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Switching Model Predictive Control for an Articulated Vehicle Under Varying Slip Angle

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***Abstract*— In this article a switching model predictive control scheme for an articulated vehicle under varying slip angles is being presented. For the non–holonomic articulated vehicle, the non–linear kinematic model that is able to take under consideration the effect of the slip angles is extracted. This model is transformed into an error dynamics model, which in the sequence is linearized around multiple nominal slip angle cases. The existence of the slip angles has a significant effect on the vehicle's path tracking capability and can significantly deteriorate the performance of the overall control scheme. Based on the derived multiple error dynamic models, the varying slip angle is being considered as the switching rule and a corresponding switching mode predictive control scheme is being designed that it is also able to take under consideration: a) the constrains on the control signals and b) the state constraints. Multiple simulation results are being presented that prove the efficacy of the overall suggested scheme.**

I. INTRODUCTION

Among the current vehicle types utilized in a mine field, articulated ones are the most characteristic vehicle's type that can be found most frequently, such as the Load Haul Dump (LHD) vehicles. In general the articulated vehicles consist of two parts, a tractor and a trailer, linked with a rigid free joint. Each body has a single axle and the wheels are all non–steerable, while the steering action is performed on the joint, by changing the corresponding articulated angle, between the front and the rear parts of the vehicle.

During the operation of an articulated vehicle, there are various external factors that degrade the overall system performance and introduce errors in the model that could be propagated with the time. Such deteriorating factors are: a) the generic interaction between the vehicle and its surrounding environment [1], b) the noise and bias in the positioning and driving sensors [2], c) the dynamic effects resulting from acceleration and braking [3], and d) the existence of variation of slip angles among the trailer and the tractor, which are also being influenced by the type of the driving ground, the fatigue and the tyres' type [4].

Among the aforementioned factors that degrade the overall performance of the articulated vehicle, the existence of slip angles is one of the most significant problem that the modeling and control approaches should face. In the relative literature there have been several research approaches for the problem of modeling articulated vehicles, based on the theory of multiple body dynamics [5–7]. Most of these methods contain simple models that are not taking under consideration the effect of the slip angles, as the complexity of the overall problem is being increased and it is more difficult to proceed to the next stage of the control scheme design based on highly non–linear articulated vehicle's dynamics.

From a control point of view, there have been proposed many traditional techniques for non–holonomic vehicles, based on error dynamics models without the presence of slip angles. More analytically, in [6, 8] linear control feedback has been applied, while in [9] a Lyapunov based approach has been presented. In [10] a control scheme based on LMIs has been presented and in [11] a pole placement technique has been applied. Moreover, in [12] the authors have presented a path tracking controller based on error dynamics, while in [13] the problem of designing a path following controller for a *n*–trailer vehicle, based on non–linear adaptive control has been derived. Finally, classical fundamental problems of motion control for articulated vehicles have been presented in [7, 14, 15].

The main contribution of this article is dual. First, to the author's best knowledge, this is the first time that an error dynamics modeling framework will be derived for the case of an articulated vehicle operating under slip angles. Until now and mainly due to the increasing complexity of the problem, only the case of non–slip angles affecting the movement of the vehicle has been considered. Secondly, the problem of controling the articulated vehicle, under the presence of varying slip angles, is being addressed by a switching Model Predictive Control (MPC) scheme.
Multiple MPC controllers are being fine tuned for specific slippage operating conditions, while the proposed control scheme has the ability to take under consideration the effect of real life constraints on the control input (articulated angle) and the environmental restrictions. In this control design approach the current estimated slip angle of the tractor is considered as the mode selector for the switching MPC. The resulting control scheme provides the optimal control for each region of slip angles, while ensuring smooth transition of the control effort as the articulated vehicle is driven over regions of different slippage.

The rest of this article is structured as it follows. In Section II the error dynamics modeling framework for the cases of with and without the effect of the slip angles is being presented. In Section III the switching model predictive controller is being analyzed, while in Section IV multiple simulation results are being presented that prove the efficacy of the proposed scheme. Finally the conclusions are draw in Section V.

