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Design of Variable-Geometry Suspension for Driver Assistance Systems

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Abstract— **The paper proposes a control design method based on a variable-geometry suspension system applied in a driver assistance system. During maneuvers an autonomous control system modifies the camber angle of the front wheels by using the variable-geometry system in order to improve road stability. The control system guarantees various crucial performances which are related to the chassis roll angle and half-track change. Moreover, by changing the camber angle of the front wheels the yaw rate of the vehicle is modified, which can be used to reduce the tracking error from the reference yaw rate. Thus, with the reconfiguration of the camber angle, the variable-geometry system can also be used as a driver assistance system. The design of a reconfigurable suspension system is based on robust LPV methods, which meet the performance specifications and guarantee robustness against model uncertainties. The operation of the control system is illustrated through different vehicle maneuvers.**

I. INTRODUCTION AND MOTIVATION

A variable-geometry suspension system is applied as a driver assistance system in vehicles. While the driver performs a maneuver by using the steering wheel, an autonomous control system modifies the camber angle of the front wheels in order to improve road stability. Since various safety and economy properties of the vehicle are determined by the suspension geometry it has significant influence on the control design. The advantages of the variable-geometry system are the simple structure, low energy consumption and low cost compared to other mechatronic solutions, such as an active front wheel steering, see [1], [2].

The height of the roll center has an important role in the roll dynamics of the vehicle. A possible way to minimize the chassis roll angle is the minimization of the height of the roll center. The roll center depends on the camber angle of the front wheels, which can be modified by the variable-geometry suspension system. The lateral movement of the contact point of the variable-geometry system is also relevant from the aspect of tire wear, when the suspension moves up and down while the vehicle moves forward, see [3]. Using an appropriately designed variable-geometry control these unnecessary movements can be eliminated. In summary, in normal cruising maneuvers the steering system focuses on trajectory tracking and the variable-geometry suspension system guarantees various performances which are related to the chassis roll angle and half-track change.

Moreover, by changing the camber angles of the front wheels the yaw rate of the vehicle is modified, which can be used to reduce the tracking error from the reference yaw rate. Thus, with the reconfiguration of the camber angles, the variable-geometry system is able to focus on trajectory tracking and assist the driver to carry out various vehicle maneuvers. Thus, in an emergency such as a sharp cornering the variable-geometry system focuses on trajectory tracking instead of the conventional performances. In this way the variable-geometry system can also be used as a driver assistance system. However, one of the properties might only be improved to the detriment of other properties. For example, if the tracking ability is enhanced at the same time the performances in terms of roll dynamics and half-track change are degraded. The conflict between different performance demands must be resolved in such a way that a balance between the performances is achieved. In the control design a parameter-dependent weighting strategy is applied.

In the paper robust LPV (Linear Parameter Varying) methods are proposed for control design. Based on the LPV modeling approaches the highly nonlinear effects can be considered in such a way that the model structure is nonlinear in the parameters, but linear in the states. The advantage of LPV methods is that the controller meets the performance specifications and guarantees robustness against model uncertainties, since the controller is able to adapt to the current operational conditions, see [4], [5].

This paper is organized as follows. In Section II the control-oriented modeling of the lateral tire forces and its formalization in the lateral dynamics of the vehicle model are presented. In Section III the nonlinear variable-geometry kinematic model of the suspension system is analyzed. The performance requirements in the control-oriented model are presented in Section IV. The design of the reconfigurable robust controller based on the parameter-dependent LPV method is presented in Section V. In Section VI the operation of the control system through different vehicle maneuvers is illustrated. Finally, in the last section the main contributions are summarized.

II. LATERAL DYNAMICS OF VEHICLE MODEL

The bicycle model illustrated in Figure 1 is used in the control design. Although the Magic form gives a highly accurate description of the lateral tire force, see e.g. [6], a simplified form is constructed for numerical reasons. The lateral tire forces in the direction of the wheel ground contact are approximated linearly to the tire side slip angles α_f, α_r

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and the wheel camber angle γ.

$$F_{yf} = C_1\alpha_f + C_\gamma\gamma, \quad (1a)$$
$$F_{yr} = C_2\alpha_r, \quad (1b)$$

where C_1, C_2 are cornering stiffnesses, C_γ is a coefficient which represents the degree of offset.

