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Fault detection & isolation for a class of hybrid systems: A dedicated switched robust observer scheme

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***Abstract*— In this paper, hybrid observers based approach for faults detection and isolation, for a class of MIMO hybrid systems, is proposed. The faults herein considered can affect the sensors or the discrete trajectory of the considered system. The proposed approach is based on a Dedicated Switched Robust Observer Scheme (DSROS) using three blocks in interaction. A block generating the mode signatures, a one for the fault signature generation and the last for the diagnosis of observed discrete events. Some LMIs conditions are provided to guarantee both of the robustness and the convergence of the proposed observers. Finally some simulation results illustrate the efficiency of the proposed approach.**

I. Introduction

HYBRID systems have been the subject of intensive study in the past few years by both of the control and the computer science communities [1] (see also previous works of authors [17, 18] for observation problem and [20] for control) . Recently, different hybrid system techniques have been applied to several industrial domains where the fault detection and isolation (FDI) are needed to guarantee reliability and safety.

Generally, the procedure of FDI can be viewed as a sequential process involving in two steps: the fault detection, and the fault diagnosis. Fault detection is concerned with extraction of relevant features that indicate the existence of a fault, whereas, fault identification refers to the dimensional, spatial and temporal locations and categorization of a fault or a set of faults. Recently, FDI techniques for hybrid systems, has gained increasing consideration world-wide. Among them, several techniques such the state estimation and/or parameters estimation have been investigated to deal with FDI for linear Hybrid Dynamic Systems (HDS) [2]. Moreover, most of the existing fault detection and localization approaches are performed by continuous or discrete abstraction of the HDS [8, 19]. However, these approaches are not efficient when both the continuous behavior and discrete events are required together to achieve the diagnosis task. To overcome that, the authors in [9] combine qualitative and quantitative strategies to efficiently identify a candidate fault set and to refine this set. More recently, a hybrid system diagnosis methodology based on hybrid automata models has been presented in [10]. Therein, the proposed architecture is composed of modules that realize both recognition and diagnosis tasks on a predictive model generating some residuals.

In addition, even if several attempts for FDI, for some particular classes, of HDS have been made [3], it seems that only few works consider the case of HDS characterized by modeling errors, noise measurements and external disturbances [4-7].

The problem addressed in this paper aims at providing a FDI approach for a class of Switched Linear Systems (SLS), subject to unknown inputs and modeling errors, without full state measurements. Two kinds of faults are herein considered. On the one hand, one deals with the detection and the isolation of the faults affecting the continuous part and more precisely those related to the sensors. On the other hand, faults causing a switching to an abnormal mode and/or to a no successor discrete mode will be considered as faults affecting the discrete part of the SLS. Hence, the main contribution of the paper consists in the proposition of a Dedicated Switched Robust Observer Scheme (DSROS) to a class of MIMO SLS for the purpose of sensor fault isolation. For that, the robust fault detection and isolation problem is considered as a H_∞ model matching, allowing a suitable trade-off between robustness to the unknown inputs and the sensitivity to faults. The proposed DSROS allows also the detection of the fault affecting the discrete trajectory thanks to a discrete event based diagnosis approach.

The paper is organized as follows: Section 2 presents the considered class of HDS and gives an overview about the proposed diagnosis approach. Section 3, details the scheme of the dedicated switched robust observer and provides the main results of the paper. Finally, a numerical example illustrates the efficiency of the approach.

II. Class of HDS and Diagnosis Approach

A. Switched Linear system

Let us consider the following class of switched linear systems:

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$$\dot{x} = A_{q(t)}x + B_{q(t)}u + B_{d_q(t)}d \tag{1}$$

$$y = C_{q(t)}x + D_{q(t)}u + D_{d_{q(t)}}d + f_{q(t)} \tag{2}$$

where $x \in \Re^n$ is the state vector, $u \in \Re^m$ is the input vector, $y \in \Re^p$ is the measurement (output) vector, $d \in \Re^m$ is the unknown input vector (including disturbance, uninterested fault as well as norm-bounded unstructured model uncertainty, noise). $f \in \Re^p$ are the faults to be detected and isolated. $q(t)$ is an index function $q:[0\infty) \to I_N = \{1,\cdots,N\}$ deciding which linear vector field is activate. $A_{q(t)}$, $B_{q(t)}$, $C_{q(t)}$, $D_{q(t)}$, $B_{d_{q(t)}}$, $D_{d_{q(t)}}$ are known constant matrices with appropriate dimensions. Without loss of generalities, d and f are assumed to be L_2-norm bounded and for the task of DSROS synthesis, $C_{q(t)}$ and $D_{q(t)}$ will be written as:

$$C_{q(t)} = \left[\left(C^1_{q(t)}\right)^T \ \cdots \ \left(C^p_{q(t)}\right)^T \right]^T, D_{q(t)} = \left[\left(D^1_{q(t)}\right)^T \ \cdots \ \left(D^p_{q(t)}\right)^T \right]^T$$

where for $i = 1\cdots p$, $C^i_{q(t)}$ and $D^i_{q(t)}$ are respectively the i^th^ row of $C_{q(t)}$ and $D_{q(t)}$.

