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Practical dwell time approach for stability analysis of hybrid systems

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Abstract— **The problem of the practical p-th mean stability of a class of stochastic nonlinear and time dependent linear hybrid systems with practically p-mean stable and unstable structures is considered. Sufficient conditions for the practical p-th mean stability under the stabilizing switching signal using Lyapunov techniques, the practical average dwell time and the practical dwell time approach, are derived. Three cases including the existence of the single Lyapunov function, the multiple Lyapunov function and the single practical Lyapunov–like function, for the practical p-th mean stability are discussed. The obtained results are illustrated by an example.**

I. INTRODUCTION

The stability of stochastic dynamic systems is one of the basic fields in the control theory. Several stability definitions and groups of methods were developed since last 60 years. Mainly the authors were used the Lyapunov definitions and methods modified for stochastic dynamic systems. Since the Lyapunov definitions are sometimes not useful for stability analysis of real physical systems La Salle and Lefschetz [1] proposed for deterministic systems so called *practical stability*. A modification of this idea, namely *finite time stability*, was first proposed for stochastic systems by Kushner [2]. Both ideas were developed by many authors since last 40 years for both deterministic and stochastic systems. The obtained results were successively reported in survey papers and monographs, for instance in [3], [4].

In the case of stochastic dynamic systems usually the practical stability in probability and the practical p-th mean stability were considered, see for instance, [10], [8], [9], [7].

Some authors in their studies [10], [8], [9], used the methodology of comparison principle, i.e. they considered parallel with the discussed nonlinear stochastic differential equation an auxiliary nonlinear scalar deterministic differential equation. They formulated sufficient conditions of the practical stability using parameters of the auxiliary system in such a way that from the practical stability of the null solution of the auxiliary system it follows the corresponding practical stability of the null solution of considered stochastic system.

Another approach was proposed by Michel and Hou [7], who considered linear stochastic differential equation. The authors found the corresponding differential equations for first and second moments and next applied the standard criteria of the practical stability for deterministic linear differential equations.

Recently the analysis of the practical stability was also developed for switched or hybrid systems. In the case of deterministic systems it was done, for instance by Xu and Zhai [5], Xu, He and G. Zhai [6], while for stochastic systems by Yin, Zhang and Zhu [11], Sathananthan and Keel [12].

In this paper, the problem of the practical p-th mean stability of a class of stochastic nonlinear and time dependent linear hybrid systems with stable and unstable structures is considered. Sufficient conditions for the p-th mean practical stability under the stabilizing switching signal, using Lyapunov techniques, the practical average dwell time and the practical dwell time approach are derived. Three cases including the existence of the single Lyapunov, the multiple Lyapunov and the single practical Lyapunov–like functions for the practical p-th mean stability are discussed.

II. MATHEMATICAL PRELIMINARIES

Throughout this paper we use the following notation. Let $|\cdot|$ be the Euclidean norm. By $\lambda(\mathbf{A})$ we denote the eigenvalue of the matrix $\mathbf{A}$, $\lambda_{\min}(\mathbf{A})$ and $\lambda_{\max}(\mathbf{A})$ denotes the smallest and the biggest eigenvalue of the matrix $\mathbf{A}$, respectively. We denote by $\mathbf{A}^T$ the transposition of matrix $\mathbf{A}$. We mark $\mathbb{T} = [t_0, \infty),\ t_0 \geq 0$. Let $\Xi = (\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying usual conditions. Let $\mathbf{w}(\mathbf{t}) = [w_1(t), \ldots, w_m(t)]^T,\ t \geq 0$, be the m - dimensional standard Wiener process defined on the probability space Ξ. Let $\sigma(\mathbf{x}(t)) : \mathbb{R}^n \to \mathbb{S}$ be the stochastic state-dependent switching rule, $\mathbb{S} = \{1, \ldots, N\}$ is the set of states. We denote switching times as $t_1, t_2, \ldots$. We also assume that there is a finite number of switches on every finite time interval. Processes $w_k(t)$ and $\sigma(\mathbf{x}(t))$ are both $\{\mathcal{F}_t\}_{t\geq 0}$ adapted. We say that a proper function $V : \mathbb{R}^n \times \mathbb{S} \to \mathbb{R}$, twice differentiable with respect to $\mathbf{x} \in \mathbb{R}^n$, is a Lyapunov function if $V(0, l) = 0$ and $V(\mathbf{x}, l) > 0\ \forall\ \mathbf{x} \neq 0$ for $l \in \mathbb{S}$. We also use a Lyapunov function $V : \mathbb{R}^n \to \mathbb{R}$, if $V(\mathbf{x}, l) = V(\mathbf{x})\ \forall l \in \mathbb{S}$. For simplicity of notation, instead of $\sigma(\mathbf{x}(t))$ we write $\sigma(\mathbf{x})$.

We consider the nonlinear hybrid system with multiplicative excitations described by the vector Itô differential equation

$$d\mathbf{x}(t) = \mathbf{f}(\mathbf{x}(t), t, \sigma(\mathbf{x}))dt + \textstyle\sum_{k=1}^{m} \mathbf{G}_k(\mathbf{x}(t), \sigma(\mathbf{x}))dw_k(t),$$

$$\mathbf{x}(t_0) = \mathbf{x}_0, \quad \sigma(\mathbf{x}_0) = \sigma_0, \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$, $t \in \mathbb{T}$, $\mathbf{f} : \mathbb{R}^n \times \mathbb{T} \times \mathbb{S} \to \mathbb{R}^n$, $\mathbf{G}_k : \mathbb{R}^n \times \mathbb{S} \to \mathbb{R}^n$, $k = 1, \ldots, m$, $\mathbf{x}_0 \in \mathbb{R}^n$ and $\sigma_0 \in \mathbb{S}$ are initial values.

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We assume that the solution $\mathbf{x}(t)$ exists and is almost surely continuous.

As a special case we consider the linear hybrid system with multiplicative excitations described by the vector Itô stochastic differential equation

$$d\mathbf{x}(t) = \mathbf{A}(t, \sigma(\mathbf{x}))\mathbf{x}(t)dt + \sum_{k=1}^{m} \mathbf{G}_k(\sigma(\mathbf{x}))\mathbf{x}(t)dw_k(t), \quad \mathbf{x}(t_0) = \mathbf{x}_0, \quad \sigma(\mathbf{x}_0) = \sigma_0, \quad (2)$$

where $\mathbf{x} \in \mathbb{R}^n$, $t \in \mathbb{T}$, $\mathbf{A} : \mathbb{T} \times \mathbb{S} \to \mathbb{R}^{n\times n}$, $\mathbf{G}_k : \mathbb{S} \to \mathbb{R}^{n\times n}$, $k = 1, \ldots, m$, $\mathbf{x}_0 \in \mathbb{R}^n$ and $\sigma_0 \in \mathbb{S}$ are initial values.

