Preprints of the 2012 20th Mediterranean Conference on Control & Automation (MED), Barcelona, Spain, July 3-6, 2012

An Optimal Control Strategy of Ballast Systems Used in Ship Stabilization

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Abstract—**This paper tackles the problematic of ship stabilization using optimal amounts of resources, in the case of loading and unloading operations on docked vessels. A mathematical model linking the ship's positioning and orientation with the forces and moments exerted upon the hull is required in order to develop an optimal control strategy for the ship's stabilizing ballast systems. This strategy is designed using graph theory concepts, and it focuses on generating an optimal command sequence in respect to minimum energy consumption taking into account the ballast system's topology.**

I. Introduction

THIS paper presents a novel approach of designing optimal and fault tolerant commands for stabilizing ships. Keeping the ship leveled is a crucial aspect in its operation. The ship's position affects both loading and unloading freight when docked [1] and maneuvering when in march [2], [3]. A variety of marine crafts need to be stabilized when docked to ensure a safe and efficient working environment for the crew. These marine crafts have in common the movement of massive weights on board while docked. Such marine crafts are oil tankers, container ships and floating docks. Floating docks are affected the most by shifting weights on board, because they are equipped with moving cranes which move around large parts, thus making them susceptible to rolling motions that could endanger the crew. A solution for ship stabilization is using a ballast system, which consists of a series of tanks located throughout the ship, interconnected with each other and the surrounding environment via a pipe and pumps transport subsystem. The idea behind the ballast system is that it counteracts onboard weight shifts using water as a counterweight. Knowing the current ship position and orientation we can determine the required ballast system configuration in order to bring the vessel back to a desired position and orientation. Another problem arises from the fact that the transition between the two states can be achieved in a various number of ways; thus, imposing a performance requirement, an optimization problem can be formulated. The performance criterion could be either the time needed to readjust the system parameters (ship position and orientation) to their reference values, either the needed amount of energy. The latter is equivalent to the amount of water transferred in the ballast system. By applying graph theory concepts on the characteristic mathematical model's equations, we can develop a recursive algorithm that generates the optimal command sequence for the transport subsystem, in respect with minimum transfers between tanks (and surrounding environment).

This paper is structured as follows: Section II aims to describe the characteristic mathematical equations for a docked marine vessel, thus introducing a static mathematical model for it; in Section III is presented the algorithm for efficient ship stabilization; the numerical results in Section IV confirm the accuracy of the proposed algorithm; final conclusions and future work are presented in Section V.

The work has been funded by the Sectoral Operational Programme Human Resources Development 2007-2013 of the Romanian Ministry of Labour, Family and Social Protection through the Financial Agreement POSDRU/88/1.5/S/61178.

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II. Mathematical modeling

To generate the control strategy we must first begin with modeling the ship's motion. A ship is a system with six degrees of freedom as defined in [4]. The motion of a ship in six degrees of freedom is considered a translation motion in three directions: surge, sway and heave; and as a rotation motion about three axes: roll, pitch and yaw. To determine the equations of motion we need to consider two reference frames, an inertial frame O and a body fixed frame O_O. The body-fixed frame is positioned in such a way that it gives us hull symmetry about the $x_O y_O$ plane, and approximate symmetry about the $y_O z_O$ plane.

