component but a different delay value. A supervisory algorithm compares the mismatch between the actual system and each candidate models and it determines, for each time interval, the one that best describes the behaviour of the real system, providing an estimation of the lead time. An additional block selects the best model for control purposes. The approach is inspired in what are called Pattern Search Algorithms [6], whose application in control is really novel. Indeed, the main advantage of the Pattern Search Algorithms is that theirs implementaction is intuitive and easy. Moreover, theirs convergence proofs are conceptually easier than those presented in the mentioned works.

The rest of the paper is formulated as follows: Section II presents the supply chain model. Section III presents the formulation of the Internal model control as a delay compensation scheme. After that, section IV presents the adopted intelligent multimodel identification scheme. Section V presents the simulation results. The paper ends with the concluding remarks in Section VI.

II. SUPPLY CHAIN MODEL

Let us consider a supply chain with three logistic echelons [7], warehouse (W), distributing center (D) and retailer (R) between the factory (F) and customer (C). Thus, denote by $j = 0,1,2,3,4$ each one of the logistic echelons of the supply chain where $j = 0$ represent the Customer (C) and $j = 4$ represents the Factory (F). According to this notation, $(j+1)$ is the immediate supplier and $(j-1)$ is the immediate customer of the echelon. For the sake of simplicity, assume

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a period base of time $T_m = 1$ which can be one day , one week or one month.

Then, let $I_j(t)$ denote the inventory level of each logistic node j at any discrete time instant $t = nT_m$ where n is a natural. The amount of orders placed by a participant j to an immediate supplier $j+1$ is denoted by $O_{j,j+1}(t)$. Finally, the amount of goods delivered by each logistic node j to the immediate customer $j-1$ is denoted by $Y_{j,j-1}(t)$. Thus, the inventory balance in each echelon is given by the difference between the goods received from the immediate supplier and the goods delivered to the immediate customer as follows [1]:

$$I_j(t) = I_j(t-1) + Y_{j+1,j}(t) - Y_{j,j-1}(t), \quad (1)$$

There is a lead time $L_j \in \mathbb{N}$ between the time an order is placed by node j^{th} and when it is received. This lead time causes problems in the control system. It is also assumed that each node has enough stock to satisfy the demand of its inferior level. In this way, the amount of goods ordered to an immediate supplier at time t will arrive at time $t + L_j$ i.e. $Y_{j+1,j}(t) = O_j(t - L_j)$. Thus, the expression Eq.(1) becomes now [1]:

$$I_j(t) = I_j(t-1) + O_{j,j+1}(t - L_j) - O_{j-1,j}(t), \quad (2)$$

described by the following transfer function in the z domain

$$I_j(z) = \underbrace{\frac{1}{1-z^{-1}}}_{p_{Lf}(z)} \underbrace{z^{-L_j}}_{p_{L_j}(z)} O_{j,j+1}(z) - \underbrace{\frac{1}{1-z^{-1}}}_{p_{Lf}(z)} O_{j-1,j}(z) \quad (3)$$

In the present work, the generation of a model for a supply chain composed by four echelons is carried out considering that the demand for a particular echelon is equal to the order generated by the downstream. Thus, the general model for this supply chain is a system of equations that can be represented in a matrix form given by [1]:

$$\mathbf{I}(z) = p_{Lf}(z) \begin{pmatrix} -1 & z^{-L_1} & 0 & 0 \\ 0 & -1 & z^{-L_2} & 0 \\ 0 & 0 & -1 & z^{-L_3} \end{pmatrix} \mathbf{O}(z) \quad (4)$$

where the vector $\mathbf{I}(z)$ contains the inventories $I_j(z)$ of all echelons while $\mathbf{O}(z)$ represents the orders of all echelons $O_{j,j+1}(z)$ and customer demand $O_{0,1}(z)$. The matrix expressed in Eq.(4) can be decomposed into a delay-free factor $\mathbf{P}_{Lf}(z)$ and a pure delay (Lead-time) term $\mathbf{P}_L(z)$ using the Schur (or component-wise) product [8], in the form:

$$\mathbf{P}(z) = \mathbf{P}_{Lf}(z) \bullet \mathbf{P}_L(z) \quad (5)$$

where

$$\mathbf{P}_{Lf}(z) = p_{Lf}(z) \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix} \quad (6)$$

and

$$\mathbf{P}_L(z) = \begin{pmatrix} 1 & z^{-L_1} & 0 & 0 \\ 0 & 1 & z^{-L_2} & 0 \\ 0 & 0 & 1 & z^{-L_3} \end{pmatrix} \quad (7)$$

The complete system is hence modeled by a multivariable system characterized by multiple delays contained in Eq. (7).