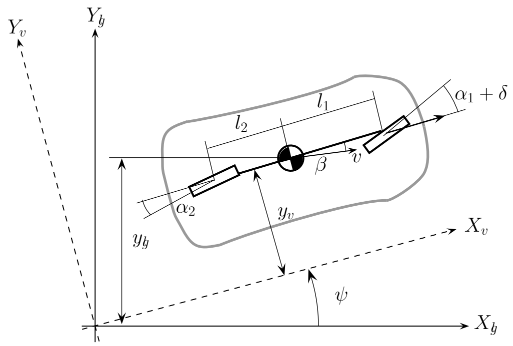


Fig. 1. Lateral model of the vehicle

Using the bicycle model the lateral dynamics of the vehicle is formalized. The first equation considers forces for the lateral dynamics, while the second one is the torque balance equation for yaw moments.

$$\begin{aligned} mv(\dot{\psi} + \dot{\beta}) &= F_{yf} + F_{yr} \\ &= C_1\alpha_f + C_2\alpha_r + C_{1,\gamma}\gamma, \quad (2a) \\ J\ddot{\psi} &= F_{yf}l_1 - F_{yr}l_2 \\ &= C_1l_1\alpha_f - C_2l_2\alpha_r + C_{1,\gamma}l_1\gamma, \quad (2b) \end{aligned}$$

where J is the yaw inertia of the vehicle, l_1 and l_2 are geometric parameters, ψ is the yaw of the vehicle, β is the side-slip angle of the vehicle. Moreover, $\alpha_f = -\beta + \delta - l_1 \cdot \dot{\psi}/v$ and $\alpha_r = -\beta + l_2 \cdot \dot{\psi}/v$ are the tire side slip angles at the front and rear, respectively, in which v denotes the longitudinal velocity. In the design of trajectory tracking control it is necessary to guarantee that the lateral position of the vehicle tracks the geometry of the road.

The required lateral motion is controlled by the difference between the actual yaw-rate of the vehicle and the yaw-rate desired by the driver. The desired yaw-rate, which is the reference signal of the system, can be computed by using the following formula [7]:

$$\dot{\psi}_{ref} = \frac{v}{l_1 + l_2}\delta \quad (3)$$

where δ is the steering angle actuated by the driver. The side-slip angles of the tires influence the actual yaw rate, which differs from the reference yaw rate. Thus, the cornering maneuver might lead to understeering or oversteering. The control goal is to compensate for the effects of side-slip angles on the vehicle.

III. Modeling of a variable-geometry suspension system

Several papers for various kinematic models of suspension systems have been published. A review of the first variable-geometry suspension systems was presented by [8]. A variable-geometry system for road stability were proposed by various papers, see [9], [10], [11]. Seeking to meet the performance requirements often leads to conflict situations and requires a compromise in terms of kinematic and dynamic properties, see e.g., [12]. The main focus of these methods is on the construction solution and the control design has received little attention. In an earlier paper of our project the simultaneous design of robust control and the construction of a variable geometry suspension system for the enhancement of vehicle stability was analyzed, see [13].

The kinematic model of the variable-geometry mechanism is presented by using the double wishbone suspension system (see Figure 2). The kinematic model contains the geometry

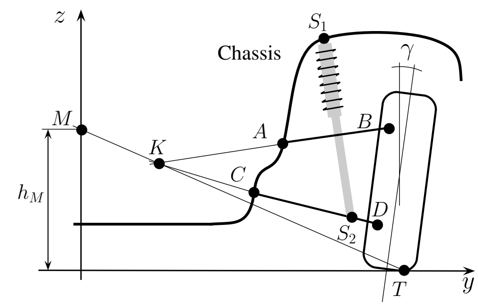


Fig. 2. Kinematic model of the suspension system

of the actuator and shows the suspension displacements.