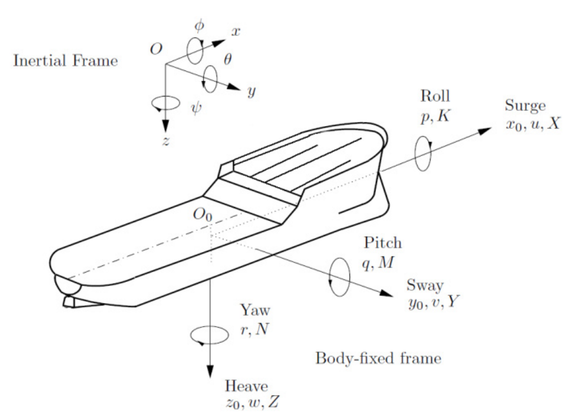

Fig. 1. Ship model with 6 degrees of freedom

The position of the origin of the z_O axis is given by the calm water surface [5]. The magnitudes describing the position and orientation of the ship are usually expressed in the inertial frame and the coordinates are noted $[x\ y\ z\ \varphi\ \theta\ \psi]^T$, whilst the forces $[X\ Y\ Z]^T$, moments $[K\ M\ N]^T$, linear velocities $[u\ v\ w]^T$, and angular velocities $[p\ q\ r]^T$ are usually expressed in the body-fixed frame [4]. If we define the position-orientation vector η with respect to the inertial frame as

$$\eta \triangleq [x\ y\ z\ \phi\ \theta\ \psi]^T \tag{1}$$

and the linear-angular velocity vector v with respect to the body-fixed frame as

$$v \triangleq [u\ v\ w\ p\ q\ r]^T, \tag{2}$$

we can define the equations of motion of a ship in the body fixed frame, using a Newtonian approach [6] as

$$M_{RB}\dot{v} = \tau(\dot{v}, v, \eta) - C_{RB}(v)v, \tag{3}$$

where M_{RB} is the matrix mass and inertia due to rigid body dynamics, the term $C_{RB}(v)v$ arises from the coriolis and centripetal forces and moments due to rigid body dynamics. The forces and moments vector τ is defined as

$$\tau \triangleq [X\ Y\ Z\ K\ M\ N]^T, \tag{4}$$

these magnitudes are generated by different phenomena and can be separated in components according to their originating effects:

$$\tau = \tau_{hydrodynamic} + \tau_{hydrostatic} + \tau_{cs} + \tau_{prop} + \tau_{ext} \tag{5}$$

where:

- $\tau_{hydrodynamic}$ represents the forces and moments that arise from the hull's movement in water
- $\tau_{hydrostatic}$ represents the forces and moments that arise from the buoyancy force and gravitational force
- τ_{cs} represents the forces and moments that arise due to the control surfaces (rudders, fins, etc.) movement
- τ_{prop} represents the forces and moments that come from the propulsion system, e.g. propellers and thrusters
- τ_{ext} represents the external forces and moments that act on the ship like waves, wind and currents

Because our goal is to stabilize a docked ship, we can make the following assumption, that $v = 0$. Also, because the ship is docked, we can neglect τ_{ext}. Since $v = 0$, the hydrodynamic forces and moments, control surfaces forces and moments and propulsion forces and moments can be assumed to be equal to zero. Thus, our equation for a docked ship becomes

$$\begin{aligned} \tau_{hydrostatic} &= 0 \\ \tau_{hydrostatic} &= g(\eta) - g_0 \end{aligned} \tag{6}$$

where $g(\eta)$ is a term that expresses the gravitational and buoyancy forces and moments, and g_0 is the ballast control vector, which represents the forces and moments generated by the ballast tanks. According to [1] we can write $g(\eta)$ as:

$$g(\eta) = \begin{bmatrix} -\rho g \int_0^z A_{wp}(\xi) d\xi \ sin\theta \\ \rho g \int_0^z A_{wp}(\xi) d\xi \ cos\theta\, sin\phi \\ \rho g \int_0^z A_{wp}(\xi) d\xi \ cos\theta\, cos\phi \\ \rho g \nabla \overline{GM_T}\ sin\phi\, cos\theta\, cos\phi \\ \rho g \nabla \overline{GM_L}\ sin\theta\, cos\theta\, cos\phi \\ \rho g \nabla (-\overline{GM_L} cos\theta + \overline{GM_T})\ sin\phi\, sin\theta \end{bmatrix} \tag{7}$$

Where $A_{wp}(\xi)$ is the water surface area at a given ξ position, ∇ is the displaced water volume, at a given ξ position, ρ is the water density, g is the gravitational acceleration, and $\overline{GM_T}$ and $\overline{GM_L}$ are the transverse and longitudinal metacentric heights, which represent the distances between the transversal metacenter and the center of gravity, and respectively the distance between the longitudinal metacenter and the center of gravity.