The objective of this work is to design a decentralized controller for the system Eq. (6) and Eq. (7) under imperfect knowledge of delays given by Eq. (7). To achieve this objective, the different delays appearing in Eq. (7) are to be identified which is a topic that has not been analyzed previously in the supply chain literature.

In order to perform the identification of the delays the following assumptions are made.

Assumption 1. The rational part of the system, i.e. Eq.(6), is known.

Assumption 2. The delay between each pair output/input lies in a known compact interval. That is, if it is considered a matrix $\mathbf{H}$ which contains the nominal delays of the system:

$$\mathbf{H} = \begin{pmatrix} 0 & L_1 & 0 & 0 \\ 0 & 0 & L_2 & 0 \\ 0 & 0 & 0 & L_3 \end{pmatrix}, \quad (8)$$

there exist two known matrices $\underline{\mathbf{H}}, \overline{\mathbf{H}}$, such that $\underline{\mathbf{H}} \leq \mathbf{H} \leq \overline{\mathbf{H}}$ ■

Notice that both Assumptions are feasible in practice for the inventory control problem in the supply chain.

Based on (3) it is possible to derive an IMC structure as a decision policy that manipulates orders $O_{j,j+1}(z)$ to maintain inventory level at a designated inventory target $I_j^t(z)$. The following section describes how this is possible.

III. INTERNAL MODEL CONTROL STRUCTURE

Since decentralized control is applied, we present the (IMC) scheme for an echelon. Then, this structure is applied echelon by echelon.

The controller structure proposed by [9] is shown in Fig. 1 where $p(z)$ is the rational part of the model defined in Eq.(3). Also, z^{-L_j} denotes the transfer function of the real lead time. However, there would be an imperfect knowledge of the delay which implies that an estimated delay $\hat{L}_j$ rather than the actual one $Ł_j$.

The proposed structure has three controllers for each echelon: $q_j^s(z)$ to stabilize $p(z)$, $q_j^t(z)$ is an IMC controller for the stabilized model $g(z)$ and $q_j^{ld}(z)$ is designed mainly to achieve internal stability and load disturbance rejection $O_{j-1,j}(z)$ in this case. Since, in this paper an online algorithm to identify the lead time is proposed to overcome the problem of uncertainty in the parameters of the system, all controllers are designed considering the perfect knowledge of them.

That is, when the lead time is perfectly known $(z^{-L_j} = z^{-\hat{L}_j})$, the equation of inventory balance for a single echelon j under the IMC scheme is represented by :

$$I_j(z) = q_j^t(z) g(z) z^{-L_j} I_j^t(z) - (1 - q_j^t(z) g(z) z^{-\hat{L}_j}) \frac{p(z)}{1 + q_j^{ld}(z) p(z) z^{-L_j}} O_{j-1,j}(z) \quad (9)$$

where

$$g(z) = \frac{z}{z(1 + q_j^s(z)) - 1} \quad (10)$$

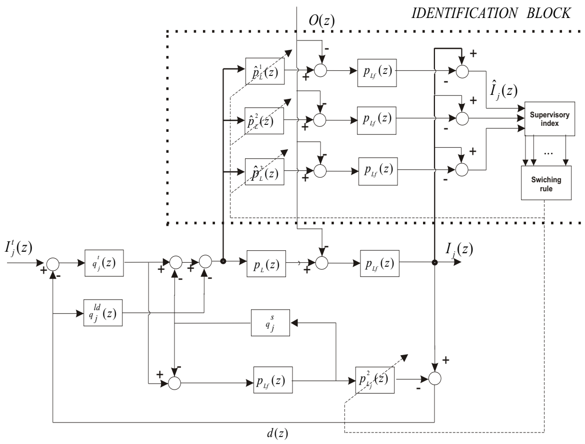

Fig. 1. Identification and control scheme for a single echelon

From (9) it follows that the lead time has disappeared from the denominator of the relation from target $I_j^t(z)$ to $I_j(z)$. Therefore, if there is no disturbance signal (the demand of the immediate customer $O_{j-1,j}(z) = 0$ in the supply chain case) the lead time has been decoupled of the rational part from the system. Thus, the tuning effort of the controller $q_j^t(z)$ can be reduced considerably because it depends only of the rational part of the system. Besides, it can be seen that $q_j^{ld}(z)$ only appears in the relation from $O_{j-1,j}(z)$ to $I_j(z)$. Thus, the problem of disturbance rejection can be tackled separately by tuning $q_j^{ld}(z)$.

