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Sliding mode control for Turbocharged Diesel Engine

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***Abstract*— For modern Diesel engines, accurate fuel-air ratio AFR and Exhaust Gas Recirculation (EGR) rates control is important for manufacturers to face a more restrictive legislation levels . To fulfill the requirements, hardware devices such (EGR) and Variable Geometry Turbochargers (VGT) valves have been introduced, and sophisticated control algorithms were designed. The main objective of the air path controller is to regulate in the intake manifold the AFR ratio and the EGR fraction rates to their desired values. Earlier EGR PID controllers needed a fastidious and time consuming calibration step for each engine operating point. Nonlinear control algorithms has quickly appeared as a promising way to provide an efficient air path controllers, since they don't need a calibration step . The main drawback of such controllers is coming from the fact that they are based upon a diesel engine model which handles parameters uncertainties and signal measurements errors, that affects the control performance. In this paper we propose a novel control scheme for controlling the diesel engine air path. Control design is carried out under the sliding mode framework. The proposed controller has been tested on the Jankovic Tubocharged Diesel Engine (TDE) model. To demonstrate the robustness of the proposed controller, simulation results showing the tracking of the compressor flow W_c and the exhaust manifold pressure p_2 variables are presented .**
Keywords: Nonlinear control, Sliding mode , Robust control, Diesel engine air path

I. INTRODUCTION

With the Euros V and the Euros VI legislation emissions laws the manufacturers face a constantly request to diminish emissions levels. To fulfill the requirements novel and sophisticated control schemes must be be conceived to provide the required engine torque under performance tradeoff between minimal fuel consumption and meeting the given exhaust gas and noise emission requirements levels . Nitrous oxide (No_x) emissions is a result of reaction between nitrogen monoxide, and oxygen in the combustion chamber. (No_x) emissions and the phenomenon of engine smoke constitute a particular concern for modern diesel engine. Indeed the authors in [1] demonstrates that controlling (No_x) depends primarily on two feedback variables namely the (EGR) and the (AFR) fraction rates in the intake manifold. The definition of the feedback variable is the first step to conceive a (No_x) reduction mechanism. Nowadays, it should be mentioned that exhaust gases, coming from (EGR) contains also oxygen. Therefore, beside (EGR) ratio, the oxygen/fuel ratio (λ_O) instead of the (AFR) are chosen as two performances variables that must be controlled to meet the legislation levels . These two feedback variables depends on complicated on the (EGR) and the (VGT) actuators.

The position of (EGR) and (VGT) actuators determines the amount of the (EGR) flow in the intake manifold and hence, control the (λ_O) and (EGR) variables. The challenge is thus, to design an efficient controller that manages the two actuators signals. The earlier EGR controllers implemented in the ECU (Electronic Control Units) uses a closed loop scheme based on PID controllers and the two actuators . The purpose is to meet the (λ_O) and the $((EGR))$ fraction rates set-point computed from the engine static data of the manufacturer. However the tuning of these PID controllers is very time consuming and must be calibrated for each engine operating point.

To overcome this difficulty the attention of the researchers has been focused on nonlinear control methods which do not need a calibration step. Several control design methods for the (TDE) air path have been proposed: Lyapunov control design, [2], model based control [3] [4], Indirect passivation, [5], Dynamic feedback linearization [6][7], Predective Control [8]. Most of these algorithms are control oriented model, i.e the control laws computed by these algorithms are based upon a model of the diesel engine air path. Discrepancies between the description model and the real system due to natural model parametric uncertainties affects the control performance. This is the case where robust nonlinear control methods are suitable in order to enhance the system robustness make it less sensitive to parametric uncertainties.

In [2] a model of diesel equipped with VGT and EGR valves based on the mean-value diesel engine models was presented. In addition the authors proposed a robust nonlinear multivariable control design, based on construction of a Lyapunov function and inverse optimal control. The control law handles parameters uncertainty and nonlinear properties of the system. However, the control design needed the construction of Control Lyapunov Function (CLF), this is a step which is not easy and may be quite restrictive.

