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Centralized inventory control approach for supply chain systems

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Abstract— **In this paper, a centralized control approach based in the two-degrees-of-freedom Internal Model Control structure is incorporated in production inventory control for a supply chain system. Analytical tuning rules for bullwhip effect avoidance are developed. The results of controller evaluations demonstrate that centralized control approach presents a good behaviour with respect to the inventory target tracking, demand rejection and bullwhip effect in the supply chain systems.**

I. INTRODUCTION

A common Supply Chain includes the necessary entities to provide goods from the factory to the customer. In this way, all entities must perform inventory replenishment policies, the aim of them is maintaining enough stock to satisfy customer demand.

Many undesirable effects may appear when an inventory replenishment policy is implemented such as the instability and the well known bullwhip effect [1]. In order to overcome these problems, decisions based on process control theory have been successfully applied. Among them, [1], [2], [3], have analyzed the effect of the replenishment policies focused on the bullwhip effect estimation and suppression. Moreover, [4] presents approaches based on Control Engineering , including proportional-integral (PI) controllers and cascade control as inventory replenishment policies. The design of these controllers is also focused on the mitigation of the bullwhip effect. These approaches present an acceptable inventory control performance with smooth information flow in an echelon. Nevertheless, the analysis of the inventory control performance of these inventory control policies on the entire supply chain is not taken into account.

[5] introduces the application of two degrees of freedom feedback and three degrees of freedom feedback-feedforward Internal Model Control, as a novel inventory replenishment policy in the supply chain, providing an important improvement in the performance of manufacturing systems with a long lead time and significant uncertainty. However, the inventory control scheme is applied to a single echelon instead of a complete supply chain composed by multiple echelons. Moreover, analytical tuning to guarantee bullwhip effect avoidance is omitted.

The supply chain is described as a multi-variable system. Therefore, the incorporation of a centralized inventory control strategy to the entire supply chain can yield insights to improve the inventory control in an overall scale. Henceforth, in this paper we focus on the development of an inventory control strategy for the complete supply chain. The inventory control will be based on Internal Model Control IMC [5], [6].

An advantage of the multi-degrees-of-freedom IMC topologies is that their performance to set point tracking (i.e meeting an inventory target), measured disturbance rejection (i.e. meeting forecasted demand), unmeasured disturbance rejection (i.e. satisfying unforecasted demand) is improved by using three independent controllers avoiding trade off between these problems. Nevertheless, the guidelines to tune these controllers for the Multi-inputs-Multi-Outputs MIMO case so as to avoid the bullwhip effect have not been explored yet. Therefore, we advocate to design a multiple degrees-of-freedom IMC scheme [5] for the entire supply chain (MIMO system) where the bullwhip effect is taken into account.

Generally, since the demand is considered completely stochastic, a feedforward degree of freedom based on the forecast of the demand does not contribute an improvement in the behaviour of the system respect to the feedback configuration [5] and this is omitted in the presented approach.

The rest of the paper is formulated as follows: Section II presents the complete supply chain model using z-transform. Section III presents the generalization of the IMC design for the entire supply chain with centralized control. The paper ends with the discussion and concluding remarks in Section V.

II. SUPPLY CHAIN MODEL

In this section, the model for a single echelon is developed. After that, the model for the entire supply chain is derived based on the model for a single echelon. For the sake of simplicity, it is assumed a period base of time $T_m = 1$ which can be one day, one week or one month according to the dynamics of the supply chain. In this model, $j = 1,2...N$ (where N is a finite integer) denotes each one of the logistic echelons of the supply chain. The customer is considered the base while the factory is on the top of the supply chain. Thus, while $j = 1$ represents the retailer, $j = N$ represents the factory. According to this notation, $(j+1)$ represents an immediate supplier and $(j-1)$ represent an immediate customer of the j^{th} echelon. logistic echelons between the factory and the customer as is shown in Fig. 1.