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II. Articulated Vehicle Modeling

A. Articulated Vehicle Kinematics Without Slip Angles

The case of an articulated vehicle under no slip angles is being presented in Figure 1.

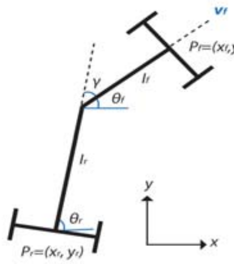

Fig. 1. Typical articulated vehicle without slipping

In this figure $(x_f,\ y_f)$ and $(x_r,\ y_r)$ denote the coordinates of the front and the rear part of the vehicle, p_f and p_r are the corresponding centers of gravity, l_f and l_r are the length of the front and the rear units, while the angles θ_f and θ_r denote the vehicle's part orientation, and the $x,\ y$ axes represent the fixed coordinating system. Moreover v_f and v_r are the linear velocities of the tractor and trailer respectively and are being defined as:

$$\dot{x}_f = v_f \cos\theta_f \quad (1)$$

$$\dot{y}_f = v_f \sin\theta_f \quad (2)$$

The articulated angle γ is being defined as the difference between the orientation angle θ_f of the front and the orientation angle θ_r of the rear part of the vehicle. By examining the vehicle's depicted geometry, as also the relation between the coordinates of p_f and p_r, the equations of motion for the vehicle's rear part, can be derived from the following equations:

$$x_r = x_f - l_f \cos\theta_f - l_r \cos\theta_r \quad (3)$$

$$y_r = y_f - l_f \sin\theta_f - l_r \sin\theta_r \quad (4)$$

When the non–holonomic constraints, acting on the front and rear axles, are taken under consideration, mainly resulting from the assumption of moving without slipping in the vehicle's wheels, the following equations (5–6) can also be extracted:

$$\dot{x}_f \sin\theta_f - \dot{y}_f \cos\theta_f = 0 \quad (5)$$

$$\dot{x}_r \sin\theta_r - \dot{y}_r \cos\theta_r = 0 \quad (6)$$

By taking the derivative of the equations (3–4) with respect to time, substituting the results in equations (5-6), and by further simplifying the results, the time derivative of the tractor's orientation angle is being defined as:

$$\dot{\theta}_f = \frac{v_f \sin\gamma + l_r \dot{\gamma}}{l_f \cos\gamma + l_r} \quad (7)$$

The angular velocities for the tractor and the trailer, which are being defined as $\dot{\theta}_f$ and $\dot{\theta}_r$ respectively, have different values when $(l_f \neq l_r)$ or the vehicle is not driving straight $(\gamma\neq 0)$.

$$\dot{\theta}_r = \frac{v_f \sin\gamma - l_f \dot{\gamma} \cos\gamma}{l_f \cos\gamma + l_r} \quad (8)$$

B. Articulated Vehicle Kinematics Under Slip Angles

In the case of slip angles, the kinematic model of the articulated vehicle can be also formulated from general geometry that includes ideal and slip behavior for the steering angles. The definition of this model has been initially based on the derivation in [16], which included the perturbed factors in the vehicle position as two slip variables β and α, being defined as the tractor's and trailer's slip angles respectively, and which are also the angles between the linear velocities of the vehicle's parts.

For the following derivation of the kinematic model, it is being assumed that the vehicle is moving on a flat surface under the influence of slip angles, while the vehicle's configuration is depicted in Figure 2. where $(r_1,\ r_2)$ are the

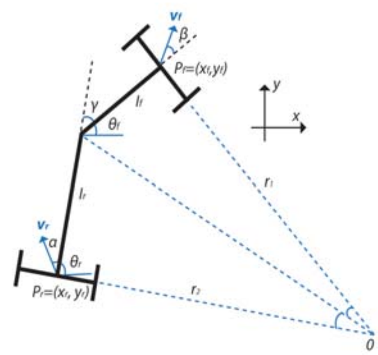

Fig. 2. Articulated vehicle modeling configuration under the influence of slip angles

instantaneous centers of velocity for the tractor and the trailer of the vehicle with different radiuses. For the initial center curvature of the trajectory it is being assumed that the vehicle is moving forward without slip conditions. In the following derivation, the unit subscript 's' denotes variables in the slip case as they have been defined previously for the non–slip one.