Sliding Mode Control (SMC), which is applicable to nonlinear systems, has become widespread and one of the most popular robust nonlinear control method (see [9][10] for a survey). In [11] the authors proposed an extension of the input-output I/O linearization technique to sliding mode control design for purpose of controlling diesel engine air path. In this work, a standard I/O linearization condition which concerns matrix inversion is needed to guarantee the existence of the control law . The aim of our work is to provide a simple air path SMC controller comparing to control law proposed in [11]. Our proposed controller does

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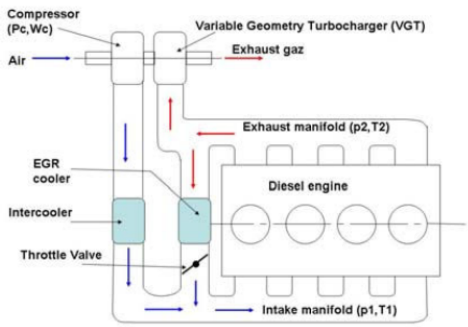

Fig. 1. Turbocharged Diesel Engine

not need a matrix inversion property . This is suitable for robustness property and ease implementation issues. The proposed (SMC) air path controller handles also parameter uncertainties and unmodeled dynamic of the system. The goal of the proposed controller is to regulate the compressor flow W_c and the exhaust manifold pressure p_2 model variables to their desired set points. A change of variable which transforms the W_c set point to the intake pressure manifold p_1 set point is introduced. A sliding mode controller which regulates the novel p_1 and p_2 set points is then proposed.

The rest of the paper is organized as follows. Section II briefly presents the TDE modeling . In section III, the air path SMC controller is described. Simulation results are given in section IV. conclusions and future works are summarized in section V.

II. DIESEL ENGINE MODEL

The schematic diagram of diesel engine is shown in Figure 1. At the top of the scheme we can see the turbocharger and the compressor mounted on the same shaft. The turbine delivers power to the compressor by transferring the energy coming from the exhaust gas to the intake manifold. Together, the mixture of air coming from the compressor and the exhaust gas coming from the *EGR* valve with the injected fuel burn and produce the torque on the crank shaft. The presented (TDE) model was outlined in [6][7]

The full-order TDE model is a seventh-order one which contains seven states: intake and exhaust manifold pressure(p_1 and p_2), oxygen mass fractions in the intake and exhaust manifolds (F_1 and F_2), turbocharger speed (ω_{tc}) and the two states describing the actuator dynamics for the two control signals (u_1 and u_2).

In order to get a simple control law, and due to the fact that the oxygen mass fraction variables are difficult to measure, the seventh-order model is reduced to a third-one.

$$\begin{cases} \dot{p}_1 = k_1(W_c + W_{egr} - k_e p_1) + \dfrac{\dot{T}_1}{T_1} p_1 \\ \dot{p}_2 = k_2(k_e p_1 - W_{egr} - W_t + W_f) + \dfrac{\dot{T}_2}{T_2} p_2 \\ \dot{P}_c = \dfrac{1}{\tau}(\eta_m P_t - P_c) \end{cases} \tag{1}$$

where the compressor (resp.the turbine) mass flow rate is related to the compressor (resp.the turbine) power as follows:

$$W_c = P_c \frac{k_c}{p_1^{\mu} - 1} \tag{2}$$

and:

$$P_t = k_t(1 - p_2^{-\mu})W_t \tag{3}$$

where $k_c = \frac{\eta_c}{c_p T_a}$ and $k_t = c_p \eta_t T_2$

Notice that the real inputs are the *EGR* valve and *VGT* valve openings. The considered inputs, in this case for the sake of simplicity, are $u_1 = W_{egr}$ and $u_2 = W_t$ (see [2]), which are respectively the air flow through the *EGR* and the *VGT* valves.