For any twice continuously differentiable function $\phi(\cdot, l)$ the l-th process of the hybrid system (1) has a generator $\mathcal{L}_l^{(1)}$ (Itô operator for the l-th subsystem of the system (1)) given by

$$\mathcal{L}_l^{(1)}\phi(\mathbf{x}, l) = \mathbf{f}^T(\mathbf{x}(t), t, l)\nabla\phi(\mathbf{x}, l) + \frac{1}{2}\sum_{k=1}^{m} tr(\mathbf{G}_k(l, \mathbf{x})\mathbf{G}_k(l, \mathbf{x})^T\nabla^2\phi(\mathbf{x}, l)), \quad l \in \mathbb{S}, \quad (3)$$

where ∇ and ∇^2 denote the gradient and Hessian, respectively. For simplicity we use the notation $\mathbf{x}(t, \sigma(\mathbf{x}(t))) = \mathbf{x}(t)$.

In particular case for the linear hybrid system (2) the generator $\mathcal{L}_l^{(2)}$ (Itô operator for the l-th subsystem of the system (2)) is given by

$$\mathcal{L}_l^{(2)}\phi(\mathbf{x}, l) = \mathbf{x}^T\mathbf{A}(t, l)^T\nabla\phi(\mathbf{x}, l) + \frac{1}{2}\sum_{k=1}^{m} tr(\mathbf{G}_k(l)\mathbf{x}\mathbf{x}^T\mathbf{G}_k(l)^T\nabla^2\phi(\mathbf{x}, l)), \quad l \in \mathbb{S}. \quad (4)$$

We use the following definitions of the p-th mean exponential and the practical stability.

Definition 1: The trivial solution $\mathbf{x} \equiv \mathbf{0}$ of the hybrid system (1) is said to be p-th mean exponentially stable if there exists a pair of positive scalars α, c such that $\forall(\mathbf{x}_0, t_0) \in \mathbb{R}^n \times \mathbb{R}_+$

$$\mathbb{E}[|\mathbf{x}(t, \mathbf{x}_0, t_0)|^p] \leq c\mathbb{E}[|\mathbf{x}_0|^p]e^{-\kappa(t-t_0)}, \quad t \geq t_0. \quad (5)$$

In the case of $p = 2$ it is called mean-square exponentially stable.

Definition 2: [7] The stochastic process $\mathbf{x}(t, \sigma(\mathbf{x}))$ satisfying equation (1) is said to be practically $p-th$ mean stable ($p > 0$) with respect to subsets $(\Lambda_1, \Lambda_2, t_0, T)$, $\Lambda_1 \subseteq \Lambda_2$ if there exists a pair of positive real parameters (δ, γ), $\delta < \gamma$ such that for any initial data $(\mathbf{x}_0, \sigma_0) \in \mathbf{R}^n \times S$ satisfying $\mathbf{x}_0 \in \Lambda_1 = \{\mathbf{x} : |\mathbf{x}|^p < \delta^p\}$

$$\sup_{t\in\mathbb{T}} E\left[|\mathbf{x}(t, \sigma(\mathbf{x}))|^p\right] \in \Lambda_2 = \{\mathbf{x} : |\mathbf{x}|^p < \gamma^p\}. \quad (6)$$

The process that is not practically p-th mean stable is said to be practically p-th mean unstable.

Definition 3: The Lyapunov function $V(\mathbf{x})$ satisfying

$$\mathcal{L}_l^{(2)}V(\mathbf{x}) \leq -\kappa V(\mathbf{x}), \quad \forall\ l \in \mathbb{S} \quad (7)$$

for a constant $\kappa \geq 0$ is a common Lyapunov function for the hybrid system (2).

Definition 4: The Lyapunov function $V(\mathbf{x})$ satisfying

$$\mathcal{L}_\sigma^{(2)}V(\mathbf{x}) \leq -\kappa V(\mathbf{x}) \quad (8)$$

for a constant $\kappa \geq 0$ and some switching rule σ is a single Lyapunov function for the hybrid system (2).

Definition 5: The Lyapunov function $V(\mathbf{x}, \sigma)$ satisfying

$$\mathcal{L}_\sigma^{(2)}V(\mathbf{x}, \sigma) \leq -\kappa V(\mathbf{x}, \sigma) \quad (9)$$

for a constant $\kappa > 0$ and some switching rule σ is a multiple Lyapunov function for the hybrid system (2).

We formulate sufficient conditions of the practical p–mean stability for the stochastic nonlinear systems using for the $l-th$ subsystem the Lyapunov function in the following form

$$V(\mathbf{x}, l) = \left[\mathbf{x}^T\mathbf{H}(l)\mathbf{x}\right]^{\frac{p}{2}}, \qquad l \in \mathbb{S}, \quad (10)$$

where $\mathbf{H}(l)$ are real valued symmetric positive-definite $n \times n$ constant matrices. Let us denote

$$\nu_l = \left(\frac{\lambda_{\max}(\mathbf{H}(l))}{\lambda_{\min}(\mathbf{H}(l))}\right)^{\frac{p}{2}} \quad (11)$$

and

$$\bar{\nu} = \left(\frac{\lambda_{MAX}}{\lambda_{MIN}}\right)^{\frac{p}{2}}, \quad (12)$$

$\lambda_{MAX} = \max_{l\in\mathbb{S}}\lambda_{max}(\mathbf{H}(l))$, $\lambda_{MIN} = \min_{l\in\mathbb{S}}\lambda_{min}(\mathbf{H}(l))$. In the case, when $V(\mathbf{x}, l) = V(\mathbf{x}) = \left[\mathbf{x}^T\mathbf{H}\mathbf{x}\right]^{\frac{p}{2}}$ for all $l \in \mathbb{S}$, we denote

$$\nu = \left(\frac{\lambda_{\max}(\mathbf{H})}{\lambda_{\min}(\mathbf{H})}\right)^{\frac{p}{2}}. \quad (13)$$

Theorem 1: [14] $l-th$ subystem of (1) is practically p-mean stable with respect to $(\Lambda_1, \Lambda_2, t_0, T)$, $\Lambda_1 \subseteq \Lambda_2$, if there exists the Lyapunov function defined by (10) and a Lebesgue integrable function $\kappa_l(t)$ on $\mathbb{T}$, satisfying conditions

(i) $\mathcal{L}_l^{(1)}V(\mathbf{x}) \leq \kappa_l(t)V(\mathbf{x}), \quad \forall t \in \mathbb{T}, \quad \mathbf{x} \in \Lambda_2$,

(ii) $\delta^p\nu_l \exp\left\{\int_{t_0}^{t}\kappa_l(s)ds\right\} \leq \gamma^p, \quad \forall t \in \mathbb{T}$, where ν_l is given by (11).