A. Controllers design

Initially, the procedure consists on stabilizing $p(z)$. For simplicity, a proportional controller $q_j^s(z) = q_j^s \in \mathbb{R}$ is a common choice in many works for this purpose [9]. Then, replacing $q_j^s(z) = q_j^s$ in Eq.(10) yields:

$$g(z) = \frac{z}{z(1+q_j^s)-1} \tag{11}$$

A discrete-time system is stable if all the roots of the characteristic equation lie within the unit open circle, this condition can be fulfilled by selecting $q_j^s > 0$.

Once the model is stabilized, the q_j^t controller is designed in the following two steps : i.

1) Design for nominal optimal performance: here, the IMC controller $q_j^t(z)$ is designed for inventory target tracking solving the following H_2-optimal problem formulated as:

$$min_{q^t(z)}||g(z)q^t(z)||_2 \tag{12}$$

Since $g(z)$ is stable, inversely stable and biproper, then its inverse $q_j^t(z) = g^{-1}(z)$ is an acceptable solution for this problem, [10].

2) For robust stability: The model of the system is considered known. Therefore, the detuning of the optimal controller $q_j^t(z)$ by adding the filter $f(z)$, which is usually done for robust stability to uncertainty in the parameters, can be done in this case for bullwhip effect mitigation. The adjusting of these controllers for bullwhip effect mitigation in the entire supply chain is subject of future research while now it is performed echelon by echelon. The structure of this filter is given by:

$$f(z) = \frac{(1-\lambda)z}{z-\lambda} \tag{13}$$

For simplicity, the controller $q^{ld}(z)$ is chosen as a constant , and its design is focused to assure the internal stability. Once the IMC scheme is formulated for a particular echelon, we can reproduce it for each echelon that belong to the supply chain. The general control system resulting for the complete supply chain is formulated as a diagonal matrix corresponding to the decentralized control case given by

$$Q^t(z) = Diag(q_1^t(z),...,q_3^t(z)) \tag{14}$$

$$Q^s(z) = Diag(q_1^s(z),...,q_3^s(z)) \tag{15}$$

$$Q^{ld}(z) = Diag(q_1^{ld}(z),...,q_3^{ld}(z)) \tag{16}$$

The presented approach to identify the lead time is based on a multi-model scheme introduced in [5] for a SISO system. The structure of the identification algorithm is described in the next section. For more details of the controllers design, see [11].

IV. PROPOSED IDENTIFICATION SCHEME

The basic structure of the proposed scheme is depicted in Fig. 1. Here, $\hat{\mathbf{P}}_L^{(l)}(z)$ with $l = 1,2,\ldots,n_e$ denotes the matrix of transfer function associated with the different delay models where n_e represents the number of candidate models taken into account. As it can be seen, the scheme is composed of three elements: a set of time-varying candidate models associated with different values for the delays, a figure of merit (performance or supervisory index) which evaluates the potential behavior of each model and a switching rule which monitors periodically this index and decides which model is the best to be used for control purposes. The switching mechanism is intended to reduce the possible mismatch between the nominal and the actual output of the system. In the following subsections the different elements of the presented architecture are considered in detail.