Since $\dot{T}_1$ and $\dot{T}_2$ have very slow variations , [2], this yields to the simplified model:

$$\begin{cases} \dot{p}_1 = k_1(W_c + W_{egr} - k_e p_1) \\ \dot{p}_2 = k_2(k_e p_1 + W_f - W_{egr} - W_t) \\ \dot{P}_c = \dfrac{1}{\tau}(\eta_m P_t - P_c) \end{cases} \tag{4}$$

When replacing W_c and P_t by their expression in (2) and (3), the simplified model can be expressed under the following Control- affine form:

$$\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2 \tag{5}$$

where $x = (p_1, p_2, P_c)^T$ and

$$f(x) = \begin{bmatrix} k_1 k_c \frac{P_c}{p_1^{\mu} - 1} - k_1 k_e p_1 \\ k_2(k_e p_1 + W_f) \\ \frac{-P_c}{\tau} \end{bmatrix} \tag{6}$$

$$g_1(x) = \begin{bmatrix} k_1 \\ -k_2 \\ 0 \end{bmatrix} \quad g_2(x) = \begin{bmatrix} 0 \\ -k_2 \\ K_o(1 - p_2^{-\mu}) \end{bmatrix} \tag{7}$$

with $K_o = \frac{\eta_m}{\tau} k_t$

We notice that the TDE model parameters $(k_1, k_2, k_c, k_e, k_t, \tau, \eta_m)$ have been identified under steady state conditions i.e (constant engine speed and constant fueling rate) and extensive mapping [2]. To detail the description of the TDE Model, the nomenclature of certain variables are given in Table I .

TABLE I

NOMENCLATURE OF DIESEL ENGINE VARIABLES

Variables	Name	Units
p_1	Intake Manifold Pressure	Pa
p_2	Exhaust Manifold Pressure	Pa
P_c	Compressor power	W
P_t	Turbine power	W
W_c	Compressor mass flow	kg/s
W_t	Turbine mass flow	kg/s
W_f	Fueling mass flow rate	kg/s
η_c	Compressor isentropic efficiency	-
η_t	Turbine isentropic efficiency	-
η_m	Turbocharger mechanical efficiency	-
T_a	Ambient temperature	K
T_1	Intake manifold temperature	K
T_2	Exhaust manifold temperature	K

Note that in [2], the authors proved that the set Ω, defined by:

$$\{\Omega = (p_1, p_2, P_c) : 1 < p_1 < P_1^{max}, \\ 1 < p_2 < p_2^{max}, 0 < P_c < P_c^{max}\}.$$

is an invariant set. This means that for all initial conditions $x(t_0) \in \Omega$, then $x(t) \in \Omega \quad \forall t \geq t_0$.

III. SYSTEM CONTROL DESIGN

In this section we will describe the design of a sliding mode controller for conventional diesel combustion mode. Since system (5) dynamics is affine with respect to control, some mathematical notations that will be used through-out this paper are introduced in what follows:

A. Vector relative degree

consider a square-MIMO $(m \times m)$ nonlinear system given by:

$$\begin{cases} \dot{x} = f(x) + \sum_{j=1}^{m} g_j(x) u_j \\ y = (h_1(x), \ldots, h_m(x)) \end{cases} \tag{8}$$

where $x \in \mathbf{R}^n$ and $u \in \mathbf{R}^m$ and $y \in \mathbf{R}^m$ represent respectively the state, control and the output vector.