Proof: For $V(\mathbf{x}, l) = (\mathbf{x}^T\mathbf{H}(l)\mathbf{x})^{\frac{p}{2}}$, $p > 0$, we have

$$\lambda_{\min}^{\frac{p}{2}}(\mathbf{H}(l))|\mathbf{x}|^p \leq V(\mathbf{x}) \leq \lambda_{\max}^{\frac{p}{2}}(\mathbf{H}(l))|\mathbf{x}|^p. \quad (14)$$

From assumption (i) it follows

$$\frac{d}{dt}\mathbb{E}[V(\mathbf{x})] \leq \kappa_l(t)\mathbb{E}[V(\mathbf{x})]. \quad (15)$$

Then

$$\mathbb{E}[V(\mathbf{x})] \leq \mathbb{E}[V(\mathbf{x}_0)]\exp\left\{\int_{t_0}^{t}\kappa_l(s)ds\right\}. \quad (16)$$

From (14) and (16) follows that

$$\mathbb{E}\left[|\mathbf{x}(t)|^p\right] \leq \mathbb{E}\left[|\mathbf{x}_0|^p\right]\left(\frac{\lambda_{\max}(\mathbf{H}(l))}{\lambda_{\min}(\mathbf{H}(l))}\right)^{\frac{p}{2}} e^{\int_{t_0}^{t}\kappa_l(s)ds}. \quad (17)$$

Hence from (17) and assumption (ii) it follows, that

$$\mathbb{E}\left[|\mathbf{x}(t)|^p\right] < \gamma^p. \quad (18)$$

It means the solution $\mathbf{x}(t)$ of the nonlinear $l-th$ subsystem (1) is practically p-th mean stable. □

Following the methodology, introduced in [16], for deterministic hybrid systems we assume that the hybrid state space is partioned into regions $\Omega_l, l \in \mathbb{S}$, and $\bigcup_{l\in\mathbb{S}} \Omega_l = \mathbf{R}^n$. In each region $\Omega_l, l \in \mathbb{S}$, a scalar function $V(\mathbf{x}, l)$ in form (10) is used.

Now, we introduce definitions and theorem which will be useful in the study of practical p-mean stability of stochastic linear hybrid systems.

For non-hybrid systems, given as follows

$$d\mathbf{x}(t) = \mathbf{A}(t)\mathbf{x}(t)dt + \sum_{k=1}^{m} \mathbf{G}_k\mathbf{x}(t)dw_k(t), \quad \mathbf{x}(t_0) = \mathbf{x}_0, \tag{19}$$

where $\mathbf{x} \in \mathbb{R}^n$, $t \in \mathbb{T}$, $\mathbf{A}$, $\mathbf{G}_k$ are $n \times n$ matrices, $k = 1, \ldots, m$, $\mathbf{x}_0 \in \mathbb{R}^n$ is an initial value, the following theorem holds.

Theorem 2: Let us assume that there exist a symmetric positive-definite $n \times n$ constant matrix $\mathbf{H}$, a Lebesgue integrable function $\alpha_1(t) \leq 0$ for all $t \in \mathbb{T}$ and positive constants α_2, α_3, such that for $(\mathbf{x}, t) \in \{\mathbf{R}^n \times \mathbb{T}\}$ the following inequalities hold

$$(i)\ \mathbf{x}^T\mathbf{H}\mathbf{A}(t)\mathbf{x} + \frac{1}{2}\sum_{k=1}^{m} \mathbf{x}^T\mathbf{G}_k^T\mathbf{H}\mathbf{G}_k\mathbf{x} \leq \alpha_1(t)\mathbf{x}^T\mathbf{H}\mathbf{x}, \tag{20}$$

$$(ii) \qquad \alpha_2\mathbf{x}^T\mathbf{H}\mathbf{x} \leq \left|\mathbf{x}^T\mathbf{H}\sum_{k=1}^{m}\mathbf{G}_k\mathbf{x}\right| \leq \alpha_3\mathbf{x}^T\mathbf{H}\mathbf{x}. \tag{21}$$

$$(iii) \qquad \delta^p\nu \exp\left\{-\int_{t_0}^{t}\kappa(s)ds\right\} \leq \gamma^p, \quad \forall t \in \mathbb{T} \tag{22}$$

where

$$\kappa(t) = \begin{cases} p\left[|\alpha_1(t)| - \left(\frac{p}{2}-1\right)\alpha_3^2\right] & 2 < p \leq 2 + 2\frac{|\alpha_1(t)|}{(\alpha_3)^2} \\ p|\alpha_1(t)| & 0 < p \leq 2 \end{cases} \tag{23}$$

and ν is defined by (13). Then the solution of the system (19) is p-th mean practically stable for p defined in (23). The proof of this theorem follows from the proof of Theorem 1 and considerations given in [14] (Corollary 4.6, p.131). □

Corollary 1: In particular, when we assume that $\nu\delta^p \leq \gamma^p$ and the Lebesgue integrable function $\alpha_1(t) \leq 0$ is bounded by a constant $\bar{\alpha}_1 \leq 0$ i.e. $\bar{\alpha}_1 < \alpha_1(t) \leq 0$, then $\kappa(t)$ defined by (23) is also a constant

$$\bar{\kappa} = \begin{cases} p\left[|\bar{\alpha_1}| - \left(\frac{p}{2}-1\right)\alpha_3^2\right] \geq 0 & \text{for } 2 < p < 2 + 2\frac{|\bar{\alpha_1}|}{\alpha_3^2} \\ p|\bar{\alpha_1}| \geq 0 & \text{for } 0 < p \leq 2 \end{cases} \tag{24}$$

and the hybrid system (2) is p-th mean practically stable. To stabilize hybrid systems in the case of multiple Lyapunov functions, some authors have introduced a special case of switching rule so called the switching with the average dwell time τ_{ADT} [13], [15].

We modify this approach to the case of practical switchings with the limited average dwell times.