A. Set of nominal models

The proposed architecture is composed of a set of candidate models running in parallel, each one associated with a different delay matrix. These models are time-varying and automatically adjusted by an algorithm. In order to describe the structure of the set of models, let us denote $M(t)$ as the set of models and its evolution through the time. It is considered a matrix $\hat{\mathbf{H}}(t)$ which contains the current nominal delays of $\hat{\mathbf{P}}_L(z)$ used for control purposes (see, Eq.(8)). Let $\Delta\hat{\mathbf{H}}^{inf}(t) \geq 0$, $\Delta\hat{\mathbf{H}}^{sup}(t) \geq 0$ and Γ be the upper and lower variations matrix, and the reduction factors matrix respectively.

$$M^-(t) = \{\hat{\mathbf{H}}(t) - \Delta\hat{\mathbf{H}}^{inf}(t)E_{i,i+1}\} \quad i = 1,2....N \tag{17}$$

$$M^{+}(t) = \{\hat{\mathbf{H}}(z) + \Delta\hat{\mathbf{H}}^{sup}(t)E_{i,i+1}\} \quad i = 1,2....N \tag{18}$$

$$M(t) = M^{-}(t) \cup \{\hat{\mathbf{H}}(t)\} \cup M^{+}(t) \tag{19}$$

where $E_{i,i+1}$ denotes the elements of the canonical basis of $\mathbb{R}^{(N-1)\times N}$ (matrices having a one in the $(i,i+1)^{th}$ entry and zeros elsewhere, [6].

From Eqs. (17)-(19) the time-dependent set of models $M(t)$ is formed by adding $\Delta\hat{\mathbf{H}}^{sup}(t)$ and subtracting $\Delta\hat{\mathbf{H}}^{inf}(t)$ to the nominal delay matrix $\hat{\mathbf{H}(t)}$ used for control purposes at time t.

B. Figure of merit

The second element of the proposed scheme is a figure of merit aimed at evaluating the behavior of each model in the set M. The suggested figure of merit is:

$$J^{(l)}(\mathbf{H}) = \sum_{t-Tres}^{t} (I(\tau) - \hat{I}_{(i)}(\tau))^{T} (I(\tau) - \hat{I}_{(i)}(\tau)) \tag{20}$$

with $l = 1,2,\ldots,n_e$ and $I(\tau)$ is the vector output of the plant at the instant $t = \tau$ while $\hat{I}(\tau)$ denotes the (vector) output of each different model. The summation (20) takes place in the time interval in which all the models act simultaneously without being modified. T_{res} is the so-called residence time and defines the window where the different models are compared. Note that the figure of merit Eq.(20) is positive, i.e. $J^{(l)} \geq 0$, $\forall l = 1,2,...,n_e$. Defining the error between the plant output and the output of the l^{th} model, in the z domain as:

$$\begin{aligned} \mathbf{E}(z)^{(l)} &= \mathbf{P}_{Lf}(z) \bullet \mathbf{P}_{L}(z) - \mathbf{P}_{Lf}(z) \bullet \hat{\mathbf{P}}_{L}^{(l)}(z) \\ &= \mathbf{P}_{Lf}(z) \bullet (\mathbf{P}_{L}(z) - \hat{\mathbf{P}}_{L}^{(l)}(z)) \end{aligned} \tag{21}$$

It is readily seen from Eq.(21) that the error is zero when $\mathbf{P}_{L}(z) - \hat{\mathbf{P}}_{L}^{(l)}(z) = 0$, that is, when each modeled delay $\hat{L}_{j}^{(l)} \in \hat{\mathbf{P}}_{L}^{(l)}(z)$ is equal to each real delay $L_j \in \mathbf{P}_{L}(z)$. Thus, the identification algorithm acts as an optimization algorithm seeking the minimum of Eq.(20). This fact motivates the algorithm in the following subsection.

C. Switching logic

The switching logic monitors the value of the figure of merit at time instants multiples of T_{res} and selects the nominal delay which is the best estimation of the real one.

The initial nominal model is selected by the designer with the (arbitrary) initialization of $\hat{\mathbf{H}}(0)$ and $\hat{H}^{sup/inf}(0)$. From this moment onwards, the switching logic can be expressed formally as the Algorithm 1.

Initially, the algorithm takes a set of data from the outputs $\mathbf{I}$(t) and the inputs $\mathbf{O}$(t) of the system. The length of this window depends on T_{res}. Then, the **New_models_calculation** function is called until $\Delta\hat{H}^{sup} > 1$ & $\Delta\hat{H}^{inf} > 1$ which implies that the Algorithm 1 has converged to the actual lead-times.

In this function, the comparisons between the different models are carried out in groups of three consisting of the nominal, plus the additive and subtractive disturbances in the direction of $E_{i,i+1}$ (line 2 of the **New_models_calculation** function). Then, the output of the three candidate models are evaluated (lines from 4 to 8). These models (which only differ in the $(i,i+1)^{th}$ component of the matrix delay) are compared to each other using the performance index (20), used in line 10.

