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Performance Analysis of Model Reference Robust Tuned 2DoF PI Controllers for Over Damped Processes

Víctor M. Alfaro and Ramon Vilanova

Abstract— **The aim of the paper is to present a closed-loop performance analysis of robust tuned two-degree-of-freedom (2DoF) proportional integral (PI) controllers for over damped (first- and second-order) plus dead-time controlled processes. A closed-loop model reference optimization method was follow with non-oscillatory and under damped response targets. The analysis shows that the controlled process model parameters (high time constants ratios or high normalized dead-times) impose a constrain on the robustness obtainable when highly oscillating closed-loop responses targets are specified. The non-oscillatory response targets produce the smoothest controller outputs. The regulatory and servo-control performance (integrated absolute error and settling time) may be improved if lightly under damping responses ($0.691 \leq \zeta \leq 0.780$) are allowed but affecting the control effort characteristics (its smoothness and maximum values).**

I. INTRODUCTION

The introduction in 1940 of the first commercially available *proportional integral derivative* (PID) controller [12], motivated Ziegler and Nichols to present the well-known tuning rules [26]. Since that date, a great number of other tuning procedures have been developed for the PID controller and its variations, as revealed in O'Dwyer's handbook [20].

At the beginning, only the control system *performance* was taken into account in the controller design, considering a step change either in the set-point, *servo-control* operation, or in the load-disturbance, *regulatory control* operation, as in the classic tuning rules of Cohen and Coon [13], López et al. [18], and Rovira et al. [23], among others [19], [21], for one-degree-of-freedom (1DoF) PI and PID controllers.

Later, the consideration of the control system relative stability, its *robustness* to the changes in the controlled process characteristics, was introduced into the controller design, considering first the control-loop gain and phase margins (A_m, ϕ_m) as in [9], [16], [17]. More recently, these classic indicators of robustness have been replaced by a single value given by the maximum of the magnitude of the sensitivity function, denoted by M_S. This approach has been used in [6], [11], [14].

The implementation of the two-degree-of-freedom (2DoF) PID controllers, proposed by Araki [8], allowed the separation of the control system design in two steps, considering in the first step, the control system stability and the regulatory control performance and in the second, the servo-control performance. See [4], [5], [25] and the references therein.

A control system with PID controllers must make use of the capabilities provided by the 2DoF controllers. Which must take into consideration several conflicting specifications: on the one hand, *performance*, or the response to the set-point and load-disturbance changes; and on the other hand, the system relative stability, or the *robustness* to the changes in the controlled process dynamics. The control signal variation and extreme values must also be taken into account. Therefore, the controller design is really a multi-objective problem [15].

In [7] using an IMC-based design procedure the servo-control overshoot and the control system robustness are directly associated: "smooth control" corresponds to $M_S = 1.38$ and 0% overshoot, whereas "thigh control" corresponds to $M_S = 1.71$ and 10% overshoot.

Following this idea, and with the aim of extending previous works of the authors [2], [3], [6], in this work the use of target models for the desired closed-loop dynamics is analyzed. At present, and with the goal of optimizing the design parameters, the target reference models where taken without oscillations and completely determined by one single parameter (in fact directly related to the closed-loop speed). See [2]. In this work, the use of more complex, under damped, target reference models is proposed. The main goal of the paper is to carry out and present the performance/robustness analysis regarding the use of such target models with reference to the over damped ones in order to determine in which situations it may be beneficial to specify a desired closed-loop response with some overshoot.

In the conducted analysis it is shown that is possible to have closed-loop control systems with the same *robustness* level but with different *performance* (critical or under damped responses). The controller parameters can be adjusted to modified the servo and regulatory control performance keeping constant its robustness.

II. MODEL REFERENCE ROBUST TUNING OF PI CONTROLLERS

For the performance analysis of the robust tuned 2DoF PI controllers, we will use a closed-loop model reference optimization procedure similar to the one proposed in [2], [3] and summarized below.