Consider a smooth scalar function $h(x)$, $h : \mathbf{R}^n \to \mathbf{R}$ and a vector function $f(x)$,$f : \mathbf{R}^n \to \mathbf{R}^n$. The projection of the gradient of h along the vector field f is referred to as the Lie derivative of h along f:

$$L_f h = \nabla h.f = \sum_{i=1}^{i=n} \frac{\partial h}{\partial x_i}(x) f_i(x) \tag{9}$$

recursively , we can define the following function:

$$L_f^i h = \nabla(L_f^{i-1} h).f (i = 1, 2, ...) \tag{10}$$

with $L_f^0 h = h$

A system of the form (8) has a vector relative degree for each output $h_i (i = 1, 2, .., m)$ $(\rho_1, \rho_2, ...\rho_m)$ if

- for any $x \in \mathbf{R}^n$, $1 < i < m <$, $1 < j < m <$ and $0 < k < \rho_i - 1$ we have

$$L_{g_j} L_f^k h_i(x) = 0$$

- The decoupling matrix $(m \times m)$ defined :

$$A = \begin{pmatrix} L_{g_1} L_f^{\rho_1 - 1} h_1(x) & \ldots & L_{g_m} L_f^{\rho_1 - 1} h_1(x) \\ \vdots & & \vdots \\ L_{g_1} L_f^{\rho_m - 1} h_m(x) & \ldots & L_{g_m} L_f^{\rho_m - 1} h_m(x) \end{pmatrix} \tag{11}$$

  is regular [12]

B. Choice of system output for conventional diesel combustion mode

A conventional diesel combustion mode is characterized by the following features detailed in [13] :

- high fresh air flow rate
- relatively low EGR rate
- high AF $(20 \sim 28)$
- high intake manifold pressure (above ambient pressure)

As shown in (4), there are three states in the reduced-order model and only two variables control inputs . To fulfill the square property of designing the MIMO tracking controller, a suitable choice of key outputs control variables is needed. Notice that in this combustion mode, intake manifold pressure, fresh air charge and EGR rate play an important role for combustion beside fueling parameter. Based on these considerations, the authors in [2] take W_c (gives fresh air) and P_2 (controls the EGR rate) as two controlled system outputs. We notice that the choice of p_2 was also motivated by the stability of the zero dynamic .

$$y_c = [W_c, p_2] \tag{12}$$

Thus the control design objective is to regulate

$$y_1 = W_c - W_{cd} \\ y_2 = p_2 - p_{2d} \tag{13}$$

to zero.

C. Feedback linearization

As mentioned previously, the control law proposed in [2] needed the construction of a CLF $V(x)$. The construction of this function was based upon feedback linearization (FL) [12] method which is one of the most important methods for nonlinear control design. The FL technique renders the input-output behavior of a multivariable nonlinear system the same as that of m chains of integrators (m is the dimension of the inputs and the output vector). For the third-order model (4) and the outputs (13) the relative degree is $\rho_1 = 1$ and $\rho_2 = 1$. With respect to time the authors in [2] differentiated y_1 and y_2 leading to the following system

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \varphi(x) + \phi(x) v \tag{14}$$

where $\phi(x)$ (being invertible) is defined as follows

$$\begin{pmatrix} -a & b \\ -k_2 & -k_2 \end{pmatrix} \tag{15}$$

where

$$a = k_1 \mu \frac{p_1^{\mu-1}(y_1 + W_{cd})}{p_1^{\mu} - 1}$$

and

$$b = \frac{1}{\tau}\eta_m \eta_c \eta_t \frac{T_2}{T_a}\frac{1 - p_2^{-\mu}}{p_1^{\mu} - 1}$$

and

$$\varphi(x) = \begin{pmatrix} -a(y_1 + W_{cd} - k_e p_1) - \frac{1}{\tau}(y_1 + W_{cd}) \\ k_2(k_e p_1 + W_f) \end{pmatrix} \quad (16)$$

By applying the feedback with w being the new input

$$v = \phi(x)^{-1}(w - \varphi(x)) \quad (17)$$

we transform (14) into the following linear decoupled system:

$$\begin{cases} \dot{y}_1 = w_1 \\ \dot{y}_2 = w_2 \\ \dot{z} = f_0(y,z) + g_0(y,z)w \end{cases} \quad (18)$$