Thus, let $y_j(t)$ denote the inventory level of each logistic node j at any discrete time instant $t = nT_m$ where n is a natural number. The amount of orders placed by a participant j to an immediate supplier $j+1$ is denoted by $o_{j,j+1}(t)$. The demand perceived by each echelon is represented by $d_j(t)$. Finally, the amount of goods delivered by each logistic node a to the node b is denoted by $\beta_{a,b}(t)$. In this way, the amount

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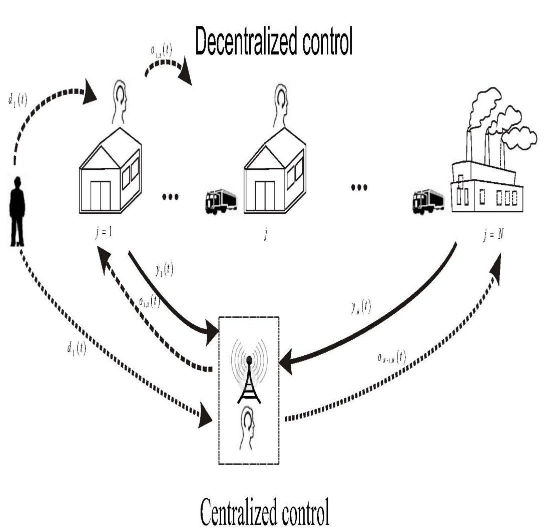

Fig. 1. The supply chain under the centralized control approach

of goods delivered by each logistic node j to the immediate customer $j-1$ is denoted by $\beta_{j,j-1}(t)$ .Thus, the inventory balance in each echelon is given by:

$$y_j(t) = y_j(t-1) + \beta_{j+1,j}(t) - \beta_{j,j-1}(t), \quad j = 1,2,...,N \tag{1}$$

In this model, a lead time $L_j \in \mathbb{N}$ is considered between the time when an order is placed by node j^{th} and the time when it is received [7]. It is also assumed that each node has enough existences to satisfy the demand of its immediate customer. In this way, the amount of goods ordered to an immediate supplier at time t will arrive at time $t + L_j$ i.e. $\beta_{j+1,j}(t) = o_{j,j+1}(t - L_j)$. Thus, the Eq. (1) that relates the inventory balance with the demand $d_j(t)$ and order $o_{j,j+1}(t)$ at node j becomes now:

$$y_j(t) = y_j(t-1) + o_{j,j+1}(t - L_j) - d_j(t), \quad j = 1,2,...,N \tag{2}$$

described by the following transfer function in the z-domain:

$$y_j(z) = \overbrace{\left[\frac{1}{1-z^{-1}}\right]}^{p^m(z)} \overbrace{z^{-L_j}}^{p^a(z)} o_{j,j+1}(z) - \overbrace{\left[\frac{1}{1-z^{-1}}\right]}^{p^m(z)} d_j(z) \tag{3}$$

which relates the z-transform of the inventory level, $y_j(z)$, with the order and the demand in a single echelon through the transfer functions $p^m(z)$ and $p^a(z)$. The generalization of a model for the complete supply chain is carried out considering that an order $o_{j-1,j}(z)$ generated by a downstream echelon $j-1$ is perceived and supplied by the immediate supplier j. In this way, the vector $\mathbf{Y}(z)$ represent the set of inventories of all echelons i.e. $\mathbf{Y}(z) = \begin{pmatrix} y_1(z) & y_2(z) & \cdots & y_N(z) \end{pmatrix}^T$ which are the controlled variables, and $\mathbf{O}(z)$ represent the vector of the set of orders i.e. $\mathbf{O}(z) = \begin{pmatrix} o_{1,2}(z) & o_{2,3}(z) & \cdots & o_{N-1,N}(z) \end{pmatrix}^T$, which are the manipulated variables of the supply chain. Finally, the unknown demand signals perceived by each echelon are represented by the vector $\mathbf{D}(z) = \begin{pmatrix} d_1(z) & d_2(z) & \cdots & d_N(z) \end{pmatrix}^T$. Thus, the complete supply chain is modeled by the matrix Eq. (4):

$$\mathbf{Y}(z) = \mathbf{P}(z)\mathbf{O}(z) - \mathbf{P}^d(z)\mathbf{D}(z) \tag{4}$$

where the transfer function matrix that relates the set of inventories $\mathbf{Y}(z)$ with the orders vector $\mathbf{O}(z)$ is given by:

$$\mathbf{P}(z) = p^m(z) \begin{pmatrix} p_1^a(z) & 0 & \cdots & 0 \\ -1 & p_2^a(z) & \ddots & \vdots \\ 0 & \ddots & \ddots & 0 \\ 0 & 0 & -1 & p_N^a(z) \end{pmatrix}_{(N)\times(N)} \tag{5}$$

