

Nonlinear Stable PID Controller Design using Local Model Networks

Christian H. Mayr^{*}, Christoph Hametner[†], Martin Kozek[‡] and Stefan Jakubek[§]

Christian Doppler Laboratory for Model Based Calibration Methodologies

at the Institute of Mechanics and Mechatronics

Vienna University of Technology

Wiedner Hauptstrasse 8-10/325/A5, 1040 Vienna, Austria

^{*}christian.mayr@tuwien.ac.at

[†]christoph.hametner@tuwien.ac.at

[‡]martin.kozek@tuwien.ac.at

[§]stefan.jakubek@tuwien.ac.at

Abstract—This paper addresses PID controller design using local model networks. The proposed method uses a common quadratic Lyapunov function to guarantee stability of the closed loop. To solve the resulting bilinear matrix inequalities (BMI) an iterative procedure is introduced which is based on state of the art linear matrix inequalities solvers (iLMI). Due to the fact that the Lyapunov approach requires a state-space model a suitable closed-loop state-space system is introduced. An example demonstrates the effectiveness of the proposed method.

Index Terms—Controller design, Lyapunov function, PID Controllers, Local area networks

I. INTRODUCTION

For nonlinear systems model-based controller design is a well established approach. For this purpose local model networks (LMN) from the family of multiple-model approaches (e.g. [1]) offer a versatile structure for the identification of nonlinear static and dynamic systems. These model architectures interpolate between different local models, each valid in a certain operating regime. Each operating regime represents a simple model, e.g. a linear regression model, where the local dynamics are usually defined as transfer functions. Due to the transparency of the LMN structure the incorporation of prior (physical) knowledge is easily possible, [2]. Another widely-used model architecture are Takagi Sugeno (TS) Fuzzy Systems, e.g. [3]. The similarity between the TS and LMN models can be easily noticed if the number of fuzzy if-then rules in the TS model equals the number of local models in an LMN, [4]. The main difference of LMN compared and TS Fuzzy Systems is that LMN are usually generated from measured input-output data. Thus, when LMNs approximate a physical process they may be more complex than usual TS Fuzzy Systems.

When controllers are designed closed-loop stability is a key issue. Basically, the closed-loop may become unstable in the large even if all local closed-loops are stable, [5].

The main contribution of this paper is the stable PID controllers design for local model networks by means of a

Lyapunov function. Due to the fact that both the local dynamics and the PID controller are usually defined as transfer functions and Lyapunov stability criteria require state-space models, the closed-loop, which consists of LMN and PID controllers, is transformed into a suitable state-space model. Controller design methods for LMN by means of a Lyapunov function basically result in BMI and cannot be solved by commonly used solvers such as [6], [7], [8]. Solving such BMI means, that the Lyapunov function *and* the controller parameters are simultaneously determined. For usual state space controllers a simple transformation is used to transform BMI into LMI, see [5], [9]. For PID controllers of LMNs such a transformation is not possible because of a rank deficiency caused by the limited controller parameters. Thus, an iterative method is introduced which uses a standard LMI solver. The basic concept is to determine *either* the Lyapunov function *or* the controller parameters in one iteration step. The used stability criterion is taken from [9], adapted for the considered closed-loop system and extended by a decay rate to provide exponential stability as well as asymptotic stability. In contrast to the existing approach in [10] the presented method is an offline approach and thus needs less calculation effort during the control procedure.

This paper is organized as follows: The architecture of local model networks is briefly described in section II. The PID controller for LMNs is introduced in section III. In section IV the transformation as well as the state-space model of the closed-loop control system is described. Section V addresses the iLMI procedure and reviews the used Lyapunov stability criterion. In section VI the effectiveness of the proposed method is shown by means of an example. The paper is concluded by some remarks in section VII.