Along this work, one assumes that only a single sensor fault can occur or a switching to an abnormal (not successor mode) can occur. Moreover, the dedicated switched observer scheme will be designed by considering that only the discrete switching sequence is known (the active mode is not known at each instant) and that all the SLS modes are observable and discernable.

B. Hybrid System Diagnosis Approach

The proposed diagnosis methodology is illustrated in Fig.1. It is based on the interaction between "normal" model of the SLS and three main blocks called Mode Signature Generator (MSG), Fault Detection and Isolation (FDI) and Mode Identification & Fault Diagnosis (MIFD). The MSG block receives the system's inputs (u) and system's outputs (y). Its task is to generate the N mode signatures related to the normal modes of the SLS. In the same way, the FDI block generates the fault signatures, which are sensitive only to a single sensor fault. These signatures are obtained by comparing the estimated and the measured outputs. Finally, MIFD block is viewed as a deterministic finite state machine that is built from the normal discrete trajectory of the considered SLS. In figure 1, we denote by σ_{r_q} the mode signature, $q \in I_N$, and σ_{z_q} the fault signature, $q \in I_P$, where $I_p = \{1,\cdots,p\}$.

III. The Proposed FDI Methodology

A. The Mode Signature Generator block

This block is used to detect a switching from a mode to another one, which can be either a normal or a faulty mode. It is based on a bank of observers, which are synthesized taking into account the unknown inputs. The MSG which will generate the N signatures corresponding the all the normal behaviors of the systems, can be, more precisely, described by a cascade of two blocks [13] (Fig. 2).

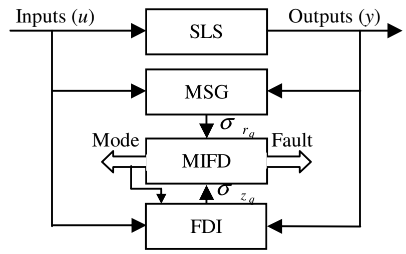

Fig. 1. Hybrid diagnosis methodology.

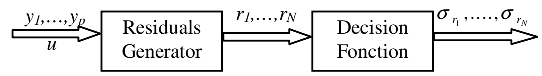

Fig. 2. MSG structure.

The first block of the MSG generates the residuals and it is based on a bank of N Luenberger observers:

$$\dot{\hat{x}} = A_{q(t)}\hat{x} + B_{q(t)}u + K_{q(t)}(y - \hat{y}) \tag{3}$$

$$\hat{y} = C_{q(t)}\hat{x} + D_{q(t)}u \tag{4}$$

$$r_{q(t)} = y - \hat{y} \tag{5}$$

where $\hat{x} \in \Re^n$ and $\hat{y} \in \Re^p$ represent respectively the state and the output estimation vectors, r_q is the so-called residual signal. $K_{q(t)}$ is the observer gain matrix (the index time will be omitted in the next when there is no ambiguity).

Let $e = x - \hat{x}$ be the observation error, then its dynamic and the residual signal can be expressed as:

$$\dot{e} = \left(A_q - K_qC_q\right)e + \left(B_{d_q} - K_qD_{d_q}\right)d \tag{6}$$

$$r_q = C_qe + D_{d_q}d \tag{7}$$

Now, in order to guarantee the asymptotic stability of r_q as well as the robustness of the observer regarding the unknown inputs, the following condition should be verified for given scalar $\gamma_q > 0$: $\|r_q\|_2^2 - \gamma_q^2\|d\|_2^2 < 0$ (8)

Finally, the observer gains K_q can be calculated using the following lemma:

Lemma 1. Consider the switched linear system (1) and (2) with the bank of N observers (3)-(5). System (6)-(7) is asymptotically stable and satisfies (8) if there exist the matrices $P_q = P_q^T > 0$, K_q such that the following LMIs hold:

$$\begin{bmatrix} A_q^TP_q + P_qA_q - M_qC_q - C_q^TM_q^T & P_qB_{d_q} - M_qD_{d_q} & C_q^T \\ * & -\gamma_q^2I & D_{d_q}^T \\ * & * & -I \end{bmatrix} \le 0 \tag{9}$$

where: $M_q = P_qK_q$.

Proof. Due to space limitation, the proof is omitted in this paper. For more details, the reader can refer to the work of [4].

On the other hand, the second block of the MSG (Fig. 2) consists of a decision function, which exploits the given signatures and delivers N binary signals indicating the active mode of the SLS. These signals are obtained as follows, for $q=1,\ldots,N$, $\sigma_{r_q}(t)$ is true $if\left\|r_q(t)\right\|\leq\varepsilon$ and $\sigma_{r_q}(t)$ is false $if\left\|r_q(t)\right\|>\varepsilon$, where ε is a designed threshold parameter.