Algorithm 1. MIMO identification

1: $\hat{H}$: *Matrix of delays.*
2: $\Delta\hat{H}^{sup}$: *Matrix upper−bound variation.*
3: $\Delta\hat{H}^{inf}$: Matrix lower-bound Variation.
4: $\underline{H},\overline{H}$ Intervals of uncertainty of delays.
5: $\Gamma = 1$: Matrix reduction factors for $\Delta\hat{H}$.
6 : $T_{res} > 0$: Residence time.
7 : $v \leftarrow 0$
8 : $M(0) = M^{-}(0) \cup \hat{H}(0)M^{+}(0)$
9 : **for** $t > 0$, At multiples of the residence time **do**.
10 : **if** $t = nT_{res}$, $m \in \mathbb{N}$ **then**
11 : $\mathbf{Y} = [I_j(t-1), I_j(t-1), ...I_j(t-Tres)]$
12 : $\mathbf{X} = [O_{j,j+1}(t-1), O_{j,j+1}(t-1), ...O_{j,j+1}(t-Tres)]$
13 : $\mathbf{Z} = [O_{j-1,j}(t-1), O_{j-1,j}(t-1), ...O_{j-1,j}(t-Tres)]$
14 : **while** $\Delta\hat{H}^{sup} > 1 \& \Delta\hat{H}^{inf} > 1$
15 : **New_models_calculation**
16 : **end while**
17 : **end for**

New_models_calculation

1: **for** $1 \leq j \leq N$**do**
2: $M_c(t) \leftarrow \{\hat{H}_{ij}(t) - \Delta\hat{H}_{ij}^{inf}(t), \hat{H}_{j,j+1}(t), \hat{H}_{j,j+1}(t) + \Delta\hat{L}^{sup}(t)\}$
3: end for
4: **for** $0<i<3, 0<h<T_{res}$
5: $I_j^{(1)}(t) = I^{(1)}(t-1) + \mathbf{X}(t - \hat{H}_{j,j+1}^{sup}) - \mathbf{Z}(t)$
6: $I_j^{(2)}(t) = I^{(2)}(t-1) + \mathbf{X}(t - \hat{H}_{j,j+1}) - \mathbf{Z}(t)$
7: $I_j^{(3)}(t) = I^{(3)}(t-1) + \mathbf{X}(t - \hat{H}_{j,j+1}^{inf}) - \mathbf{Z}(t)$
8: **end for**
9: **for** $1 \leq j \leq N$ **do**
10: $pos \leftarrow argmin_{1 \leq k \leq 3} J_{ij}^{(k)}$
11: $v \leftarrow v + (-2 + pos)E_{ij}$
12: if Mc = 1 **then**
13: $\Delta\hat{H}_{i,(i+1)}^{inf}(t) \leftarrow \left(2\Delta\hat{H}_{i(i+1)}^{inf}(t) + \gamma_{j(j+1)}\right)$
14: $\Delta\hat{H}_{j,(j+1)}^{sup}(t) \leftarrow \left(\Delta\hat{H}_{i(i+1)}^{sup}(t) + \gamma_{i(i+1)}\right)$
15: **end if**
16: if Mc = 2 **then**
17: $\Delta\hat{H}_{i,(i+1)}^{inf}(t) \leftarrow \left(\Delta\hat{H}_{i(i+1)}^{inf}(t) + \gamma_{i(i+1)}\right)$
18: $\Delta\hat{H}_{i(i+1)}^{sup}(t) \leftarrow \left(\Delta\hat{H}_{i(i+1)}^{inf}(t) + \gamma_{i(i+1)}\right)$
19: **end if**
20: if Mc = 3 **then**
21: $\Delta\hat{H}_{j,(j+1)}^{inf}(t) \leftarrow \Delta\hat{H}_{i(i+1)}^{inf}(t) + \gamma_{j,(j+1)}$
22: $\Delta\hat{H}_{i(i+1)}^{sup}(t) \leftarrow \left(2\Delta\hat{H}_{i(i+1)}^{sup}(t) + \gamma_{i(i+1)}\right)$
23: **end if**
24: **end for**
25: $\hat{H}(t) = proy_{\underline{H}_{i(i+1)}, \bar{H}_{i(i+1)}}(\hat{H}(t) + v)$
26: $M(t) = M^{-}(t) \cup \{\hat{H}(z)\} \cup M^{+}(t)$
27: **Return to Algorithm 1**