For IMC design, $\mathbf{P}(z)$ must be factored into a portion $\mathbf{P}^A(z)$ that includes the delays of the system, [6]:

$$\mathbf{P}^A(z) = \begin{pmatrix} p_1^a(z) & 0 & \cdots & 0 \\ p_2^a(z)-1 & p_2^a(z) & 0 & \vdots \\ \vdots & \ddots & \ddots & 0 \\ p_N^a(z)-1 & \cdots & p_N^a(z)-1 & p_N^a(z) \end{pmatrix} \tag{6}$$

and a mimimum-phase portion given by:

$$\mathbf{P}^M(z) = p^m(z) \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ -1 & 1 & 0 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{pmatrix} \tag{7}$$

such that

$$\mathbf{P}(z) = \mathbf{P}^A(z)\mathbf{P}^M(z) \tag{8}$$

Finally, the transfer function matrix that relates the set of inventories $\mathbf{Y}(z)$ with the set of demands $\mathbf{D}(z)$ is represented by:

$$\mathbf{P}^d(z) = p^m(z)\mathbf{I} \tag{9}$$

where $\mathbf{I}$ is the identity matrix.
On the one hand, the rational part of the system describes the balance of material carried out in each echelon. On the other hand, the lead time can be determined by (on-line) identification algorithms [7], [8]. Therefore, for control purposes, the model of the system Eq. (7) and Eq. (6) is considered known.

The objective of this work is to design a centralized IMC control using the MIMO model of the supply chain represented by Eqs. (4),(9) . The design is oriented to improve the inventory target tracking and the demand rejection, avoiding aggressiveness in the orders (bullwhip effect). In the rest of the paper we will formulate the IMC scheme for inventory control in the entire supply chain in order to satisfy the system requirements.

III. CENTRALIZED CONTROL STRATEGY

The centralized control design is based on the IMC scheme shown in Fig. 2 where the model formulated for the entire supply chain, Eqs. (6 - 9), is taken into account. Thus, the vector of inventories, $\mathbf{Y}(z)$, is given by:

$$\mathbf{Y}(z) = \mathbf{P}(z)\mathbf{Q}^t(z)\mathbf{R}(z) - (\mathbf{I} - \mathbf{P}(z)\mathbf{Q}^d(z))\mathbf{P}^d(z)\mathbf{D}(z) \quad (10)$$

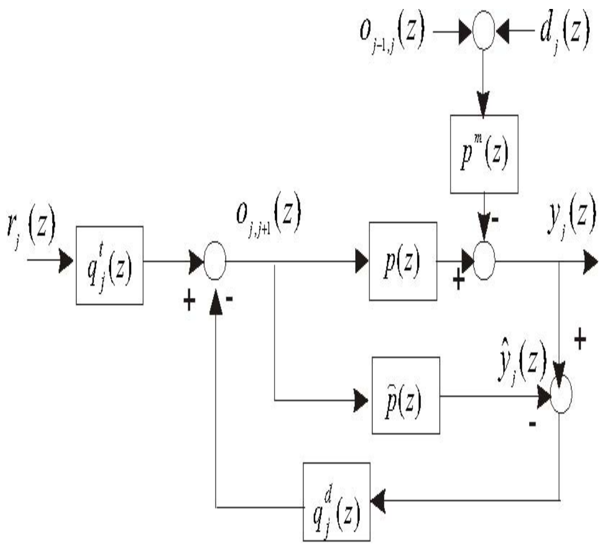

Fig. 2. Two-degrees-of-freedom-feedback IMC scheme

In the centralized control strategy, $\mathbf{Q}^t(z)$ and $\mathbf{Q}^d(z)$ can be designed for inventory target tracking and disturbance (demand) rejection respectively. This design is carried out by using the IMC guidelines for multivariable (MIMO) systems.

