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Optimal Gains Tuning of PI-Fuzzy Controllers

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Abstract— **This paper presents a new methodology for tuning the gains of fuzzy proportional-integral controllers where the closed loop system performance is explicitly taken into account. The gains are found by solving a nonlinear constrained optimization problem considering the system's dynamics described by a nonlinear model, and including a set of constraints on gains, control actions and on the system outputs. Experimental results ilustrate the relevance of using the proposed tuning technique.**

I. Introduction

Standard Proportional-Integral-Derivative (PID) control techniques are still in these days largely used in the process industry, mainly due to its recognized simplicity, not only in terms of the underlying supporting theory in the case of linear systems but also owing to available straightforward tuning methods. However, in the case of nonlinear systems it is difficult to get satisfied closed loop performance over extended operating regimes by using nonadaptive classical linear controllers.

Fuzzy Logic Control (FLC) techniques are inherently nonlinear approaches since they incorporate three main sources of nonlinearities, namely, the rule base, the inference engine and the fuzzification and defuzzification modules.

This control paradigm based on fuzzy logic [1] has proven to be a successful approach in controlling many nonlinear systems and it has been suggested as an alternative approach to conventional control techniques [2]. FLCs are more robust than conventional controllers and their performance are less sensitive to parametric variations of systems [3] or to unmeasured disturbances. Furthermore, recent applications of FLC methodologies have shown great potential in the context of complex ill-defined systems that can effectively be controlled by a skilled operator without the explicit knowledge of the underlying system dynamics [4].

Nevertheless the great potential of fuzzy control applications in a wide range of applications, finding a good setting for a particular controller, by choosing appropriate linguistic variables, membership functions, rules and scaling factors, and subsequently tune it, is still a challenge subject due to the lack of a systematic framework. When the system to be controlled is a MIMO (Multi-Input/Multi-Output) system this is even more complex because one or more manipulated variables might be affecting the other controlled variables due to coupling effects. As a result the controller performance ends up compromised.

In literature there are multiple proposals for how to select FLC parameters. They include heuristics based approaches (see e.g. [5]), methodologies relying on a pseudo-equivalence of digital fuzzy PID controllers with linear PID controllers [6] or those based on evolutionary computation [7], as the application of evolutionary algorithms for solving optimization and search problems related with fuzzy systems, obtaining genetic fuzzy systems and he use of fuzzy tools and fuzzy logic-based techniques for modelling different evolutionary algorithm components and adapting evolutionary algorithm control parameters, with the goal of improving performance.

The present paper proposes a new conceptual methodology for tuning PI-fuzzy scaling factors or gains, although not exclusively restricted to this topology, by solving a nonlinear optimization problem subject to a set of explicit constraints on scaling factors, control actions and outputs, and assuming the systems dynamics described by a nonlinear model. Because the gains are found by solving an offline constrained optimization problem the closed loop system response might be considered, to some extent, for the reference signal adopted in the optimization problem, as an optimal trajectory provided the certainty equivalence principle holds.

In order to demonstrate the feasibility of the proposed approach PI-Fuzzy based control architecture controller is applied to a SISO (Single-Input/Single-Output) system and to a MIMO system.

II. Fuzzy Logic Control Systems

Fuzzy Logic can be defined as a theory of vagueness and uncertainties. This theory provides an approximate yet effective mean of describing the behavior of systems, which can be too complex and/or ill-defined to permit a precise mathematical analysis or an analytical description under the form of a quantitative model.

The basic structure of FLC (Fig. 1) consists of four conceptual components, namely, the knowledge base, the fuzzification module, the inference engine and the defuzzification module. The knowledge base system contains all the knowledge of the fuzzy control system. It comprises a fuzzy control rule base, i.e. the procedural part of the knowledge, and a data base comprising facts, terms and concepts. The inference engine is a reasoning mechanism that performs inference procedures upon the fuzzy control rules and given conditions to derive reasonable control actions. It is the kernel of a fuzzy control system. The fuzzification module