Consider the closed-loop control system in Fig. 1 where $P(s)$ and $C(s)$ are the controlled process model and the controller transfer functions respectively. In the system, $r(s)$ is the set-point, $u(s)$ is the controller output signal, $d(s)$

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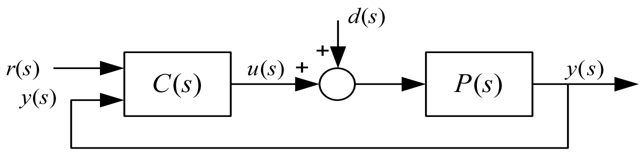

Fig. 1. Closed-Loop Control System

is the load-disturbance and $y(s)$ is the process controlled variable.

The closed-loop control system output, $y(s)$, to a change in its inputs, $r(s)$ and $d(s)$, is given by

$$y(s) = M_{yr}(s)r(s) + M_{yd}(s)d(s), \tag{1}$$

where $M_{yr}(s)$ is the *servo-control* closed-loop transfer function, and $M_{yd}(s)$ is the *regulatory control* closed-loop transfer function.

A. PI_2: 2DoF Proportional Integral Controller

The process will be controlled with a two-degree-of-freedom proportional integral (PI_2) controller [10] whose output is

$$u(s) = K_p \left\{ \beta r(s) - y(s) + \frac{1}{T_i s}\left[r(s) - y(s)\right] \right\}, \tag{2}$$

where K_p is the controller *proportional gain*, T_i the *integral time constant* and β the *set-point proportional weight*.

Controller output (2) will be rewrite for the analysis (not for the implementation) as

$$u(s) = C_r(s)r(s) - C_y(s)y(s), \tag{3}$$

where

$$C_r(s) = K_p \left(\beta + \frac{1}{T_i s} \right), \tag{4}$$

is the PI_2 controller part applied to the set-point r, the *set-point controller* transfer function, and

$$C_y(s) = K_p \left(1 + \frac{1}{T_i s} \right), \tag{5}$$

is the PI_2 controller part applied to the feedback signal y, the *feedback controller* transfer function.

The servo-control and the regulatory control closed-loop transfer functions in (1) are now

$$M_{yr}(s) = \frac{C_r(s)P(s)}{1 + C_y(s)P(s)}, \tag{6}$$

and

$$M_{yd}(s) = \frac{P(s)}{1 + C_y(s)P(s)}, \tag{7}$$

which are related by

$$M_{yr}(s) = C_r(s)M_{yd}(s). \tag{8}$$

B. Over Damped Controlled Process Models

The over damped controlled processes will be represented by a linear model given by the transfer function

$$P(s) = \frac{Ke^{-Ls}}{(Ts+1)(aTs+1)},\ \tau_o = \frac{L}{T}, \tag{9}$$

where K is the model gain, T the main time constant, a the ratio of its two time constants ($0 \leq a \leq 1.0$), L the dead-time, and τ_o the model *normalized dead-time* ($0.1 \leq \tau_o \leq 2.0$).

The parameters of the controlled process model (9), $\overline{\theta}_p = \{K, T, a, L, \tau_o\}$, may be identified from the process reaction curve [1].

The transfer function (9) allows the representation of first-order plus dead-time (FOPDT) processes ($a = 0$), over damped second-order plus dead-time (SOPDT) processes ($0 < a < 1$), and dual-pole plus dead-time (DPPDT) processes ($a = 1$) modeling a wide range of self-regulating controlled processes [22], [24].

C. Closed-Loop Transfer Functions Targets

In [2], [3] the regulatory control closed-loop transfer function target was selected non-oscillatory; for a smooth response; and with no steady-state error, given by

$$M_{yd}^t(s) = \frac{(T_i/K_p)s\mathrm{e}^{-Ls}}{(\tau_c Ts+1)^2(a\tau_c Ts+1)}, \tag{10}$$

where τ_c is the dimensionless design parameter that represents the ratio between closed-loop system time constant and the controlled process dominant time constant.

Using (10) and (4) in (8) the obtained servo-control closed-loop transfer function was

$$M_{yr}(s) = \frac{(\beta T_i s+1)\mathrm{e}^{-Ls}}{(\tau_c Ts+1)^2(a\tau_c Ts+1)}. \tag{11}$$

Then, to have a response to a set-point step change without oscillation and overshoot, and with no steady-state error, the servo-control closed-loop transfer function target was selected as

$$M_{yr}^t(s) = \frac{\mathrm{e}^{-Ls}}{(\tau_c Ts+1)(a\tau_c Ts+1)}. \tag{12}$$

Equation (12) implies that $\beta \to \tau_c T/T_i$.