II. LOCAL MODEL NETWORKS

The architecture of dynamic local model networks is depicted in Fig. 1. First, an ordered set for the indices of the local models is defined:

$$\mathcal{I} = (i \in \mathbb{N} | 1 \leq i \leq I) \quad (1)$$

where I denotes the number of local linear models. Local model networks have an input vector $\mathbf{r}(k)$ with past inputs

This work was supported by Christian Doppler Research Association and AVL List GmbH

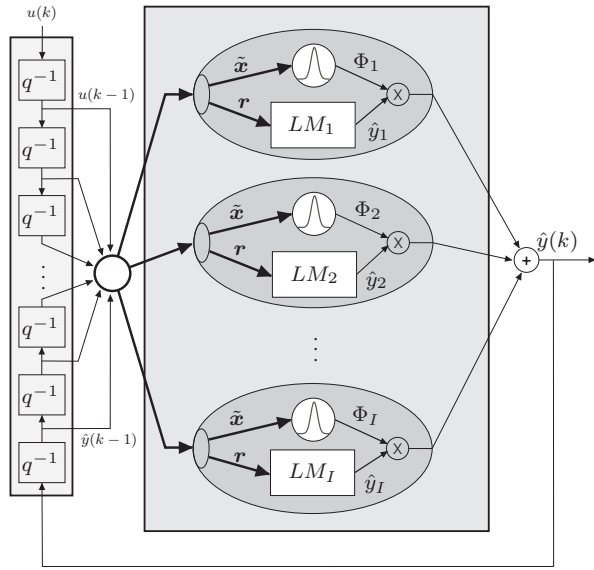


Fig. 1. Architecture of a local model network with external dynamics

and outputs according to Fig. 1:

$$\mathbf{r}^T(k) = [u(k-1) \dots u(k-m) \hat{y}(k-1) \dots \hat{y}(k-n)], \mathbf{r}^T(k) \in \mathbb{R}^{1 \times O} \quad (2)$$

where m denotes the numerator order and n denotes the system order. All local model outputs

$$\hat{y}_i(k) = \mathbf{r}^T(k) \boldsymbol{\theta}_i \quad (3)$$

with the local parameter vectors

$$\boldsymbol{\theta}_i = [b_1^{(i)} \dots b_m^{(i)} a_1^{(i)} \dots a_n^{(i)}]^T, \boldsymbol{\theta}_i \in \mathbb{R}^{O \times 1} \quad (4)$$

are used subsequently to form the global model output $\hat{y}(k)$ by weighted aggregation, see Fig. 1:

$$\hat{y}(k) = \sum_{\mathcal{I}} \Phi_i(\tilde{\mathbf{x}}(k)) \hat{y}_i(k), \quad (5)$$

where the validity functions are constrained to form a partition of unity:

$$\sum_{\mathcal{I}} \Phi_i(\tilde{\mathbf{x}}(k)) = 1, \forall k \in \mathbb{N}^+ \quad (6)$$

$$0 \leq \Phi_i(\tilde{\mathbf{x}}(k)) \leq 1, \forall i \in \mathcal{I}, \forall k \in \mathbb{N}^+. \quad (7)$$

From Fig. 1 it becomes obvious that the input vector $\tilde{\mathbf{x}}(k)$ for the validity functions $\Phi_{\mathcal{I}}$, which lies in the so-called partition space, can be chosen differently to the input vector \mathbf{x} for the local models. This is an important feature of LMN.

III. PID-CONTROLLER FOR LOCAL MODEL NETWORKS

The discrete-time PID control law is as follows, [11]:

$$u(k) = K_P \left[e(k) + \frac{T_S}{T_N} \sum_{i=0}^{k-1} e(i) + \frac{T_V}{T_S} (e(k) - e(k-1)) \right] \quad (8)$$

where T_S denotes the sampling time. The control error is

$$e(k) = w(k) - \hat{y}(k) \quad (9)$$

where $w(k)$ denotes the reference signal. Rewriting (8) for $(k-1)$ gives:

$$u(k-1) = K_P \left[e(k-1) + \frac{T_S}{T_N} \sum_{i=0}^{k-2} e(i) + \frac{T_V}{T_S} (e(k-1) - e(k-2)) \right] \quad (10)$$

Subtracting (10) from (8), one gets the PID control algorithm:

$$u(k) = u(k-1) + d_0 e(k) + d_1 e(k-1) + d_2 e(k-2), \quad (11)$$

with the following coefficients:

$$d_0 = K_P \left[1 + \frac{T_V}{T_S} \right], \quad (12)$$

$$d_1 = K_P \left[\frac{T_S}{T_N} - \frac{2T_V}{T_S} - 1 \right], \quad (13)$$

$$d_2 = K_P \frac{T_V}{T_S}. \quad (14)$$

Eq. (11) can be reformulated by inserting (9) in (11)

$$u(k) = u(k-1) + \underbrace{\mathbf{k}_{PID}^T(k) \mathbf{w}(k) - \mathbf{k}_{PID}^T(k) \hat{\mathbf{y}}(k)}_{\Delta u(k)} \quad (15)$$

with

$$\mathbf{k}_{PID}^T(k) = [d_2(k) \quad d_1(k) \quad d_0(k)], \quad \hat{\mathbf{y}}(k) = \begin{bmatrix} \hat{y}(k-2) \\ \hat{y}(k-1) \\ \hat{y}(k) \end{bmatrix}, \quad \mathbf{w}(k) = \begin{bmatrix} w(k-2) \\ w(k-1) \\ w(k) \end{bmatrix} \quad (16)$$