B. The Fault Detection & Isolation block

This section extends our previous results [4] on the fault detection where the SLS were characterized by known switching conditions. Now, we deal with MIMO SLS with unknown switching conditions. Moreover, we consider the more challenging problem of the fault localization and isolation. The proposed FDI approach is based on the design of a bank of DSROS illustrated in Fig. 3. The latter can be viewed as an extension of the dedicated observer scheme originally proposed in [14] to handle with sensor fault isolation for LTI systems. The idea behind that is quite simple. In fact, under the assumption that p sensor faults have to be detected and isolated, p residual generators are then constructed, and each one of them is driven by only one output.

$$\begin{bmatrix} z_1 & \cdots & z_p \end{bmatrix}^T = \begin{bmatrix} F_1(u,y_1) & \cdots & F_p(u,y_p) \end{bmatrix}^T \tag{10}$$

where $F_i(u,y_i)$, $i=1,\ldots,p$, stands for a function of the inputs and i^{th} output y_i. It is obvious that the i^{th} residual, z_i, will only be influenced by the i^{th} sensor fault f_i, and it thus can guarantee a sensor fault isolation.

Thus for the case of a MIMO SLS, the i^{th} robust hybrid DSROS for the system (1)-(2) can be defined as:

$$\dot{\hat{x}} = A_{\hat{q}}\hat{x} + B_{\hat{q}}u + L_{\hat{q}}^i\left(y_i-\hat{y}_i\right) \tag{11}$$

$$\hat{y}_i = C_{\hat{q}}^i\hat{x} + D_{\hat{q}}^i u \tag{12}$$

$$z_i = y_i - \hat{y}_i \tag{13}$$

As depicted in Fig. 3, the estimated discrete state issued from the MIFD block (Fig. 2) and called "Mode", is used by the DSROS to construct an estimate of the continuous state.

However, the estimation of the discrete state is not instantaneous and requires a delay. Thus, when the system state evolves in the mode q and the observer in the mode $\hat{q}$, the dynamics of the observation error and the residual signal can be expressed as:

$$\dot{e} = \bar{A}_{\hat{q}}^i e + \Delta\bar{A}_{q,\hat{q}}^i x + \Delta\bar{B}_{q,\hat{q}}^i u + \bar{B}_{d_q}^i d - L_{\hat{q}}^i f_q^i \tag{14}$$

$$z_i = C_{\hat{q}}^i e + D_{d_q}^i d + \Delta C_{q,\hat{q}}^i x + \Delta D_{q,\hat{q}}^i u + f_q^i \tag{15}$$

with: $\bar{A}_{\hat{q}}^i = A_{\hat{q}} - L_{\hat{q}}^i C_{\hat{q}}^i$, $\Delta\bar{A}_{q,\hat{q}}^i = \Delta A_{q,\hat{q}} - L_{\hat{q}}^i \Delta C_{q,\hat{q}}^i$, $\Delta C_{q,\hat{q}}^i = C_q^i - C_{\hat{q}}^i$, $\Delta\bar{B}_{q,\hat{q}}^i = \Delta B_{q,\hat{q}} - L_{\hat{q}}^i \Delta D_{q,\hat{q}}^i$, $\Delta B_{q,\hat{q}} = B_q - B_{\hat{q}}$, $\Delta D_{q,\hat{q}}^i = D_q^i - D_{\hat{q}}^i$, $\bar{B}_{d_q}^i = B_{d_q} - L_{\hat{q}}^i D_{d_q}^i$, $\Delta A_{q,\hat{q}} = A_q - A_{\hat{q}}$.

One can remark that the dynamics of the residual signal depend on f_q^i, d and also on the states x and the inputs u. Hence according to (14), equation (15) can be rewritten as:

$$z_i(t) = z_i^x(t) + z_i^u(t) + z_i^d(t) + z_i^f(t) \tag{16}$$

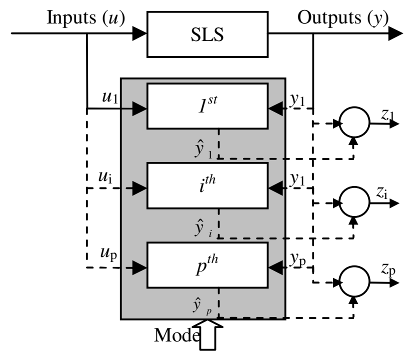

Fig. 3: Scheme of the proposed DSROS

The design of a robust hybrid DSROS, consists in deriving matrices $L_{\hat{q}}^i$ such that $\left(A_{\hat{q}} - L_{\hat{q}}^i C_{\hat{q}}^i\right)$ are asymptotically stable. The proposed design approach will provide the conditions, the boundedness of both z_i^x and z_i^u and the asymptotic stability of z_i^d and z_i^f.