The kinematic equations, for the motion under the effect of slip angles, for the front part can be formulated as:

$$\dot{x}_{fs} = v_f \cos(\theta_f + \beta) \quad (9)$$

$$\dot{y}_{fs} = v_f \sin(\theta_f + \beta) \quad (10)$$

In this case, the vehicle motion depends not only on vehicle's speeds, the articulated angle and the vehicle's lengths, but also on the slip angles, while the resulting vehicle's heading is provided by a combination of the slip angles with the orientation angles. The rate of the orientation $\dot{\theta}_{rs}$ can be defined as function of the steering angle γ and both slip angles. Based on the assumption that the vehicle develops a steady–state motion turning, this rate can be provided by the utilization of a virtual center of rotation, depending on the

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velocity as a function of $(cos(\beta),\ sin(\beta))$ [2]. With respect to the local coordinate axis origin, the velocities are being defined as:

$$v_{fs} = \begin{bmatrix} cos\beta & 0 \\ sin\beta & -l_f \end{bmatrix} \begin{bmatrix} v_{fs} \\ \dot{\theta} fs \end{bmatrix} \tag{11}$$

$$v_{rs} = \begin{bmatrix} cos\alpha & 0 \\ sin\alpha & l_r \end{bmatrix} \begin{bmatrix} v_{rs} \\ \dot{\theta} rs \end{bmatrix} \tag{12}$$

By utilizing rigid body principles, the velocity of the tractor, with respect to the velocity of the trailer, can be calculated with respect to the articulated angle as it follows [17]:

$$v_{rs} = \begin{bmatrix} cos\alpha & 0 \\ sin\alpha & l_r \end{bmatrix}^{-1} \begin{bmatrix} cos\gamma & -sin\gamma \\ sin\gamma & cos\gamma \end{bmatrix} \begin{bmatrix} cos\beta & 0 \\ sin\beta & -l_r \end{bmatrix} \begin{bmatrix} v_{fs} \\ \dot{\theta} fs \end{bmatrix} \tag{13}$$

By replacing equations (11-12) in (13), the angular speed for the tractor can be defined as:

$$\dot{\theta}_{fs} = \frac{v_f\ sin(\gamma+\beta-\alpha) + l_r\ \dot{\gamma}\ cos\ \alpha}{l_f\ cos(\gamma-\alpha) + l_r\ cos\ \alpha} \tag{14}$$

It should be noted that equation (14) is more accurate if the slip angles are known a priori, a task that is quite difficult as the slip angles are dependent of the vehicle's speed, mass, tire–terrain interaction and articulation angle, in a highly non–linear way. Moreover, in calculating the angular velocity of the tractor, the primary sources of the errors, are due to the time varying parameters $\gamma, \dot{\gamma}, \beta$ and α, as errors in these parameters propagate directly to the states. The variables γ and $\dot{\gamma}$ can be measured with a great accuracy, while an estimation algorithm or a look up table should be utilized for determining the value of the slip parameters.

C. Error Dynamics Modeling

Before proceeding with the design of the control scheme, an appropriate model should be initially derived. In the sequel, the error dynamics modeling procedure will be presented. while it should be highlighted that this is the first time in the relevant scientific literature that such a modeling framework, with the ability to take under consideration the effect of slip angles, is being presented.

Based on the assumption that the vehicle develops a steady–state motion turning, or $\dot{\gamma} = 0$, the rates of the orientation change are being provided by: $r_1 = \frac{v_f}{\dot{\theta}_f}$ and $r_2 = \frac{v_r}{\dot{\theta}_r}$. In Figure 3 it is depicted an overview of how the displacement, heading and curvature errors are being defined, between the actual path of the articulated vehicle and the desired one. In this figure, the distance from the vehicle to the reference path displacement, as the angle between the vehicle and the reference are also being displayed.