$$\overline{GM_T} = \overline{BM_T} - \overline{BG} \tag{8}$$

$$\overline{GM_L} = \overline{BM_L} - \overline{BG} \tag{9}$$

$\overline{BG}$ represents the distance between the center of buoyancy and the center of gravity. The metacenter is a theoretical point at which an imaginary vertical line through the center of buoyancy intersects another imaginary vertical line through a new center of buoyancy when the ship is displaced or tilted in the water.

Given the working conditions, we can safely assume that ϕ, θ and z are small, thus we obtain:

$$\begin{aligned} sin\theta \approx \theta, \quad & cos\theta \approx 1 \\ sin\phi \approx \phi, \quad & cos\phi \approx 1 \\ \int_0^z A_{wp}(\xi) d\xi \approx & A_{wp}(0) z \end{aligned} \tag{10}$$

Under these assumptions we can say that

$$g(\eta) \approx G\eta \tag{11}$$

where

$$G = G^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -Z_z & 0 & -Z_\theta & 0 \\ 0 & 0 & 0 & -K_\phi & 0 & 0 \\ 0 & 0 & -M_z & 0 & -M_\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{12}$$

$$-Z_Z = \rho g A_{wp}(0) \tag{13}$$

$$-Z_\theta = \rho g \int_{A_{wp}} x dA$$
$$-M_z = -Z_\theta$$
$$-K_\phi = \rho g \nabla \overline{GM_T}$$
$$-M_\theta = \rho g \nabla \overline{GM_L}$$

The ballast control vector, g_0, can be computed, for a marine craft with n ballast tanks, with a volume V_i, located in respect to O_O in $[x_i\ y_i\ z_i]^T$ as follows. For each ballast tank we can define the contained water volume as:

$$V_{i=1,\ldots,n}(h_i) = \int_0^{h_i} A_i(h)\, dh \approx A_i h_i$$
$$A_i(h) = constant \tag{14}$$

The gravitational forces in heave are expressed as:

$$Z_{ballast} = \rho g \sum_{i=1}^{n} V_i \tag{15}$$

The restoring moments are:

$$K_{ballast} = \rho g \sum_{i=1}^{n} y_i V_i$$
$$M_{ballast} = -\rho g \sum_{i=1}^{n} x_i V_i \tag{16}$$

Using these expressions for the gravitational forces and restoring moments, we can write:

$$g_0 = \begin{bmatrix} Z_{ballast} \\ K_{ballast} \\ M_{ballast} \end{bmatrix} = \mathrm{H} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}$$
$$H = \rho g \begin{bmatrix} 1 & \ldots & 1 & 1 \\ y_1 & \ldots & y_{n-1} & y_n \\ -x_1 & \ldots & -x_{n-1} & -x_n \end{bmatrix} \tag{17}$$

Finally, the volumes $[V_1\ V_2\ \ldots\ V_n]^T$ from the ballast control vector are determined using the pseudo-inverse of the H matrix.

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = H^T (HH^T)^{-1} g_0 \tag{18}$$

III. BALLAST SYSTEM CONTROL ALGORITHM

The stabilization problem formulation, as depicted in *Figure 2*, consists in generating a suitable command sequence for the pipe and pumps subsystem, in order to readjust the ship's position and orientation to their pre-specified reference values.

The first step consists in evaluating, using the previously described mathematical model, the forces and moments vector, $[Z\ K\ M]^T$, needed to counterbalance the effects of weight shifts during onboard loading or unloading operations. Once this vector is obtained, the ballast equations (18) give an expression for the ballast tanks configuration necessary to ensure the ship's stability in the desired reference coordinates, $[z_0\ \theta_0\ \varphi_0]^T$. The next step is to generate a minimum energy consuming transfer strategy for reaching the desired tank levels.

