# State Controllabillity Analysis and Re-design for a Wastewater Treatment Plant

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*Abstract*— **In this paper a analysis of state controllability for the dynamical behavior of a wastewater treatment plant is presented. This plant is a large non-linear system subject to large disturbances. Based on a state controllability analysis a redesign for the plant is proposed. State controllability analysis is done by using set theory. This allows checking the controllability limits for the system. Indeed it allows including disturbance limits and constraints on control inputs. In this way, nominal design parameters determine the system controllability while other parameters would improve this system property. Finally, improvements in controllability issues for the system are presented.**

## I. Introduction

Water pollution is one of the most serious environmental problems faced today by man [9, 19]. A biological wastewater treatment is used to keep the ecological balance of the environment and now it is frequently used for industrial and domestic wastewaters. The increasing concern about environmental and health risks demands a more rigorous control of wastewaters, promoting the development of new control strategies for plants operating such that neither operating costs nor pollutant discharge values are out of constraints.

The usual practice of considering process control issues as a sequential second step after process design ignores the interaction of design and control. During the last two decades, the importance of a simultaneous approach, considering operability together with the economic issues, has been recognized ([16] and the references cited therein). Due to this several equipments and plants are designed without consider control and dynamical issues using only static optimizations methods to design. One form to include dynamical issues in the design stage is by mean of the concept of controllability.
It is a relevant property in the analysis of the dynamical systems. This property indicates if it is possible that the system can be steered from a given initial state to a given final state. Note that, the Controllability links control issues with the system states. In Control Theory, the controllability property plays a crucial role in many control problems, such as stabilization of unstable systems by feedback, or optimal control.

In 1960 R. E. Kalman [12] defined the concept of controllability for a linear continuous system invariant time. In subsequent work, this concept has been generalized for nonlinear dynamic systems [8], [10], [17], [18]. Nonlinear controllability is a complex property. In fact, it becomes necessary to define other terms like local and global accessibility and local and global controllability. On the other hand, for linear systems these additional concepts become equivalent.

At 70's Bertsekas and Rhodes [3] used the concept of set to define the Reachabibilty. This view has been widely used in Control Theory. Then with the work of Blanchini, a survey about Set Invariance Control, the set theory is again considered especially in MPC [5], [14], [15]. Only recently the first book that relates the Control Theory and the Set Theory [4] appears. In this approach the authors define several sets with a specific property, where perhaps the most important set is the invariant set.

One advantage of the set theory is that it allows considering constrains on control actions and system states. In the computation of some sets, as example, reachable set, controllable set it is possible to include explicitly the time and constrains on control actions and system states. However, the results are numerical and therefore particular.

By computing these sets it is possible to relate them with the system controllability and consider it as an indicative for system controllability. In this way controllability issues can be taken into account since design or re-design stages.

In this paper second section describes the method to compute reachable sets. Third section describes the study case used, a biological wastewater treatment plant and the results obtained by computing reachable sets at several times. Finally, conclusions about design considerations and

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future works are discussed. Redesign suggested parameter was selected as the aerobic section for the biological reactor. An increase of 10% on the volume of the aerated section was proposed as re-design hypothesis.

## II. Method Description

Since 70's when Bertsekas and Rodes [3] stated that controllability problem can be formulated as determining sets in the state space of a system that can be achieved with a sequence of constrained control actions, several works have dealt with computation of these sets [2], [13], [6]. Intuitively, if a set of differential equation describes the temporal evolution of each state in a space state dynamic model, every solution of these equations with each one of all possible inputs trajectories ($u(t) \epsilon U$) give us all the possible states that can be reached in a defined time. In the same way by solving a system by a backward approach (if it is possible), one computes all the possible states from it is possible to reach a defined state or set of states in a defined time (controllable set).

A set is a collection of objects; each one is called a member or element. Sets theory is the branch of mathematics that studies the relation among different sets and elements. This section aims to lay the fundamentals of set theory and its application in state controllability. It is necessary to clarify some concepts and to establish a toolbox in order to operate successfully with sets and its combinations.