In the following derivation, three errors are defined as: a) e_d is the displacement error, b) e_h is the heading error, and c) e_c is the curvature error. Based on [5], the displacement error e_d is the difference between the coordinates of the tractor and the coordinates of the desired circular path. From the triangle (abd), it can be defined that $\theta = \arctan\frac{l}{r}$ and from the triangle (dbc) $e_h = \arctan(\frac{e_d}{l})$. If it is assumed that ϕ and

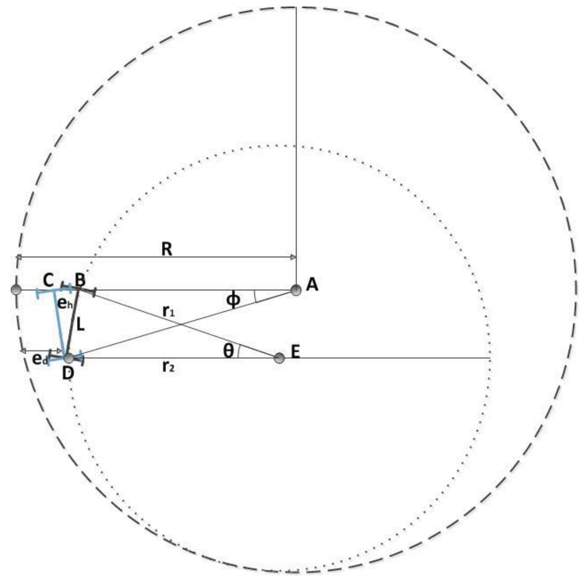

Fig. 3. Articulated vehicle following error transformation

θ are small, l can be defined as $l = r\ \theta$ and $e_d = r\theta e_h$, and by taking the time derivative of this equation yield:

$$\dot{e}_d = v\ e_h \tag{15}$$

The heading error e_h is the orientation difference between the centers of the vehicle and the circular path. A change in the heading error is being defined as $e_h = \theta - \phi$, while the relation between θ and ϕ with respect to l, after small assumptions, is defined as: $(\theta\ r = \phi\ R)$. From this equation it can be derived that $e_h = \frac{\theta\ R - \theta\ r}{R}$, and by taking the time derivative results in: $\dot{e}_h = \dot{\theta} r(\frac{1}{r} - \frac{1}{R})$. The curvature error e_c measures the difference between the vehicle's path circle and the curvature of the trajectory path, and it can be defined as: $e_c = (\frac{1}{r} - \frac{1}{R})$. By utilizing the relations in above equations, the curvature error is being defined as:

$$\dot{e}_h = v\ e_c \tag{16}$$

With the assumption that the velocity and the curvature of the trajectory are constant and by differentiating the curvature equation with respect to time, the rate of the curvature error is being defined as:

$$\begin{aligned} \dot{e}_c = & \frac{\dot{\gamma}\ l_r cos(\gamma+\beta-\alpha) cos\ \alpha + \dot{\gamma} l_f\ cos(\gamma+\beta-\alpha)\ cos(\gamma-\alpha)}{(l_r\ cos\alpha + l_f\ cos(\gamma-\alpha))^2} + \\ & \frac{\dot{\gamma}\ v\ l_f\ sin(\gamma+\beta-\alpha)\ sin(\gamma-\alpha) - \dot{\gamma}^2\ l_f\ l_r\ cos\ \alpha)\ sin(\gamma-\alpha)}{v(l_r\ cos\alpha + l_f\ cos(\gamma-\alpha))^2} - \\ & \frac{\ddot{\gamma}\ l_f\ l_r\ cos(\gamma-\alpha)\ sin(\alpha)\dot{\alpha} + (l_r^2\ cos\alpha\ sin\alpha\ \dot{\alpha})}{v(l_r\ cos\alpha + l_f\ cos(\gamma-\alpha))^2} \end{aligned} \tag{17}$$

Linearizing the error dynamics in equation (17) around the reference path γ yields:

$$\dot{e}_c = \dot{\gamma}\ \frac{(l_f + l_r) + l_f(\gamma^2 + \gamma\beta - 2\gamma\alpha) + l_f(\alpha^2 - \alpha\beta)}{(l_r + l_f)^2} \tag{18}$$

where it can be observed that $\dot{e_c}$ is dependant on the slippage and the rate of change of articulation angles. By performing the following change of variable:

$$\acute{e}_c = \dot{e}_c \ - \dot{\gamma}\ \frac{l_f(\gamma^2+\gamma\beta-2\gamma\alpha)}{(l_r+l_f)^2} \tag{19}$$