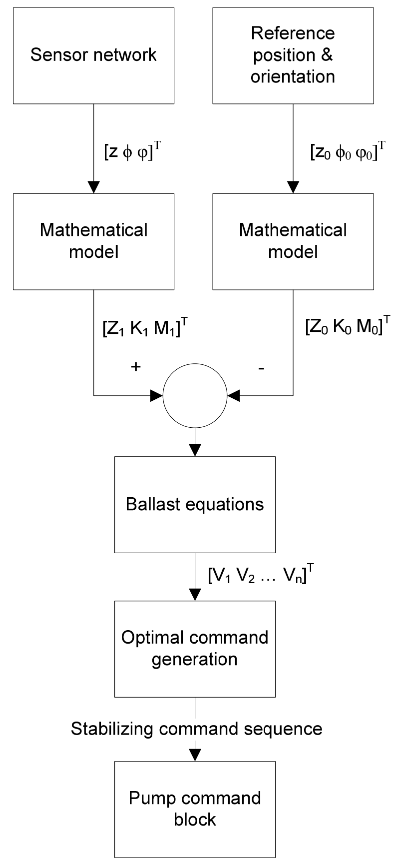

Fig. 2. Control algorithm

We have developed a graph theory [7] based algorithm, which is described in the next paragraphs.

The algorithm assumes that from the starting configuration of tank levels we can reach the target configuration through more than one way of transferring water volumes. The intermediate states, along with the final and initial states, can be regarded as vertices in a directed graph. The arcs between the vertices represent water transfers between the tanks. The cost of an arc can be computed in a number of ways. In this paper the cost of an arc is the total amount of water transferred between two states. The cost function of an arc can also be considered as the time needed to make the transition to the next state or in can be the variation of the ships position. The cost function can also be a linear combination of the criteria mentioned above. After we generate the graph, we need to find the shortest path between the initial and final state.

The ballast system is considered to be a set of n tanks, T_{tanks}, connected through p pumps. For each pump we define a $(n+1)\times(n+1)$ matrix, $P_{con_i}, i=1{:}p$, that defines how the tanks are connected through a pump. If the term $P_{con_i}[j][k]=1$ then tank T_j is connected to tank T_k through pump i. If the term $P_{con_i}[k][j]=1$, it means that the transfer is bidirectional. The terms $P_{con_i}[n+1][k], k=1{:}n+1$ and $P_{con_i}[k][n+1]$ tell us if a tank is connected to the surrounding environment. To generate the graph, we devised a recursive procedure that for a given node returns all the states that can be reached from it. The first step of the recursive procedure is to assess the number of tanks that have volumes different than the target volumes. This will generate a set of tanks T_{source} which is a subset of T_{tanks}. In order to maximize the use of the pumps and minimize the number of intermediate states between the initial state and the final state we must have *np* operations, $np=min(p, card(T_{source}))$ operations, meaning that we either use all the pumps available (p) or as many pumps as tanks that need to be brought to the target level exist ($card(T_{source})$). Once we have T_{source} we need to generate all the possible sources for the *np* transfers. A transfer source is an array with *np* elements, each element taking all the values in T_{source}. The total number of source arrays is $card\ (T_{source})^{np}$. For each source we need to generate all the possible destinations that can be reached by using the available pumps. For a transfer, we can use the pumps in multiple ways, the total number of ways in which the pumps can be used is $A_p^{\min(p,card(T_{source}))}$. For each source tank T_s, $s=1{:}card(T_{source})$, using the corresponding pump j, and P_{con_j} we can obtain a set of valid destinations, T_d^s. By combining the sets T_d^s in all the possible ways we can generate all the subsequent vertices of the current vertex and the arcs that connect them. If for a given *np* there are no subsequent vertices we decrease *np* and restart the process from the generation of the possible sources. To obtain the actual state of the tanks in the subsequent vertices and the total amount of water transferred between the tanks, we need to establish a few rules about how water is transferred through the ballast system. These rules also ensure that all the paths will reach the vertex corresponding to the final state of the tanks, and that no infinite loops or redundant paths are included in the graph. The conditions that a source-destination pair of tanks must comply are as follows:

$-\ T_S \cup T_D \neq T_S$; meaning that there is at least one destination tank not included in T_s. This condition is needed to eliminate infinite loops from the graph.