Definition 6: We assume that the intervals between switchings are bounded by a practical interval $t_{prac} > 0$ called practical dwell time, i.e.

$$t_{i+1} - t_i \geq t_{prac}, \quad i = 1, 2, \ldots \tag{25}$$

Let $N_\sigma^{prac}(s,t)$ be the practical switching number of σ on the interval $[s,t)$, then

$$N_\sigma^{prac}(s,t) \leq \frac{t-s}{t_{prac}}. \tag{26}$$

Definition 7: For a given $t_{prac} > 0$ and any given σ, $t > s \geq t_0$, let $N_\sigma^{prac}(s,t)$ be the practical switching number of σ on interval $[s,t)$. For any given N_0, $0 \leq N_0 < N_{max}$ and $\tau_{ADT_{prac}} > 0$ set $\mathcal{S}[\tau_{ADT_{prac}}, N_0] = \{\sigma : N_\sigma^{prac}(s,t) \leq N_0 + \frac{t-s}{\tau_{ADT_{prac}}}, \forall s,t \in \mathbb{T},\ t > s\}$. Then $\tau_{ADT_{prac}}$ and N_0 are called the practical average dwell time and the chatter bound of $\mathcal{S}[\tau_{ADT_{prac}},\ N_0]$, respectively. For a given $\sigma^{prac} \in \mathcal{S}[\tau_{ADT_{prac}},\ N_0]$, the constant $\tau_{ADT_{prac}}^\sigma$ determined by

$$\tau_{ADT_{prac}}^{\sigma^*} = max\left\{t_{prac},\ \tau_{ADT}^\sigma\right\} \tag{27}$$

is called the practical average dwell time of σ^{prac}, where

$$\frac{1}{\tau_{ADT}^\sigma} = \sup_{s\geq t_0}\sup_{t>s}\sup_{N_\sigma^{prac}(s,t)>N_0} \frac{N_\sigma^{prac}(s,t) - N_0}{t-s}. \tag{28}$$

Remark 1: In fact, for any given $0 \leq N_0 < N_{max}$, the average dwell time $\tau_{ADT} > 0$ in Definition 7 is the infimum of the average dwell time τ_{ADT}^σ of all $\sigma \in \mathcal{S}[\tau_{ADT}, N_0]$ and we obtain

$$N_\sigma^{prac}(s,t) \leq N_0 + \frac{t-s}{\tau_{ADT}^\sigma}. \tag{29}$$

III. MAIN RESULTS

Now we discuss three approaches of the determination of sufficient conditions for the p-th mean practical stability, using Lyapunov techniques, the practical average dwell time and the practical dwell time approach. In particular we consider three cases including the existence of the single Lyapunov, the multiple Lyapunov and the single practical Lyapunov–like functions.

A. Single Lyapunov functions

In this case we use a single function V i.e. $V(\mathbf{x}, l) = V(\mathbf{x})$ for all $l \in \mathbb{S}$.

Criterion 1: Let us assume that the following conditions are satisfied

(i) there exist a common symmetric positive-definite constant matrix $\mathbf{H}$, a Lebesgue integrable function $\alpha_1(t) \leq 0$ for all $t \in \mathbb{T}$ and positive constants α_2, α_3, such that

$$\mathbf{x}^T\mathbf{H}\mathbf{A}(t,l)\mathbf{x} + \frac{1}{2}\sum_{k=1}^{m}\mathbf{x}^T\mathbf{G}_k^T(l)\mathbf{H}\mathbf{G}_k(l)\mathbf{x} \leq \alpha_1(t)\mathbf{x}^T\mathbf{H}\mathbf{x}, \tag{30}$$

$$\alpha_2\mathbf{x}^T\mathbf{H}\mathbf{x} \leq |\mathbf{x}^T\mathbf{H}\sum_{k=1}^{m}\mathbf{G}_k(l)\mathbf{x}| \leq \alpha_3\mathbf{x}^T\mathbf{H}\mathbf{x}, \tag{31}$$

for $\mathbf{x} \in \Omega_l, \ \forall\ l \in \mathbb{S},\ t \in \mathbb{T}$,

(ii) $\bigcup_{l\in\mathbb{S}} \Omega_l = \mathbb{R}^n$,

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(iii) $\delta^p \nu \exp\left\{-\int_{t_0}^{t} \kappa(s)ds\right\} \le \gamma^p, \quad \forall t \in \mathbb{T}$, where $\kappa(t) \ge 0, t \in \mathbb{T}$, and ν are defined by (23) and (13), respectively.

Then the control given by the stabilizing switching rule

$$\sigma^* \in \Sigma = \{\sigma : (\sigma(\mathbf{x}(t)) = l) \Rightarrow (\mathbf{x}(t-) \in \Omega_l)\} \quad (32)$$

together with the practical dwell time t_{prac} makes the hybrid system (2) p-th mean practically stable for p defined in (23).

Proof: For the Lyapunov function $V(\mathbf{x}) = (\mathbf{x}^T\mathbf{H}\mathbf{x})^{\frac{p}{2}}$ applying the Itô operator to the l-th subsystem one can obtain

$$\mathcal{L}_l^{(2)}V(\mathbf{x}) = p(\mathbf{x}^T\mathbf{H}\mathbf{x})^{\frac{p}{2}-1} \times \left(\mathbf{x}^T\mathbf{H}\mathbf{A}(l)\mathbf{x} + \frac{1}{2}\sum_{k=1}^{m}\mathbf{x}^T\mathbf{G}_k^T(l)\mathbf{H}\mathbf{G}_k(l)\mathbf{x}\right) +p\left(\frac{p}{2}-1\right)(\mathbf{x}^T\mathbf{H}\mathbf{x})^{\frac{p}{2}-2}|\mathbf{x}^T\mathbf{H}\sum_{k=1}^{m}\mathbf{G}_k(l)\mathbf{x}|^2, \quad (33)$$

where the operator $\mathcal{L}_l^{(2)}(\cdot)$ is given by (4). Let us take $\sigma \equiv \sigma^*$ defined in (32). Then simple calculations gives us

$$\mathcal{L}_{\sigma^*}^{(2)}V(\mathbf{x}) \le -p\left(|\alpha_1(t)| - \left(\frac{p}{2}-1\right)\alpha_3^2\right)V(\mathbf{x}), \quad l \in \mathbb{S}. \quad (34)$$

Hence we obtain

$$\frac{d}{dt}\mathbb{E}[V(\mathbf{x})] \le -\kappa(t)\mathbb{E}[V(\mathbf{x})], \quad (35)$$

where $\kappa(t), t \in \mathbb{T}$, is given by (23).

Further proof is similar to the proof of Theorem 1. Function V is a single Lyapunov function for the hybrid system (2) under the switching $\sigma^* \in \Sigma$. □

Remark 2: Note that if the following condition

$$\Omega_l = \mathbb{R}^n \quad (36)$$

is satisfied for some $l \in \mathbb{S}$, then l-th subsystem of (2) is p-th mean practically stable for corresponding p. In a particular case when condition (36) is satisfied for all $l \in \mathbb{S}$ all subsystems of (2) are practically stable for some p. Since the switching $\sigma^* \in \Sigma$ is any switching rule and V is a common Lyapunov function for system (2), the hybrid system (2) is practically stable for any switching rule for some p.