As a result, the following outcomes are possible. If $Mc = 1$ means that the model with the subtractive disturbance has been selected while Mc = 2 stands for the previous nominal one and $Mc = 3$ for the model with the additive disturbance. In this way the element associated with the lowest value of the figure of merit is obtained for each delay component, while the vector v is the vector sum necessary to convert the nominal model into the best model of $M(t)$.
At the end, the vector v results into the optimum direction change vector, allowing the update of the current nominal model so as to make it the best possible one. This vector is added to the nominal model to obtain the corresponding new nominal model from which a battery of models that will operate in parallel during the next interval of residence is generated again through Eq. (17)-(19). To rebuilt the set M(t), the search patterns $\hat{H}^{sup}_{i,i+1}(t), \hat{H}^{inf}i, i+1(t)$ are adjusted (lines 12 to 22 of the function **New_models_calculation**), to ensure the convergence to the true delay taking a reduction factor that depends on the new nominal model value and reduction matrix Γ. This guarantees that all the search patterns converge to the same value and hence the scheme tends to a time invariant system with the real delays. Once the lead time is identified by **New_models_calculation** for a time window, the function returns to the algorithm 1 for a new period of identification.

Each element of the lead times matrix is estimated at each integer multiple of the residence time (line 9 of Algorithm 1).

For more details on the identification scheme and convergence proofs, see [11].

V. SIMULATION RESULTS

The basic supply chain composed by three echelons which was represented in Eqs. (6) and (7) is simulated in order to show the feasibility and usefulness of the proposed schemes. It is considered that the real lead-time values L_j change in several time instants in order to demonstrate that the Algorithm 1 converges to the true lead time quickly, so that the control scheme is adapted on time and thereby oscillations in the system are minimal.

A. Several changes in the Lead times under a stochastic demand

In this simulation at t=0 the modeled lead times coincide with the real lead times i.e $\hat{L}_1 = L_1 = 4$, $\hat{L}_2 = L_2 = 6$, $\hat{L}_3 = L_3 = 3$. Then, multiples changes in the lead times in different periods of time are contemplated. It is better illustrated in Fig. 2. A stochastic variability in the customer demand $O_{0,1}(z)$ is also applied to the system from time instant $t = 10$ and onwards, in order to provide reality to the simulation. Customer demand is formulated as a normal function, with an average equal to 20 and a variance equal to 4, i.e. $\in N(20,4)$. All real lead times involved in the supply chain are identified with accuracy as is shown in Fig. 2. Fig. 3 proves that the system is robust to changes in the lead time, and at the same time is able to present a good performance to a stochastic customer demand. There are some fluctuations while there is a mismatch in the lead time but once the lead time is identified the scheme correct timely these fluctuations. Fig. 4 evidence that the faster tracking of the lead time allows that its changes do not affect the behavior of the orders in the complete supply chain.

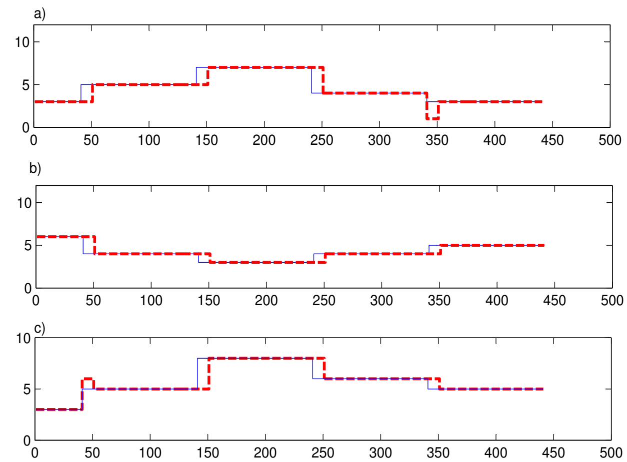

Fig. 2. a) solid line: Real L_1. Dash line: Identified $\hat{L}_1$. b) solid line: Real L_2. Dash line: Identified $\hat{L}_2$. c) solid line: Real L_3. Dash line: Identified $\hat{L}_3$