The suspension system is analyzed in a local coordinate system, whose center point is C. Point A in the variable-geometry suspension system is able to move only in a horizontal direction. In the variable geometry system the change of point A in the direction y is the real input of the mechanism, which is denoted by a_y. Two further points B and D are marked on the tire, which move both in directions y and z. Their movements are denoted by b_y, b_z, d_y, d_z. T is the road-wheel contact point, which moves as a function of the road irregularities, i.e., t_y, t_z. The aim is to formalize the relationship between the input a_y and the wheel camber output γ. In the following only the main results are summarized. More details are found in [13].

Let us introduce the following vector of variables: $\eta = \begin{bmatrix} b_y & b_z & d_y & d_z & t_y \end{bmatrix}^T$. The relationship between η and a_y is formalized in the following form:

$$A_\eta(\eta)\ \eta = K(t_z) + B_\eta(a_y)\ a_y \quad (4)$$

where t_z can be considered as the disturbance. In the equation η is unknown, the variables A_η and $K(t_z)$ change as a function of η and t_z, and B_η depends on a_y. The vector η is expressed from (4):

$$\eta = A_\eta(\eta)^{-1}[K(t_z) + B_\eta(a_y)a_y] \quad (5)$$

where $A_\eta(\eta)$ is invertible. The output of the system is the camber of the wheel: $\gamma = \arccos\{(B_z - D_z)/L_{BD}\}$. Thus, the output equation is expressed: $C_\eta \eta = D_\eta$, where $C_\eta = \begin{bmatrix} 0 & 1 & 0 & -1 & 0 \end{bmatrix}^T$ and $D_\eta = L_{BD}\cos(\gamma) + D_z - B_z$. Consequently, the input of the mechanism a_y is expressed in the following form:

$$a_y = [C_\eta A_\eta(\eta)^{-1} B_\eta(a_y)]^{-1}[D_\eta - C_\eta A_\eta(\eta)^{-1} K(t_z)] \quad (6)$$

Equation (6) gives the relationship between γ and a_y. It is a parameter-varying expression, which depends on η, t_z and a_y.

The relationship between a_y and γ as a function of t_z based on the numerical solution of (6) is shown in Figure 4(a). An analysis shows that it is possible to approximate it with linear functions in the following form:

$$\gamma = \kappa + \xi_1 t_z + \varepsilon_1 a_y \quad (7)$$

The static components of the lateral forces are approximately equal, thus in the next computations constant κ is omitted from (7).

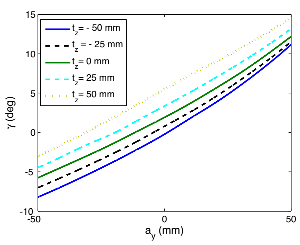


Fig. 3. a_y - γ characteristics

IV. Performance requirements in the control design

In normal cruising maneuvers the steering control assists the driver in following the trajectory, while the variable-geometry suspension control focuses on two performances. It minimizes the chassis roll angle by modifying the roll center of the vehicle. Moreover, the half-track change can also be minimized by using the variable-geometry suspension system. In the driver assistance system an additional performance requirement is also applied. The variable-geometry system is able to focus on trajectory tracking and assist the driver in carrying out various vehicle maneuvers. Thus, in an emergency, the variable-geometry suspension control also focuses on trajectory tracking. Consequently, the performance requirements are related to the yaw-rate tracking, the roll angle and the half track change.

In the trajectory tracking control the vehicle must follow the reference yaw-rate, which is approximated by (3). The difference between the yaw-rate of the vehicle and the reference yaw-rate must be minimized:

$$z_1 = |\dot{\psi}_{ref} - \dot{\psi}| \quad (8)$$

It has been shown that the roll center depends on both a_y and t_z. The height of the roll center has an important role in the roll dynamics of the vehicle [7]:

$$(I_{xx} + m\Delta h^2)\ddot{\phi} = mg\Delta h\phi + mv\Delta h(\dot{\beta} + \dot{\psi}) - \\ - B_i \sum F_{susp,i} \quad (9)$$

where Δh is the difference between the height of the center of gravity and the height of the roll center ($\Delta h = h_{CG} - h_M$), ϕ is the chassis roll angle, I_{xx} is the inertia of the chassis, B_i is the half track and $F_{susp,i}$ are the vertical forces of suspension.