- For all the destination tanks the current water volume must be lower than the target value, because the transfers are interpreted as the pumping of water from the source tank to the destination tank, to bring the volume in the latter to the target value. This condition is needed to ensure that all the paths in the graph reach the final state.

- If a tank is a multiple source, the transfers that originate from it are meant to bring the destination tanks to the target value.

- If a tank is a multiple destination, the transfers are meant to bring the sources to the target value.

If a source-destination pair complies with the conditions above, the corresponding vertex of the destination state is added, along with the arc between the corresponding vertices. The process continues using the new vertices as sources until the final destination is reached.

Using the recursive procedure described above we obtain an oriented graph containing multiple routes between the starting and final position. Now the problem is to find the shortest route between the two. Finding the shortest route in a graph between two vertices is a well known problem in graph theory, and it can be solved by using a number of algorithms. We choose Dijktra's algorithm [8] for our application. After the shortest path is found, the last step consists in transmitting the calculated command sequence to the pumps, thus ensuring the pre-specified stabilization conditions.

IV. Case Study

In order to illustrate how the control strategy will work in a given situation, we will consider the following particular case. The studied ship is a barge with a volume that can be approximated by a block $L\times B\times H$. The barge's dimensions are as follows $L=100m$ (length), $B=30m$ (width), $H=10m$ (height). The ship's mass is considered uniformly distributed and the origin of the body fixed coordinate system is considered to be the center of gravity of the ship, with no additional weight on board located at *(L/2, B/2, H/2)*. The origin of the inertial coordinate system is considered to be the same point. The barge is equipped with four identical ballast tanks. The ballast tanks have a volume of $1125m^3$ *(25m x 9m x 5m)*. Their position in respect to the body fixed coordinate system in the $x_O y_O$ plane is: T_1 *(37,5m -10,5m)*; T_2 *(37,5m 10,5m)*; T_3 *(-37,5m -10,5m)*; T_4 *(-37,5m 10,5m)*. The ballast tanks are connected through a system of pipes and two pumps as shown in the following figure.

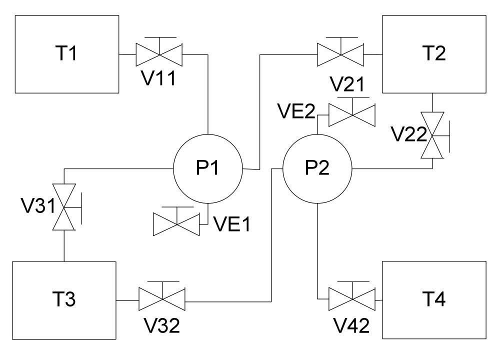

Fig. 3. Ballast system pump configuration

Each pump has a connection with the outside environment thus allowing each tank to exchange water with it. The pump connection matrixes have the following form:

$$P_{con_1} = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$$P_{con_2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \tag{19}$$

The ship is in a state η_1, and we need to bring it back to the reference position of η_0. The current state is $\eta_1=[-0.91\ \pi/180\ \pi/180]^T$ and the reference state is $\eta_0=[-0.75\ 0\ 0]^T$. The water volumes in the ballast tanks are $V_1=500m^3$; $V_2=400m^3$; $V_3=700m^3$; $V_4=650m^3$.