The selection of the closed-loop transfer functions targets (10) and (12) seek also to obtain, as a side effect, a smooth controller output.

III. Under Damped Closed-Loop Responses Targets

In order to analyze if it is possible to modify the control system performance to a load-disturbance and set-point step changes without affecting its robustness the regulatory control closed-loop transfer function target is now selected with two under damped dominant poles given by

$$M_{yd}^t(s) = \frac{(T_i/K_p)s\mathrm{e}^{-Ls}}{(\tau_c^2 T^2 s^2 + 2\zeta\tau_c Ts+1)(a\tau_c Ts+1)}, \tag{13}$$

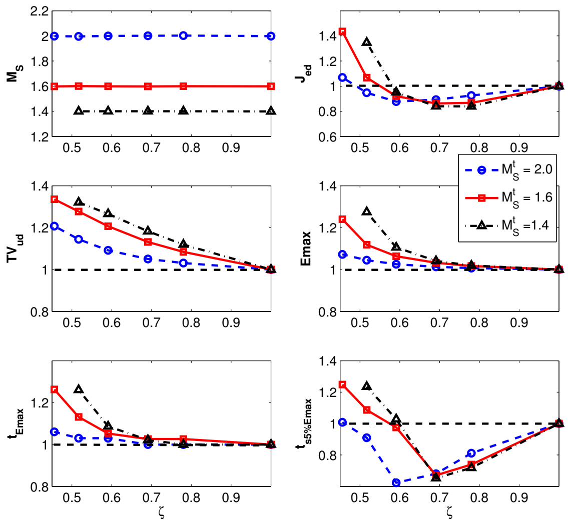

Fig. 2. Robustness and Regulatory Control Performance ($a = 0$, $\tau_o = 0.1$)

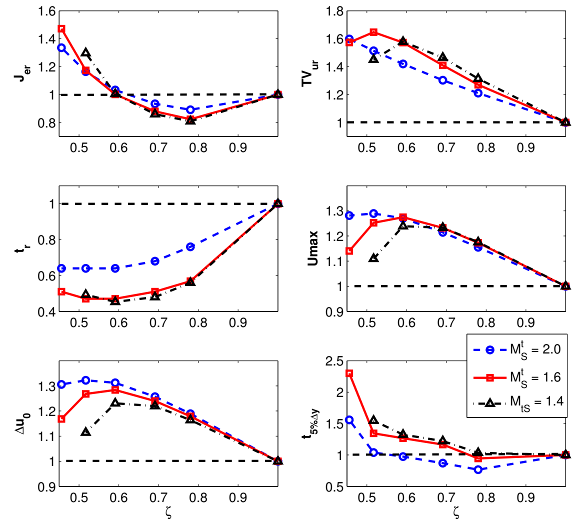

Fig. 3. Servo Control Performance ($a = 0$, $\tau_o = 0.1$)

Using (13) and (4) in (8) the servo-control closed-loop transfer function is

$$M_{yr}(s) = \frac{(\beta T_i s + 1)\mathrm{e}^{-Ls}}{(\tau_c^2 T^2 s^2 + 2\zeta\tau_c T s + 1)(a\tau_c T s + 1)}. \quad (14)$$

Now, using $\beta = \tau_c T/T_i$ as in (12) the servo-control closed-loop transfer function target is selected as an under damped system given by

$$M_{yr}^t(s) = \frac{(\tau_c T s + 1)\mathrm{e}^{-Ls}}{(\tau_c^2 T^2 s^2 + 2\zeta\tau_c T s + 1)(a\tau_c T s + 1)}. \quad (15)$$

Then, the new global control system output target $y^t(s)$ is computed as

$$y^t(s) = \frac{(\tau_c T s + 1)\mathrm{e}^{-Ls}}{(\tau_c^2 T^2 s^2 + 2\zeta\tau_c T s + 1)(a\tau_c T s + 1)} r(s) + \frac{(T_i/K_p) s e^{-Ls}}{(\tau_c^2 T^2 s^2 + 2\zeta\tau_c T s + 1)(a\tau_c T s + 1)} d(s). \quad (16)$$

The control system has now two design parameters, the *closed-loop relative speed* τ_c and the *damping ratio* ζ.