Corollary 2: In particular, when we assume that $\nu\delta^p \le \gamma^p$ and the Lebesgue integrable function $\alpha_1(t) \le 0$, $t \in \mathbb{T}$, is bounded by a constant $\bar{\alpha}_1 \le 0$ i.e. $\bar{\alpha}_1 \le \alpha_1(t) \le 0$, $t \in \mathbb{T}$, then $\kappa(t)$ defined by (23) is also a constant given by (24) and the hybrid system (2) is p-th mean practically stable.

B. Multiple Lyapunov functions

Similarly to the previous case we can find the p-th mean multiple Lyapunov function for the hybrid system (2) under the stabilizing switching rule σ^*.

We can formulate the following criterion.

Criterion 2: Let us assume that the following conditions are satisfied

(i) there exist symmetric positive-definite constant matrices $\mathbf{H}(l), l \in \mathbb{S}$, a real Lebesgue integrable function $\alpha_1(t) < 0$ for all $t \in \mathbb{T}$ and positive constants α_2, α_3, such that

$$\mathbf{x}^T\mathbf{H}(l)\mathbf{A}(t,l)\mathbf{x} + \frac{1}{2}\sum_{k=1}^{m}\mathbf{x}^T\mathbf{G}_k^T(l)\mathbf{H}(l)\mathbf{G}_k(l)\mathbf{x} \le \le \alpha_1(t)\mathbf{x}^T\mathbf{H}(l)\mathbf{x} \quad \text{for } \mathbf{x} \in \Omega_l, \quad \forall\, l \in \mathbb{S}, t \in \mathbb{T}, \quad (37)$$

$$\alpha_2\mathbf{x}^T\mathbf{H}(l)\mathbf{x} \le \left|\mathbf{x}^T\mathbf{H}(l)\sum_{k=1}^{m}\mathbf{G}_k(l)\mathbf{x}\right| \le \le \alpha_3\mathbf{x}^T\mathbf{H}(l)\mathbf{x} \quad \text{for } \mathbf{x} \in \Omega_l, \quad \forall\, l \in \mathbb{S}, t \in \mathbb{T}, \quad (38)$$

(ii) $\bigcup_{l\in\mathbb{S}} \Omega_l = \mathbb{R}^n$

(iii) $\delta^p \bar{\nu}^{N_0}\, min\left\{\bar{\nu}^{\frac{t-t_0}{t_{prac}}} e^{\int_{t_0}^{t} -\kappa(s)ds},\ 1\right\} \le \gamma^p$, where $\kappa(t) > 0$ for all $t \in \mathbb{T}$ and $\bar{\nu}$ are defined by (23) and (12), respectively.

Then the control given by the stabilizing switching rule

$$\sigma^* \in \Sigma = \{\sigma : (\sigma(\mathbf{x}(t)) = l) \Rightarrow (\mathbf{x}(t-) \in \Omega_l)\} \quad (39)$$

together with the practical average dwell time $\tau_{ADT_{prac}}^{\sigma^*}$ given by (27) makes the hybrid system (2) p-th mean practically stable for p defined in (23).

Proof: Let us consider the Lyapunov function as follows

$$V(\mathbf{x},\sigma) = (\mathbf{x}^T\mathbf{H}(\sigma)\mathbf{x})^{\frac{p}{2}}, \quad (40)$$

which in general is not continuous in switching times t_k. We have the following inequality

$$\lambda_{MIN}^{\frac{p}{2}}|\mathbf{x}|^p \le V(\mathbf{x},\sigma) \le \lambda_{MAX}^{\frac{p}{2}}|\mathbf{x}|^p. \quad (41)$$

From (41) it follows that

$$\mathbf{H}(i) \le \bar{\nu}\mathbf{H}(j) \quad \forall i \ne j \in \mathbb{S}. \quad (42)$$

Let us take the switching rule of the form $\sigma \equiv \sigma^*$ given by (39). When $t \in [t_i, t_{i+1})$ and $\sigma^*(\mathbf{x}(t)) = l^*$, $V(\mathbf{x},\sigma^*) = V(\mathbf{x}, l^*)$ in $[t_i, t_{i+1})$ is continuous and the following inequality holds

$$\mathbb{E}[V(\mathbf{x}(t),\sigma^*(\mathbf{x}(t)))] \le \mathbb{E}[V(\mathbf{x}(t_i),\sigma^*(\mathbf{x}(t_i)))]e^{\int_{t_0}^{t} -\kappa(s)ds}, \quad (43)$$

where $\kappa(t)$ is the same for all $l \in \mathbb{S}$ and is given by (23).

From (42) it follows that at switching times t_k the following inequality holds

$$V(\mathbf{x}(t_k),\sigma^*(\mathbf{x}(t_k))) \le \bar{\nu}V(\mathbf{x}(t_k-),\sigma^*(\mathbf{x}(t_k-))). \quad (44)$$

These observations gives us

$$\mathbb{E}[V(\mathbf{x}(t_{i+1}),\sigma^*(\mathbf{x}(t_{i+1}))] \leq$$
$$\leq \bar{\nu}\mathbb{E}[V(\mathbf{x}(t_{i+1}-),\sigma^*(\mathbf{x}(t_{i+1}-)))]$$
$$\leq \bar{\nu}\mathbb{E}[V(\mathbf{x}(t_i),\sigma^*(\mathbf{x}(t_i)))]\exp\left\{\int_{t_i}^{t_{i+1}} -\kappa(s)ds\right\}$$
$$\leq \bar{\nu}^2\mathbb{E}[V(\mathbf{x}(t_i-),\sigma^*(\mathbf{x}(t_i-)))]\exp\left\{\int_{t_i}^{t_{i+1}} -\kappa(s)ds\right\}$$
$$\leq \bar{\nu}^2\mathbb{E}[V(\mathbf{x}(t_{i-1}),\sigma^*(t_{i-1}))]\exp\left\{\int_{t_{i-1}}^{t_i} -\kappa(s)ds\right\}$$
$$\exp\left\{\int_{t_i}^{t_{i+1}} -\kappa(s)ds\right\} \leq \ldots \leq$$
$$\bar{\nu}^{N_{\sigma^*}(t_0,t)}\mathbb{E}[V(\mathbf{x}(t_0),\sigma^*(\mathbf{x}_0))]\exp\left\{\int_{t_0}^{t} -\kappa(s)ds\right\}.$$