**Definition 1. Reachable Set.** Given the set Ω, the reachable set $R_T(\Omega)$ from Ω at time T > 0 is the set of all the state space vectors $\boldsymbol{x}$, for which exists $\boldsymbol{x_0} \in \Omega$ and $\boldsymbol{u}(\cdot) \in$ U such that $\boldsymbol{x}(T) = x$.

The reachable set is integrated by all those state space vectors that can be reached from the evolution of the dynamic system under the effect of some admissible control action.

**Definition 2. Controllable Set.** Given the set Ω, the controllable set $C_T(\Omega)$ to Ω at time T > 0 is the set of all the state space vectors $\boldsymbol{x}$, for which exists $\boldsymbol{u}(\cdot) \in$ U such that if $\boldsymbol{x}(0) = \boldsymbol{x}$ then $\boldsymbol{x}(T) \in \Omega$.

The controllable set is integrated by all those state space vectors that are influenced by some admissible control action, and can arrive at certain time from the evolution of the system to be inside the set Ω.

The methodology presented here to compute reachable or controllable sets is an extension of the methodology presented by [7]. In this reference a nonlinear discrete systems as is shown in (1) is considered. Where $k \in \mathbb{Z}$, $\boldsymbol{u}_k \in U \subset \mathbb{R}^m$ and $\boldsymbol{x}_k \in X \subset \mathbb{R}^n$ are the discrete time, control input and state vector respectively.

$$\boldsymbol{x}_{k+1} = \boldsymbol{f}(\boldsymbol{x}_k, \boldsymbol{u}_k) \quad (1)$$

By considering dynamic discrete model in (1), it becomes intuitive that if one can apply each control input in a discrete step starting from a defined $\boldsymbol{x}_0$, one could find all the states that the system achieves from $\boldsymbol{x}_0$ in one step (Definition 1). Now, $U$ is a continuous set of $\mathbb{R}^m$ so it is not possible to apply each one of the control inputs. Then, it becomes necessary to obtain a countable representative set for $U$. At this point, Randomized Algorithms (RA), especially Monte Carlo type, arise as an effective tool to face this problem. The continuous set $U$ can be uniformly sampled taken a defined number of independent samples $\boldsymbol{n}$. Then one can solve (1) as many times as samples $\boldsymbol{n}$ taking a defined $\boldsymbol{x}_0$. Then one obtain $\boldsymbol{n}$ states $\boldsymbol{x}_{1-i}$ for $i = 1, \dots, n$. These set is an approximation for the reachable set at one step or at time $k$ if the initial time was stated to 0. In this way it is possible to think in an extension of this methodology for $z$ steps computing recursively one-by-one step reachable sets. Definition 1 and Definition 2 establish that it is possible to compute such sets for a set $\boldsymbol{\Omega}$ more than just for a defined state $\boldsymbol{x}_0$. For this, in [7] is established that for these cases the set $X$ must be uniformly sampled too.

Now, suggested expansion consists in the recursive character to compute reachable and controllable sets for $z$ steps as mentioned above. It becomes clear that major of nonlinear continuous time dynamic models as (2), where $t \in \mathbb{R}$, $\boldsymbol{u}(t) \in U \subset \mathbb{R}^m$ and $\boldsymbol{x}(t) \in X \subset \mathbb{R}^n$, must be solved by using a numerical routine of integration. These routines "discretize" the model in order to approximate the derivative value. Then it is possible to state this "discretization" or "control switching" time by considering real operation conditions (such as controller action time, or system time constants). A randomized sequence for $\boldsymbol{u}(t)$ is generated by uniformly to sample the set $U$ at each step, generating $\boldsymbol{n}$ piece-wice randomized functions as Fig. 1 shows. This figure is not associated with any defined example, it just attempts to clarify the construction of signals $\boldsymbol{u}(.)$. Both signals in Fig. 1 (solid an dotted lines) are for the same $u_i$, in this case $\boldsymbol{n}$ is equal to two, but if $\boldsymbol{n} \to \infty$, $U$ is totally covered. Finally, each randomized sequence is used in the system's solution with any numerical integration routine.