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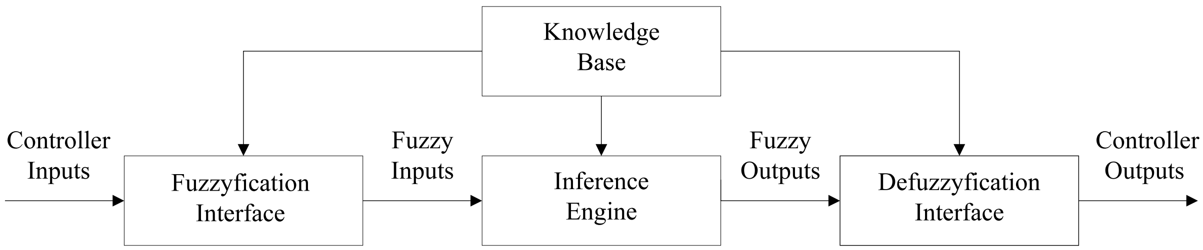

Fig. 1. General structure of a Mamdani-type Fuzzy Logic System.

defines a mapping from a real-value space to a fuzzy space, while the defuzzification module implements a mapping from a fuzzy space defined over an output universe of discourse to a real-valued space [2].

One of the major issues regarding fuzzy control design refers to how to explicitly reflect in the tuning stage the concept of closed loop performance. This would lead to an improved fuzzy control systems behavior over extended operation regimes, while enabling a balance between the tracking error, energy consumption and actuators wearing, expressed by the variance in the control action increments. The present work proposes an effective structured automatic framework to deal with the problem of PI-fuzzy controller gains tuning based on the minimization of a user defined cost function. This approach not only makes straightforward the tuning procedure, avoiding the ever cumbersome and subjective trial-and-error based heuristics approaches, but also promotes a more efficient and rational usage of resources, including energy, contributing to sounding improvements in terms of overall efficiency and longevity of control systems.

A. PI-Fuzzy Controller Architecture

With no loss of generality let consider a two-dimension PI-fuzzy controller structure, in which error e (1) and change in error Δe (2) are selected as input words, while the output from the fuzzy logic system is chosen as the increment of control action.

$$e(k) = r(k) - y(k) \tag{1}$$

$$\Delta e = e(k) - e(k-1) \tag{2}$$

with k the current discrete time, y the system output and r the reference signal. Fig. 2 shows the PI-Fuzzy controller

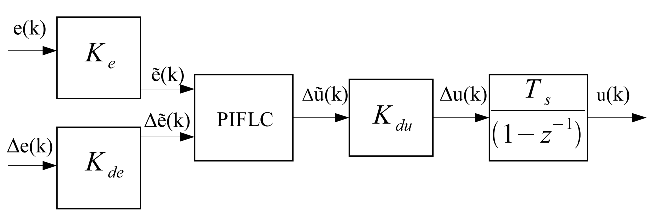

Fig. 2. PI-Fuzzy Logic Control schematics.

structure where the scaling factors K_e, $K_{\Delta e}$ and $K_{\Delta u}$ are explicitly represented. The normalized error $\tilde{e}$, change in error $\Delta\tilde{e}$ and the denormalized increment of control action Δu are given by:

$$\tilde{e}(k) = K_e.e(k) \tag{3}$$

$$\Delta\tilde{e} = K_{\Delta e}.\Delta e(k) \tag{4}$$

$$\Delta u = K_{\Delta u}.\Delta\tilde{u} \tag{5}$$

where $\Delta\tilde{u}$ is the normalized increment of control action.

Upon selecting a given setting for the fuzzy controller follows the tuning of its parameters. The next section presents the proposed methodology for tuning the FLC scaling factors by solving a constrained nonlinear optimization problem.

B. Optimal Tuning of Scaling Factors

Optimization methods are classified according to the type of the search space $A \subseteq R^n$ and objective functions. The simplest method is the *linear programming*, which concerns the case where the objective function $L(\cdot)$ to minimized is linear and the restriction space A is specified using only linear equality and inequality constraints [8]. However, in general the objective function and/or the constraints contain nonlinearities, leading to a *nonlinear programming* problem where multiple minima may exist in virtue of the nonconvexity of the optimization problem.

When the optimization problem contains multiple objective functions the functional vector constrained minimization is performed in terms of an aggregating function that combines individual cost function values into a single utility value, prior the optimization stage. The decision maker is then presented by the optimizer with a set of candidate non-inferior solutions, before expressing any preferences under the form of a compromise solution [9]. At each step, a partial preference information is supplied by the decision maker to the optimizer that generates improved solutions.