A. Bullwhip effect formulation

In order to formulate the bullwhip effect, the transfer function matrix that relates the orders vector $\mathbf{O}(z)$ with the demand vector $\mathbf{D}(z)$, considering no changes in the set point is obtained:

$$\mathbf{O}(z) = \mathbf{P}^d(z)\mathbf{Q}^d(z)\mathbf{D}(z) \quad (11)$$

The magnitude ratio of orders to successive nodes Γ under a centralized strategy can be expressed as:

$$\Gamma = |\mathbf{P}^d(z)\mathbf{Q}^d(z)| = |\mathbf{P}^d(e^{i\omega})\mathbf{Q}^d(e^{i\omega})| \quad (12)$$

where the magnitude ratio is calculated component-wise and $\omega \in [0, 2\pi)$. Thus, the demand signals perceived in the supply chain will not be amplified if:

$$\Gamma_{ij} \leq 1 \quad i, j = 1, 2, ..., N \quad (13)$$

In this case the bullwhip effect implies that each demand signal represented by d_j introduced in the system is not amplified to subsequents suppliers represented by $o_{j,j+1}(z)$. Therefore, the bullwhip effect can be analysed component-wise since each component of Γ contains the relation between each couple $d_j(z), o_{j,j+1}(z)$. Thus, in the centralized control approach multiples demand signals are taken into account i.e $d_j(z) \neq 0 \;\; j = 1, 2, ..., N$. After formulating the inventory control system in a centralized control way, the controller matrices $\mathbf{Q}^t(z)$ and $\mathbf{Q}^d(z)$ will be design in subsection III-B.

B. Controllers design

Notice from Eq. (10) that $\mathbf{Q}^t(z)$ only affects the relation between $\mathbf{Y}(z)$ and $\mathbf{R}(z)$ as well as $\mathbf{Q}^d(z)$ only affects the relation between $\mathbf{Y}(z)$ and $\mathbf{D}(z)$. Therefore, these controllers can be designed separately. The design procedure is presented below:

1) Inventory target tracking design: For inventory target tracking $\tilde{\mathbf{Q}}^t(z)$ is designed to solve the H_2-optimal MIMO problem given by

$$min_{Q^t(z)} \|[\mathbf{I} - \mathbf{Q}^t(z)\mathbf{P}^A(z)\mathbf{P}^M(z)]\mathbf{R}(z)\|_2 \quad (14)$$

where the vector $\mathbf{R}(z)$ contains the set of inventory targets $r_j(z)$ for the entire supply chain. Assuming a step change in the inventory target, the optimal controller obtained by IMC design is:

$$\tilde{\mathbf{Q}}^t(z) = (p^m(z))^{-1} \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 1 & \cdots & 1 & 1 \end{pmatrix} \quad (15)$$

The optimal controller $\tilde{\mathbf{Q}}^t(z)$ for inventory target tracking guarantees fast response and no offset for Type-1(step) inventory targets changes in the control system but in consequence, aggressive orders are generated in all supply chain which is an undesirable situation for factory manager. Therefore, the fast response of this controller to changes in the inventory targets is degraded, obtaining less aggressive orders. Then, this controller must be enhanced with a low-pass filters bank given by:

$$\mathbf{F}^t(z) = Diag[f_1(z), f_2(z), ..., f_N(z)] \quad (16)$$

where each one type 1 filter $f_j(z)$ is defined by:

$$f_j^t(z) = \frac{(1-\lambda_j^t)z}{z-\lambda_j^t} \quad (17)$$

Thus, the final IMC controller is:

$$Q^t(z) = \tilde{Q}^t(z)\mathbf{F}^t \quad (18)$$

2) Design for disturbance rejection: The two-degrees-of-freedom-IMC scheme allows us to specify the system response to demand changes by using the $\mathbf{Q}^d(z)$ controller. As a result of the integrating nature of the inventory process, a step change in demand becomes a Type-2(ramp) disturbance. Therefore the design procedure relies on solving the H_2-optimal control given by

$$min_{\mathbf{Q}^d(z)} \|[1 - \mathbf{P}^M(z)\mathbf{P}^A(z)\mathbf{Q}^d(z)]\mathbf{P}^d(z)\mathbf{D}(z)\|_2 \quad (19)$$

whose IMC solution is:

$$\tilde{\mathbf{Q}}^d(z) = z(\mathbf{P}_M(z))^{-1}\{z^{-1}\mathbf{P}_A^{-1}(z)\mathbf{D}\}_*\mathbf{D}^{-1} \tag{20}$$

where the $\{.\}_*$ operator denotes that after a partial fraction expansion of the operand $\{.\}$, all the terms involving the poles of $(P^A(z))^{-1}$ are omitted.