In addition, the robustness of the DSROS can be guaranteed if the following conditions as satisfied for given scalars $\bar{\gamma}_{\hat{q}} > 0$ and $\bar{\beta}_{\hat{q}} > 0$:

$$\left\|z_i^d\right\|_2^2 - \bar{\gamma}_{\hat{q}}^2\left\|d\right\|_2^2 < 0 \tag{17}$$

$$\left\|z_i^f\right\|_2^2 - \bar{\beta}_{\hat{q}}^2\left\|f_q\right\|_2^2 > 0 \tag{18}$$

Note that the first condition (17) concerns the component z_i^d and is used to attenuate the effect of the disturbances and the unknown signals. While the second condition (18), concerns the component z_i^f and is used as a measurement in the worst-case fault sensitivity of the residuals signals.

1) Boundedness study of z_i^x and z_i^u

According to (14) and (15), the following relations yield:

$$\begin{cases} \dot{e}_x = \bar{A}_{\hat{q}}^i e_x + \Delta\bar{A}_{q,\hat{q}}^i x \\ z_i^x = C_{\hat{q}}^i e_x + \Delta C_{q,\hat{q}}^i x \end{cases} \tag{19}$$

and

$$\begin{cases} \dot{e}_u = \bar{A}_{\hat{q}}^i e_u + \Delta\bar{B}_{q,\hat{q}}^i u \\ z_i^u = C_{\hat{q}}^i e_u + \Delta D_{q,\hat{q}}^i u \end{cases} \tag{20}$$

We can remark that z_i^x and z_i^u are bounded when $e_x(t)$, $e_u(t)$, $x(t)$ and $u(t)$ are bounded.

Note that, in the sequel of the paper, $(.)^{\dagger}$ will denote the pseudo-inverse of $(.)$, and the symbol $(*)$ in the matrix at position (i,j) denotes the transpose of the matrix element at the position (j,i). In addition, we define by e_w the sum of e_x and e_u.

Theorem 1. Consider the switched linear system (1) and (2) with i^{th} DSROS (11)-(13) and suppose that for $T_0 > 0$ we have $\sup_{t>T_0}\|x(t)\| \le x_{\max}$ and $\sup_{t>T_0}\|u(t)\| \le u_{\max}$.

Assume there exist matrices $P_{\hat{q}}^i = \left(P_{\hat{q}}^i\right)^T \ge 0$, $L_{\hat{q}}^i$, and positive scalars ε, α, ξ, $v_{\hat{q},q} \ge 0$ and $\mu_{\hat{q},q} \le 1$, **such that** the following set of matrix inequalities is satisfied:

1. $\alpha I \le P_{\hat{q}}^i \le \xi I \qquad \hat{q} \in I_N,\ i \in I_p \qquad (21)$
2. $\begin{bmatrix} \Gamma_{\hat{q},q}^{11} & P_{\hat{q}}^i \Delta\bar{A}_{q,\hat{q}}^i \\ * & -\varepsilon^2 v_{\hat{q},q} I \end{bmatrix} \le 0 \quad (\hat{q},q) \in I_s, i \in I_p \qquad (22)$
3. $\begin{bmatrix} \Gamma_{\hat{q},q}^{11} & P_{\hat{q}}^i \Delta\bar{B}_{q,\hat{q}}^i \\ * & -\varepsilon^2 v_{\hat{q},q} I \end{bmatrix} \le 0 \quad (\hat{q},q) \in I_s,\ i \in I_p \qquad (23)$
4. $P_q^i - \mu_{\hat{q},q} P_{\hat{q}}^i < 0 \qquad (\hat{q},q) \in I_s,\ i \in I_p \qquad (24)$

with: $\Gamma_{\hat{q},q}^{11} = \left(\bar{A}_{\hat{q}}^i\right)^T P_{\hat{q}}^i + P_{\hat{q}}^i \bar{A}_{\hat{q}}^i + I + v_{\hat{q},q} I.$

Then, the state estimation error e_w is bounded by:

$$\lim_{t\to\infty}\sup\|e_w(t)\| \le \sqrt{\frac{\bar{v}}{1+\bar{v}}}\sqrt{\frac{\xi}{\alpha}}\varepsilon\left(x_{\max} + u_{\max}\right) \qquad (25)$$

where $\bar{v}$ is the largest $v_{\hat{q},q}$, $(\hat{q},q) \in I_s$ and $R_{\hat{q}}^i \in \Re^{n\times n}$ is a symmetric positive definite matrix.

Proof: due to space limitation, the proofs are omitted. Nevertheless, some key points can be found in [4].

2) Convergence analysis and robustness of the DSROS

In this section, we will study separately the components z_i^d and z_i^f of the residual. Then, we will formulate the DSROS design problem as a model-matching one and solve it.

According to (14) and (15), the following relations yield:

$$\begin{cases} \dot{e}_d = \bar{A}_{\hat{q}}^i e_d + \bar{B}_{d_q}^i d \\ z_i^d = C_{\hat{q}}^i e_d + D_{d_q}^i d \end{cases} \qquad (26)$$

and

$$\begin{cases} \dot{e}_f = \bar{A}_{\hat{q}}^i e_f - L_{\hat{q}}^i f_q^i \\ z_i^f = C_{\hat{q}}^i e_f + f_q^i \end{cases} \qquad (27)$$

Regarding the component z_i^d, one proposes the use the following lemma (bounded real lemma) to ensure the asymptotic stability of (26) and to reduce the impact of the unknown inputs vector.