In this paper we propose the following feedback control law which stabilize vector $(y_1, y_2) = (0,0)^T$:

$$\begin{cases} w_1 = -\sigma_1 y_1 \\ w_2 = -\sigma_2 y_2 \end{cases} \quad (19)$$

$\sigma_1, \sigma_2 > 0$ z describes the so called zero dynamic of the system [12]. The zero dynamic of system (18) must be stable. This means that while controls w_1, w_2 maintains $y_1, y_2 = 0$ the dynamic of z must not escape to infinity. This the case for system (18), since the zero dynamic of the system is on the state p_1, ie $z = p_1$, and the authors proved in [2] that it is stable.

D. Robust SMC controller for diesel combustion mode

Now consider the TDE model

$$\begin{cases} \dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2 \\ y = \begin{pmatrix} W_c & p_2 \end{pmatrix}^T \end{cases} \quad (20)$$

System (20) handles some parametric uncertainties. For instance, the turbine and the compressor isentropic efficiencies (η_t,η_c) vary with operating conditions (temperature,engine speed) and so do other parameter (k_c,k_e,...). It is well known that sliding mode control is insensitive and robust to system parametric uncertainties and unmodeled dynamics,[10][14][15] which make it suitable for engine control. For tracking control, ideal SMC can bring the system tracking errors to the origin with in finite time and maintain it thanks to the structure of the SMC control law which contains two terms the equivalent and the discontinuous control [10]. The design of our air path SMC controller requires the definition of sliding surface S_i chosen as follows:

$$S_i = \{e = y_i - y_{id} : e = 0\} \quad (21)$$

Where y_i is the outputs of system (4) to be controlled.

E. System output for air path SMC controller

A natural choice of vector output for our SMC controller was to keep the same one as in (20) i.e $y_1 = W_c$, $y_2 = p_2$. Motivated by the fact that p_2 allows a stable zero dynamic for system (20)[2], we keep it as a component of vector output and we propose to replace the compressor flow W_c set point by the pressure manifold p_1 set point computed from the reduced third order model (4). A simple computation from (2) gives:

$$p_1 = (\frac{k_c P_c}{W_c} + 1)^{\frac{1}{\mu}} \quad (22)$$

From (22), we have for P_c bounded:

$$p_{1ds} = (\frac{k_c P_c}{W_{cd}} + 1)^{\frac{1}{\mu}} \quad (23)$$

F. Definition of the sliding surfaces for the SMC controller

Now we are able to define the following sliding surfaces:

$$\begin{aligned} S_1 &= p_1 - p_{1ds} \\ S_2 &= p_2 - p_{2d} \end{aligned} \quad (24)$$

Next we will propose a vector control law which guaranties that sliding mode will be enforced on all surfaces $S_i = 0$ in finite time.

G. SMC law for diesel engine air path

Before deriving the control law the following assumption must be verified:

Assumption 1: The zero dynamic exists and is asymptotically stable

Proof 1: Since vector output in (24) is $y = \begin{pmatrix} p_1 & p_2 \end{pmatrix}^T$ the zero dynamic is on the state P_c which is a stable mode for the TDE model. Hence the zero dynamic of system (24) is asymptotically stable

The attractiveness condition for each surface S_i is defined as follows:

$$\dot{S}_i S_i < 0 \quad (25)$$

This condition is fulfilled if we choose

$$\dot{S}_i = -\lambda_i sign(S_i) \quad (26)$$

With $\lambda_i > 0$

where $sign(S)$ is the sign function defined as follows:

$$sign(x) = \begin{cases} -1 & if \quad x < 0 \\ [-1;1] & if \quad x = 0 \\ 1 & if \quad x > 0 \end{cases}$$

From (24) we have

$$\begin{cases} \dot{S}_1 = \dot{p}_1 - \dot{p}_{1ds} \\ \dot{S}_2 = \dot{p}_2 - \dot{p}_{2d} \end{cases} \quad (27)$$