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t)) \quad (2)$$

Main drawback with these randomized methods is the probability of error in the solution. Although the solution will be correct it is necessary to determine the relation among the estimation error, the risk of failure and the sample size. In [7] it was proved the relation among the three aspects mentioned above for discrete systems as (1) with sets computed at one step.

The extension proposed here implies that $u(t)$ is defined as a piecewise function and not as a constant function. Then $u(t)$ can take constrained values, and remain constant during a time interval (named above as “discretization” or “control switching” time). The original method based on Monte-Carlo methods attempts to cover uniformly all the possible static values of the subspace $U$ containing all the possible control actions vectors. This extension attempts to cover all the possible functions $u(t)$. With a number of samples large enough the subspace $U$ is fulfilled at any time.

Finally it must be mentioned the procedure to compute the controllable set. In [18] the time reversal of a continuous time dynamic system as (2) is defined as follows.

**Definition 3. Time reversal system.** The system with right side $-\boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t))$ is called the reversal time system of (2), if $\boldsymbol{f}$ is k-times continually differentiable and $U$ is an open subset of $\mathbb{R}^m$.

Then, by Definition 3, if the system (2) has a defined reversal time system, controllable set is obtained by computing the reachable set for the respective time reversal system.

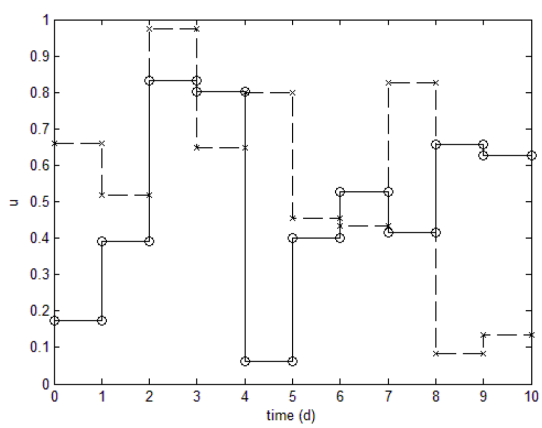


Fig. 1. Examples for $u(t)$ trajectories. Dotted and continuous lines correspond each one to one $u_i(t)$ signal.

## III. Applications to Wastewater Plant

### A. Plant Description

Waste Water Treatment Plant (WWTP) used in this paper is the presented in [1] as BSM2 benchmark. It has seven pieces of equipment called: biological reactor, secondary clarifier, primary clarifier, thickener, anaerobic digester, dewatering, and storage tank.

Primary clarifier is fed with influent wastewater and some internal recycles. But if influent is bigger than WWTP capacity some flow quantity is bypassed to secondary clarifier. This piece of equipment attempts for a first clarification. Biological reactor is divided in five compartments, only for modeling purposes, in which first and second ones are anaerobic and others are consider aerobic. In secondary clarifier sludge sedimentation is done. For modeling purposes it is divided into 10 perfect mixed transversal sections. Thickener and dewatering improve consistency of sludge from secondary clarifier and anaerobic digester respectively. Both are includes as algebraic maps in the global WWTP’s model. Finally storage tank is taking into account for accumulation effects. Fig. 2 clarifies the plant configuration (inputs, outputs and mass recycles).

