

# Study on Observer-Based Robust Control of the Vehicle Lateral Dynamics via TS Model with Unmeasurable Premise Variables

N. Daraoui, O. Pagès and A. El Hajjaji

**Abstract**—This paper introduces a fuzzy observer-based control approach for the vehicle lateral dynamics. The Takagi-Sugeno (TS) fuzzy model is utilized to describe the dynamics of a nonlinear time-varying lateral system. Based on the fuzzy model, a fuzzy observer is developed to estimate the side slip angle using the yaw velocity measurement. The objective of this study is twofold: first, the observer-based controller is designed to improve the stability of the vehicle although perturbations related to the nonlinearities of the contact force models, the variations of the adherence coefficients, etc. Second, the proposed strategy takes into account the unmeasurable membership functions of the TS formulation in the design of the observer.

**Keywords** Takagi-Sugeno fuzzy model, lateral dynamics, vehicle skid control, unmeasurable premise functions, side slip angle estimation, LMI.

## I. INTRODUCTION

In the last decade, research on road safety has received great attention. Several driver assistance systems such as active safety systems like ABS (Anti-lock Bracking System) and ESP (Electronic Stability Program), and passive safety systems such as airbags, have been developed, [25]-[16]. Other research is made to understand in detail the mechanisms of accidents [16]. This understanding goes through the knowledge of the vehicle dynamic behavior in critical situations. Among these studies, researchers have focused on the improvement of the vehicle lateral dynamics in critical situations, i.e, road state variation, emergency braking, skid in cornering, etc. We refer the reader, for example to [17]-[6]-[22]-[14]. The effectiveness of a vehicle dynamic control system is based on the accurate knowledge of the vehicle states, particularly the yaw rate as well as the side slip angle. While the yaw rate is measured in passenger vehicles with inexpensive sensors, the side slip angle must be estimated by advanced observation techniques. This problem has been addressed by many researchers, [3]-[20]. The most difficult step for the side slip angle estimation methods is how to deal with the nonlinear nature of the vehicle dynamics. This nonlinear behaviour is due mainly to the tire force saturation in large tire slip angle regions, determined by the tires, the road adherence coefficients and the steering angle. Using the approximation of the tire forces, the nonlinear vehicle lateral dynamics are expressed by a TS fuzzy model which is largely applied to solve control and estimation problems met in the nonlinear systems [18]-[24]-[1]-[22]. A fuzzy observer is designed to estimate the side slip angle by only considering

the yaw rate measurement. In this fuzzy observer, the nonlinear tires characteristics are preserved, which gives more accurate results in different operating situations compared to linear observers. This idea was already addressed in some papers, [20]-[2], in which the authors do not take into account the influence of the state variable (side slip angle) on the membership functions of the TS model. The observer design problem for TS systems with unmeasurable membership functions is studied by [12]-[5]. In the following study, we consider the active rear steering for controlling the vehicle lateral dynamics while leaving the front steering angle under direct control of the driver. We propose a method to design a fuzzy observer-based controller for the vehicle lateral model with unmeasurable membership functions. The fuzzy controller and observer design is obtained by one step procedure. The proposed observer-based controller approach is represented in terms of LMI (Linear Matrix Inequalities) which can be efficiently solved by using the existing LMI solvers.

The paper is organized as follows: the description of the nonlinear vehicle lateral model with its TS approximation is presented in section II. The design procedure for developing the fuzzy controller and observer will be presented in section III. In section IV, the computer simulations of the obtained theoretical results for the vehicle lateral dynamics are given. Finally, some conclusions are drawn in section V.

The notations used throughout the paper are quite standard:  $\mathbb{R}^n$  is the  $n$ -dimensional real-valued space. For a real-valued matrix  $P$ ,  $P^T$  denotes its transpose,  $P = P^T > 0$  means that  $P$  is symmetric positive-definite.  $*$  is used to denote the transposed elements in the symmetric positions.  $\mathbb{I}, \mathbf{0}$  denote, respectively, the identity and zero matrices with compatible dimensions. For any nonsingular square matrix  $X$ ,  $X^{-1}$  is its inverse.

## II. VEHICLE MODEL ANALYSIS

### A. Nonlinear model of the vehicle lateral dynamics

As we are interested only in the lateral control, we consider the mathematical model, named bicycle model, describing the vehicle lateral dynamics with the nonlinear tire characteristics as introduced in [8](see figure 1):

$$\begin{cases} m v \dot{\beta} = 2(F_f + F_r) - m v \dot{\psi}, \\ I_z \ddot{\psi} = 2(a_f F_f - a_r F_r) \end{cases} \quad (1)$$

where  $\beta$  denotes the side slip angle,  $\dot{\psi}$  is the yaw velocity,  $F_f, F_r$  are the cornering forces of the front and rear tires respectively and  $v$  is the vehicle velocity assuming constant.  $I_z$  is the yaw moment of inertia,  $m$  is the vehicle mass,  $a_f$  is

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the distance from the front axle to the center of gravity and  $a_r$  is the distance from the rear axle to the center of gravity. In this work, we use Pacejka's formulation [7] to express