IV. Closed-Loop Performance Analysis

For the control systems performance analysis we select six servo-control overshoot levels (or damping ratios): 0% ($\zeta = 1.0$), 2% ($\zeta = 0.780$), 5% ($\zeta = 0.691$), 10% ($\zeta = 0.591$), 15% ($\zeta = 0.517$), and 20% ($\zeta = 0.456$).

During the optimization process, for each damping ratio above, the closed-loop relative speed parameter τ_c is selected in such a way that the robustness level of the resulting closed-loop system met a specific target M_S^t in the range from 1.4 to 2.0.

For all the controller parameter sets obtained both the control system robustness and performance are evaluated.

The regulatory control response performance indices evaluated are the integrated absolute error (J_{ed}), the controller output total variation (TV_{ud}), the maximum error (E_{max}), the time to reach the maximum error (t_{max}), and the settling time ($t_{5\%Emax}$). For the servo-control response the performance indices evaluated are the integrated absolute error (J_{er}), the controller output total variation (TV_{ur}), the rise time (t_r), the control effort maximum value (U_{max}), the controller output instant change (Δu_0), and the settling time ($t_{5\%\Delta y}$). These performance indices are defined in the Appendix.

As the model normalized dead-time τ_o is in the range from 0.1 to 2.0 and the time constants ratio a in the range from 0 to 1.0 we first analyze some extreme cases.

A. FOPDT Model with Low Normalized Dead-Time

Fig. 2 and Fig. 3 show the robustness and the normalized regulatory control performance indices, and the normalized servo-control performance indices, respectively, for the ($a = 0$, $\tau_o = 0.1$) case. All the performance indices have been normalized using their corresponding values for the non-oscillating target ($\zeta = 1$).

As can be seen from Fig. 2 all the robustness level targets are perfectly accomplished but it is not possible to have a highly under damped system ($\zeta = 0.456$) with high robustness ($M_S^t = 1.4$) at the same time.

Allowing control system outputs with small oscillations ($0.691 \leq \zeta \leq 1.0$) it is possible to improve the regulatory control performance; a reduction on J_{ed} and $t_{s5\%Emax}$ values; but with a deterioration of the control effort smoothness (TV_{ud}), while the effect over E_{max} and t_{Emax} is negligible.

At the servo-control side (Fig. 3) small reductions in the damping ratio will improve the J_{er}, t_r and $t_{s5\%\Delta y}$ indices, but will deteriorate the control effort characteristics, TV_{ur}, U_{max}, and Δu_0.

In order to take into account both responses (for servo and regulatory control), the normalized combined indices for the

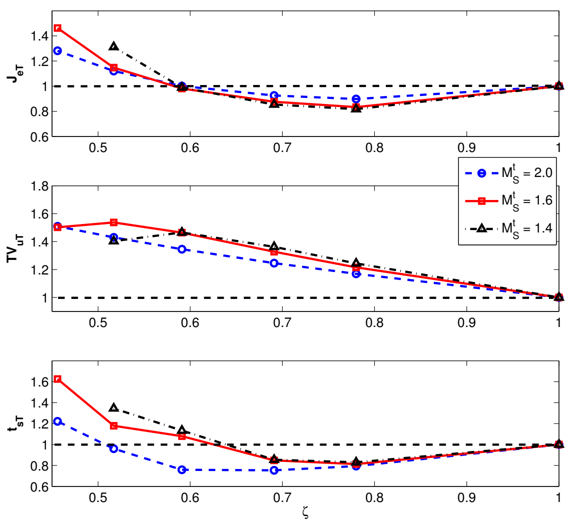

Fig. 4. Servo/Regulatory Control Combined Performance ($a = 0$, $\tau_o = 0.1$)