The following state space error dynamics description for the articulated vehicle is being extracted:

$$\begin{bmatrix} \dot{e}_d \\ \dot{e}_h \\ \acute{e}_c \end{bmatrix} = \begin{bmatrix} 0 & v & 0 \\ 0 & 0 & v \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_d \\ e_h \\ \acute{e}_c \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{l_f(1+\beta-2\alpha)}{(l_f+l_r)^2} \\ \frac{l_r+l_f(1+\alpha^2-\alpha\ \beta)}{(l_f+l_r)^2} \end{bmatrix} \dot{\gamma} \tag{20}$$

In the case that the articulated vehicle is being driven over terrains that are characterized by different slip angles, multiple larger operating sets of slip angles, with the same characteristics, $a_i \in \mathscr{L}_1$ and $b_i \in \mathscr{L}_2$, with $\mathscr{L}_1,\ \mathscr{L}_2 \in \Re^2$ and $i \in \mathbf{Z}^+$, can be defined, leading to a switching state space description for the model in (20) as it follows:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_i\dot{\gamma} \tag{21}$$

where:

$$\mathbf{x} = \begin{bmatrix} e_d \\ e_h \\ \acute{e}_c \end{bmatrix},\ \mathbf{A} = \begin{bmatrix} 0 & v & 0 \\ 0 & 0 & v \\ 0 & 0 & 0 \end{bmatrix},\ \ \mathbf{B}_i = \begin{bmatrix} 0 \\ \frac{l_f(1+\beta_i-2\alpha_i)}{(l_f+l_r)^2} \\ \frac{l_r+l_f(1+\alpha_i^2-\alpha_i\ \beta_i)}{(l_f+l_r)^2} \end{bmatrix}$$

and $\mathbf{x} \in \mathscr{X} \subseteq \Re^3$ is the state vector, $\dot{\gamma} \in \mathscr{U} \in \Re$ is the control action, $\mathbf{A} \in \Re^{6\times 6}$, $\mathbf{B_i} \in \Re^{3\times 1}$, and full state feedback is being considered, or $\mathbf{C} = \mathbf{I}_{3\times 3}$.

III. Switching Model Predictive Control Design

MPC is a highly effective control scheme that is able to take under consideration multiplicative system model descriptions, uncertainties, nonlinearities and physical and mechanical constraints in the system model parameters or in the control signals [18]. In the current research effort, the MPC schemes is able to predict future values of the vehicles error dynamics based on the present available information and the current constraints [19]. The MPC control action, which is the rate of articulation angle, is based on a finite horizon continue time minimization of predicted tracking error with constraints on the control inputs and the state variables.

The overall block diagram of the proposed closed loop system is depicted in Figure 4. For efficiently controlling the articulated vehicle, the controller utilizes the current state of motion of the vehicle as well as the next target points of the reference trajectory. The trajectory planner is generating the desired path, while in the sequel this path (planar coordinates) are being translated to displacement, heading and curvature coordinates that act as the reference input for the MPC controller. In the presented methodology for the design of the MP–controller scheme, the mode selector signal of the MPC is the estimated slip angle for the front part of the vehicle using estimation approaches (like extended Kalman filter). The formulation of the MPC is based on: a) the current full state feedback, b) the active constrains on the system, c) the estimated or measured slip angles, and d) the mode selector signal, applies the necessary switching optimal control action.