This work considers the case in which the performance index is described by a single objective function subject to a set of constraints. The underlying optimization problem can be formulated as follows:

$$\min_K \mathrm{V} \equiv \min_K \sum_{k=1}^{N_p} L_k(e(k), u(k-1), K) \tag{6}$$

subject to:

$$\begin{array}{l} y(k) - g(\varphi(k)) = 0 \\ \phi(y(k), u(k), K) = 0 \\ \psi(y(k), u(k), K) \leq 0 \end{array} \tag{7}$$

where $V \in R^+$ is the objective function to be minimized, $N_p \in N^+$ the prediction horizon, $y \in R^p$ the output vector, $u \in R^m$ the control trajectory, $K \in R^q$ the vector of scaling factors, $g(\cdot)$ the nonlinear system dynamics model, $\phi(\cdot)$ and $\psi(\cdot)$ functions defining equality and inequality constraints and $\varphi(k)$, the regressor vector given by:

$$\varphi(k) = [u(k-1) \quad \ldots \quad u(k-n_u) \quad y(k-1) \quad \ldots \quad y(k-n_u)] \tag{8}$$

In what the optimization of PI-Fuzzy controller gains is concerned it is here carried out using the MATLAB function *fmincon*() available in the *Optimization Toolbox*. The algorithm relies on the Hessian of the Lagrangian (9) and uses a merit function in the search of the optimized gains. In each iteration the Hessian matrix is calculated based on a quasi-Newton approximation.

$$\bigtriangledown^2_{xx} L(x,\lambda) = \bigtriangledown^2 f(x) + \sum \lambda_i \bigtriangledown^2 c_i(x) + \sum \lambda_i \bigtriangledown^2 ceq_i(x) \tag{9}$$

where f is the objective function, c the nonlinear inequality constraint vector, ceq the nonlinear equality constraint vector and λ_i the Lagrange multipliers.

III. Case Studies

This section shows some experimental results carried out on two different didactic setups, namely, on the Feedback® Process Trainer PCT 37-100 and on the AMIRA® DTS 200, the Three-Tank system. The experiments aim to compare the optimal gains PI-fuzzy control system performance against a PI-fuzzy control system where the scaling factors are tuned based on classical PI-control control approach[10].

A. PI-Fuzzy Controller Design

The PI-fuzzy controller comprises two inputs, namely, the control error and the change in error, and one output, under the form of control action increment (see Fig. 2).

Regarding the normalized universe of discourse for $\tilde{e}$ and $\Delta\tilde{e}$ they were chosen to be $[-1.5 \;\; , \;\; 1.5]$ and partioned into seven fuzzy sets, namely, $\{NB, NM, NS, ZE, PS, PM, PB\}$ (NB, Negative Big; NM, Negative Medium; NS, Negative Small; ZE, Zero; PS, Positive Small; PM, Positive Medium; PB, Positive Big). For the fuzzy controller output, $\Delta\tilde{u}$, the corresponding universe of discourse was defined in the range $[-1.0 \;\; , \;\; 1.0]$, and assuming a partition similar to those assumed for $\tilde{e}$ and $\Delta\tilde{e}$.

The fuzzy inference considered is the Mamdani-type inference (10), while the controller output is generated by the Centroid Defuzzification technique.

The membership function of $\tilde{e}$ and $\Delta\tilde{e}$ are presented in Fig. 3 and the membership function of $\Delta\tilde{u}$ are in Fig. 4.

$$\mu_{e/\Delta e}(e, \Delta e) = min(\mu_A(e), \mu_B(\Delta e)) \tag{10}$$

with μ the membership value. Concerning the rule base used in this work it comprises forty-nine rules, as shown in Table I.