Hence, for any $t \geq t_0$ we obtain the following inequality

$$\begin{gathered}\mathbb{E}[V(\mathbf{x}(t),\sigma^*(\mathbf{x}(t)))] \leq \\ \mathbb{E}[V(\mathbf{x}_0,\sigma_0^*)]\bar{\nu}^{N_{\sigma^*}(t_0,t)}\exp\left\{\int_{t_0}^{t} -\kappa(s)ds\right\}.\end{gathered} \tag{45}$$

We assume that instead of $N_{\sigma^*}(t_0,t)$ we consider $N_{\sigma^*}^{prac}(t_0,t)$ and we assume that

$$N_{\sigma^*}^{prac}(t_0,t) \leq N_0 + \frac{t-t_0}{\tau_{ADT_{prac}}^{\sigma^*}}, \tag{46}$$

where $\tau_{ADT_{prac}}^{\sigma^*}$ was defined in Definition 7, i.e.

$$\tau_{ADT_{prac}}^{\sigma^*} = max\left\{t_{prac},\ \tau_{ADT}^{\sigma}\right\}.$$

Let us choose an integer $N_0 \geq 0$, $\theta = \bar{\nu}^{N_0}$ such that

$$\tau_{ADT}^{\sigma} = \frac{\ln\bar{\nu}(t-t_0)}{\int_{t_0}^{t}\kappa(s)ds}.$$

Then

$$\begin{aligned}&e^{N(t_0,t)\ln\bar{\nu}-\int_{t_0}^{t}\kappa(s)ds} \leq \\ &e^{N_0\ln\bar{\nu}} min\left\{e^{\frac{(t-t_0)\ln\bar{\nu}}{t_{prac}}},\ \ e^{\int_{t_0}^{t}\kappa(s)ds}\right\}e^{\int_{t_0}^{t}-\kappa(s)ds} \\ &= \theta\ min\left\{\bar{\nu}^{\frac{t-t_0}{t_{prac}}}e^{\int_{t_0}^{t}-\kappa(s)ds},\ \ 1\right\} = \widetilde{\theta}(t_0,t,t_{prac})\end{aligned} \tag{47}$$

and

$$\mathbb{E}[V(\mathbf{x}(t),\sigma^*(\mathbf{x}(t)))] \leq \mathbb{E}[V(\mathbf{x}_0,\sigma_0^*)]\widetilde{\theta}(t_0,t,t_{prac}). \tag{48}$$

Since

$$\lambda_{MIN}^{\frac{p}{2}}|\mathbf{x}|^p \leq V(\mathbf{x},\sigma^*) \leq \lambda_{MAX}^{\frac{p}{2}}|\mathbf{x}|^p, \tag{49}$$

we obtain

$$\lambda_{MIN}^{\frac{p}{2}}\mathbb{E}[|\mathbf{x}|^p] \leq \mathbb{E}[V(\mathbf{x},\sigma^*)] \leq \lambda_{MAX}^{\frac{p}{2}}\mathbb{E}[|\mathbf{x}|^p]. \tag{50}$$

Then using (49)-(50) we have

$$\mathbb{E}[|\mathbf{x}(t)|^p] \leq \widetilde{\theta}(t_0,t,t_{prac})\bar{\nu}\mathbb{E}[|\mathbf{x}_0|^p]. \tag{51}$$

Hence and from assumption (2) it follows the practical p-th mean stability of the linear hybrid system (2) with the practical average dwell time $\tau_{ADT_{prac}}^{\sigma^*}$ for p defined in (23) under the stabilizing switching rule σ^*. Function V is a multiple Lyapunov function for the hybrid system (2) under the switching $\sigma^* \in \Sigma$. □

Corollary 3: In particular, when we assume that $\bar{\nu}^{N_0+1}\delta^p \leq \gamma^p$ and the Lebesgue integrable function $\alpha_1(t) < 0$, $t \in \mathbb{T}$, is bounded by a constant $\bar{\alpha}_1 < 0$ i.e. $\bar{\alpha} < \alpha_1(t) \leq 0$, $t \in \mathbb{T}$, then $\kappa(t)$ defined by (23) is also a constant given by (24)

If in Definition 7 in formula (27) we assume that

$$\tau_{ADT}^{\sigma} = \frac{\ln\bar{\nu}}{\kappa}. \tag{52}$$

Then

$$\tau_{ADT_{prac}}^{\sigma^*} = max\left\{t_{prac},\ \frac{\ln\bar{\nu}}{\kappa}\right\}. \tag{53}$$

Hence, if $\tau_{ADT_{prac}}^{\sigma^*} = t_{prac}$ inequality (48) takes the form

$$\mathbb{E}[V(\mathbf{x}(t),\sigma^*(\mathbf{x}(t)))] \leq \mathbb{E}[V(\mathbf{x}_0,\sigma_0^*)]\theta e^{-\rho(t-t_0)}, \tag{54}$$

where $\rho = \kappa - \frac{\ln\bar{\nu}}{t_{prac}}$. From inequality (54) follows exponential p-th mean stability and hence the practical p-th mean stability.

In the case when $\tau_{ADT_{prac}}^{\sigma^*} = \frac{\ln\bar{\nu}}{\kappa}$ inequality (48) takes the form

$$\mathbb{E}[V(\mathbf{x}(t),\sigma^*(\mathbf{x}(t)))] \leq \mathbb{E}[V(\mathbf{x}_0,\sigma_0^*)]\theta. \tag{55}$$

From inequalities (49) and (50) follows that

$$\mathbb{E}[|\mathbf{x}(t)|^p] \leq \bar{\nu}^{N_0+1}\mathbb{E}[|\mathbf{x}_0|^p] < \bar{\nu}^{N_0+1}\delta^p \leq \gamma^p. \tag{56}$$

In both approaches, i.e. in single and multiple Lyapunov functions approach, the obtained criteria give too strong conditions. It means, one can find examples of practically p-stable stochastic hybrid systems even the sufficient conditions are not satisfied. In particular for the multiple Lyapunov function approach it is difficult to find the corresponding matrices $\mathbf{H}(l)$. To omit these difficulties we present the next approach based on a modified single practical Lyapunov-like function.

C. Single practical Lyapunov-like functions

Let us assume in this case that the region Ω_l of the hybrid system residence in the l-th subsystem, $l \in \mathbb{S}$, consists of three subregions Ω_l^{as}, Ω_l^{s} and Ω_l^{us}, $\Omega_l = \Omega_l^{as} \cup \Omega_l^{s} \cup \Omega_l^{us}$, $\Omega_l^{us} \cap \Omega_l^{s} = \emptyset$. We also denote by τ_{as} the total time of the hybrid system residence in asymptotic stable regions Ω_l^{as}, by τ_s the total time of the hybrid system residence in stable but not asymptotic stable regions $\Omega_l^{s}\backslash\Omega_l^{as}$ and by τ_{us} the total time of the hybrid system residence in unstable regions Ω_l^{us}.