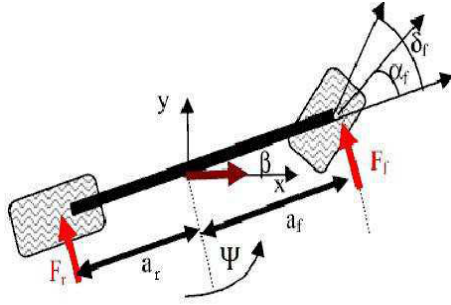


Fig. 1. Vehicle lateral model

the cornering forces  $F_f$  and  $F_r$ . These forces are given as functions of tire slip angles as follows:

$$\begin{aligned} F_f &= D_f(\lambda) \sin [C_f(\lambda) U_{1f}], \\ F_r &= D_r(\lambda) \sin [C_r(\lambda) U_{1r}]. \end{aligned} \quad (2)$$

with  $U_{1i} = \arctan [U_{2i}(\lambda) \alpha_i + E_i(\lambda) \arctan \{B_i(\lambda) \alpha_i\}]$ ,  $U_{2i}(\lambda) = B_i(\lambda)(1 - E_i(\lambda))$ ,  $i = f, r$ . The coefficients  $B_f, B_r, C_f, C_r, D_f, D_r$  depend on the tire characteristics and the road adhesion coefficient  $\lambda$ .  $\alpha_f$  and  $\alpha_r$  respectively denote the slip angle at the front and rear tires, which are given by:

$$\begin{cases} \alpha_f = \delta_f - \beta - \arctan \left( \frac{a_f}{v} \dot{\psi} \cos(\beta) \right), \\ \alpha_r = \delta_r - \beta + \arctan \left( \frac{a_r}{v} \dot{\psi} \cos(\beta) \right). \end{cases} \quad (3)$$

where  $\delta_f$  and  $\delta_r$  are respectively the front and rear steer angles.

The model parameters (2)-(3) are summarized in the table I. To match the purpose of this paper, the tire parameters are

TABLE I  
VEHICLE PARAMETERS

Parameters	$I_z$	m	$a_f$	$a_r$	$v$
Values	3000kg.m <sup>2</sup>	1200 kg	1.2 m	1.45 m	18m/s

considered for low friction roads and high friction roads as given in Table II. In this sequel, the rear steer angle is the only manipulated variable.

TABLE II  
TIRE PARAMETERS

		High friction road	Low friction road
Front	$B_f$	6.7651	11.275
	$C_f$	1.3	1.56
	$D_f$	6436.8	2574.7
	$E_f$	-1.99	-1.999
Rear	$B_r$	9.0051	18.631
	$C_r$	1.3	1.56
	$D_r$	5430	1749.7
	$E_r$	-1.7908	-1.7908
Adhesion coefficient		0.7	0.35

## B. Takagi-Sugeno model of the vehicle lateral dynamics

The goal of this section is to approximate the nonlinear behaviour of the cornering forces  $F_f, F_r$  by a TS fuzzy model in order to describe the entire operating regions of these forces.

Let us consider two fuzzy sets  $M_1$  and  $M_2$  defined for two slip regions, i.e.,  $M_1$  is the fuzzy symbol for the high slip region,  $M_2$  for the low slip region. The front and rear cornering forces can be described by the following rules:

$$\begin{aligned} \text{If } |\alpha_f| \text{ is } M_1 \text{ then } F_f &= C_{f1}(\lambda) \alpha_f, F_r = C_{r1}(\lambda) \alpha_r, \\ \text{If } |\alpha_f| \text{ is } M_2 \text{ then } F_f &= C_{f2}(\lambda) \alpha_f, F_r = C_{r2}(\lambda) \alpha_r. \end{aligned}$$

The membership function parameters and the stiffness coefficients  $C_{fi}, C_{ri}, i = 1, 2$ , depend on the adhesion coefficient  $\lambda$  and are obtained using the identification method based on the Levenberg-Marquardt's algorithm [13]. For the road adhesion coefficient  $\lambda = 0.7$ , the membership functions are defined as follows:

$$h_1(\alpha_f) = \frac{w_1(\alpha_f)}{w_1(\alpha_f) + w_2(\alpha_f)}, h_2(\alpha_f) = 1 - h_1(\alpha_f).$$

where,  $w_i(\alpha_f) = \frac{1}{\left(1 + \left|\frac{\alpha_f - a_1}{b_1}\right|\right)^{c_1}}$ ,  $i = 1, 2$ ,

and

$$\begin{cases} a_1 = 3.1893, b_1 = 0.5077, c_1 = 0.9496, \\ a_2 = 0.5633, b_2 = 5.3907, c_2 = 0.8712 \end{cases}$$

The overall cornering forces are written as follows:

$$\begin{cases} F_f = h_1(\alpha_f) C_{f1}(\lambda) \alpha_f + h_2(\alpha_f) C_{f2}(\lambda) \alpha_f, \\ F_r = h_1(\alpha_f) C_{r1}(\lambda) \alpha_r + h_2(\alpha_f) C_{r2}(\lambda) \alpha_r. \end{cases} \quad (4)$$

For more details, the reader can refer to [1]-[2]-[17].