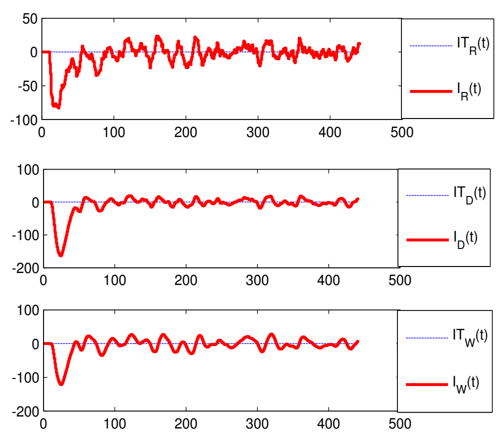

Fig. 3. Inventories in all echelons

The presented approach presents an improvement in the inventory control in comparison to previous works.

VI. CONCLUSIONS

Simulation examples have shown that the algorithm identifies the complete set of lead times of the supply chain timely improving the performance of the inventory control. The IMC delay compensation control structure complemented with the delay identification scheme has been applied in a serial supply chain with good results.

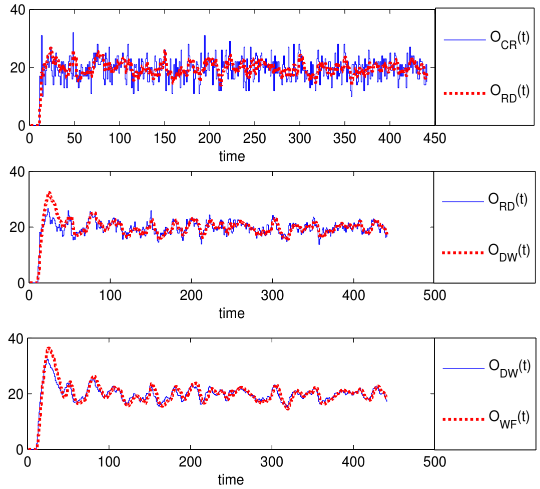

Fig. 4. Orders in all echelons

VII. ACKNOWLEDGMENTS

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REFERENCES

[1] J. Dejonckheere, S. Disney, M. Lambrecht, D. Towill, The impact of information enrichment on the bullwhip effect in supply chains: A control engineering perspectiva, European Journal of Operating Research 153 (2) (2004) 727–750.

[2] P. Lin, D. S.-H. Wong, S. Jang, S. Shieh, J. Chu, Controller design and reduction of bullwhip for a model supply chain system using z-transform analysis, Journal of Process Control 14 (2004) 487–499.

[3] D. Schwartz, D. Rivera, A process control approach to tactical inventory management in production-inventory systems., International Journal of Production Economics 125 (2) (2010) 111–124.

[4] E. Aggelogiannaki, H. Sarimveis, Design of a novel adaptive inventory control system based on the online identification of lead time, International Journal of Production Economics 176 (4) (2007) 162–172.

[5] J. Herrera, A. Ibeas, S. Alcantara, d. l. S. M, Multimodel-based techniques for the identification and adaptive control of delayed mimo systems, [IET] control theory and applications 5 (2011) 188–202.

[6] T. A. Sriver, J. W. Chrissis, M. A. Abramson., Pattern search ranking and selection algorithms for mixed variable simulation-based optimization., European journal of operational research 198 (2009.) 878–890.

[7] J. Dejonckheere, S. Disney, M. Lambrecht, D. Towill, Measuring and avoiding the bullwhip effect: A control theoretic approach, European Journal of Operating Research 147 (2002) 567–590.

[8] S. B. Dennis, Matrix mathematics, Princeton University Press 4 (2005) 1251–1266.

[9] W. Tan, H. J. Marquez, T. Chen, Imc design for unstable processes with time delays, Journal of Process Control 31 (2) (2003) 157 – 167.

[10] M. Morari, E. Zafiriou, Robust Process Control., Prentice-Hall International, 1989.

[11] C. Garcia, A. Ibeas, J. Herrera, R. Vilanova, Inventory control for the supply chain: An adaptive control approach based on the identification of the lead-time., Omega: The International Journal of Management Science 40 (2012) 314–327.