The solution of Eq. (20) is explicitly given by:

$$\tilde{\mathbf{Q}}^d(z) = \frac{1}{p^m}\begin{pmatrix} \frac{\frac{(1+L_1)}{L_1}z-1}{z} & 0 & \cdots & 0 \\ \frac{\frac{(1+L_1+L_2)}{L_1+L_2}z-1}{z} & \frac{\frac{(1+L_2)}{L_2}z-1}{z} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \frac{\frac{(1+\Sigma_1^N L_j)}{\Sigma_1^N L_j}z-1}{z} & \cdots & \frac{\frac{(1+L_{N-1}+L_N)}{L_{N-1}+L_N}z-1}{z} & \frac{\frac{(1+L_N)}{(L_N)}z-1}{z} \end{pmatrix} \tag{21}$$

Since the optimal controller for disturbance rejection yields strong variability in the orders, the bullwhip effect is also a restriction to be taken into account in the $Q^d(z)$ controller design. *Therefore, novel guidelines to detune $\tilde{Q}^d(z)$ controller to satisfy bullwhip effect constraint are shown below.*

3) Detuning of $\tilde{\mathbf{Q}}_j^d(z)$ for bullwhip effect avoidance: Since each demand signal is related with each order, the detuning must be done component-wise with a low-pass filter. Each one of the low-pass filters appearing in Eq. (22) is defined by Eq. (23). Thus, the filter matrix is given by:

$$\mathbf{F}^d(z) = \begin{pmatrix} f_{1,1}^d(z) & 0 & \cdots & 0 \\ f_{2,1}^d(z) & f_{2,2}^d(z) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ f_{N,1}^d(z) & \cdots & f_{N,N-1}^d(z) & f_{N,N}^d(z) \end{pmatrix} \tag{22}$$

Each one of the low-pass filters appearing in Eq. (22) is defined by:

$$f^d(z) = \frac{(\alpha_1 z - \alpha_2)(1-\lambda^d)z}{(z-\lambda^d)^2} \tag{23}$$

Therefore, the final controller can be obtained using the Schur product (or component-wise product) [9], in the form:

$$\mathbf{Q}^d(z) = \tilde{\mathbf{Q}}^d(z) \bullet \mathbf{F}^d \tag{24}$$

From (12), the bullwhip effect depends on the lead times L_j, included in $Q^d(z)$ and the $Q^d(z)$ parameters (λ_{ij}^d, α_j^1 and α_j^2). Since the λ_j^d parameter modifies the bandwidth, this is selected so as to satisfy the bullwhip effect condition for a determined L_j value while the parameters α_j^1 and α_j^2 are adjusted to guarantee internal stability for this λ_j^d value.

In this way, [4] has suggested to tune the controller of the system such that the frequency response of the ratio of orders Γ_j to satisfy the following two constraints in closed-loop:

1) **Bandwidth**: the frequency at which the magnitude ratio is reduced to below 0.7. A wide bandwidth indicates a faster response but poorer bullwhip mitigation.
   Note that we are dealing with a discrete-times systems. Therefore, the highest frequency is at $\omega = \pi/T_m = \pi$ since $T_m = 1$. Thus, we can define a term Γ_j^π as the magnitude ratio given by Eq. (13) at $\omega = \pi$ i.e $\Gamma_j^\pi = \Gamma_j(\omega = \pi)$.
   Since a higher Γ_j^π implies a wider bandwidth and a faster response, it results in more severe bullwhip. Therefore, Suitable setting of Γ_j^π ranges from 0.7 to 1.
2) **Resonance peak** (σ_j): the highest value of the amplitude ratio. A higher resonance peak indicates a faster response but the closed-loop response may be more oscillatory. Suitable setting of σ ranges from 1.5 to 2.0.

According to these criterias, we chose a controller setting with Γ_j^π in 1; and σ_j in 1.8.

Fig. 3 presents the tuning of the λ_{ij} for the complete supply chain according to this tuning criterion. Notice that each component of Fig. 3 represents the magnitude ratio of an order $o_{j,j+1}(z)$, $\forall j = 1,2,...,N$ respect each demand signal $d_j(z)$, $\forall j = 1,2,...,N$. In this case it is also considered the actual lead times known i.e $\hat{L}_1 = L_1 = 3$, $\hat{L}_2 = L_2 = 3$, $\hat{L}_3 = L_3 = 3$.