Under the assumption of small angles, the linear approximations of  $\alpha_f$  and  $\alpha_r$  are given by:

$$\begin{cases} \alpha_f \cong \delta_f - \beta - \frac{a_f}{v} \dot{\psi} \\ \alpha_r \cong \delta_r - \beta + \frac{a_r}{v} \dot{\psi} \end{cases} \quad (5)$$

Taking account of the expressions (5), the nonlinear model (1) can be approximated by the following TS fuzzy model:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^2 h_i(\alpha_f) (A_i x(t) + B_i u(t) + F_i \delta_f), \\ y(t) = Cx(t). \end{cases} \quad (6)$$

where,  $x(t) = \begin{bmatrix} \beta \\ \dot{\psi} \end{bmatrix}$  is the state vector,  $u(t) = \delta_r$  is the control input,  $y(t) = \dot{\psi}$  is the output vector and the matrices  $A_i, B_i, F_i, i = 1, 2$  are given in [2].

Contact forces change according to the road environment caused by the variation of the friction between the tires and the road. Thus, the stiffness coefficients change according to the road adhesion coefficient  $\lambda$ . Taking into account these variations, we describe the stiffness coefficients by:

$$C_{fi} = C_{fi0}(1 + z_i \Delta_i), C_{ri} = C_{ri0}(1 + z_i \Delta_i), i = 1, 2.$$

where  $|\Delta_i| \leq 1$  and  $z_i$  depend on the road environment and describe the derivation magnitude from the nominal value.

Therefore, taking into account these uncertainties, TS fuzzy model (6) can be rewritten as:

$$(\Sigma) : \begin{cases} \dot{x}(t) = \sum_{i=1}^2 h_i(\alpha_f) [A_{di}x(t) + B_{di}u(t) + F_{di}\delta_f(t)], \\ y(t) = Cx(t). \end{cases}$$

with,  $A_{di} = A_i + \Delta A_i$ ,  $B_{di} = B_i + \Delta B_i$ ,  $F_{di} = F_i + \Delta F_i$ . The structured uncertainties considered here are norm-bounded in the form, for each rule:

$$\Delta A_i = H_{ai}\Delta_i(t)E_{ai}, \Delta B_i = H_{bi}\Delta_i(t)E_{bi}, \Delta F_i = H_{fi}\Delta_i(t)E_{fi}$$

where  $\Delta_i(t)$  is an unknown matrix function satisfying:

$$\Delta_i^T(t)\Delta_i(t) \leq \mathbb{I}, i = 1, 2$$

$H_{ai}, H_{bi}, H_{fi}, E_{ai}, E_{bi}, E_{fi}$  are known real constant matrices given in [2].

The following lemmas are useful to establish our main results.

*Lemma 2.1:* [10] Given constant matrices  $D$  and  $E$ , symmetric constant matrix  $S$  and unknown constant matrix  $F$  of appropriate dimension satisfying the constraint  $F^T F < R$ . The following two propositions are equivalent:

- (i)  $S + DFE + E^T F^T D^T < 0$ ;
- (ii)  $S + \begin{bmatrix} E^T & D \end{bmatrix} \begin{bmatrix} \varepsilon^{-1}R & 0 \\ 0 & \varepsilon\mathbb{I} \end{bmatrix} \begin{bmatrix} E \\ D^T \end{bmatrix} < 0$  for some  $\varepsilon > 0$ .

*Lemma 2.2:* [21] Considering a negative definite matrix  $\Xi < 0$ , a given matrix  $Z$  and a scalar  $\mu > 0$ , the following holds:

$$Z^T \Xi Z < -\mu (Z^T + Z) - \mu^2 \Xi^{-1}.$$

### III. OBSERVER-BASED CONTROLLER DESIGN

The side slip angle measurement is necessary to the control design. However, since the side slip angle sensor is very expensive, we need to estimate this value using the output measurement. We furthermore remark that the membership functions depend on the state variable  $\beta$  since it depends on  $\alpha_f$  (3). Thus let us denote  $\hat{\alpha}_f$  the estimate of  $\alpha_f$ . The system  $(\Sigma)$  can be rewritten in the following form:

$$(\Sigma_1) : \begin{cases} \dot{x}(t) = \sum_{i=1}^2 h_i(\hat{\alpha}_f) [A_{di}x(t) + B_{di}u(t) + w(t)], \\ y(t) = Cx(t). \end{cases}$$

with,  $w(t) = (F_i + \Delta F_i)\delta_f(t) + v(t)$ , and

$$v(t) = \sum_{i=1}^2 (h_i(\alpha_f) - h_i(\hat{\alpha}_f)) [A_{di}x(t) + B_{di}u(t) + F_{di}\delta_f(t)].$$

Based on the structure of the TS model  $(\Sigma_1)$ , the following fuzzy state observer is proposed:

$$\dot{\hat{x}}(t) = \sum_{i=1}^2 h_i(\hat{\alpha}_f) [A_i\hat{x}(t) + B_iu(t) + G_i(y(t) - \hat{y}(t))] \quad (7)$$

The matrices  $G_i, i = 1, 2$  are observer gains to be determined. To stabilize this class of systems, we use the PDC (Parallel Distributed Compensation) observer-based controller [15] defined as:

$$\begin{cases} \text{If } |\hat{\alpha}_f| \text{ is } M_1 \text{ then } u(t) = -K_1\hat{x}(t), \\ \text{If } |\hat{\alpha}_f| \text{ is } M_2 \text{ then } u(t) = -K_2\hat{x}(t). \end{cases}$$