The first step is to compute the *G* matrix. The *G* matrix has terms that are independent of the values of η, and values that are dependent of it. The independent values are $-Z_Z$ and $-Z_\theta$.

$$-Z_Z = \rho g A_{wp}(0) = \rho g BL = 10^3 \cdot 10 \cdot 100 \cdot 30 = 3 \cdot 10^7 \tag{20}$$

$$-Z_\theta = \rho g \int_{A_{wp}} x dA = \int_{-L/2}^{L/2} \int_{-B/2}^{B/2} x\, dy\, dx = \int_{-L/2}^{L/2} Bx\, dx = 0$$
$$-M_z = -Z_\theta = 0 \tag{21}$$

Note that the terms $-Z_\theta$ and $-M_z$ are different than zero only when the ship doesn't have fore aft symmetry. The next step in computing the *G* matrix is to evaluate the terms that are dependent of η, more precisely dependent of *z*. These terms depend of *z* through the terms ∇ and $\overline{GM_T}$ respectively $\overline{GM_L}$, and are strictly connected with the ship's design. For ∇ we have a straight forward approach, the displaced water volume can be computed in respect to the initial stable state as follows:

$$\nabla = \left(\frac{H}{2} - z\right) BL \tag{22}$$

To compute $\overline{GM_T}$ and $\overline{GM_L}$ we need to define $\overline{BG}$, $\overline{BM_T}$ and $\overline{BM_L}$

$$\overline{BG} = \frac{\frac{H}{2} - z}{2} \tag{23}$$

$$\overline{BM_T} = \frac{I_T}{\nabla} = \frac{\int_{A_{wp}} y^2 dA}{\nabla} = \frac{LB^3}{12\nabla}$$
$$\overline{BM_L} = \frac{I_L}{\nabla} = \frac{\int_{A_{wp}} x^2 dA}{\nabla} = \frac{BL^3}{12\nabla} \tag{24}$$

The corresponding G matrixes for the η_1 and η_0 states are:

$$G_{\eta_1} = 10^6 \begin{bmatrix} 30 & 0 & 0 \\ 0 & 1725 & 0 \\ 0 & 0 & 24475 \end{bmatrix}$$
$$G_{\eta_0} = 10^6 \begin{bmatrix} 30 & 0 & 0 \\ 0 & 1754 & 0 \\ 0 & 0 & 24504 \end{bmatrix} \tag{25}$$

Next we can compute the ballast control vectors, and from them we can derive the tank volumes needed to bring the ship back in the reference position.

$$g_{01} = G_{\eta_1} \cdot \eta_1 = 10^4 \begin{bmatrix} -2750 \\ 3011 \\ 42717 \end{bmatrix}$$

$$g_{00} = G_{\eta_0} \cdot \eta_0 = 10^4 \begin{bmatrix} -2250 \\ 0 \\ 0 \end{bmatrix} \tag{26}$$

$$g_{0\Delta} = g_{01} - g_{00}$$

$$\Delta V = H^T (HH^T)^{-1} g_{0\Delta} \tag{27}$$

$$H = 10^4 \begin{bmatrix} 1 & 1 & 1 & 1 \\ -10.5 & 10.5 & -10.5 & 10.5 \\ 37.5 & 37.5 & -37.5 & -37.5 \end{bmatrix} \tag{28}$$

$$\Delta V = \begin{bmatrix} 88.0747 \\ 231.4319 \\ -481.4819 \\ -338.1247 \end{bmatrix} \tag{29}$$

Using ΔV we can obtain the target volumes in the ballast tanks: $V_1=588m^3$; $V_2=631m^3$; $V_3=219m^3$; $V_4=312m^3$. Having the volumes of the initial state and the final state we can run the graph generation algorithm and we will obtain a graph like in the following figure.

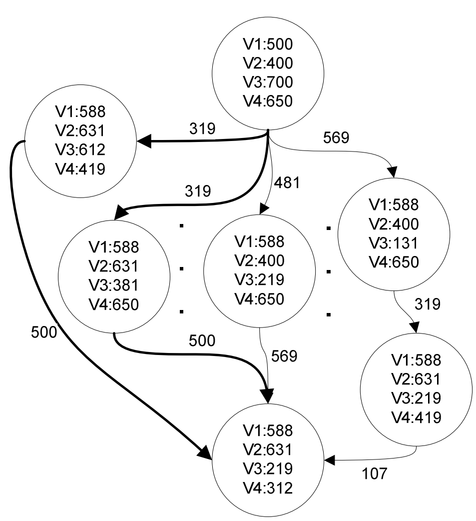

Fig. 4. Oriented graph presenting some of the paths from the start vertex to the destination vertex.