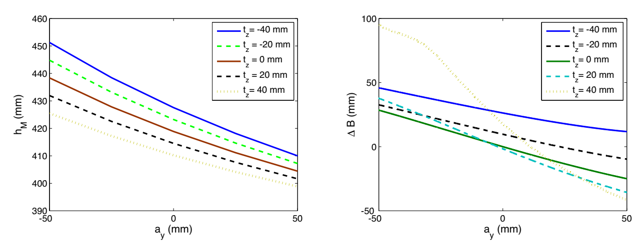
In order to minimize the chassis roll angle, the dynamic displacement of the height of the roll center based on (11) must be minimized:

$$z_2 = |\Delta h_M| \quad (10)$$

The construction of suspension determines the height of the roll center of the chassis, h_M. The intersection of the arms (A, B) and (C, D) is marked by K. The intersection of the line (T, K) and the vertical centerline of the chassis is the roll center itself. The relationship between a_y and h_M as a function of t_z based on the numerical solution of (6) is shown in Figure 4(b). The height of the roll center can be divided into static and dynamic components as follows: $h_M = h_{M,st} + \Delta h_M$. Component $h_{M,st}$ represents the height of the roll center of a stationary vehicle, while Δh_M represents the change of height during traveling. The dynamic component is expressed in the following linear form:

$$\Delta h_M = \xi_2 t_z + \varepsilon_2 a_y \quad (11)$$

Thus, the performance criterion is formalized in the following form: $z_2 = |\zeta_2 t_z + \epsilon_2 a_y| \rightarrow min.$



(a) Height of roll center (b) Half track change

Fig. 4. Properties of the variable-geometry suspension system

The fulfillment of this performance can be reached by increasing actuation a_y. However, in practice Δh_M has a physical limit since actuation a_y also has a limitation. Therefore, a signal $h_{ref}(< h_{CG} - h_{M,ST})$ is introduced instead of Δh_M and applied as a reference signal for the tracking task.

An additional important economy parameter is the half track change. Using a linear approximation and based on (13) the performance criterion is formalized in the following form:

$$z_3 = |\Delta B| \quad (12)$$

During traveling the half track change ΔB is also an important economical dynamic parameter of the suspension system, since it is related to tire wear. The relationship between a_y and ΔB as a function of t_z based on the numerical solution of (6) is shown in Figure 4(c). The figure shows that the relationship between a_y and ΔB can be expressed linearly in the following form:

$$\Delta B = \xi_3 t_z + \varepsilon_3 a_y \tag{13}$$

Thus, the performance criterion is formalized in the following form: $z_3 = |\xi_2 t_z + \varepsilon_3 a_y| \rightarrow min$.

Note that the performance requirements are in conflict, thus a balance must be achieved between them. The aim of the control design is to achieve a balance between performances.

V. Robust control of the variable-geometry suspension system

The control design of the variable-geometry suspension system is based on the control-oriented bicycle model:

$$\dot{x} = Ax + B_1 w + B_2 u \tag{14a}$$

$$z = C_1(\rho)x + D_{11}(\rho)w + D_{12}(\rho)u \tag{14b}$$

$$y = C_2 x \tag{14c}$$

where the state vector of the system contains the yaw-rate and the side-slip angle $x = \begin{bmatrix}\dot{\psi}, & \beta\end{bmatrix}^T$. The control input of the system is $u = a_y$, the disturbances are $w = \begin{bmatrix}\delta, & t_z\end{bmatrix}^T$, while the performances are $z = \begin{bmatrix}z_1, & z_2, & z_3\end{bmatrix}^T$.