Control design for LMN mostly involves the design of many local controllers, each of which is associated to a particular local model, yielding a so called local controller network (LCN), [12]. In view of the actual nonlinear system which the LMN is supposed to represent the local model partitioning then represents a meaningful scheduling strategy. In this context there is an associated set of PID parameters $\mathbf{k}_{PID,i}^T$ for each local model which are aggregated to the global time-varying matrix $\mathbf{k}_{PID}^T(k)$ as follows:

$$\mathbf{k}_{PID}^T(k) = \sum_{\mathcal{I}} \Phi_i(\tilde{\mathbf{x}}(k)) \mathbf{k}_{PID,i}^T, \quad \mathbf{k}_{PID,i}^T = [d_2^{(i)} \quad d_1^{(i)} \quad d_0^{(i)}], \forall i \in \mathcal{I} \quad (17)$$

IV. CLOSED-LOOP STATE-SPACE MODEL

A. Basic Concept

To investigate controller design by means of Lyapunov's direct method the system has to be transformed into a suitable state-space notation, see Fig. 2. Thus, the LMN as well as the integrator of the PID-controller are transferred

into a non-minimal state space system. According to Fig. 2 and (15) the PID control law is divided into three parts:

- 1) past input $u(k-1)$ (integrator)
- 2) "filtered" reference signal $v(k)$
- 3) feedback $\mathbf{k}_{PID}^T \hat{\mathbf{y}}(k)$

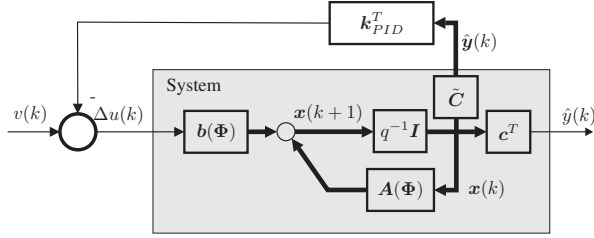


Fig. 2. PID controller in state-space

The system can be generally defined as

$$\mathbf{x}(k+1) = \mathbf{A}(\Phi)\mathbf{x}(k) + \mathbf{b}(\Phi)\Delta u(k) \quad (18)$$

$$\hat{\mathbf{y}}(k) = \mathbf{c}^T \mathbf{x}(k) \quad (19)$$

where the state vector contains time shifted in- and outputs as follows

$$\mathbf{x}(k) = \begin{pmatrix} u(k-o) \\ \vdots \\ u(k-1) \\ \hline \hat{\mathbf{y}}(k-p+1) \\ \vdots \\ \hat{\mathbf{y}}(k-1) \\ \hat{\mathbf{y}}(k) \end{pmatrix}, \quad \mathbf{x} \in \mathbb{R}^{o+p \times 1}. \quad (20)$$

The system matrices are as follows

$$\mathbf{A}(\Phi) = \sum_{\mathcal{I}} \Phi_i(\tilde{\mathbf{x}}(k)) \mathbf{A}_i, \quad \mathbf{A} \in \mathbb{R}^{o+p \times o+p} \quad (21)$$

$$\mathbf{b}(\Phi) = \sum_{\mathcal{I}} \Phi_i(\tilde{\mathbf{x}}(k)) \mathbf{b}_i, \quad \mathbf{b} \in \mathbb{R}^{o+p \times 1} \quad (22)$$

$$\mathbf{c}^T = [0 \ \dots \ 0 \ 1], \quad \mathbf{c}^T \in \mathbb{R}^{1 \times o+p} \quad (23)$$

with

$$o = \max(m-1, 1) \quad (24)$$

$$p = \max(n, 3). \quad (25)$$

B. Notation of \mathbf{A}_i and \mathbf{b}_i

Rewriting (3) for $(k+1)$:

$$\begin{aligned} \hat{\mathbf{y}}_i(k+1) = & u(k)b_1^{(i)} + u(k-1)b_2^{(i)} + \\ & + \dots + u(k-m+1)b_m^{(i)} + \\ & + \hat{\mathbf{y}}(k)a_1^{(i)} + \dots + \hat{\mathbf{y}}(k-n+1)a_n^{(i)} \end{aligned} \quad (26)$$

and inserting $u(k)$ of (15) in (26) the outputs of the local models are as follows:

$$\begin{aligned} \hat{\mathbf{y}}_i(k+1) = & u(k-1)(b_1^{(i)} + b_2^{(i)}) + \Delta u(k)b_1^{(i)} + \\ & + \dots + u(k-m+1)b_m^{(i)} + \\ & + \hat{\mathbf{y}}(k)a_1^{(i)} + \dots + \hat{\mathbf{y}}(k-n+1)a_n^{(i)} \end{aligned} \quad (27)$$

The feedback loop of the input $u(k-1)$ is integrated in the system matrix to provide a feedback matrix with only three entries:

$$\mathbf{A}_i = \hat{\mathbf{A}}_i + \mathbf{b}_i \cdot [\mathbf{0}_{1 \times o-1} \ 1 \ \mathbf{0}_{1 \times p}], \quad \mathbf{A}_i \in \mathbb{R}^{o+p \times o+p} \quad (28)$$

with $\hat{\mathbf{A}}_i$ according to (29) (see next page). The local input matrices \mathbf{b}_i are as follows:

$$\mathbf{b}_i = \begin{bmatrix} \mathbf{0}_{o-1 \times 1} \\ 1 \\ \mathbf{0}_{p-1 \times 1} \\ b_1^{(i)} \end{bmatrix}, \quad \mathbf{b}_i \in \mathbb{R}^{o+p \times 1} \quad (30)$$

Remark 1: The notation of \mathbf{A}_i according to (28) and \mathbf{b}_i according to (30) provides that (27) and the last row of (18) are equal.

C. Feedback Loop

The input vector $\hat{\mathbf{y}}(k)$ of the feedback matrix is calculated as follows:

$$\hat{\mathbf{y}}(k) = \tilde{\mathbf{C}} \mathbf{x}(k) \quad (31)$$

with

$$\tilde{\mathbf{C}} = \begin{bmatrix} 1 & 0 & 0 \\ \mathbf{0}_{3 \times o+p-3} & 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \tilde{\mathbf{C}} \in \mathbb{R}^{3 \times o+p}$$

V. PID PARAMETER DETERMINATION USING iLMI

This section reviews Lyapunov based stability criterion and describes the iterative LMI (iLMI) procedure to determine the PID controller parameters for local model networks.

A. Lyapunov Stability

The global stability of a PID controlled LMN can be proved by Lyapunov's direct method. This general approach is based on a state space formulation of the system, such as (18), and one has to find a suitable Lyapunov function $V(\mathbf{x}) : \mathbb{R}^{o+p} \rightarrow \mathbb{R}$, [13].

Definition 5.1: A Lyapunov function basically has to satisfy four properties to provide *asymptotic* stability of a discrete-time system:

- i) $V(\mathbf{x}(k)) = 0$
- ii) $V(\mathbf{x}(k)) > 0$ for $\mathbf{x}(k) \neq 0$
- iii) $V(\mathbf{x}(k))$ approaches infinity as $\|\mathbf{x}(k)\| \rightarrow \infty$
- iv) $\Delta V(\mathbf{x}(k)) = V(k+1) - V(k) < 0, \forall k \in \mathbb{N}^+$.

Lemma 1: For *exponential* stability, the Lyapunov function must satisfy i)-iii) of Definition 5.1 and decrease strictly monotonically over time k with a decay rate α , $0 < \alpha < 1$:

$$V(k+1) - \alpha^2 V(k) \leq 0, \quad \forall k \in \mathbb{N}^+ \quad (32)$$

The proof of Lemma 1 is given in [14].