Lemma 2. Consider the switched linear system (1) and (2) with i^{th} DSROS (11)-(13), system (26) is asymptotically stable and satisfies (17) if there exist matrices $P_{\hat{q}}^i = \left(P_{\hat{q}}^i\right)^T > 0$ and $L_{\hat{q}}^i$ such that the following LMIs hold:

$$\begin{bmatrix} A_{\hat{q}}^T P_{\hat{q}}^i + P_{\hat{q}}^i A_{\hat{q}} - M_{\hat{q}}^i C_{\hat{q}}^i - \left(C_{\hat{q}}^i\right)^T \left(M_{\hat{q}}^i\right)^T & P_{\hat{q}}^i B_{d_q} - M_{\hat{q}}^i D_{d_q}^i & \left(C_{\hat{q}}^i\right)^T \\ * & -\left(\bar{\gamma}_{\hat{q}}^i\right)^2 I & \left(D_{d_q}^i\right)^T \\ * & * & -I \end{bmatrix} \le 0 \qquad (28)$$

with: $M_{\hat{q}}^i = P_{\hat{q}}^i L_{\hat{q}}^i$.

In the same way, dealing with the component z_i^f, the following lemma ensures the asymptotically stability of the system (27) and increases significantly the sensitivity of the residual signal to the faults.

Lemma 3. Consider the switched linear (1) and (2) with the i^{th} DSROS (11)-(13), system (27) is asymptotically stable and satisfies (18) if there exist matrices $P_{\hat{q}}^i = \left(P_{\hat{q}}^i\right)^T > 0$ and $L_{\hat{q}}^i$ such that the following inequality holds:

$$\begin{bmatrix} A_{\hat{q}}^T P_{\hat{q}}^i + P_{\hat{q}}^i A_{\hat{q}} - M_{\hat{q}}^i C_{\hat{q}}^i - \left(M_{\hat{q}}^i C_{\hat{q}}^i\right)^T - \left(C_{\hat{q}}^i\right)^T C_{\hat{q}}^i & \left(C_{\hat{q}}^i\right)^T + M_{\hat{q}}^i \\ * & \left(\bar{\beta}_{\hat{q}}^i\right)^2 I - I \end{bmatrix} \le 0 \qquad (29)$$

Finally, one can state the following theorem ensuring both the convergence and the robustness of the synthesized DSROS.

Theorem 2. Consider the switched linear system (1) and (2) with the i^{th} DSROS (11)-(13). The residual (15) is bounded and satisfies (17) and (18) if there exist matrices $P_{\hat{q}}^i = \left(P_{\hat{q}}^i\right)^T > 0$ and $L_{\hat{q}}^i$ such that LMIs (21), (22), (23), (24), (28) and (29) hold. ■

Remark: For the task of residual evaluation, this work adopts the so-called norm based residual evaluation approach. Thus, faults can be detected using the following criterion, $for\ i = 1,\dots,p$, $\sigma_{z_i}(t)$ is true (alarm), $if\ \|z_i(t)\|_{2,T} > J_{th}$, and $\sigma_{z_i}(t)$ is false (no alarm), $if\ \|z_i(t)\|_{2,T} \le J_{th}$. ($\|z_i(t)\|_{2,T} = \sqrt{\int_{t_1}^{t_2} z_i(t)^T z_i(t)\,dt}$)

where: $\|z_i\|_{2,T} = \left\|z_i^x(t) + z_i^u(t) + z_i^d(t) + z_i^f(t)\right\|_{2,T}$

with $T = t_2 - t_1, J_{th} = J_{th,d} = \sup_{x,u,d\in L_2} \left\|z_i^c(t)\right\|_{2,T}, z_i^c(t) = z_i(t)|_{f^i=0}$.

C. Mode Identification & Fault Diagnosis block

The proposed MIFD is a discrete event based block where the mode and the fault signatures are abstracted in terms of discrete events ([16], for example). These discrete events are merged into the original model describing the discrete dynamic of the considered MIMO SLS. The MIFD block, which guarantee the detection and the localization of sensor faults as well as the transition to a no successor mode can be viewed as a finite state machine defined as a quintuple given by: $D_{MIFD} = \left(\hat{Q}, \Sigma, \Psi, Init, E, \gamma\right)$.

- $\hat{Q} = \hat{Q}_N \cup \hat{Q}_d \cup \hat{Q}_s$ is the set of discrete states with :

$\hat{Q}_N=\{\hat{q}_N,\hat{q}_N\in I_N\}$ is the set of discrete state corresponding to a normal behavior. $\hat{Q}_d=\{\hat{q}_d,\hat{q}_d\in I_N\}$ is a set of discrete state that is used, for the diagnosis task, to represent abnormal switching from a normal mode to another normal one but which doesn't respect the normal discrete trajectory. $\hat{Q}_s=\{\hat{q}_s,\hat{q}_s\in I_p\}$ is the set of discrete state corresponding to the occurrence of the sensor fault.