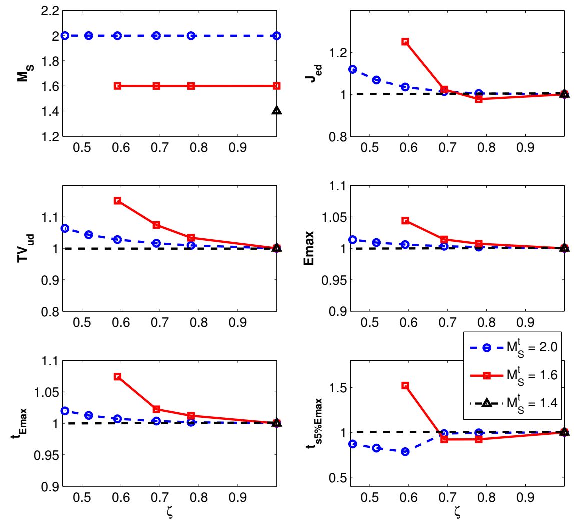

Fig. 5. Robustness and Regulatory Control Performance ($a = 1.0$, $\tau_o = 2.0$)

integrated absolute error ($J_{eT} = J_{er} + J_{ed}$), the controller output variation ($TV_{uT} = TV_{ur} + TV_{ud}$), and the settling time ($t_{sT} = t_{s5\%\Delta y} + t_{s5\%Emax}$) are computed and shown in Fig. 4.

From the combined performance information it is seen that a good balance of the (J_{eT}, t_{sT}) versus TV_{uT} trade-off is obtained for damping ratios ζ in the range from 0.7 to 0.8.

B. DPPDT Model with High Normalized Dead-Time

Fig. 5 and Fig. 6 show the robustness and the normalized regulatory control performance indices, and the normalized servo-control performance indices, respectively, for the ($a = 1$, $\tau_o = 2.0$) case.

As can be seen from Fig. 5 it is not possible to obtain under damped responses with high robustness ($M_S^t = 1.4$). In this case for $\zeta = 0.78$ the maximum robustness obtained is $M_S = 1.48$.

From Fig. 5 (regulatory control), Fig. 6 (servo-control), and Fig. 7 (combined performance) it can be seen that in this case for the low robustness system ($M_S^t = 2.0$) the improvement in performance that can be obtained allowing some oscillation in the responses is negligible. For the intermediate robustness level ($M_S^t = 1.6$) a small performance improvement can be obtained reducing the damping ratio to $\zeta = 0.78$.

C. Additional Performance Evaluations

In addition to the two extreme cases above the control system performance is evaluated for following controlled processes: ($a = 0$, $\tau_o = 2.0$), ($a = 1$, $\tau_o = 0.1$), and ($a = 0.75$, $\tau_o = 0.50$).

From the information obtained, not showed here for space reasons, it can be concluded that high values of the model time constants ratio a (models tend to be of dual pole), or of

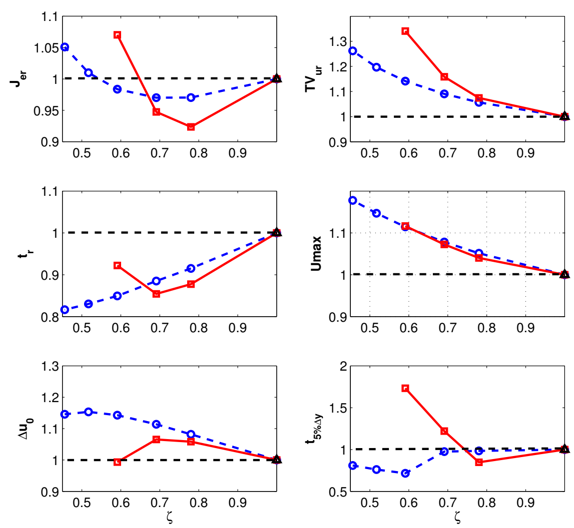

Fig. 6. Servo Control Performance ($a = 1.0$, $\tau_o = 2.0$)

the normalized dead-time τ_o (models tend to be dead-time dominant) impose a constraint on the robustness of the highly under damped responses. For example for the model ($a = 0$, $\tau_o = 2.0$) the highest robustness obtained were $M_S = 1.47$ for $\zeta = 0.780$ and $M_S = 1.67$ for $\zeta = 0.591$; and for the model ($a = 1$, $\tau_o = 0.1$) the robustness limit was $M_S = 1.42$ for $\zeta = 0.517$.