The overall fuzzy controller is represented by:

$$u(t) = -\sum_{i=1}^2 h_i(\hat{\alpha}_f) K_i \hat{x}(t). \quad (8)$$

Let us denote the estimation error by:

$$e(t) = x(t) - \hat{x}(t).$$

and the perturbation terms on the membership functions by:

$$\begin{aligned} \mu_{1i}(t) &= h_i(\alpha_f)x(t) - h_i(\hat{\alpha}_f)\hat{x}(t), \\ \mu_{2i}(t) &= (h_i(\alpha_f) - h_i(\hat{\alpha}_f))u(t), \\ \tilde{h}_i &= h_i(\alpha_f) - h_i(\hat{\alpha}_f). \end{aligned} \quad (9)$$

*Assumption 3.1:* We assume that, for each  $i = 1, 2$ , the functions  $\mu_{1i}$  and  $\mu_{2i}$  are Lipschitz and verify:

- $\|\mu_{1i}(t)\| \leq \theta_{1i} \|x(t) - \hat{x}(t)\|$
- $\|\mu_{2i}(t)\| \leq \theta_{2i} \|x(t) - \hat{x}(t)\|$

*Remark 3.1:* Under this assumption, we can easily verify that the disturbance vector  $v(t)$  is bounded. In fact, the term  $v(t)$  can be rewritten as:

$$v(t) = \sum_{i=1}^2 [A_{di}\mu_{1i}(t) + B_{di}\mu_{2i}(t) - h_i(\hat{\alpha}_f)A_{di}e(t)] + \sum_{i=1}^2 \tilde{h}_i F_{di} \delta_f(t).$$

and using the membership function characteristics, we have:

$$-1 \leq \tilde{h}_i = h_i(\alpha_f) - h_i(\hat{\alpha}_f) \leq 1$$

The estimation error dynamics are given by:

$$\begin{aligned} \dot{e}(t) &= \sum_{i=1}^2 \sum_{j=1}^2 h_i(\hat{\alpha}_f) h_j(\hat{\alpha}_f) [(A_i - G_i C + \Delta B_i K_j) e(t) \\ &\quad + (\Delta A_i - \Delta B_i K_j) x(t) + (F_i + \Delta F_i) \delta_f(t) + v(t)] \end{aligned}$$

Taking into account the expression of the control law (8), the augmented system  $\tilde{x}(t) = \begin{bmatrix} x(t) & e(t) \end{bmatrix}^T$  can be written as follows:

$$\dot{\tilde{x}}(t) = \sum_{i=1}^2 \sum_{j=1}^2 h_i(\hat{\alpha}_f) h_j(\hat{\alpha}_f) (\tilde{A}_{ij} \tilde{x}(t) + \tilde{H}_i \tilde{w}(t)). \quad (10)$$

with,

$$\begin{aligned} \tilde{A}_{ij} &= \begin{bmatrix} A_i - B_i K_j + \Delta A_i - \Delta B_i K_j & (B_i + \Delta B_i) K_j \\ \Delta A_i - \Delta B_i K_j & A_i - G_i C + \Delta B_i K_j \end{bmatrix}, \\ \tilde{H}_i &= \begin{bmatrix} H_i \\ H_i \end{bmatrix}, \tilde{w}(t) = \begin{bmatrix} \delta_f(t) \\ v(t) \end{bmatrix}, H_i = \begin{bmatrix} F_i + \Delta F_i & \mathbb{I} \end{bmatrix}. \end{aligned} \quad (11)$$

To guarantee the global asymptotic stability of the equilibrium point zero of (10), the following theorem is established where the sets of matrices  $K_i$  and  $G_i$  are designed in one step procedure.

*Theorem 3.1:* For given scalars  $\gamma, \mu$  and  $\varepsilon_{ij}, i, j = 1, 2$ , if there exist symmetric matrices  $X > 0, Y > 0$ , matrices  $V_j$  and  $W_j, j = 1, 2$ , solutions of the following LMIs:

$$\begin{cases} \Psi_{ii} \leq 0, & i = 1, 2, \\ \Psi_{ii} + \frac{1}{2} (\Psi_{ij} + \Psi_{ji}) \leq 0, & i \neq j \leq 2. \end{cases} \quad (12)$$

where  $\Psi_{ij} = \begin{bmatrix} \Gamma_{1ij} & \Upsilon_{1ij} & \mathbf{0} \\ * & \Gamma_{2ij} & \Upsilon_{2ij} \\ * & * & \Gamma_{3ij} \end{bmatrix}$ ,

and,

$$\Gamma_{1ij} = \begin{bmatrix} \mathbb{N}_{ij} & XE_{ai}^T & V_j^T E_{bi}^T \\ * & -0.5\varepsilon_{ij}\mathbb{I} & \mathbf{0} \\ * & * & -0.5\varepsilon_{ij}\mathbb{I} \end{bmatrix},$$

$$\Gamma_{2ij} = \begin{bmatrix} -2\mu X & \mathbf{0} & \mathbf{0} & V_j^T E_{bi}^T \\ * & -2\mu\mathbb{I} & \mathbf{0} & \mathbf{0} \\ * & * & -2\mu\mathbb{I} + 2\varepsilon_{ij}^{-1}E_{fi}^T E_{fi} & \mathbf{0} \\ * & * & * & -0.5\varepsilon_{ij}\mathbb{I} \end{bmatrix},$$