The control design is formalized through a closed-loop interconnection structure, see Figure 5. Input and output weighting functions are selected to the specifications of disturbances, inputs and outputs.

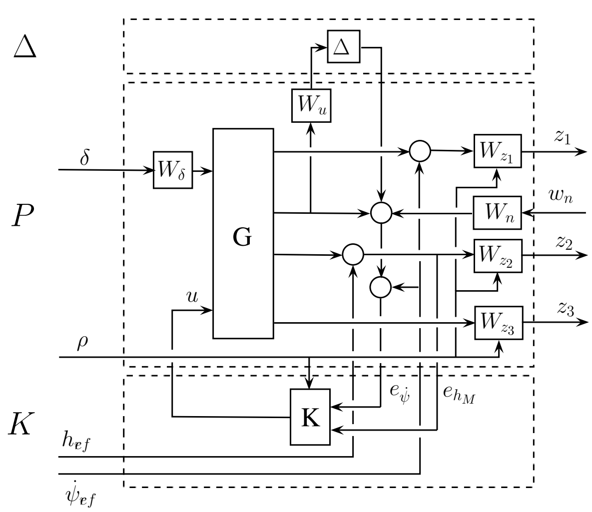


Fig. 5. Closed-loop interconnection structure

The crucial point of the control design is the selection of the weighting functions for performances. In the control design parameter-dependent weighting functions are applied to the performances, i.e., the yaw-rate tracking, the roll angle and the half track change:

$$W_{z_1} = \frac{\rho}{e_{\dot{\psi}_{max}}} \tag{15a}$$

$$W_{z_2} = \frac{1-\rho}{\phi_{max}} \tag{15b}$$

$$W_{z_3} = \frac{1-\rho}{\Delta B_{max}} \tag{15c}$$

where $\rho \in [0;1]$ is a scheduling variable, $e_{\dot{\psi}_{max}}$ is the possible maximum of the yaw-rate error, ϕ_{max} is the maximum of the chassis roll angle and ΔB_{max} is a maximum of the half-track change.

The role of parameter ρ in the weighting functions is to create a balance between the different performances. For example, in the case of $\rho = 1$ the control system prefers the yaw-rate tracking, while in the case of $\rho = 0$ the control system focuses on both the roll angle and half-track change. Note that the reference height of the roll moment h_{ref} is also used in the control design.

In normal cruising maneuvers the control of the variable-geometry suspension system guarantees two performances which are related to the chassis roll angle and the half-track change. In an emergency the variable-geometry system must focus on trajectory tracking and assist the driver in carrying out various vehicle maneuvers. Thus the control design must be extended with a reconfigurable structure.

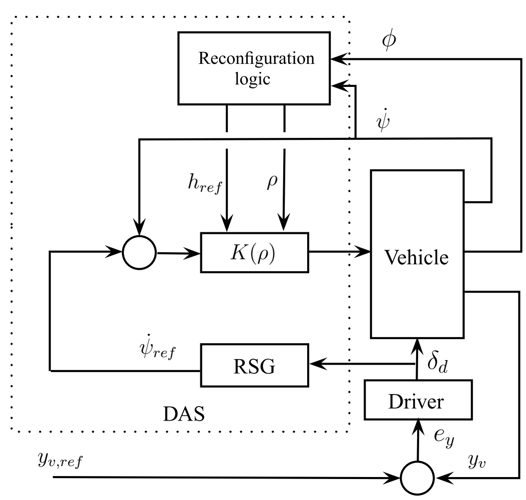


Fig. 6. Architecture of control systems

Figure 6 illustrates the architecture of the reconfigurable control system. It contains several components besides the vehicle model and the driver model, such as the suspension control $K(\rho)$, the generator of the reference yaw-rate signal (RSG) and the block of the reconfiguration logic. The role of the variable-geometry suspension control $K(\rho)$ is to minimize the chassis roll angle and the half-track change and, in an emergency, to reduce the yaw-rate error. Since these performances are in conflict, a reconfiguration logic is applied. The variable-geometry suspension control is activated by using two signals, ρ and h_{ref}.