Criterion 3: Let us assume that the following conditions are satisfied

(i) there exist a common symmetric positive-definite constant matrix $\mathbf{H}$ and real Lebesgue integrable functions $\alpha_1^{as}(t) < 0, \alpha_1^{us}(t) > 0$ for all $t \in \mathbb{T}$ such that

$$\mathbf{x}^T \mathbf{H}\mathbf{A}(t,l)\mathbf{x} + \frac{1}{2}\sum_{k=1}^{m} \mathbf{x}^T \mathbf{G}_k^T(l)\mathbf{H}\mathbf{G}_k(l)\mathbf{x} \leq \alpha_1^{as}(t)\mathbf{x}^T\mathbf{H}\mathbf{x}, \quad \text{for } \mathbf{x} \in \Omega_l^{as}, \ \ \forall\, l \in \mathbb{S},\ t \in \mathbb{T} \quad (57)$$

$$\mathbf{x}^T \mathbf{H}\mathbf{A}(t,l)\mathbf{x} + \frac{1}{2}\sum_{k=1}^{m} \mathbf{x}^T \mathbf{G}_k^T(l)\mathbf{H}\mathbf{G}_k(l)\mathbf{x} \leq 0, \quad (\alpha_1^{s} \equiv 0) \quad \text{for } \mathbf{x} \in \Omega_l^{s}, \ \ \forall\, l \in \mathbb{S},\ t \in \mathbb{T} \quad (58)$$

$$0 < \mathbf{x}^T \mathbf{H}\mathbf{A}(t,l)\mathbf{x} + \frac{1}{2}\sum_{k=1}^{m} \mathbf{x}^T \mathbf{G}_k^T(l)\mathbf{H}\mathbf{G}_k(l)\mathbf{x} \leq \alpha_1^{us}(t)\mathbf{x}^T\mathbf{H}\mathbf{x}, \quad \text{for } \mathbf{x} \in \Omega_l^{us}, \ \ \forall\, l \in \mathbb{S},\ t \in \mathbb{T} \quad (59)$$

(ii)

$$\bigcup_{l\in\mathbb{S}} \Omega_l = \mathbb{R}^n, \quad \Omega_l = \Omega_l^{as} \cup \Omega_l^{s} \cup \Omega_l^{us} \quad (60)$$

(iii) $\delta^p \nu \leq \gamma^p, \quad \forall t \geq t_0$ where ν is defined by (13).

(iv) there exists a constant $\kappa \geq 0$ such that the set $\Sigma \cap \mathcal{T} \neq \emptyset$, where

$$\Sigma = \{\sigma : (\sigma(\mathbf{x}(t)) = l) \Rightarrow (\mathbf{x}(t-) \in \Omega_l)\} \quad (61)$$

$$\mathcal{T} = \left\{\sigma : \int_{\tau_{us}} \kappa_{us}(t)dt - \int_{\tau_{as}} \kappa_{as}(t)dt \leq -\kappa(\tau_{as} + \tau_s + \tau_{us})\right\}, \quad (62)$$

$$\kappa_\beta = p|\alpha_1^\beta| \quad \text{for} \quad \beta \in \{as, s, us\}. \quad (63)$$

Then the control given by the stabilizing switching rule $\sigma^{**} \in \Sigma \cap \mathcal{T}$ together with the practical dwell time t_{prac}, defined by (25), makes the hybrid system (2) p-th mean practically stable for $0 < p \leq 2$.

Proof: Let us consider Lyapunov function $V(\mathbf{x}) = \left(\mathbf{x}^T\mathbf{H}\mathbf{x}\right)^{\frac{p}{2}}$ and let us choose $\sigma = \sigma^* \in \Sigma$ given by (61). Then

$$\mathcal{L}_{\sigma^*}^{(2)} V(\mathbf{x}) \leq \begin{cases} -\kappa_{as}V(\mathbf{x}), & \text{for } \ \mathbf{x} \in \Omega_{\sigma^*}^{as}, \\ \kappa_{s}V(\mathbf{x}), & \text{for } \ \mathbf{x} \in \Omega_{\sigma^*}^{s} \backslash \Omega_{\sigma^*}^{as}, \\ \kappa_{us}V(\mathbf{x}), & \text{for } \ \mathbf{x} \in \Omega_{\sigma^*}^{us}. \end{cases} \quad (64)$$

It follows from the proof of Criterion 1. For $t \in [t_i, t_{i+1})$, $i \in \mathbb{N}$, we obtain

$$\mathbb{E}[V(\mathbf{x}(t))] \leq \begin{cases} \mathbb{E}[V(\mathbf{x}(t_i))]e^{-\int_{t_i}^{t}\kappa_{as}(s)ds}, & \text{for } \mathbf{x} \in \Omega_{\sigma^*}^{as}, \\ \mathbb{E}[V(\mathbf{x}(t_i))], & \text{for } \mathbf{x} \in \Omega_{\sigma^*}^{s} \backslash \Omega_{\sigma^*}^{as}, \\ \mathbb{E}[V(\mathbf{x}(t_i))]e^{\int_{t_i}^{t}\kappa_{us}(s)ds}, & \text{for } \mathbf{x} \in \Omega_{\sigma^*}^{us}. \end{cases} \quad (65)$$

Since the function $V(\mathbf{x}) = (\mathbf{x}^T\mathbf{H}\mathbf{x})^{\frac{p}{2}}$ is continuous inequalities (65) are satisfied also for $t = t_{i+1}$. Using previous computations we obtain

$$\mathbb{E}[V(\mathbf{x}(t))] \leq \mathbb{E}[V(\mathbf{x}_0)]e^{-\int_{\tau_{as}}\kappa_{as}(s)ds}e^{\int_{\tau_{us}}\kappa_{us}(s)ds}. \quad (66)$$

Let us choose the switching $\sigma^{**} \in \Sigma \cap \mathcal{T}$ with the practical dwell time t_{prac}. We note that

$$\tau_{as} \geq n_{as}t_{prac}, \quad \tau_s \geq n_s t_{prac}, \quad \tau_{us} \geq n_{us}t_{prac} \quad (67)$$

where n_{as}, n_s and n_{us} are numbers of switchings from any subspaces to Ω_l^{as}, Ω_l^{s} and Ω_l^{us}, respectively. From (62), (66) it follows, that

$$\mathbb{E}[V(\mathbf{x}(t))] \leq \mathbb{E}[V(\mathbf{x}_0)]. \quad (68)$$

Further proof is similar to the proof of Theorem 1. Hence we obtain the thesis.