Let us also note $\Phi_{ns}(\hat{q})\subset\hat{Q}_d$ all the sates that are not successors to $\hat{q}$ and $\Phi_s(\hat{q})\subset\hat{Q}_N$ all the normal successors of the discrete states $\hat{q}$,

- $\Sigma=\Sigma_M\cup\Sigma_F\cup\{\varepsilon_e\}$ is the set of discrete events; with:

Σ_M is the set of discrete events corresponding to the occurrence of the mode change. Σ_F is the set of discrete events corresponding to the occurrence of the sensor fault. ε_e is the "null" event. Let us note $\Sigma_s(\hat{q})\subset\Sigma_M$ the set of discrete events representing the transition to the successor states of $\hat{q}$ and $\Sigma_{ns}(\hat{q})\subset\Sigma_M$ the set of discrete events representing abnormal switching from the active mode $\hat{q}$.

- Ψ : is the set of discrete outputs.
- $Init\subset\hat{Q}$ denotes the set of discrete initial states.
- $E\subset\hat{Q}\times\Sigma\times\hat{Q}$ is a collection of discrete transition.
- $\gamma:E\rightarrow\Psi$ is a mapping that associates a discrete output to each discrete transition.

For the diagnosis task, one supposes that the generated residuals have already reached their steady state.

Without loss of generalities, the construction of the diagnoser can be summarized as follows. The MIMO SLS starts initially in a normal state $q_{0N}\in Init$ and the diagnoser D_{MIFD} is initialized in $\hat{q}_{0N}$. Then, after the occurrence of an observed event $\sigma\in\Sigma$, three cases can be distinguished. The first one corresponds to the occurrence of a discrete event σ, which belongs to the set $\Sigma_s(\hat{q}_{0N})$. The observation of this event, which corresponds to a normal behavior, will switch the diagnoser to another mode $\hat{q}_{1N}\in\Phi_s(\hat{q}_{0N})$. Thus, the output generated by the diagnose after the occurrence of the transition $(\hat{q}_{0N},\sigma,\hat{q}_{1N})$ will be null indicating that the systems is in its normal trajectory.

The second case corresponds to the occurrence of a discrete event σ, which belongs to the set $\Sigma_{ns}(\hat{q}_{0N})$. Thus, the discrete state of the diagnoser D_{MIFD} switches to the discrete state $\hat{q}_{1d}$ following the transition $(\hat{q}_{0N},\sigma,\hat{q}_{1d})$, where $\hat{q}_{1d}\in\Phi_{ns}(\hat{q}_{0N})$. In this case, the diagnoser generates a discrete output $o_d\in\Psi$ indicating the presence of a discrete fault.

The third case corresponds to the occurrence of a discrete event σ, which belongs to the set Σ_F. Thus, the discrete state of the diagnoser D_{MIFD} switches to discrete state $\hat{q}_{1s}$ following the transition $(\hat{q}_{0N},\sigma,\hat{q}_{1s})$, where $\hat{q}_{1s}\in\hat{Q}_s$. In this case, the diagnoser generates a discrete output $o_s\in\Psi$ indicating the presence of a sensor fault.

IV. SIMULATION AND RESULTS

Let us consider the following MIMO SLS with three discrete modes.

Mode q_1: $A_1=\begin{bmatrix}-20.7 & -0.46\\ 21.2 & -27.3\end{bmatrix}$, $B_1=\begin{bmatrix}10.1\\ 63.7\end{bmatrix}$, $B_{d_1}=\begin{bmatrix}-9.8\\ 0\end{bmatrix}$,

$C_1=\begin{bmatrix}-144 & 3.74\\ 0 & 1\end{bmatrix}$, $D_1=\begin{bmatrix}71\\ 0\end{bmatrix}$, $D_{d_1}=\begin{bmatrix}-1.4\\ 0\end{bmatrix}$.

Mode q_2: $A_2=\begin{bmatrix}-9.66 & -0.88\\ 21.2 & -12.7\end{bmatrix}$, $B_2=\begin{bmatrix}4.73\\ 63.7\end{bmatrix}$, $B_{d_2}=\begin{bmatrix}-0.7\\ 0\end{bmatrix}$,

$C_2=\begin{bmatrix}-145 & 1.31\\ 0 & 1\end{bmatrix}$, $D_2=\begin{bmatrix}71\\ 0\end{bmatrix}$, $D_{d_2}=\begin{bmatrix}-9.8\\ 0\end{bmatrix}$.