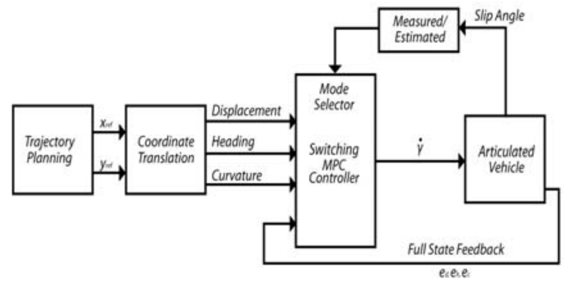

Fig. 4. Switching MPC scheme block diagram

The construction of the MP–controller is based on the system description defined in equation (21). The mode selector signal $i \in \mathscr{S}$ with $\mathscr{S} \triangleq \{1,2,\cdots,s\}$ is a finite set of indexes and s denotes the number of switching sub–systems in (25). For polytopic description, Σ is the polytope $\Sigma:\ Co\{[\mathbf{A}\ \mathbf{B_1}],\cdots,[\mathbf{A}\ \mathbf{B_s}]\}$, Co denotes the convex hull and $[\mathbf{A},\mathbf{B_i}]$ are the vertices of the convex hull. Any $[\mathbf{A},\ \mathbf{B_i}]$ within the convex set Σ is a linear combination of the vertices $\sum_{j=1}^{s}\mu_j[\mathbf{A}\ \mathbf{B_i}]$with $\sum_{j=1}^{s}\mu_j = 1,\ 0 \le \mu_j \le 1$. In the presented methodology for the design of the MPC scheme, the mode selector signal is the estimated slip angle. For defining the switching instances, the sets $\mathscr{L}_1$ and $\mathscr{L}_2$ have been discretized into equal operating subspaces. The discretization of the the slip angles operating set, can be formulated by defining multiple nominal values $\alpha_0,\ \beta_0$ and allowing them to take values into neighboring regions of lengths ξ_i and ψ_i. This can be formulated as:

$$\begin{aligned} \mathscr{L}_{1,i} &= \alpha_{0,i}^{\min} = \alpha_{0,i} - \xi_i \le \alpha_{0,i} \le \alpha_{0,i}^{\max} + \xi_i = \alpha_{0,i}^{\max} \\ \mathscr{L}_{2,i} &= \beta_{0,i}^{\min} = \beta_{0,i} - \psi_i \le \beta_{0,i} \le \beta_{0,i}^{\max} + \psi_i = \beta_{0,i}^{\max} \end{aligned}$$

and by assuming that both slip angles are taking the same values at the same time instances, due to the fact that the length of the articulated vehicle is being considered small, results in ξ_i and ψ_i and for the switching regions:

$$\mathscr{L}_1 = \mathscr{L}_2 = \bigcup \mathscr{L}_{1,i} = \bigcup \mathscr{L}_{2,i}$$

The sets $\mathscr{X}$ and $\mathscr{U}$ specify state and input constraints. Let the set $\mathscr{X}$ contain the $\mathbf{x}$ states that satisfy the following bounding inequality:

$$\mathbf{x}^{\min} = \mathbf{x} - \Delta_1 \le \mathbf{x} \le \mathbf{x} + \Delta_1 = \mathbf{x}^{\max}$$

where $\Delta_1 \in \Re_+^{(3,1)}$ is the vector containing the selecting state boundary conditions. The control input bounding set $\mathscr{U}$ can be derived by taking under consideration the mechanical and the physical constraints of the articulated vehicle, as also the preference on aggressive or not maneuvers. These constraints can be also formulated as presented in (22).

$$u^{\min} = u - \Delta_2 \leq u \leq u + \Delta_2 = u^{\max}$$

where $\Delta_2 \in \Re_+$ is the vector containing the selecting control boundary conditions. Let the matrix $\mathbf{H_i}$ be a zeroed 2×2 matrix with its i–th column equal to $[1, -1]^T$, and the other $\mathbf{0}_{2,1}$ i.e. for $i = 2$:

$$\mathbf{H_i} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$$

Then the previous bounds can be cast in a more compact form as:

$$\begin{bmatrix} \mathbf{H_1} \\ \mathbf{H_2} \end{bmatrix}_{4\times 2} \cdot \begin{bmatrix} \mathbf{x} \\ -- \\ u \end{bmatrix}_{2\times 1} \leq \begin{bmatrix} \mathbf{x}^{\max} + \Delta_1 \\ ----- \\ u^{\max} + \Delta_2 \end{bmatrix}_{4\times 1}$$

These constraints are embedded in the Model Predictive Control computation algorithm in order to compute an optimal controller that counts for the physical and mechanical constraints that restrict the articulated vehicle's motion.