$$\Gamma_{3ij} = \begin{bmatrix} \mathbb{N}_{2ij} & YH_{ai} & YH_{bi} & YH_{fi} & YF_i & Y \\ * & -\varepsilon_{ij}^{-1}\mathbb{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & -0.5\varepsilon_{ij}^{-1}\mathbb{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\varepsilon_{ij}^{-1}\mathbb{I} & \mathbf{0} & \mathbf{0} \\ * & * & * & * & -\gamma^2\mathbb{I} & \mathbf{0} \\ * & * & * & * & * & -\gamma^2\mathbb{I} \end{bmatrix},$$

$$\Upsilon_{1ij} = \begin{bmatrix} B_i V_j & F_i & \mathbb{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\Upsilon_{2ij} = \begin{bmatrix} \mu\mathbb{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mu\mathbb{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\mathbb{N}_{1ij} = A_i X - B_i V_j + X A_i^T - V_j^T B_i^T + \varepsilon_{ij} H_{ai} H_{ai}^T + 2\varepsilon_{ij} H_{bi} H_{bi}^T + \varepsilon_{ij} H_{fi} H_{fi}^T,$$

$$\mathbb{N}_{2ij} = Y A_i - W_i C + A_i^T Y - C^T W_i^T + \mathbb{I}.$$

then the uncertain fuzzy system (10) is globally asymptotically stable. The observer and the controller gains (7-8) are given by :  $K_j = V_j X^{-1}$  and  $G_i = Y^{-1} W_i$ , respectively, for  $i, j = 1, 2$ .

*Proof 1:* The proof is given in Appendix 1.

*Remark 3.2:* Theorem 3.1 has an advantage to give both the controller and the observer gains in one step [18]. The theorem provides a stabilization conditions in LMI form, which are solvable using different toolboxes (Matlab LMI toolbox, SeDuMi, etc.). If necessary, the number of the LMIs (12) can be easily reduced by setting  $\varepsilon_{ij} = \varepsilon, \forall i, j$ .

#### IV. SIMULATION RESULTS

The proposed observer based controller is evaluated through computer simulations for a vehicle travelling at a constant velocity  $v = 18m/s$  and doing a lane change manoeuvre with a front steer angle pattern as shown in Fig. 2. The considered nominal stiffness coefficients are given in Table III, for the dry road friction coefficient  $\lambda = 0.7$ . For

TABLE III  
NOMINAL STIFFNESS COEFFICIENTS

Stiffness coefficient	$C_{f10}$	$C_{f20}$	$C_{r10}$	$C_{r20}$
Nominal values (N/rad)	71946	11213	67847	7868

the simulation purpose, we consider  $z_i = 0.1$ , which means that the designed controller will ensure stability although the stiffness coefficients change until 10% from their nominal values.

Using the toolbox YALMIP for the resolution of (12) with

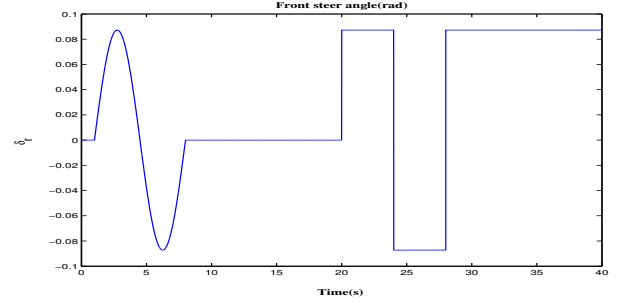


Fig. 2. Front steer angle  $\delta_f$

$\gamma = 10^{-3}$ , we obtain:

$$K_1 = \begin{bmatrix} 0.8364 & -2.1358 \end{bmatrix}, G_1 = \begin{bmatrix} 48.469 \\ 463.617 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 0.0316 & -0.0716 \end{bmatrix}, G_2 = \begin{bmatrix} 37.733 \\ 368.105 \end{bmatrix}.$$

All simulations are carried out on the nonlinear model given by (1) with the road friction coefficient equal to 0.7. We can

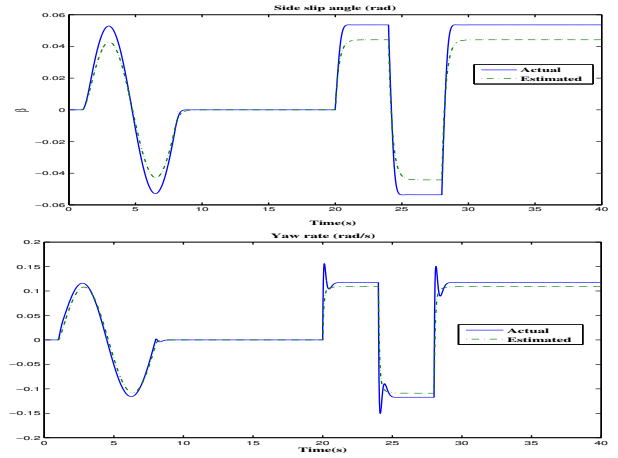


Fig. 3. Behaviour of the state vector and its estimation, solid curve: state vector; and dash-dotted curve: its estimation for  $\lambda = 0.7$

see the effectiveness of the proposed fuzzy observer-based controller, i.e, the estimated state variables are compared to the vehicle state variables for the same front steering angle given by Fig.2. The Fig.4 shows the rear steering angle  $\delta_r$ , considered as the control vector. Fig.5 shows a comparison between three cases: the open loop case, the linear controller which it is obtained by resolving (12) for  $K_i = K, i = 1, 2$  and the proposed fuzzy controller. To show the robustness of our approach, the simulations, as given in Fig. 6, are also done with the adhesion coefficient equal to  $\lambda = 0.35$ , with the parameters given in Table II. Fig. 6 (b) shows the vehicle trajectories with the fuzzy controller and without control for the same front steer angle 2. We can see that the fuzzy observer-based controller gives better performances than the linear controller as well as the open loop case.