When LMN are considered it is common to restrict the search to the class of quadratic Lyapunov functions

$$V(k) = \mathbf{x}^T(k) \mathbf{P} \mathbf{x}(k) > 0, \quad \mathbf{P} \succ 0 \quad (33)$$

$$\hat{\mathbf{A}}_i = \left[\begin{array}{c|c} \mathbf{0}_{o-1 \times 1} & \mathbf{I}_{o-1 \times o-1} \\ \hline & \mathbf{0}_{1 \times o} \\ \mathbf{0}_{p-1 \times o} & \mathbf{0}_{o \times p} \\ \hline b_{o+1}^{(i)} \cdots b_s^{(i)} \cdots b_2^{(i)} & \mathbf{0}_{p-1 \times 1} \quad \mathbf{I}_{p-1 \times p-1} \\ a_p^{(i)} \cdots a_q^{(i)} \cdots a_1^{(i)} & \end{array} \right], \quad \begin{array}{l} \hat{\mathbf{A}}_i \in \mathbb{R}^{o+p \times o+p}, \\ \text{if } o = 1 : b_{o+1}^{(i)} = 0, \\ \forall q > n : a_q^{(i)} = 0, \\ \forall i \in \mathcal{I} \end{array} \quad (29)$$

where \mathbf{P} is a positive definite matrix, [5].

The following theorem provides a statement on exponential stability of a PID controlled LMN using a common quadratic Lyapunov function:

Theorem 1: The equilibrium of the PID controlled dynamic LMN (5) is asymptotically ($\alpha = 1$) or exponentially ($0 < \alpha < 1$) stable via the control law (15) in the large if there exist symmetric matrices \mathbf{P} and \mathbf{X}_{ij} and a decay rate α such that

$$\mathbf{P} \succ 0 \quad (34)$$

$$\mathbf{\Lambda}_{ii}^T \mathbf{P} \mathbf{\Lambda}_{ii} + \mathbf{X}_{ii} \prec \alpha^2 \mathbf{P} \quad (35)$$

$$\mathbf{\Lambda}_{ij}^T \mathbf{P} \mathbf{\Lambda}_{ij} + \mathbf{X}_{ij} \preceq \alpha^2 \mathbf{P} \quad (36)$$

$$\tilde{\mathbf{X}} = \begin{pmatrix} \mathbf{X}_{11} & \mathbf{X}_{12} & \cdots & \mathbf{X}_{1I} \\ \mathbf{X}_{12} & \mathbf{X}_{22} & \cdots & \mathbf{X}_{2I} \\ \vdots & & \ddots & \vdots \\ \mathbf{X}_{1I} & \mathbf{X}_{2I} & \cdots & \mathbf{X}_{II} \end{pmatrix} \succ 0 \quad (37)$$

$\forall i \in \mathcal{I}, \forall i < j \leq I$

with

$$\begin{aligned} \mathbf{\Lambda}_{ii} &= \mathbf{G}_{ii}, \quad \mathbf{\Lambda}_{ij} = \frac{\mathbf{G}_{ij} + \mathbf{G}_{ji}}{2}, \\ \mathbf{G}_{ij} &= \mathbf{A}_i - \mathbf{b}_i \mathbf{k}_{PID,j}^T \tilde{\mathbf{C}}. \end{aligned} \quad (38)$$

The proof of Theorem 1 directly follows from [9] and Lemma 1.

Remark 2: The BMI of Theorem 1 arise from the product of \mathbf{P} and $\mathbf{k}_{PID,i}^T$ which follow from inserting (38) in (35), (36), respectively.

B. iLMI Procedure

Due to the fact that the BMI of Theorem 1 are not solvable by state of the art LMI solvers an iterative method is introduced (iLMI). The basic idea is to use *either* the parameters of the Lyapunov function *or* the PID parameters as decision variables for the LMI solver in one iteration step. The remaining parameters are fixed as calculated in the previous iteration step. The iLMI method described as pseudocode is as follows:

Initial Design:

set $\mathbf{k}_{PID,i}^T, \forall i \in \mathcal{I}$ by an arbitrary method

e.g.: local tuning, LQR, random

$k \leftarrow 0$

Iterative LMI design:

repeat

$k \leftarrow k + 1$

if k is odd **then**

$\mathbf{P} \leftarrow$ Theorem 1 with $\mathbf{k}_{PID,i}^T$ fixed from previous iteration step

set $\mathbf{P} \succ 0$ by using eigendecomposition and setting the negative eigenvalues to a small positive ones {optional}

else

$\mathbf{k}_{PID,i}^T \leftarrow$ Theorem 1 with \mathbf{P} fixed from previous iteration step