In the graph in *Figure 4*, not all the paths have been represented. In bold we have the shortest path between the initial and final state given by the Dijkstra algorithm. As we can see the path is not unique. The cost of the shortest path is 819m^3 and is composed from two arcs. One of the pump command sequences is, for the first arc, using pump 1 to move 231m^3 from T3 to T2 and pump 2 to move 88m^3 from T3 to T1. The command sequences corresponding to the second arc are using pump 1 to pump out 162m^3 from T3 and pump 2 to pump out 338m^3 from T4.

The cost of the shortest path is greatly impacted by the connections between the tanks, especially by the connections with the surrounding environment. If we consider the same initial states as in the previous example, and the same pump configuration, except from the connection with the surrounding environment which we will consider only available for T4.

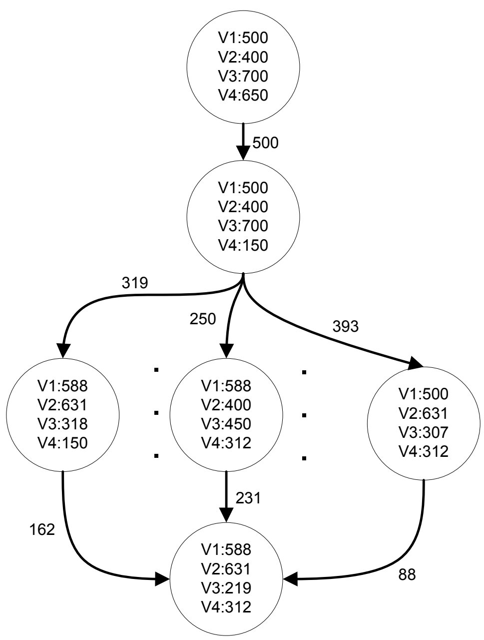

Fig. 5. Oriented graph presenting some of the paths from the start vertex to the destination vertex, in the case when only one tank is available for transfer with the exterior.

As we can see now the total cost is 981m^3, which is an increase in the total amount of water pumped to reach the final state. This was to be expected since all the water that needs to be pumped in or out of the system goes through only one tank.

V. CONCLUSIONS

The problematic of docked ships stabilization during loading and unloading procedures, by using interconnected ballast tanks, is formulated throughout this paper. A mathematical model is used to describe the codependence between the ship's position and orientation (and their desired values) and the volumes of water needed to be transferred in the ballast system in order to ensure ship stability during the weight shifting operations. This paper offers an optimal solution, in respect to the amount of energy consumption required to stabilize the vessel. Deriving from graph theory, an algorithm for solving the optimization problem has been developed, regarding the emerging constraints; its functionality has been proved by numerical results.

Also, conceiving this paper has opened new paths that will be investigated in the near future. The above presented algorithm finds an optimal solution for adjusting the ship's position and orientation to pre-specified reference values. However, situations in which critical weight shifts that do not permit stabilization using the reference values for the position and/or orientation coordinates, could find an optimal stabilizing solution by replacing the fixed draft reference value with a whole interval. Thus, a new optimization problem emerges, namely finding the minimum draft value (in an interval of possible solutions, limited by natural constraints, such as minimum ship draft or navigation conditions) which can stabilize the ship using the least amount of energy. Another idea has to be optimizing the ballast system transfers in respect to another important factor, duration. It's worth mentioning that, due to multiple pumps usage, minimum energy costs do not imply minimum operating time; the latter can be obtained by exploiting the pipes and pumps subsystem's parallel structure. Also, a study concerning possible intermediary perilous states (which could lead to ship sinking) is needed to be carried out, in order to eliminate such transitions.

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