Furthermore, the assumption 3.1 is verified by simulation and the obtained fuzzy observer-based controller yields satisfac-

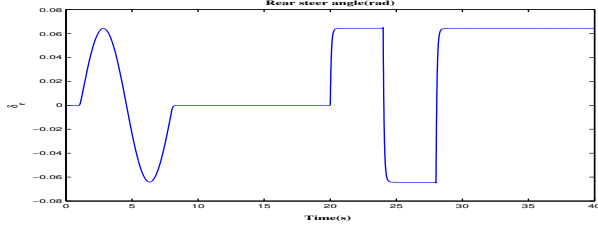


Fig. 4. Control signal evolution  $u(t)$

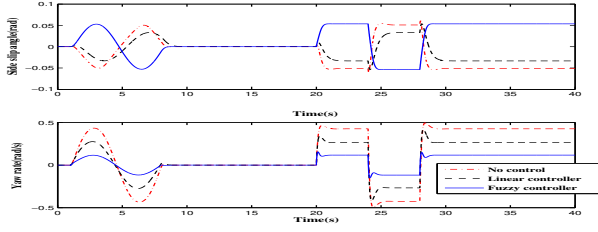


Fig. 5. Vehicle response for the lane change with  $\lambda = 0.7$

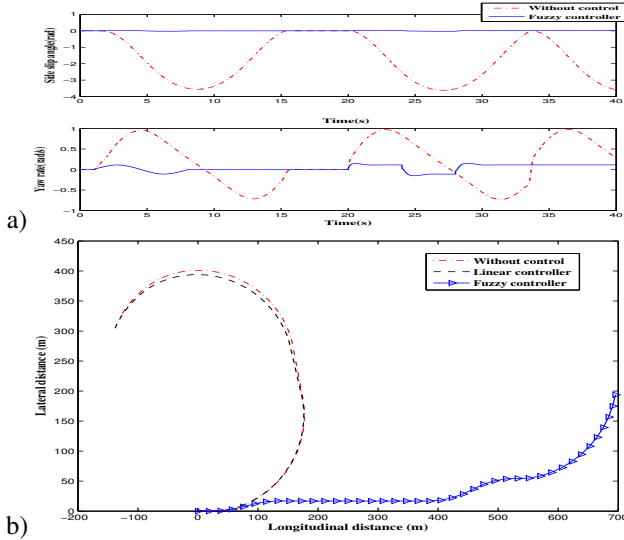


Fig. 6. Vehicle response for the lane change with  $\lambda = 0.35$

tory results with the vehicle simulator CarSim [19]. These obtained results are omitted due to lack of space.

## V. CONCLUSION

In this study, an observer-based controller for the vehicle lateral dynamics has been developed. The dynamics are expressed by an uncertain TS fuzzy model. The main objectives have been to design an observer-based controller without using the separation principle, but by taking into account the state variable dependence on the membership functions of the TS model. To achieve these objectives, Lyapunov's theory is used, and stabilization conditions in LMI form are given. The simulation results have shown the effectiveness of this proposed scheme. Future work will be extended to apply the pole placement technique to improve the observer

dynamics while eliminating the Lipschitz assumption on the membership functions.

## VI. APPENDIX

Consider the following Lyapunov function candidate:

$$V(\tilde{x}(t)) = \tilde{x}^T(t)P\tilde{x}(t), \quad P = P^T > 0. \quad (13)$$

The time derivative of  $V(\tilde{x}(t))$  along the trajectory of (10) is given by:

$$\begin{aligned} \dot{V}(\tilde{x}(t)) &= \sum_{i,j=1}^2 h_i(\hat{\alpha}_f)h_j(\hat{\alpha}_f) \left[ \tilde{x}^T(t) \left( P\tilde{A}_{ij} + \tilde{A}_{ij}^T P \right) \tilde{x}(t) + \right. \\ &\quad \left. \tilde{x}^T(t)P\tilde{H}_i\tilde{\omega}(t) + \tilde{\omega}^T(t)\tilde{H}_i^T P\tilde{x}(t) \right] \end{aligned}$$

The objective is to design the observer-based controller i.e. to find  $K_i, G_i, i = 1, 2$  so that the estimation error converges to zero. The observer convergence is studied taking into account the dependence between the estimation error and the exogenous signals of (10). Thus, we seek to satisfy the following robust performance under zero initial conditions:

$$\frac{\|e(t)\|_2}{\|\tilde{\omega}(t)\|_2} \leq \gamma, \quad \text{for } \tilde{\omega}(t) \neq 0 \quad (14)$$

where  $\gamma$  is the desired disturbance attenuation parameter. This objective is guaranteed by ensuring the following equality:

$$\dot{V}(\tilde{x}) + e^T(t)e(t) - \gamma^2 \tilde{\omega}^T(t)\tilde{\omega}(t) < 0 \quad (15)$$