end if

until $k \geq k_{max}$ or Theorem 1 satisfied

if Theorem 1 satisfied **then**

$\mathbf{k}_{PID,i}^T$ from last iteration may be used

else

stabilization not possible

end if

It must be noted that the proposed method does not guarantee convergence and the achievable result strongly depends on the initial PID parameters. Thus, the initial PID parameters should be a choice which satisfactory controller performance. Usually BMI are solved by a combination of a Genetic Algorithm (GA) with an usual LMI solver. The main advantage of the proposed approach compared to the common one is the lower numerical effort. In general, the success of the proposed method strongly depends on the number of local models and the system order. The basic concept of the iLMI procedure is illustrated in Fig. V-B.

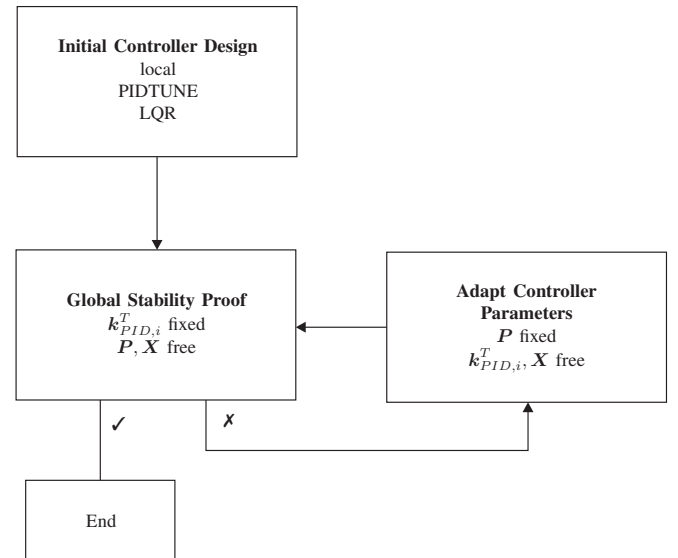


Fig. 3. iLMI basic scheme

VI. EXAMPLE

A stable second order Wiener model is considered. It consists of a dynamic linear block with a normalized transfer

function $G_L(z) = V(z)/U(z)$ in cascade with a static nonlinearity $f(v)$ at the output with v as the intermediate variable at the output of the linear block. For the present simulation results $G_L(z)$ and $f(v)$ where chosen as

$$G_L(z) = \frac{0.0187z^{-1} + 0.0175z^{-2}}{1 - 1.64z^{-1} + 0.6929z^{-2}} \quad (39)$$

$$y(k) = f(v(k)) = \arctan(v(k)). \quad (40)$$

The structure of Wiener systems enable a simple representation of nonlinear systems. The nonlinearity $f(v)$ has full impact on the output and stability analysis can become challenging, in particular when the nonlinearity has a saturation character like in the present example, [15].

In this example the input $u(k)$ is bounded to the interval $[-3, 3]$.

A local model network comprising six local models was generated by the algorithm presented in [16], where the local models are constructed using an axis oblique decomposition of the partition space. Fig. 4 shows the identification data as well as a contour plot of the validity functions where its input vector is as follows:

$$\tilde{x}(k) = [u(k-1) \hat{y}(k-1)]. \quad (41)$$

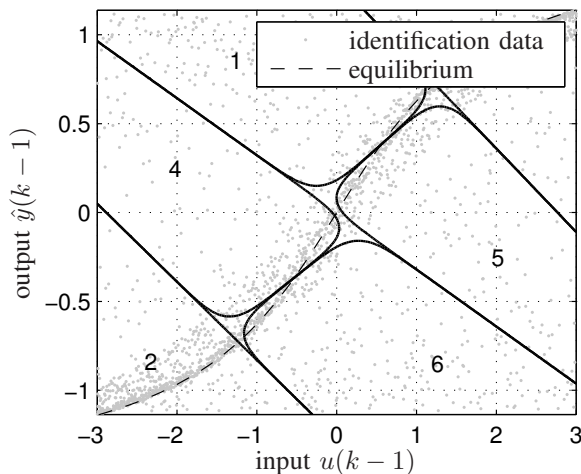


Fig. 4. Contour plot of the validity functions and identification data sequence

Fig. 5 illustrates the outputs of the the Wiener model $y(k)$ and the local model network $\hat{y}(k)$ for the same input sequence $u(k)$. Thus, the good approximation capability of the local model network is illustrated.