Several simulations based on the method described in section 2 were carried out using the BSM2 mathematical model for plant described above. Previous simulations were achieved in order to define the steady-state conditions, see Table I, and to remove start-up conditions at the beginning of simulations. A control action switching time was set as one day based on the nominal hydraulic retention time for the plant ($\tau_{\text{plant}}$ = 22 hours). This value gives appropriate controllability values. It is not necessary to increase this frequency due the large plant response time. Obviously, in real plants the control actions is taken at short times, for example 4 hours. Fig. 1 exemplify how at each “control switching time” interval, a random value for each $u_i^{min} \leq u_i \leq u_i^{max}$ is fixed during all time interval. In this paper only one control action is considered for illustrative purposes. The $K_La$ of the aerated zone was the control action selected for analysis. It is important to highlight that more detailed analysis can be done by considering other possible control actions. Although in practice $K_La$ is not easily manipulated due to bubble coalescence, in this work it will be assumed that a direct relationship between air flow and $K_La$ there exists. It was assumed to vary between 0 and 300 days$^{-1}$, based on its nominal value of 200 days$^{-1}$.

### B. Simulation Results

In this section some reachable sets for WWTP are presented. Just reachable sets are presented in order to highlight the plant “maneuverability”. Simulation results show places where the plant can be steered by available control actions from a defined nominal operational point in a defined time. This simulation time was set in 180 days. As a first approach for design just reachable sets were computed and presented here. But, considerations since design stage about disturbance rejection can be taking into account by computing controllable sets. As the definition for controllable set stated, these sets provide information about the system ability to reject disturbances (or to return the system to its nominal operation point after a disturbance). Simulation time is considered a large enough time to appreciate several dynamic behaviors due to the retention time suggested in [1] for this kind of plants. Redesign suggested parameter was selected as the aerobic section for the biological reactor. An increase of 10% on the volume of the aerated section was proposed as re-design hypothesis. Results are focused on comparative graphics that enable us to conclude about redesign hypothesis.

Simulations to compute reachable sets were achieved with nominal and modified parameter. A number of $u(t)$ trajectories was fixed in 10000. Simulations were run in

Matlab® and ODE23tb routine was used due the model's stiffness character. Time required for these computations is relatively large. Two quad-core processors at 3.0 GHz were used and approximately 5 hours were required to obtain final reachable sets for each either $V_{nom}$ or $1.3V_{nom}$. Under considering that this is a off-line redesign task, this long simulation time could be considered as viable.

TABLE I
SOME STATE VARIABLES, INPUTS AND MODEL PARAMETERS

| Symbol | Quantity | Nominal Value |
|---|---|---|
| | EFLUENT VARIABLES | |
| $S_{NO}$ | Soluble biodegradable organic nitrogen | 9.1948 (g-N.m$^{-3}$) |
| $S_{NH}$ | $NH_4^+ + NH_3$ nitrogen | 0.1585 (g-N.m$^{-3}$) |
| $S_O$ | Dissolved oxygen | 1.3748 (g-COD.m$^{-3}$) |
| TSS | Total Suspended Solids | 14.3 (g.m$^{-3}$) |
| | INFLUENT VARIABLES | |
| $S_{NO}$ | Soluble biodegradable organic nitrogen | 0 (g-N.m$^{-3}$) |
| $S_{NH}$ | $NH_4^+ + NH_3$ nitrogen | 23.8594 (g-N.m$^{-3}$) |
| $S_O$ | Dissolved oxygen | 0 (g-COD.m$^{-3}$) |
| TSS | Total Suspended Solids | 380.3 (g.m$^{-3}$) |
| Q | Influent caudal | 20648.3 (m$^3$.day$^{-1}$) |
| | PARAMETERS | |
| V | Aerated section Reactor volume | 9000 (m$^3$) |
| $K_La$ | Oxygen transfer coefficients | 120 (day$^{-1}$) |

These are some values for state variables, inputs and parameters. Complete model include 208 state variables, more than 20 inputs, and more than 100 parameters. Only variables included in the analysis are mentioned here. Further information can be founded at [1].