TABLE I
Format of the rule base.

e Δe	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NB	NB	NB	NM	NS	ZE
NM	NB	NB	NB	NM	NS	ZE	PS
NS	NB	NB	NM	NS	ZE	PS	PM
ZE	NB	NM	NS	ZE	PS	PM	PB
PS	NM	NS	ZE	PS	PM	PB	PB
PM	NP	ZE	PS	PM	PB	PB	PB
PB	ZE	PS	PM	PB	PB	PB	PB

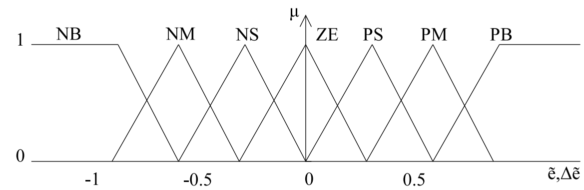

Fig. 3. Membership functions of $\tilde{e}$ and $\Delta\tilde{e}$.

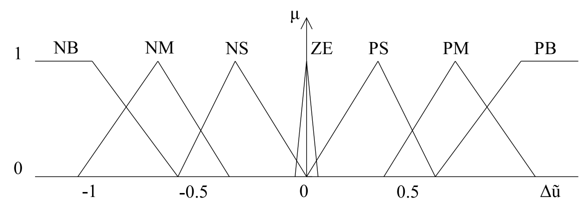

Fig. 4. Membership functions of $\Delta\tilde{u}$.

B. Experiments on a SISO System

The Process Trainer PCT 37-100 represented in Fig. 5 comprises a variable-speed axial fan, regulated via a potentiometer that circulates an airstream along a polypropylene tube. The airflow rate is heated by means of a heating element controlled by a thyristor circuit. A thermistor detector is incorporated in the setup to sensing the temperature at the insertion point.

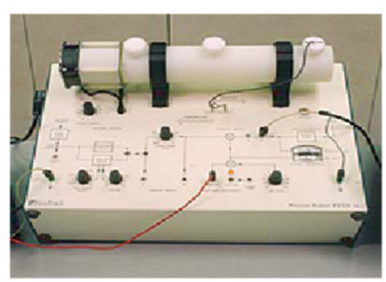

Fig. 5. Feedback® Process Trainer PCT 37-100.

The experiments concern the temperature control of the airstream by manipulating the input voltage (the control variable) to the heater grid. The default fan speed was adjusted to position 5, while the sampling period was set as 0.1 second.

Fig. 6 shows the closed loop response and input signals for the conventional based PI-FLC tuning approach. As can be inferred from this figure, the closed loop system response is rather oscillatory as a result of an aggressive controller, whose behavior is reflected in the control signal variance.

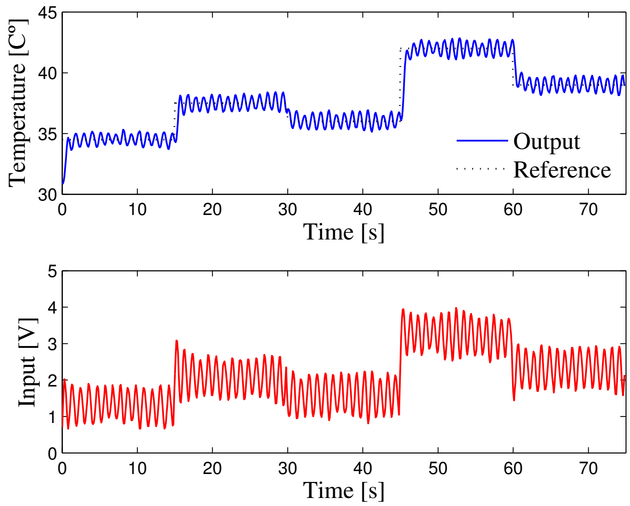

Fig. 6. SISO system controlled with a conventional based PI-Fuzzy controller.

The next experiment was carried out using a PI-Fuzzy controller, where its gains were found by solving a nonlinear constrained optimization problem. The cost function was chosen as a quadratic performance index involving the penalized control error and the control action increment, and assuming the system dynamics described by a nonlinear model. The underlying problem can be written as follows:

$$\min_{K}\left\{\sum_{k=0}^{N_P}\left[\,y(k)-r(k)\right]^2+0.8\left[u(k)-u(k-1)\right]^2\right\} \tag{11}$$

subject to the system dynamics and to

$$0\leq y(k)\leq 60, k=1,...,N_P \tag{12}$$

$$0\leq u(k)\leq 5, k=0,...,N_P-1 \tag{13}$$

$$K_e\geq 0 \tag{14}$$

$$K_{\Delta e}\geq 0 \tag{15}$$

$$K_{\Delta u}\geq 0 \tag{16}$$

with $K=[K_eK_{\Delta e}K_{\Delta u}]^{\mathrm{T}}$ the vector of gains and $u(k)$ given according to,