Mode q_3: $A_3=\begin{bmatrix}-7.24 & -0.93\\ 21.2 & -9.57\end{bmatrix}$, $B_3=\begin{bmatrix}3.55\\ 63.7\end{bmatrix}$, $B_{d_3}=\begin{bmatrix}-0.5\\ 0\end{bmatrix}$,

$C_3=\begin{bmatrix}-145 & 1.31\\ 0 & 1\end{bmatrix}$, $D_3=\begin{bmatrix}71\\ 0\end{bmatrix}$, $D_{d_2}=\begin{bmatrix}-9.8\\ 0\end{bmatrix}$.

Applying lemma 1, one can guarantee the robustness and the convergence of the MSG block observer's with: $K_1=\begin{bmatrix}0.0778 & -10.6348\\ -0.1532 & -26.4310\end{bmatrix}$, $K_2=\begin{bmatrix}0.0619 & 0.5514\\ -0.1211 & -11.0301\end{bmatrix}$ and $K_3=\begin{bmatrix}0.0463 & 0.1794\\ 0.1265 & -7.8068\end{bmatrix}$ for $\gamma_1=77.6308$, $\gamma_2=9.8298$ and $\gamma_3=9.8773$.

In order to illustrate the efficiency of the proposed methodology, an unknown input d is assumed to be band-limited white noise with power 0.01. The evolution of the mode signature r_1 of the first subsystem of the SLS is illustrated by Fig. 4. We can note that both residues, corresponding to both output of the switched system, converge to zero when the switched system evolves in mode 1. These residuals leave the zero when the SLS switches to another mode. This evolution can be stated in the same way for the modes 2 and 3.

Now, for the localization task, one exploit the results provided by theorem 2, and can obtain the following gain matrices: $L_1^1=\begin{bmatrix}0.0739\\ -0.1369\end{bmatrix}$, $L_2^1=\begin{bmatrix}0.0518\\ -0.1327\end{bmatrix}$, $L_3^1=\begin{bmatrix}-0.0430\\ -0.1364\end{bmatrix}$,

$L_1^2=\begin{bmatrix}5.8190\\ -12.8523\end{bmatrix}$, $L_2^2=\begin{bmatrix}3.2011\\ -8.3446\end{bmatrix}$, $L_3^2=\begin{bmatrix}10.6888\\ 9.3300\end{bmatrix}$.

Finally, in order to illustrate the sensitivity of the FDI block when a fault occurs, a fault signal f_q is considered from 90 to 100 sec. This fault is simulated as a pulse of amplitude 1 affecting the 1st output when the SLS evolves in the 2nd mode. The evolution of fault signature for both system's outputs when the SLS evolves in mode 2 is illustrated in the Fig. 5. This figure shows that the components residual converge to zero when the SLS evolves in mode 2. After that, only the first component of the residue departs significantly from zero the fault occurs. Note that other simulations (not presented here) were performed regarding the faults affecting the discrete trajectory. The obtained results provided similar detection performances.

V. Conclusion

This paper considers the problem of fault detection and isolation for a class of MIMO switched linear systems. A dedicated switched robust observer scheme has been proposed to deal with sensor fault as well as abnormal switching from a discrete mode to another one. The proposed scheme is based on three blocks in interaction an MSG block that generates the N mode signatures related to the normal modes of the SLS. An FDI block generating the fault signatures, which are sensitive only to a single sensor fault and a MIFD block built from the normal discrete trajectory of the considered SLS. LMI conditions have been provided to guarantee both of the robustness and the convergence of the proposed scheme. Future works aims at considering the problem of fault tolerant control using the proposed scheme.

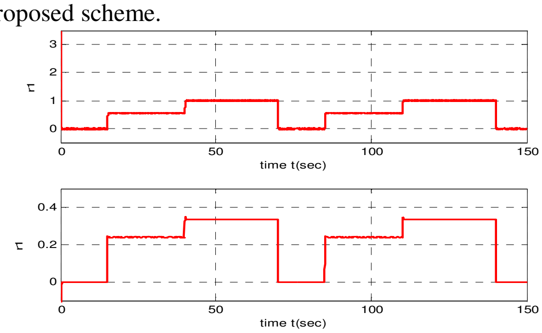

Fig. 4. The mode signature evolution of sub-system 1.

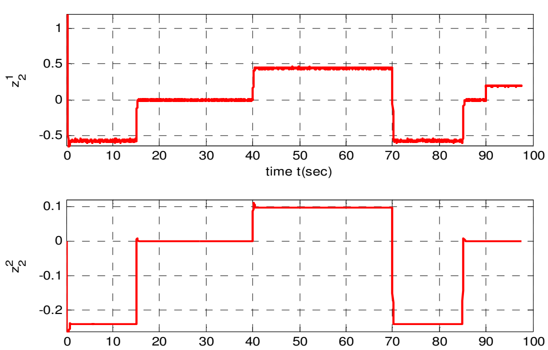

Fig. 5. The fault signature evolution of sub-system 2.