Combining (4)(26)(27) we get

$$\begin{cases} -\lambda_1 sign(S_1) = k_1(W_c + u_1 - k_e p_1) - \dot{p}_{1ds} \\ -\lambda_2 sign(S_2) = k_2(k_e p_1 + W_f - u_1 - u_2) - \dot{p}_{2d} \end{cases} \quad (28)$$

The control law can then be easily derived:

$$\begin{cases} u_1 = \dfrac{-\lambda_1 sign(S_1) + \dot{p}_{1ds} - k_1 W_c + k_1 k_e p_1}{k1} \\ u_2 = \dfrac{\lambda_2 sign(S_2) + k_2(k_e p_1 + W_f) - \dot{p}_{2d}}{k_2} - u_1 \end{cases} \quad (29)$$

IV. SIMULATIONS RESULTS

A. Tracking of W_c and p_2

To demonstrate the validity of the proposed control algorithm a number of simulations is performed on the TDE model under Simulink software. The characteristics of the simulations are: the solver is "Ode 45" with step size and tolerances both under the position "auto". The TDE model was initialized with following values X_0=($P_1 = 1.32$ bar,$P_2 = 1.35$ bar,$P_c = 5.605$ W). The simulation time is 40s. The vector set point of the two controlled variables W_c, P_2 is depicted respectively in Figure 2 and Figure 3.

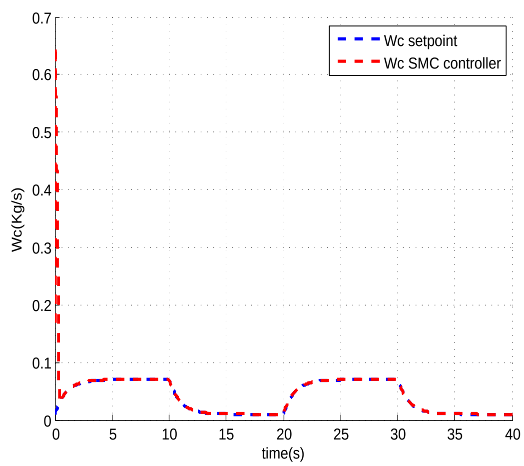

Fig. 2. Tracking of variable W_c

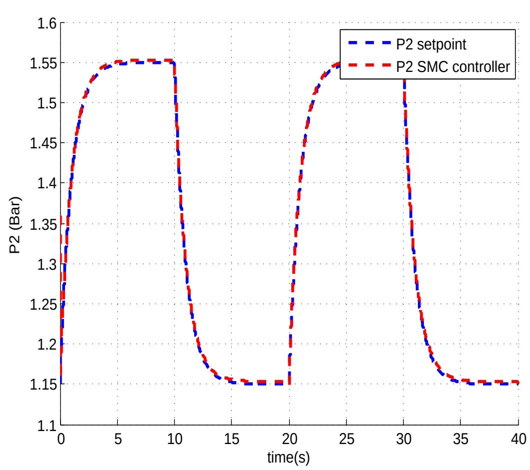

Fig. 3. Tracking of variable P_2

In the simulation we choose λ_1 and λ_2 sufficiently large so that the sliding mode is enforced in the manifold of interest. The choice is also motivated by the fact that we need to select