The responses of the ($a = 0.75$, $\tau_o = 0.50$) model control system to a 20% set-point step change followed by a 10% load-disturbance step-change with 2DoF PI controllers for $M_S^t \in \{1.4, 2.0\}$ are shown in Fig. 8 and Fig. 9.

For the minimum robustness level ($M_S^t = 2.0$) it is

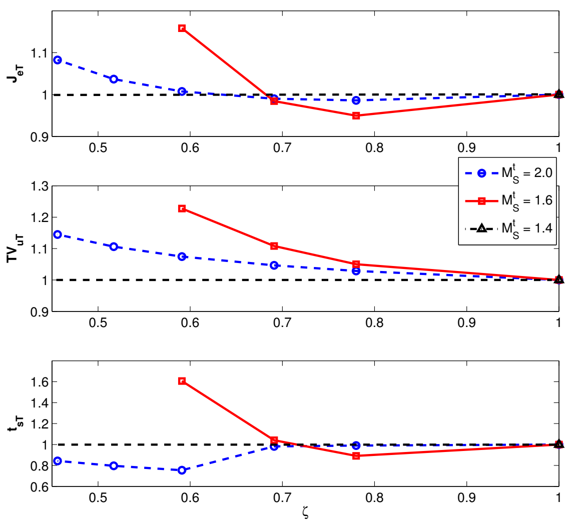

Fig. 7. Servo/Regulatory Control Combined Performance ($a = 1.0$, $\tau_o = 2.0$)

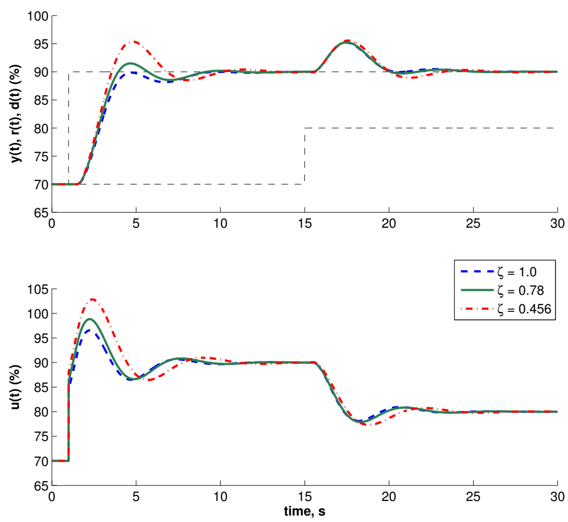

Fig. 8. Closed-Loop Control System Output, $M_S^t = 2.0$, ($a = 0.75$, $\tau_o = 0.50$))

possible to obtain responses with a damping ratio in the range from 1.0 to 0.456. In this case if a damping ratio $\zeta = 0.78$ is specified a 3.26% improvement is obtaned in the combined integrated absolute error J_{eT} corresponding to a 5.56% reduction in J_{er} with only a 0.28% increment in J_{ed} respect to the corresponding performances indices for the non-oscillatory response ($\zeta = 1.0$).

The high robustness level corresponding to $M_S^t = 1.4$ may be obtained only for $\zeta \geq 0.691$. At this robustness level responses with a $\zeta = 0.78$ target improves J_{eT} by 7.62%, corresponding to a 9.12% reduction in J_{er} with at the same time a 1.19% increment in J_{ed}.

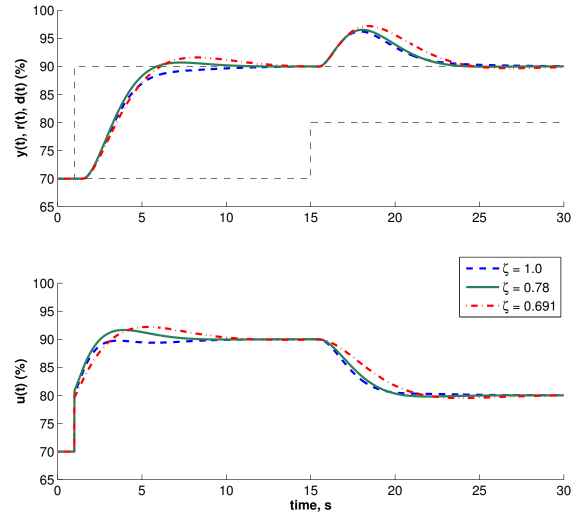

Fig. 9. Closed-Loop Control System Output, $M_S^t = 1.4$, ($a = 0.75$, $\tau_o = 0.50$))