$$u(k)=u(k-1)+K_{\Delta u}\times\Delta u(k) \tag{17}$$

The nonlinear model of the system is described by a three-layered feedforward neural network with sigmoidal activation functions in the hidden layer and linear activation functions for the output layer. In order to capture the underlying system's dynamics the neural network was trained offline using the Levenberg-Marquardt algorithm, available in the MATLAB Neural Network Toolbox, and subsequently validated on a different data set. Equation (18) describes the system dynamics embedded in the training data set.

$$y(k)=\begin{bmatrix}-1.81\\-0.05\\25.39\\25.15\\-5.92\end{bmatrix}^T\times tangh\left(\begin{bmatrix}6.25 & -7.74 & -1.16\\15.45 & -13.38 & -2.54\\7.50 & 15.35 & -5.26\\-11.50 & 2.20 & 6.53\\-0.30 & 0.13 & -0.01\end{bmatrix}\times\begin{bmatrix}y(k-1)\\y(k-2)\\u(k-1)\end{bmatrix}+\begin{bmatrix}-2.28\\1.67\\13.74\\-14.52\\0.35\end{bmatrix}\right) \tag{18}$$

In Fig. 7 it is shown the closed loop system response for the optimal gains PI-fuzzy controller. As can be observed, the closed loop system response follows adequately the reference signal, and outperforms the classical based tuning PI-Fuzzy controller.

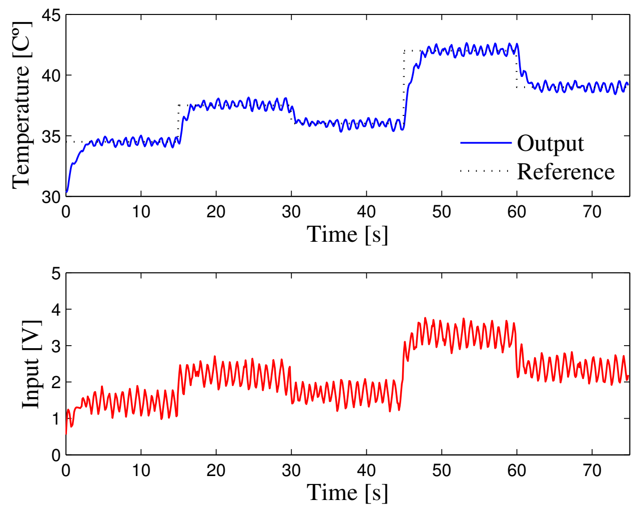

Fig. 7. SISO system controlled with an optimal PI-Fuzzy controller.

The superior behavior of the optimal gains PI-Fuzzy controller is corroborated by two metrics presented in Table II, namely, the root mean squared of error (RMSE) given according to (19) and the root mean squared of control action increment (RMSI) given according to (20).

$$RMSE=\sqrt{\frac{\sum_{k=1}^{N}\left[y(k)-r(k)\right]^T\left[y(k)-r(k)\right]}{\sum_{k=1}^{N}r(k)^Tr(k)}} \tag{19}$$

$$RMSI=\frac{\sqrt{\left[u(k)-u(k-1)\right]^T\left[u(k)-u(k-1)\right]}}{N} \tag{20}$$

C. Experiments on a MIMO System

The three-tank system (see Fig. 8) consists of three plexiglas cylindrical tanks with identical cross-section supplied with distilled water. The liquid levels, h_1 , h_2 and, h_3 are measured by piezoresistive transducers. The middle tank T_3 is connected to the other two tanks by means of circular

TABLE II
PERFORMANCE METRICS FOR THE FEEDBACK® PROCESS TRAINER PCT 37-100.

Controller	RMSE	RMSI
Conventional based PI-Fuzzy controller	0.0861	0.0058
Optimal PI-Fuzzy controller	0.0937	0.0095

cross section pipes provided with manually adjustable ball valves. In the tank T_2 it is located the main outlet of the system, which is directly connected to the collecting reservoir by means of a circular cross-section pipe provided with an outflow ball valve.