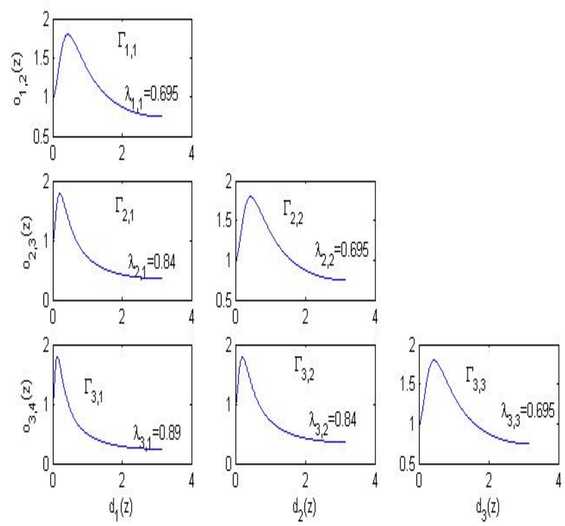

Fig. 3. Tuning of λ_{ij}^d

For a branched multi-echelon (rather than linear multi-echelon)production and distribution network, the model matrices Eqs. (5-9), the $Q^t(z)$ and $Q^d(z)$ controllers become complete matrices. Hence, the design procedure can be directly extended.

IV. Simulation results

Since the system decouples the inventory tracking from the demand rejection and bullwhip effect avoidance, the simulations are performed in two different subsections. Subsection IV-A is oriented to evaluate the inventory target tracking and subsection IV-B evaluates the rejection to demand. In this last situation, the inventory target si set to zero.

A. Evaluation of the inventory target tracking

The inventory target tracking of the supply chain under a centralized control is shown in Fig. 4 as well as the orders in response to these changes are shown in Fig. 5. In this simulation, a step change of 200 unities is considered at $t = 20$ and onwards. Notice that, by modifying λ^t from 0 to 1, a tradeoff between a faster response of inventory to a step change in the inventory target and a smoothing of orders can be done. Fig. 4 shows that, the centralized control approach does not exhibit overshoot for any λ^t values when it is subjected to inventory target changes in the entire supply chain. Orders changes in response to inventory target changes can be less abrupt, consequently decreasing inventory holding costs, smoothing factory operations, and improving profitability.

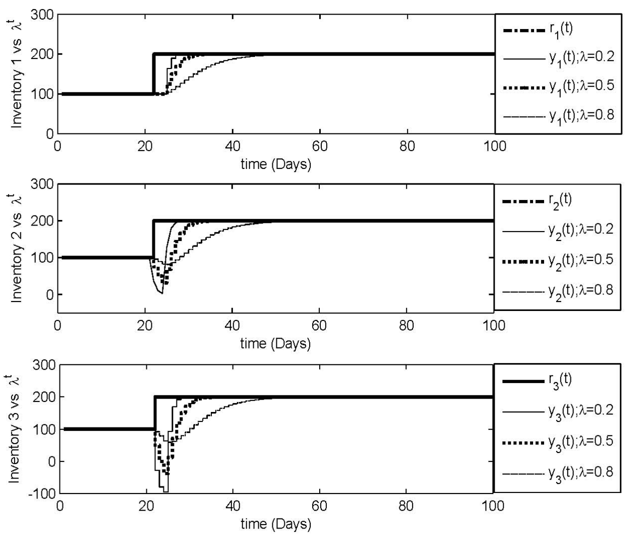

Fig. 4. Inventory responses of echelon 1 2 and 3 to step changes in the inventory targets.