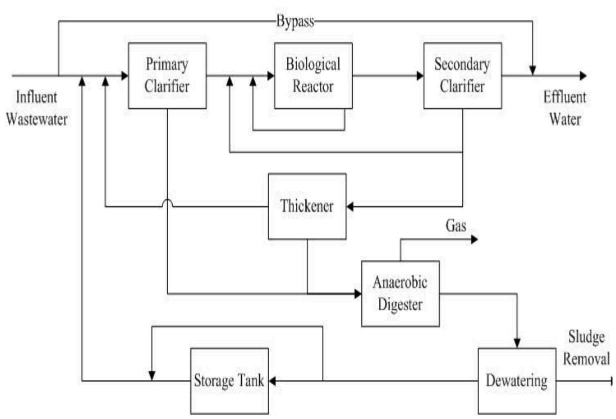


Fig. 2. Block diagram of the plant.

Fig. 3 shows that an increment in aerated section deteriorates removal of NO (nitrate and nitrite nitrogen). Reachable set computed with nominal value in aerated section is located totally at the left of the reachable set computed with modified parameters. Indeed size of reachable set with modified parameters is greater than reachable with nominal parameters. This means that with a larger aerated section and a given oxygen concentration, nitrogen will have more time to react oxidation state, and then NO concentration has a bigger variation range.

Fig. 4 just confirms previous analysis from equations. It is to say that for a bigger NO concentration it is improved the NH ($NH_4^+$ and $NH_3$ nitrogen) removal. This is due to inhibitions in biological mechanism that involve heterotrophic organisms. If this analysis is combined with Fig. 5 one can say that decrease in NH is due to autotrophic and heterotrophic biomass generation (represented by Total Suspended Solids TSS).

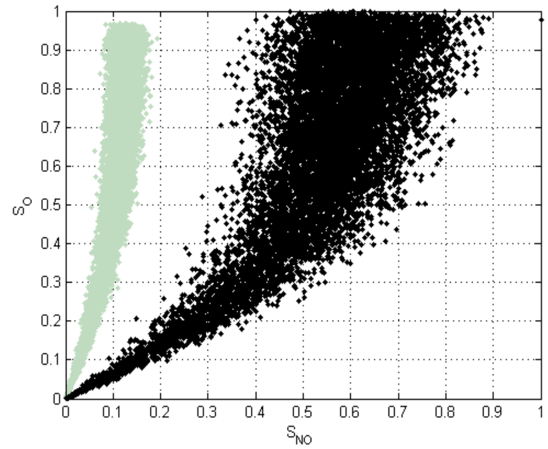


Fig. 3. Effluent Normalized concentrations of dissolved NO and Oxigen ($V_{nom}$= Gray, 1.3*$V_{nom}$= Black).

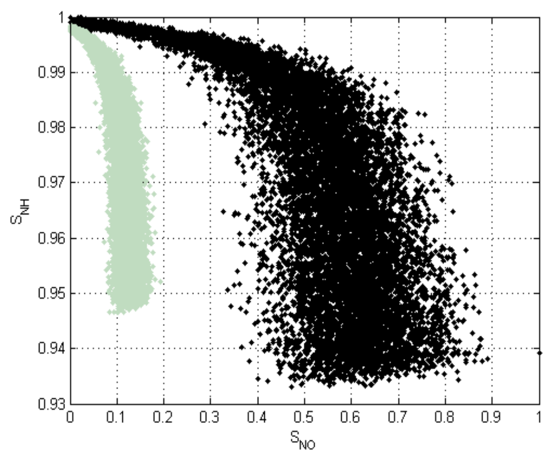


Fig. 4. Effluent Normalized concentrations of dissolved NO and NH ($V_{nom}$= Gray, 1.3*$V_{nom}$= Black)

Fig. 6 shows that it is not possible to achieve simultaneously small values in TSS and NH with the nominal volume for aerated zone. It will be possible to obtain lower TSS and NH concentrations if aerated zone is increased. TSS presents an interesting point at $S_{NH}$ = 0.965. This suggests the existence of a NH concentration value that deteriorates critically the suspended solids removal.