The function V, which satisfies conditions (64), is called the single practical Lyapunov-like function for the hybrid system (2) under the switching $\sigma^* \in \Sigma$. We note that the single practical Lyapunov-like function is a generalization of a single Lyapunov-like function introduced in [16] for $\Omega_{\sigma^*}^{s} = \Omega_{\sigma^*}^{as}$. □

Example 1: Let us consider a particular case of the hybrid system (2) with two structures ($\mathbb{S} = \{1, 2\}$) described by

$$d\mathbf{x}(t) = \mathbf{A}(\sigma)\mathbf{x}(t)dt + \mathbf{G}(\sigma)\mathbf{x}(t)dw(t), \quad l = 1,2$$
$$\mathbf{x}(0) = \mathbf{x}_0 \in \mathbb{R}^2, \quad \sigma(\mathbf{x}_0) = \sigma_0 \in \{1, 2\},$$
$$\mathbf{x} = [x_1, x_2]^T \in \mathbb{R}^2 \quad (69)$$

where

$$\mathbf{A}(1) = \begin{bmatrix} -0.02 & -9.5 \\ 9 & -0.02 \end{bmatrix} \quad \mathbf{G}(1) = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$$

$$\mathbf{A}(2) = \begin{bmatrix} -0.105 & -9.5 \\ 10 & 0.35 \end{bmatrix} \quad \mathbf{G}(2) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

Let us choose a Lyapunov function of a form $V(\mathbf{x}) = \mathbf{x}^T\mathbf{x}$. We have

$$\mathbf{A}(1) + \frac{1}{2}\mathbf{G}^T(1)\mathbf{G}(1) = \begin{bmatrix} 0 & -9.5 \\ 9 & 0 \end{bmatrix} \quad (70)$$

and

$$\mathbf{A}(2) + \frac{1}{2}\mathbf{G}^T(2)\mathbf{G}(2) = \begin{bmatrix} -0.1 & -9.5 \\ 10 & 0.355 \end{bmatrix} \quad (71)$$

Regions Ω_l^β, where $l = 1, 2$, $\beta \in \{as, s, us\}$ are chosen as follows

$$\Omega_1^{as} = \{\mathbf{x} \in \mathbb{R}^2 : -0.5x_1x_2 \leq -0.1(x_1^2 + x_2^2)\}$$
$$\Omega_1^{s} = \{\mathbf{x} \in \mathbb{R}^2 : -0.5x_1x_2 \leq 0\}$$
$$\Omega_1^{us} = \emptyset$$
$$\Omega_2^{as} = \{\mathbf{x} \in \mathbb{R}^2 : -0.1x_1^2 + 0.5x_1x_2 + 0.355x_2^2 \leq -0.1(x_1^2 + x_2^2)\}$$
$$\Omega_2^{s} = \{\mathbf{x} \in \mathbb{R}^2 : -0.1x_1^2 + 0.5x_1x_2 + 0.355x_2^2 \leq 0\}$$
$$\Omega_2^{us} = \{\mathbf{x} \in \mathbb{R}^2 : 0 < -0.1x_1^2 + 0.5x_1x_2 + 0.355x_2^2 \leq 0.355(x_1^2 + x_2^2)\}$$

and we note that (60) is satisfied. Parameters: $t_{prac} = 0.001, \kappa_{as} = 0.1p, \kappa_{us} = 0.355p, \kappa_s = 0, \kappa = 0, \nu = 1, \delta = \gamma = 7, p \in (0, 2]$.

Then $V(\mathbf{x}) = \mathbf{x}^T\mathbf{x}$ is the single practical Lyapunov-like function for system (2) under the stabilizing switching signal $\sigma^{**} \in \Sigma \cap \mathcal{T}$ given as follows

$$\sigma^{**}(\mathbf{x}(t)) = \begin{cases} 1 & if \ \ \mathbf{x}(t-) \in \Omega_1 = \{\mathbf{x} \in \mathbb{R}^2 : x_1x_2 \leq 0\} \\ 2 & if \ \ \mathbf{x}(t-) \in \Omega_2 = \{\mathbf{x} \in \mathbb{R}^2 : x_1x_2 \geq 0\} \end{cases} \quad (72)$$

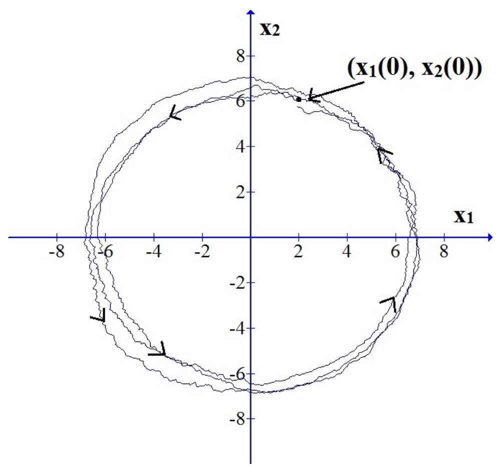

Fig. 1. Exemplary simulation of a practically p-th stable path ($0 < p \leq 2$) under switching $\sigma^{**} \in \Sigma \cap \mathcal{T}$

From Criterion 3 follows that the hybrid system (2) is p-th mean practically stable for $0 < p \leq 2$.

Remark 3: Note that even if $\alpha_1(t) > 0$, $t \in \mathbb{T}$, in the inequality (30) or (37) the hybrid system (2) still can be p-th mean practically stable for $p < 2 - 2\alpha_1(t)/\alpha_2^2$ if $\alpha_1(t) < \alpha_2^2$ for all $t \in \mathbb{T}$, where α_2 is given by (31) or (38), respectively. It follows directly from previous considerations and from [14]. The corresponding single or multiple Lyapunov function can be constructed in a similar way as it was done for $\alpha_1(t) < 0, t \in \mathbb{T}$. However, this requires some additional algebraic calculations.

IV. Conclusions

In this paper linear hybrid systems parametrically excited by a white noise, consisted of unstable subsystems described by Itô stochastic differential equations, have been analyzed. To find sufficient conditions for the practical p-th mean stability the Lyapunov function techniques, the practical average dwell time approach, the practical dwell time approach and the hybrid control theory have been used. To stabilize stochastic linear hybrid systems, that have been discussed, we have used the stabilizing switching rule, which is constructed on the basis of the knowledge of the regions of decreasing of Lyapunov functions for subsystems and ensures the existence of the single/multiple Lyapunov or single practical Lyapunov-like functions.

The obtained results have been illustrated by an example. Proposed criteria of the practical p-th stability can be generalized to hybrid systems parametrically excited by Gaussian colored and non-Gaussian noises. Also other Lyapunov -like functions approach can be used.

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