Therefore, we have,

$$\begin{aligned} \dot{V}(\tilde{x}) + e^T(t)e(t) - \gamma^2 \tilde{\omega}^T(t)\tilde{\omega}(t) &= \\ &= \sum_{i,j=1}^2 h_i(\hat{\alpha}_f)h_j(\hat{\alpha}_f) \begin{bmatrix} \tilde{x}(t) \\ \tilde{\omega}(t) \end{bmatrix}^T \left( \frac{\Omega_{ij} + \Omega_{ji}}{2} \right) \begin{bmatrix} \tilde{x}(t) \\ \tilde{\omega}(t) \end{bmatrix} \end{aligned}$$

with,

$$\Omega_{ij} = \begin{bmatrix} P\tilde{A}_{ij} + \tilde{A}_{ij}^T P + L^T L & P\tilde{H}_i \\ * & -\gamma^2 \mathbb{I} \end{bmatrix}, \quad L = \begin{bmatrix} \mathbf{0} & \mathbb{I} \end{bmatrix}.$$

By using the relaxation scheme from [11], (15) holds if the following conditions are satisfied:

$$\Omega_{ii} < 0, \quad i = 1, 2 \quad (16)$$

$$\Omega_{ii} + \frac{1}{2}(\Omega_{ij} + \Omega_{ji}) < 0, \quad i \neq j \leq 2 \quad (17)$$

Let us consider the particular form of  $P$

$$P = \begin{bmatrix} P_1 & \mathbf{0} \\ \mathbf{0} & P_2 \end{bmatrix}, \quad P_1 = P_1^T > 0, \quad P_2 = P_2^T > 0 \quad (18)$$

By substituting (11), the matrix  $\Omega_{ij}$  can be expressed by:

$$\Omega_{ij} = S_{ij} + \mathbb{W}_{ij} + \mathbb{W}_{ij}^T.$$

with,

$$\begin{aligned} \mathbb{W}_{ij} &= \begin{bmatrix} P_1(\Delta A_i - \Delta B_i K_j) & \begin{bmatrix} P_1 \Delta B_i K_j + \\ (\Delta A_i - \Delta B_i K_j)^T P_2 \\ K_j^T \Delta B_i^T P_2 \\ \Delta F_i^T P_2 \end{bmatrix} & P_1 \Delta F_i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \\ S_{ij} &= \begin{bmatrix} \begin{bmatrix} P_1(A_i - B_i K_j) + \\ (A_i - B_i K_j)^T P_1 \end{bmatrix} & P_1 B_i K_j & P_1 F_i & P_1 \\ * & \begin{bmatrix} P_2(A_i - G_i C) + \\ (A_i - G_i C)^T P_2 + \mathbb{I} \end{bmatrix} & P_2 F_i & P_2 \\ * & * & -\gamma^2 \mathbb{I} & \mathbf{0} \\ * & * & * & -\gamma^2 \mathbb{I} \end{bmatrix} \end{aligned}$$

According to the definition of matrices  $\Delta A_i$ ,  $\Delta B_i$ ,  $\Delta F_i$ , and using the lemma 2.1, the inequalities (16)-(17) hold if there exist real scalars  $\varepsilon_{ij}$  satisfying the following condition:

$$\tilde{\Omega}_{ii} < 0, i = 1, 2 \quad (19)$$

$$\tilde{\Omega}_{ii} + \frac{1}{2} \left( \tilde{\Omega}_{ij} + \tilde{\Omega}_{ji} \right) < 0, i \neq j \leq 2 \quad (20)$$

with

$$\tilde{\Omega}_{ij} = \begin{bmatrix} \Pi_{ij} & \mathbb{M}_{ij} \\ * & \Phi_{ij} \end{bmatrix} \quad (21)$$

and,

$$\begin{aligned} \Phi_{ij} &= \begin{bmatrix} \Theta_{ij} & P_2 F_i & P_2 \\ * & -\gamma^2 \mathbb{I} + 2\varepsilon_{ij}^{-1} E_{fi}^T E_{fi} & \mathbf{0} \\ * & * & -\gamma^2 \mathbb{I} \end{bmatrix}, \\ \mathbb{M}_{ij} &= [ P_1 B_i K_j \quad P_1 F_i \quad P_1 ], \\ \Pi_{ij} &= P_1 (A_i - B_i K_j) + (A_i - B_i K_j)^T P_1 + \varepsilon_{ij} P_1 H_{ai} H_{ai}^T P_1 + 2\varepsilon_{ij} P_1 H_{bi} H_{bi}^T P_1 + \varepsilon_{ij} P_1 H_{fi} H_{fi}^T P_1 + 2\varepsilon_{ij}^{-1} E_{ai}^T E_{ai} + 2\varepsilon_{ij}^{-1} K_j^T E_{bi}^T E_{bi} K_j, \\ \Theta_{ij} &= P_2 (A_i - G_i C) + (A_i - G_i C)^T P_2 + \varepsilon_{ij} P_2 H_{ai} H_{ai}^T P_2 + 2\varepsilon_{ij} P_2 H_{bi} H_{bi}^T P_2 + \varepsilon_{ij} P_2 H_{fi} H_{fi}^T P_2 + 2\varepsilon_{ij}^{-1} K_j^T E_{bi}^T E_{bi} K_j + \mathbb{I}. \end{aligned}$$