The basic idea of MPC is to calculate a sequence of future control actions in such a way that it minimizes a cost function defined over a predefined prediction horizon. The index to be optimized is the expectation of a quadratic function measuring the distance between the predicted system's output and some predicted reference sequence over the horizon in addition to a quadratic function measuring the control effort. More specifically, the (i)–th MP–controller's objective is to optimize the quadratic cost in (22), while the (i)–th linearized system is within Σ.

Special care must be provided in order to correctly tune the prediction N_p and the control N_c horizon. A long prediction horizon increases the predictive ability of the MPC controller but on the contrary it decreases the performance and demands more computations. The control horizon must also be fine–tuned since a short control horizon leads to a controller that tries to reach the set–point with a few conservative motions, a method that might lead to significant overshoots. On the contrary, a long control horizon produces more aggressive changes in the control action that tend to lead to oscillations. Obviously the tuning of prediction and control horizon is a coupled process. For the aforementioned reasons the control horizon must be chosen short enough compared to the prediction horizon. Additionally, the response of the system can be also shaped using weight matrices on the system outputs, the control action and the control rates.

Each model predictive controller V^i_{MPC} corresponds to the i-th error dynamic model of the articulated vehicle, obtained by solving the following optimization problem $\min J(k)$ with respect to the control moves variations Δu and to the error coordinates, k the discrete time sample index and $J(k)$ defined as:

$$\begin{aligned} J(k) \quad = \quad & \sum_{n=N_w}^{N_p} [\hat{\mathbf{y}}(k+n|k) - \mathbf{r}(k+n|k)]^T \mathbf{Q} [\hat{\mathbf{y}}(k+n|k) - \mathbf{r}(k+n|k)] + \\ & + \sum_{n=0}^{N_c-1} [\Delta u^T(k+n|k) \mathbf{R} \Delta u(k+n|k)] \qquad (22) \\ & + \sum_{n=N_w}^{N_p} [u(k+n|k) - \mathbf{s}(k+n|k)]^T \cdot \mathbf{N} [u(k+n|k) - \mathbf{s}(k+n|k)] \end{aligned}$$

where, n is the index along the prediction horizon, N_w is the beginning of the prediction horizon, $\mathbf{Q}$ is the output error weight matrix, $\mathbf{R}$ is the rate of change in control weight matrix, $\mathbf{N}$ is the control action error weight matrix, $\hat{\mathbf{y}}(k+n|k)$ is the predicted system's output at time $k+n$, given all measurements up to including those at time k, $\mathbf{r}(k+n|k)$ is the output set–point profile at time $k+n$, given all measurements up to including those at time k, $\Delta u(k+n|k)$ is the predicted rate of change in control action at time $k+n$, given all measurements up to including those at time k, $u(k+n|k)$ is the predicted optimal control action at time $k+n$, given all measurements up to and including those at time k, and $\mathbf{s}(k+n|k)$ is the input set–point profile at time $k+n$, given all measurements up to and including those at time k.

Once all $V^i_{MPC}(k)$ controllers are computed, the total switching MP–controller is constructed by implementing a switching among the difference controllers in relation with the estimated/measured values of slip angles α and β. The objective of the $i \in \mathscr{S}$ controller is to stabilize the i–th system, while if s increases, then the approximation of the error dynamic modeling is more accurate for a larger part of slip angles and thus allowing the development of more efficient flight control algorithms.

IV. Simulation Results

For simulating the efficacy of the proposed control scheme for the problem of path following for an articulated vehicle, over a terrain with varying slip angles, the following vehicle's characteristics have been considered: $l_f = 0.6m$, $l_r = 0.8m$ and constant speed $u = 2m/sec$, values that have been extracted from the real articulated vehicle in Figure 1. Three operating sets for the slip angles have been defined as: $0 \leq |\beta_1| \leq 0.02$, $0.02 < |\beta_2| \leq 0.04$, and $0.04 < |\beta_3| \leq 0.09$, while the bounds on the articulated angle have been defined as $-0.785 \leq \gamma \leq 0.785$, and the bounds on the dynamic errors were $-0.2 \leq \mathbf{x} \leq 0.2$. The proposed controller has been evaluated in a closed circle reference path. The corresponding predictive horizon has been set to $N_p = 10$ and the control horizon as $N_c = 5$ with control interval $0.2sec$.