Fig. 8. Three-tank system.

The results for the classical based PI-fuzzy control system are presented in Fig. 9 and Fig. 10. As can be observed, the closed loop response is very fast, without any overshoot, but the increment in control actions is very stringent in both subsystems.

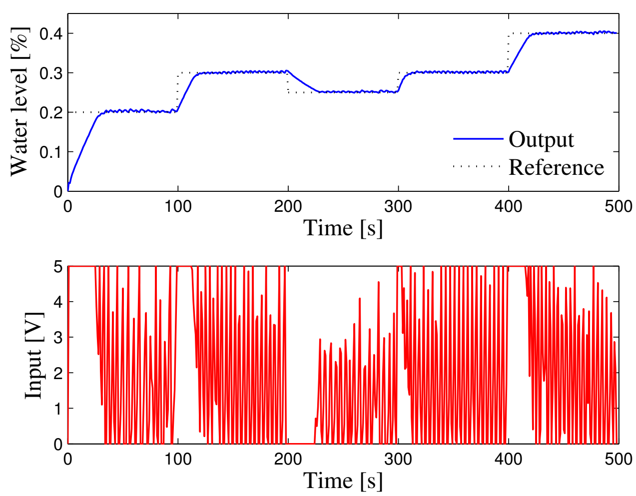

Fig. 9. Tank 1 of a MIMO system controlled with a conventional based PI-Fuzzy controller.

The next experiment was carried out using a Optimal PI-Fuzzy controller where the nonlinear model of the three-Tank system is given by:

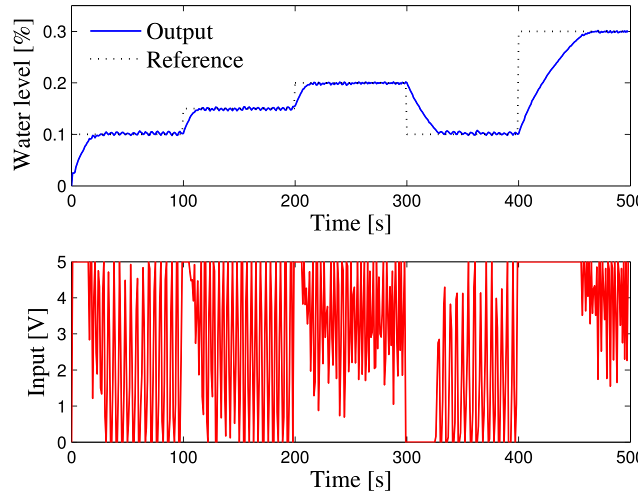

Fig. 10. Tank 2 of a MIMO system controlled with a conventional based PI-Fuzzy controller.

$$y(k) = \begin{bmatrix} 0.327 & 0.540 \\ 0.059 & 0.036 \\ -0.625 & -1.466 \\ 0.132 & 0.217 \\ 0.001 & -0.002 \\ 0.004 & 0.008 \\ -0.002 & -0.002 \\ -0.001 & -0.005 \end{bmatrix}^T \times tangh\left(\begin{bmatrix} 6.24 & -2.45 \\ 2.87 & -1.90 \end{bmatrix} \times \begin{bmatrix} y_1(k-1) \\ y_1(k-2) \\ u_1(k-1) \\ y_2(k-1) \\ y_2(k-2) \\ u_2(k-1) \end{bmatrix} + \begin{bmatrix} -0.511 \\ -0.435 \end{bmatrix} \times 10^{-}3 \right) \tag{21}$$

The underlying constrained optimization problem can be written as (22).

$$\min_K \left\{ \begin{array}{c} \sum_{k=0}^{N_P} 20 \left[h_1(k) - r_1(k) \right]^2 \\ +20 \left[h_2(k) - r_2(k) \right]^2 + 1.4 \left[u_1(k) - u_1(k-1) \right]^2 \\ +1.4 \left[u_2(k) - u_2(k-1) \right]^2 \end{array} \right\} \tag{22}$$

where $K = [K_{e_1}, K_{\Delta e_1}, K_{\Delta u_1}, K_{e_2}, K_{\Delta e_2}, K_{\Delta u_2}]$ subject to the system dynamics and to