In order to demonstrate the effectiveness of the proposed method two different PID controllers are compared. Controller A is the conventional controller where the controller parameters are locally determined by using auto-pid command from the Toolbox of MATLAB[®]. The parameters of controller A are used as initial set for the iLMI procedure. Controller B is the guaranteed stable controller. The parameters of which were determined by means of the iLMI method presented in section V-B. The parameters sets of the two controllers are given in Table I and Table II, respectively.

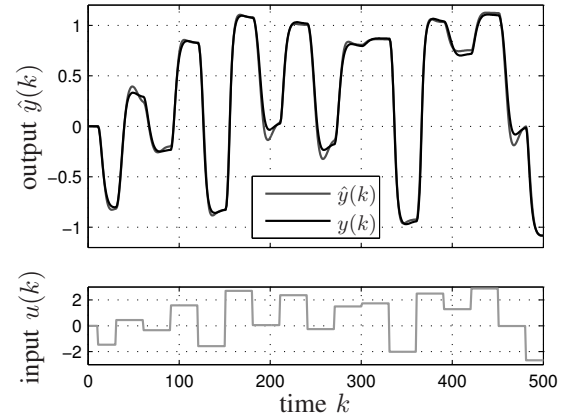


Fig. 5. Comparison of the open-loop behavior of the process and the local model network

TABLE I
PARAMETERS OF CONTROLLER A

Model #	K_p	T_N	T_V
1	9.481	79.817	1.039
2	5.778	4.043	0.649
3	5.519	4.029	0.649
4	2.878	7.418	0.722
5	2.934	7.603	0.726
6	9.240	79.762	0.986

TABLE II
PARAMETERS OF CONTROLLER B

Model #	K_p	T_N	T_V
1	0.664	4.538	1.908
2	0.073	1.638	18.588
3	0.092	2.046	14.834
4	0.830	4.973	1.947
5	0.821	4.883	1.947
6	0.637	4.393	2.011

In Fig. 6 the oscillatory behavior using controller A shows a poor controller performance and the strongly different damping indicates that a stability proof will be difficult. Nevertheless, the closed-loop with controller A may be stable although a stability proof fails because Lyapunov stability criteria are sufficient rather than necessary conditions. The closed-loop stability of the local model network controlled by controller B is guaranteed because of the usage of Theorem 1. The controller performance in the time domain looks good as well, see Fig. 6. The closed loop performance also depends on the approximation capability of the local model network. This becomes visible when the local controller network is applied to the actual plant rather than to the local model network, see Fig. 7.

VII. CONCLUSION AND OUTLOOK

In this article a method to determine PID controller parameters for local model networks was proposed which provides exponential stability of the closed loop. First, a suitable state-space model was introduced to offer the pos-

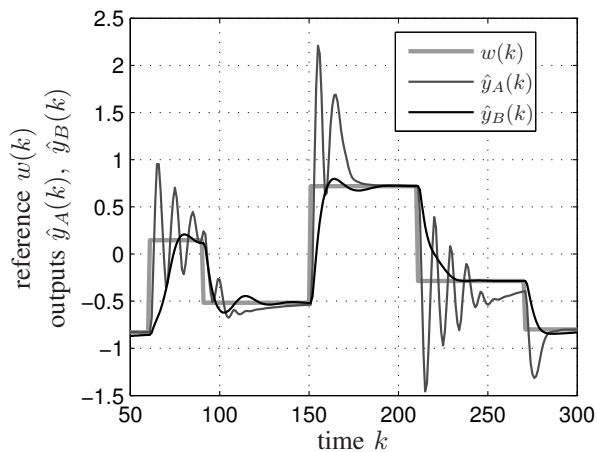


Fig. 6. Comparison of the closed loop performance of the local model network with two PID controllers

Controller A: initial design; no stability assertion possible
 Controller B: iLMI stabilized; exponentially stable, $\alpha = 0.98$