V. CONCLUSIONS

A performance analysis of robust tuned two-degree-of-freedom proportional integral controllers (PI_2) was conducted using a closed-loop model reference optimization design procedure with servo and regulatory control response targets for damping ratios ζ in the range from 1.0 to 0.456 with a robustness M_S constrain in the range from 2.0 to 1.4.

Its shows that all the controllers obtained with the non-oscillatory response target ($\zeta = 1.0$) provide the smoothest control efforts with an integrated absolute error near the lower obtainable value for the corresponding robustness level target.

An improvement in the control system performance (integrated absolute error and settling time); specially for the servo-control; may be obtained if the closed-loop transfer function damping ratio target is selected in the range from 0.7 to 0.8 but adversely affecting the control effort characteristics.

VI. ACKNOWLEDGMENTS

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APPENDIX

For the controller *performance* evaluation, we select, as the main control system performance metric, the integrated absolute error (IAE) given by the following:

$$J_e \doteq \int_0^\infty |e(t)|\,\mathrm{d}t = \int_0^\infty |r(t) - y(t)|\,\mathrm{d}t. \qquad (17)$$

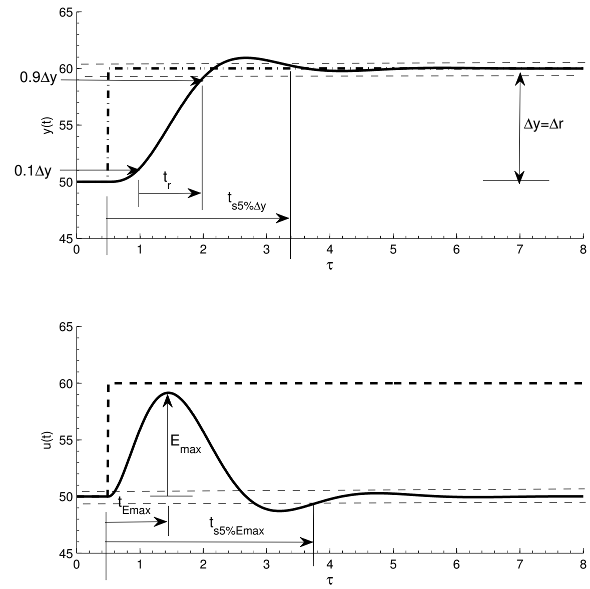

Fig. 10. Servo and Regulatory Control Performance Indices

The performance measure (17) will be evaluated by set-point and load-disturbance changes, J_{er}, J_{ed}. As complementary performance metrics we will use the following indices shown in Fig. 10, :

1) Servo control response
   - Rise-time, t_r (the time required to the control system output to change from $10\%\Delta y$ to $90\%\Delta y$).
   - Settling-time, $t_{s5\%\Delta y}$ (the time required to the control system output to rise and settle within a $\pm 5\%\Delta y$ band around its final value).
2) Regulatory control response
   - Peak error, E_{max} (the magnitude of the maximum error),
   - Time to the maximum error, $t_{E_{max}}$,
   - Settling-time, $t_{s5\%E_{max}}$ (the time required to the control system output to decrese and setttle within a $\pm 5\%E_{max}$ error band).

All the above indices can be obtained independently if the responses are over or under damped.

For the evaluation of the *control effort* the control signal total variation TV_u given by

$$TV_u \doteq \sum_{k=1}^{\infty} |u_{k+1} - u_k|\,, \qquad (18)$$

will be used as main indication of the *smoothness* of the control action for both input changes, TV_{ur} and TV_{ud}.

As complementary measurements of the control effort use we will consider the controller output instant change to a set-point step change (the “proportional kick”) given by

$$\Delta u_0 \doteq \beta K_p \Delta r \qquad (19)$$

and the maximum control effort, U_{max}.