$$0 \leq y(k) \leq 1, k = 1, ..., N_P \tag{23}$$

$$0 \leq u(k) \leq 5, k = 0, ..., N_P - 1 \tag{24}$$

$$K_e \geq 0 \tag{25}$$

$$K_{\Delta e} \geq 0 \tag{26}$$

$$K_{\Delta u} \geq 0 \tag{27}$$

with $u(k)$ given by:

$$u(k) = u(k-1) + K_{\Delta u} \times \Delta u(k) \tag{28}$$

The outcome from this optimization problem is the set of suboptimal gains, in the case of non-global solutions, for both PI-Fuzzy controllers, assuming the interaction effects between tanks as a non-measurable disturbance.

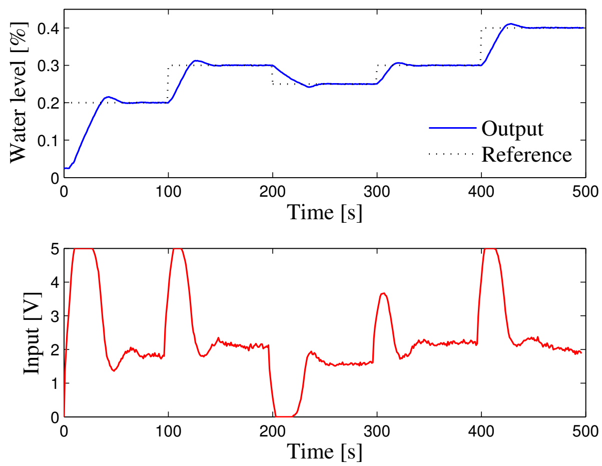

Fig. 11. Tank 1 of a MIMO system controlled with a optimal PI-Fuzzy controller.

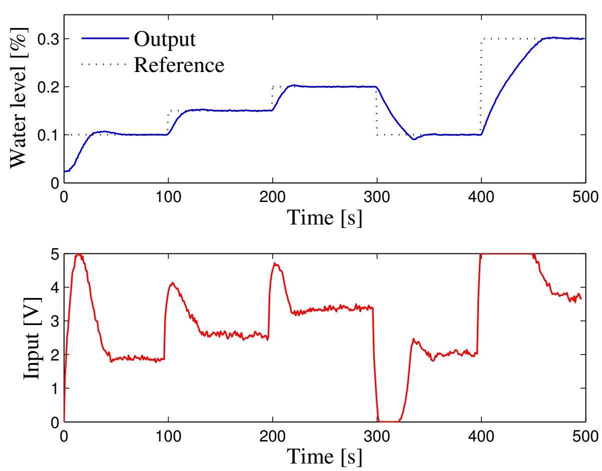

Fig. 12. Tank 2 of a MIMO system controlled with a optimal PI-Fuzzy controller.

Fig. 11 and Fig. 12 show the closed loop system response in a context where the gains were found in accordance with the proposed offline optimization scheme. As can be observed, the closed system response is quite satisfactory, exhibiting short settling time with a tiny overshoot. Furthermore, in what the control actions is concerned, they show a smoother behavior than the classical based tuning counterpart. In order to get a clear picture of the comparative performance between these two PI-Fuzzy controllers two metrics, as in the previous case study, were computed and presented in the following table.

As can be inferred from the table above, both control systems responses show a similar small RMSE, which are

TABLE III

PERFORMANCE METRICS FOR THE THREE-TANK SYSTEM.

Controller	**Tank**	**RMSE[%]**	**RMSI**
Conventional based PI - Fuzzy controller	T_1	0.1047	0.1047
	T_2	0.2082	0.0917
Optimal PI-Fuzzy controller	T_1	0.1257	0.0068
	T_2	0.2126	0.0082

in line with the corresponding performances in terms of steady state error. However, with regard to RMSI the results favor the tuning approach based on the constrained nonlinear optimization.

IV. CONCLUSIONS

This paper addressed the problem of tuning the gains of Proportional Plus Integral Fuzzy Controllers. The proposed approach takes into account the closed loop control system performance and is formulated in terms of a constrained nonlinear optimization problem. Results from two case studies demonstrated the effectiveness and pertinence of the proposed methodology, not only in terms of easing and automating the tuning task, but also in what the closed loop system performance is concerned.

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