Dead zones are evidenced in Figures 3 to 5. These zones are product of sets projections on two dimensional planes. It is possible that sets seem to be completely separated but different projection shows the opposite (see Fig. 3 and Fig. 6), this fact is analyzed in detail in next section.

Finally, it must be clarified to reader than reachable sets presented here were computed for time t = 90 days. Cases about behavior in intermediate times must be analyzed individually. The nonlinear model plant states that linear interpolation is not going to be a good choice to analyze intermediate times. But numerical procedure computes sets at several intermediate times, and it can be used to analyze that behavior and observe sets evolution.

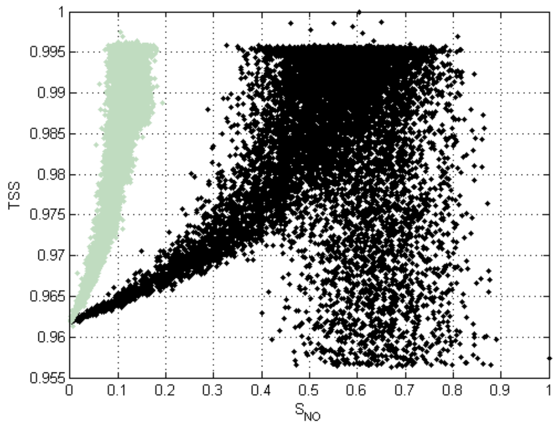

Fig. 5. Effluent Normalized concentration of dissolved NO and total suspended solids ($V_{nom}$= Gray, 1.3*$V_{nom}$= Black)

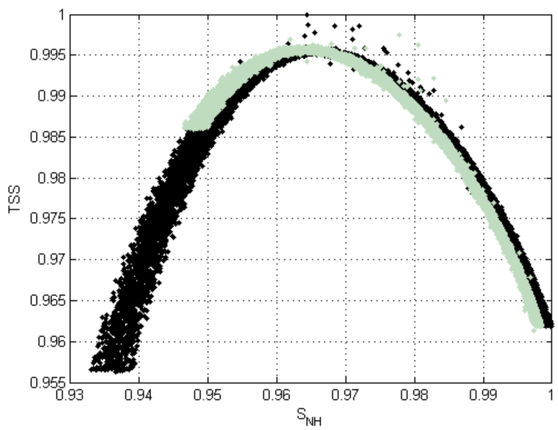

Fig. 6. Effluent Normalized concentration of dissolved NH and total suspended solids ($V_{nom}$= Gray, 1.3*$V_{nom}$= Black)

## IV. Conclusions and Future Works

It becomes clear that modifying the volume of aerated section enlarge the variation range of relevant effluent variables, becoming the system more flexible and controllable. But these changes are small compared with the 10% variation in volume.

Modifying aerated section an increment in system's controllability is achieved due to controllable sets are larger than unmodified controllable sets. But it is relevant to think in the natural evolution of the system. It is the sets are displaced significantly. This implies that stationary conditions would be modified; indeed it is possible that admissible control inputs do not be enough to achieve the minimal requirements, as in this case. In figures it was evidenced that some dead zones when nominal and modified sets are included in the same figure. These zones make evident that by manipulating aeration it is not enough to achieve the same final results. A solution could be to increase simultaneously the volume and the values of control action.

It is important to remember that in these work just one parameter was analyzed. But this model presents a large list of design parameter that would be candidates to modifications. In this sense a most formal strategy for Simultaneous Process and Control Design must be applied. For example, in [20] several steps are conducted before controllability analysis. The novelty proposed in the present work is to use set theory for controllability proof and quantification.

Analysis presented here were purely qualitative. Size and shape for sets must be quantified in order to assign a metric for state controllability; this work is currently being conducted based on vector approach.

Finally, as a key point it must be noted that set theory gives a complementary view of controllability considering Lie algebra approach as the basement. In this sense, any set measurement will be useful for determining increasing or decreasing on process controllability. This fact will be profited to improve process design.

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