In Figure 5 the results from path tracking by utilizing switching and non-switching MP–controller under varying slip angles are being presented. As it can been observed there is a significant tracking error, due to the effect of slippage angles deteriorating the overall controller performance.

In Figure 6 the error dynamics and in Figure 7 the articulation angle and articulation angle's rate for the two cases are being presented. It should be noted that: a) in the presented simulation results, harsh switching and big slippage angles have been considered, that simulate the translation of the vehicle under heavy road conditions, and b) the same switching angles have been utilized as the testing scenario in the depicted simulation results.

V. Conclusions

In this article a switching model predictive control scheme for an articulated vehicle under varying slip angles has

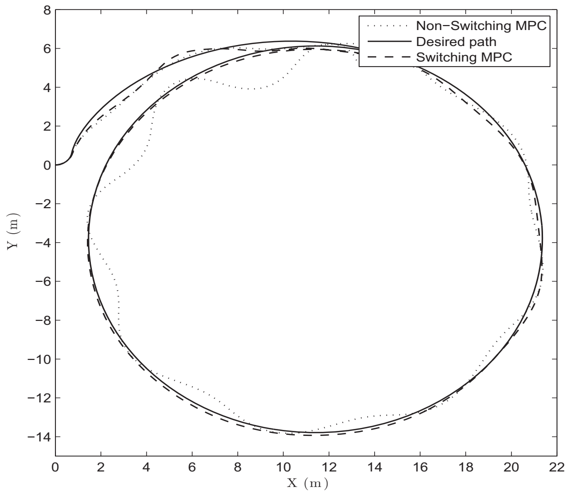

Fig. 5. Circle path tracking based on switching MP-controller and non-switching MP–controller under varying slip angles

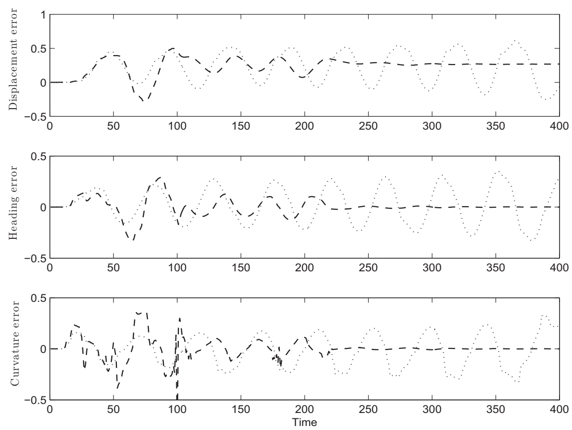

Fig. 6. Time evolution of the system error dynamics for both case studies

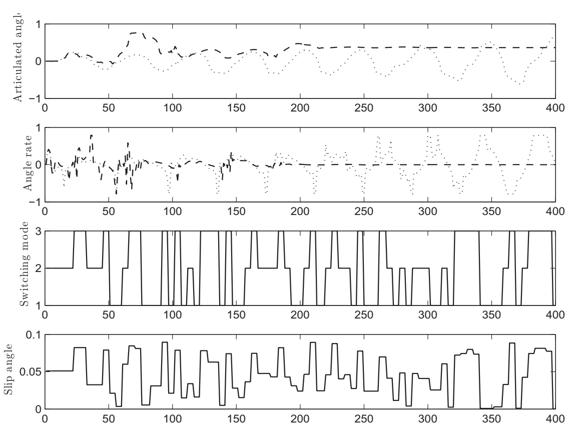

Fig. 7. The articulated angle as a control signal, switching mode selector and the slip angle

been presented. The non-linear kinematic model that is able to take under consideration the effect of the slip angles was extracted and been transformed into an error dynamics model, which in the sequence has been linearized around multiple nominal slip angle cases. The varying slip angle has been considered as the switching rule and a corresponding switching mode predictive control scheme was designed. The simulation results have been presented that prove the efficacy of the overall suggested scheme. Future work includes the application of the proposed scheme in experimental studies.

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