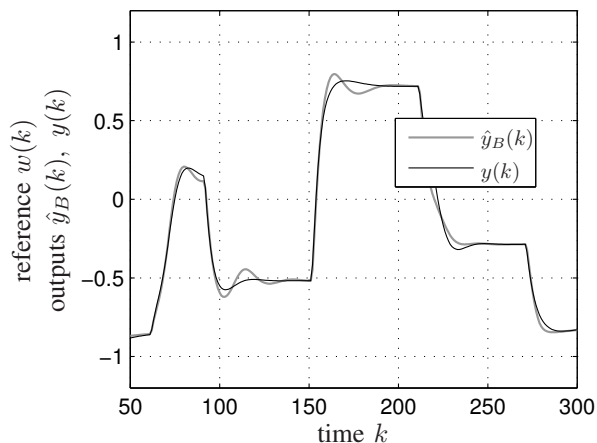


Fig. 7. Comparison of the local model network output $\hat{y}_B(k)$ and the output of the original process $y(k)$ controlled by the globally stable controller

sibility to investigate closed-loop stability with a Lyapunov based approach. Second, a decay rate was introduced and implemented into a common stability criterion. Because state of the art LMI solver cannot simultaneously determine the Lyapunov function and the PID controller parameters an iterative method was introduced to solve this issue. A simulation example highlights the effectiveness of the proposed method. The introduced state-space architecture offers a versatile closed-loop description of PID controlled local model networks. Thus, it is possible to relax the conservatism by applying a more advanced Lyapunov criterion (e.g. Fuzzy Lyapunov approach).

VIII. ACKNOWLEDGMENTS

This work was supported by Christian Doppler Research Association and AVL List GmbH.

REFERENCES

- [1] R. Murray-Smith and T. A. Johansen, "Multiple Model Approaches to Modelling and Control," *Taylor & Francis, London*, 1997.
- [2] C. Hametner and S. Jakubek, "New Concepts for the Identification of Dynamic Takagi-Sugeno Fuzzy Models," pp. 185–190, June 2006.
- [3] T. Takagi and M. Sugeno, "Fuzzy Identification of Systems and its Applications to Modeling and Control," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 15, no. 1, pp. 116–132, 1985.
- [4] G. Gregorcic and G. Lightbody, "Nonlinear System Identification: From Multiple-Model Networks to Gaussian Processes," *Engineering Applications of Artificial Intelligence*, vol. 21, no. 7, pp. 1035–1055, 2008.
- [5] G. Feng, "A Survey on Analysis and Design of Model-based Fuzzy Control Systems," *IEEE Transactions on Fuzzy Systems*, vol. 14, no. 5, pp. 676–697, 2006.
- [6] Y. Nesterov and A. Nemirovskii, *Interior-Point Polynomial Algorithms in Convex Programming*, siam ed., ser. Theory and Application. Philadelphia: Society for Industrial Mathematics, Jan. 1994.
- [7] J. Sturm, "Using SeDuMi 1. 02, a MATLAB toolbox for optimization over symmetric cones," *Optimization Methods and Software*, 1999.
- [8] K. Toh and M. Todd, "SDPT 3- A MATLAB Software Package for Semidefinite Programming," *Optimization Methods and Software*, Sept. 1999.
- [9] E. Kim and H. Lee, "New Approaches to Relaxed Quadratic Stability Condition of Fuzzy Control Systems," *IEEE Transactions on Fuzzy Systems*, vol. 8, no. 5, pp. 523–534, 2000.
- [10] W. Chang, R. Hwang, and J. Hsieh, "A self-tuning PID control for a class of nonlinear systems based on the Lyapunov approach," *Journal of Process Control*, vol. 12, pp. 233–242, 2002.
- [11] K. Ogata, *Discrete-Time Control Systems*, 3rd ed. Prentice Hall India, 2006.
- [12] K. J. Hunt and T. A. Johansen, "Design and analysis of gain-scheduled control using local controller networks," *Internat. J. Control*, vol. 66, no. 5, pp. 619–651, 1997.
- [13] G. Feng, *Analysis and Synthesis of Fuzzy Control Systems*, ser. A Model Based Approach. CRC, Feb. 2010.
- [14] M. Bernal and P. Husek, "Non-Quadratic Performance Design for Takagi-Sugeno Fuzzy Systems," *International Journal of Applied Mathematics and Computer Science*, vol. 15, no. 3, pp. 383–391, 2005.
- [15] M. Kozek and N. Jovanovic, "Identification of Hammerstein/Wiener nonlinear systems with extended Kalman filters," pp. 969–974, 2002.
- [16] C. Hametner and S. Jakubek, "Neuro-Fuzzy Modelling Using a Logistic Discriminant Tree," pp. 864–869, July 2007.