Pre-post multiplying the equality (21) by the matrix  $\text{diag}(X, X, \mathbb{I}, \mathbb{I})$ , with  $X = P_1^{-1}$ , we obtain:

$$\text{diag}(X, X, \mathbb{I}, \mathbb{I}) \tilde{\Omega}_{ij} \text{diag}(X, X, \mathbb{I}, \mathbb{I}) = \begin{bmatrix} \bar{\Pi}_{ij} & \bar{\mathbb{M}}_{ij} \\ * & \bar{\Phi}_{ij} \end{bmatrix} \quad (22)$$

with,

$$\begin{aligned} \bar{\Pi}_{ij} &= A_i X - B_i V_j + X A_i^T - V_j^T B_i^T + \varepsilon_{ij} H_{ai} H_{ai}^T + 2\varepsilon_{ij} H_{bi} H_{bi}^T + \varepsilon_{ij} H_{fi} H_{fi}^T + 2\varepsilon_{ij}^{-1} X E_{ai}^T E_{ai} X + 2\varepsilon_{ij}^{-1} V_j^T E_{bi}^T E_{bi} V_j, \\ \bar{\mathbb{M}}_{ij} &= [ B_i V_j \quad F_i \quad \mathbb{I} ], \\ \bar{\Phi}_{ij} &= \begin{bmatrix} X \Theta_{ij} X & X P_2 F_i & X P_2 \\ * & -\gamma^2 \mathbb{I} + 2\varepsilon_{ij}^{-1} E_{fi}^T E_{fi} & \mathbf{0} \\ * & * & -\gamma^2 \mathbb{I} \end{bmatrix}, \\ \bar{\Theta}_{ij} &= P_2 A_i - W_i C + A_i^T P_2 - C^T W_i^T + \varepsilon_{ij} P_2 H_{ai} H_{ai}^T P_2 + 2\varepsilon_{ij} P_2 H_{bi} H_{bi}^T P_2 + \varepsilon_{ij} P_2 H_{fi} H_{fi}^T P_2 + 2\varepsilon_{ij}^{-1} K_j^T E_{bi}^T E_{bi} K_j + \mathbb{I}. \end{aligned}$$

with the matrices  $V_j, W_i$  are given by:

$$V_j = K_j X, W_i = Y G_i \text{ and } Y = P_2.$$

Matrices  $\bar{\Phi}_{ij}$  can be rewritten in the following form:

$$\bar{\Phi}_{ij} = \mathbb{T}_{ij} + \text{diag}(X, \mathbb{I}, \mathbb{I}) \bar{\mathfrak{T}}_{ij} \text{diag}(X, \mathbb{I}, \mathbb{I}),$$

with,  $\mathbb{T}_{ij} = \text{diag}(2\varepsilon_{ij}^{-1} V_j^T E_{bi}^T E_{bi} V_j, 2\varepsilon_{ij}^{-1} E_{fi}^T E_{fi}, \mathbf{0})$ ,  $\bar{\mathfrak{T}}_{ij} = \begin{bmatrix} \mathfrak{L}_{ij} & Y F_i & Y \\ * & -\gamma^2 \mathbb{I} & \mathbf{0} \\ * & * & -\gamma^2 \mathbb{I} \end{bmatrix}$ ,  $\mathfrak{L}_{ij} = Y A_i - W_i C + A_i^T Y - C^T W_i^T + \varepsilon_{ij} Y H_{ai} H_{ai}^T Y + 2\varepsilon_{ij} Y H_{bi} H_{bi}^T Y + \varepsilon_{ij} Y H_{fi} H_{fi}^T Y + \mathbb{I}$

Using the lemma 2.2, there exists a positif scalar  $\mu > 0$ , such that:

$$\bar{\Phi}_{ij} \leq \mathbb{T}_{ij} - 2\mu \text{diag}(X, \mathbb{I}, \mathbb{I}) - \mu^2 \bar{\mathfrak{T}}_{ij}^{-1}$$

Finally, the expression (22) can be rewritten in the following form:

$$\text{diag}(X, X, \mathbb{I}, \mathbb{I}) \tilde{\Omega}_{ij} \text{diag}(X, X, \mathbb{I}, \mathbb{I}) \leq \begin{bmatrix} \bar{\Pi}_{ij} & \bar{\mathbb{M}}_{ij} \\ * & \mathfrak{S}_{ij} - \mu^2 \bar{\mathfrak{T}}_{ij}^{-1} \end{bmatrix}$$

with,

$$\mathfrak{S}_{ij} = \begin{bmatrix} 2\varepsilon_{ij}^{-1} V_j^T E_{bi}^T E_{bi} V_j - 2\mu X & \mathbf{0} & \mathbf{0} \\ * & -2\mu \mathbb{I} + 2\varepsilon_{ij}^{-1} E_{fi}^T E_{fi} & \mathbf{0} \\ * & * & -2\mu \mathbb{I} \end{bmatrix}$$

Using the Schur's complement several times, sufficient conditions of Theorem 3.1 are established. ■

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