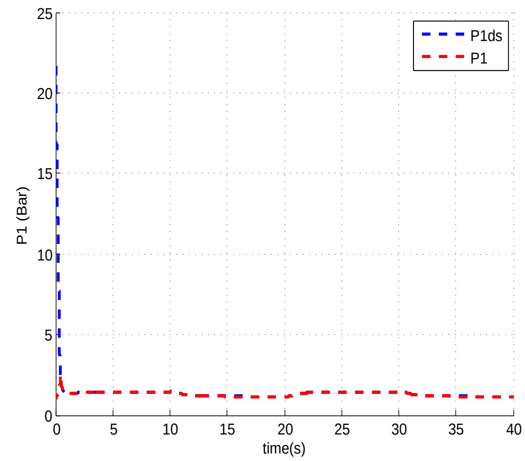

Fig. 4. Tracking of variable P_{1ds}

an appropriate gain to overcome all uncertainties coming from parameters variations, signal measurements. As we can see the tracking of the two output variables in Figures (2) and (3) gives us a good results. Figure (4) shows the tracking of the set point p_{1ds} computed from (23).

B. Control law robustness facing parameters variations

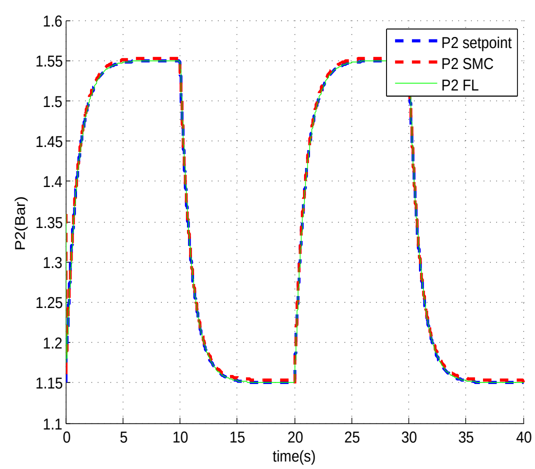

Fig. 5. Tracking of variable P_2 with SMC and FL controller/ without parametric uncertainty

In order to test the robustness of the proposed control law, we perform a variation over each TDE model parameter $K = (k_1, k_2, k_c, k_e, k_t, \tau, \eta_m)$. The formalization of the variations is stated as follows:

$$k_i \subset K, k_i = k_{oi} + \delta k_{oi}$$

where $k_{oi}(1 \leq i \leq 7)$ is the nominal value of the parameter, δk_{oi} the uncertainty on the parameter. We test the robustness of the proposed controller by adding an uncertainty value over each parameter nominal value of the TDE model. We keep the parameters nominal values for the SMC and the FL controller, while uncertainties was reported in the air path model. The simulations results in Figures (5) and (6) shows

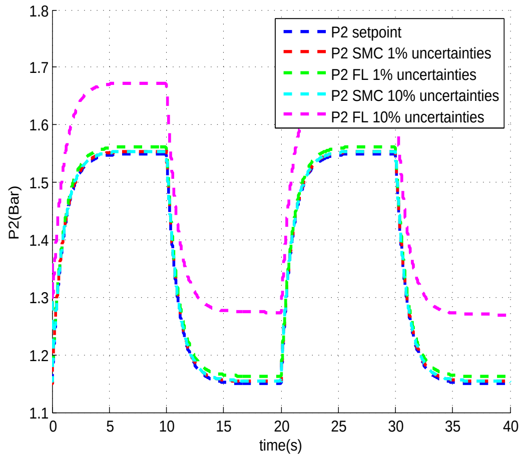

Fig. 6. Tracking of variable P_2 with SMC and FL controller/ 1%, 10% parametric uncertainty

the performance of our SMC controller and FL controller (17-19) to tracks the output variable P_2. From Figure (6) we can see that for different values of parametric uncertainties, the performance of the FL control law is gradually affected by parameters variations while the SMC controller keep the same level of performance. We can also observe that for SMC controller, 10% parametric uncertainties lead to the appearance of the so called chattering phenomenon which is undesirable for control performance.

V. CONCLUSION AND FUTURE WORKS

The simulation results show the performance of our proposed controller. In our future work :
We are planning to reduce the chattering phenomenon,by investigating higher order sliding mode framework. We will propose a fault tolerant control algorithm for the TDE, combining a fault estimator and the proposed air path SMC controller.

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