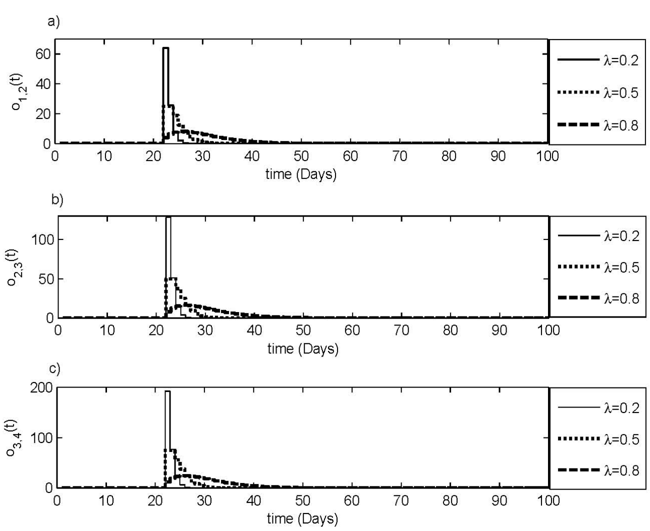

Fig. 5. Orders of the echelons 1 2 and 3 in response to changes in the inventory target.

B. Evaluation of Disturbance (demand) rejection and bullwhip effect avoidance

In this subsection the $Q^d(z)$ behavior to demand signal is evaluated. In this simulation, an stochastic demand (20,4) is considered at $t = 20$ and onwards. Fig. 6 shows fast recuperation of inventory without overshoot in the first echelon. The second and third echelons present sluggish recuperation of inventory, that is due to the detuning for bullwhip avoidance. Fig. 7 shows that the centralized control design allows as a perfect control over the bullwhip effect avoidance. Therefore, a tradeoff between a faster response to the demand and bullwhip effect avoidance is available in the entire supply chain by the selection of the respective λ_{ij}^d values.

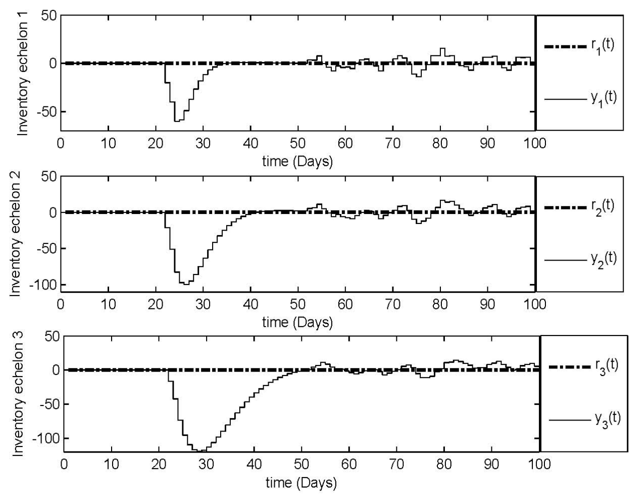

Fig. 6. Inventory behavior to changes in the demand assuming no changes in the inventory target

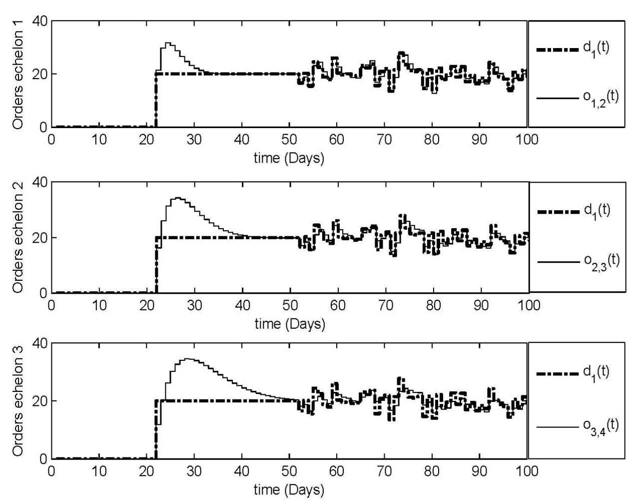

Fig. 7. Orders to changes in the demand assuming no changes in the inventory target

V. CONCLUSIONS

The IMC scheme allows us to tackle the two problems of inventory target tracking and demand rejection with two controllers separately. That is an advantage to others replenishment inventory policies based in control theory proposed in the literature [3], [10], [1]. Two trade-off must be taken into account in the design of these controllers for inventory management in the supply chain, (Inventory target tracking vs aggressive orders avoidance for the $\mathbf{Q}^t(z)$ design) and (Demand rejection vs bullwhip effect avoidance for the $\mathbf{Q}^d(z)$ design). These trade-off can be performed easily for the entire supply chain by the filters included in the $\mathbf{Q}^t(z)$ and $\mathbf{Q}^d(z)$ controllers.

VI. ACKNOWLEDGMENTS

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