References

[1] A. Balluchi, L. Benvenuti, M. D. Di Benedetto, C. Pinello, and A. L. Sangiovanni-Vincenelli, "Automative engine control and hybrid systems: Challenges and opportunities," in *Proc. IEEE, 88, "Sepcial Issue on Hybrid Systems" (invited paper)"*, pp. 888-912, July 2000.

[2] V. Cocquempot, M. Staroswiecki, T. El Mezyani, "Switching time estimation and fault detection for hybrid systems using structured parity residuals," in *Proc. 5th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes*, pp. 681–686, 2003.

[3] E. R. Loures and J. C. Pascal, "Detection and diagnosis of hybrid dynamic systems based on time Petri nets," in *Proc of IEEE International Conference on Systems, Man and Cybernetics,* Piscataway, NJ, USA, 2004.

[4] D. E. C. Belkhiat, N. Messai, N. Manamanni, " Design of robust fault detection based observer for linear switched systems with external disturbances," *Nonlinear Analysis: Hybrid Systems,* doi: 10.1016/j.nahs. 2010. 10. 009. 2010.

[5] D. E. C. Belkhiat, N. Messai, N. Manamanni, " Robust fault detection based-observer for linear switched systems, " in *Proc. IFAC Conference on Analysis and Design of Hybrid Systems,* Zaragoza, Spain, Septembre 2009, pp. 392-397.

[6] W. Wang, D. H. Zhou, Z. Li, "Robust state estimation and fault diagnosis for uncertain hybrid systems," *Nonlinear Analysis,* vol. 65, pp. 2193-2215, 2006.

[7] W. Wang, L. Li, D. Zhou, K. Liu, "Robust state estimation and fault diagnosis for uncertain hybrid nonlinear systems," *Nonlinear Analysis: Hybrid Systems*, vol. 1, pp. 2-15, 2007.

[8] P. J. Mosterman, G. Biswas, "Diagnosis of continuous valued systems in transient operating regions," *IEEE Trans on systems, Man and Cybernetics,* vol. 29, pp. 554-565, 1999.

[9] S. Narasimhan, G. Biswas, G. Karsai, T. Pasternak and F. Zhao, "Building observer to address fault isolation and control problems in hybrid dynamic systems,"in *Proc. IEEE Int. Conf. on Systems, Man, and Cybernetics,* Nashville, TN, USA, 2000, pp 2393-2398.

[10] J. Vento, V. Ping and R. Sarrate, "Fault detection and isolation of hybrid system using diagnosers that combine discrete and continuous dynamics," in *Proc. of Conference and Fault Tolerant Systems,* Nice, France, 2010, pp. 149-154.

[11] M. Bayoudh, L. Travé-Massuyès and X. Olive, "Towards active diagnosis of hybrid systems," in *Proc. of DX'2008,*Leura, NSW, Australia, 2008.

[12] S. Tabatabaeipour, A. P. Ravn, R. Izadi-zamabadi and T. Bak, "Active diagnosis of hybrid systems- A model predictive approach," in *Proc. of IEEE international conference on control and automation,* Christchurch, New Zealand, 2009, pp 465-470.

[13] A. Balluchi, L. Benvenuti, M. Di Benedetto and A. Vincentelli, " A hybrid observer for the driveline dynamics," in *Proc. of the European control coference,* Porto, Portugal, September 2001.

[14] R. N. Clark, "Instrument fault detection," *IEEE trans. Aerospace Electron. Syst.,* vol. 14, 1978, pp. 456-465.

[15] S. Pettersson, "Switched state jump observers for switched systems," in *Proc. 16th IFAC World Congress,* Prague, Czech Republic, 2005.

[16] M. Bayoudh, L. Travé-Massuyès and Xavier Olive, "Hybrid systems diagnosability by abstracting faulty continuous dynamics," in *Proc. of 17th international Workshop on Principles of Diagnosis DX'06,* Burgos Spain, 2006, pp. 9-15.

[17] M. Defoort, M. Djemai, T. Floquet and W. Perruquetti, Robust finite time observer design for multicellular converter, Special Issue of Int. Journal of Systems Sciences on VSS methods for Hybrid Systems, Vol. 42, (11), November 2011, pages 1859-1868,

[18] H. Saadaoui, N. Manamanni, M. Djemaï, J.P. Barbot and T. Floquet, Exact differentiation and sliding mode observers for switched Lagrangian systems, in Nonlinear Analysis : Theory, Methods & Applications, Ed. Elsivier, Vol. 65, Issue 5, Pages 1050-1069, 2006.

[19] T. Floquet, J.P. Barbot, W. Perruquetti and M. Djemaï, `` On the robust fault detection via sliding mode perturbation observer," Int. Journal of Control, Vol. 77, No 7, May 10, pp. 622-629,, Mai 2004.

[20] B. Mansouri, N. Manamanni, K. Guelton, M. Djemaï, "Robust pole placement controller design in LMI region for uncertain and disturbed switched systems", In Nonlinear Analysis: Hybrid Systems Volume 2, Issue 4, Pages 1136-1143, November 2008.