In the following various vehicle scenarios are distinguished.

- Half-track change minimization: In normal cruising the goal of the variable-geometry suspension control is to minimize half-track change ΔB. This configuration is achieved by the selection $\rho = 0$, $h_{ref} = h_M$.
- Roll angle minimization: When the roll angle ϕ increases significantly, the variable-geometry suspension control must minimize the roll angle. This configuration is achieved by the selection $\rho = 0$, $h_{ref} = h_{ref,max}$.
- Balance between half-track change and roll angle minimization: It is possible to achieve vehicle maneuvers in which there is a balance between the two performances, i.e., the reduction of the half-track change and that of the roll angle. In these configurations $\rho = 0$ and h_{ref} is selected in an interval $h_M < h_{ref} < h_{ref,max}$.
- Yaw-rate tracking: When the suspension system must focus on trajectory tracking the scheduling variable ρ is selected greater than 0.
- In an emergency maneuver, when there has been an extreme increase in the yaw rate error, the suspension system must focus on the tracking error $e_{\dot{\psi}}$ instead of the conventional two performances. This configuration is achieved by the selection $\rho = 1$ and $h_{ref} = h_M$.

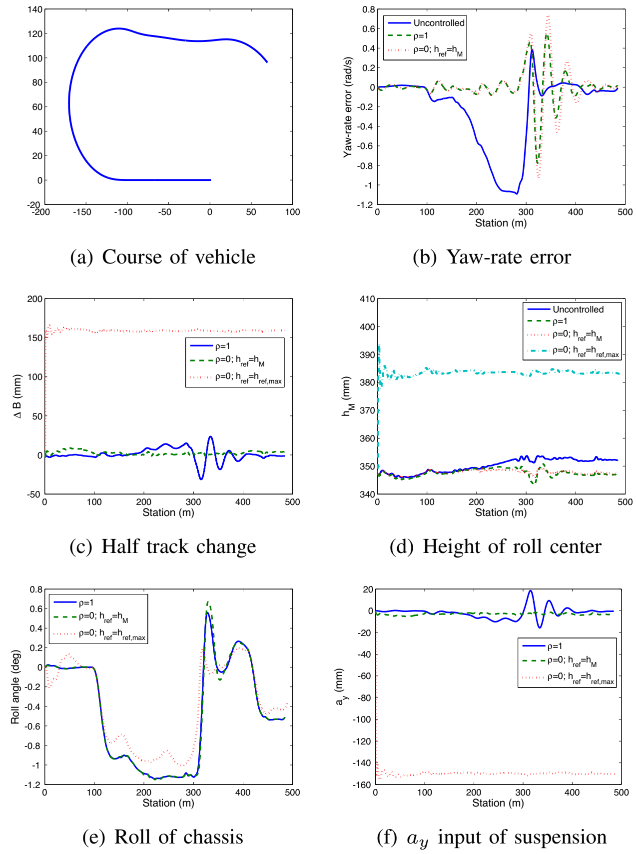
The control design is based on the LPV method that uses parameter-dependent Lyapunov functions, see [4], [5]. The quadratic LPV performance problem is to choose the parameter-varying controller in such a way that the resulting closed-loop system is quadratically stable and the induced $\mathcal{L}_2$ norm from the disturbance and the performances is less than a predefined value.

VI. Simulation results

In the simulation examples the vehicle is traveling along a predefined road, while the variable-geometry suspension system assists the driver in carrying out maneuvers. The control design is performed by using the Matlab/Simulink while the verification of the designed controller is performed by using the CarSim simulation software. The efficiency of the proposed driver assistance system is illustrated by using the CarSim simulation software.

In the first simulation example the interaction between performance specifications is presented. The vehicle is traveling along a course, which is depicted in Figure 7(a).

Figure 7(b) shows the tracking of the yaw rate error of the control systems with different configurations and without any control. In the uncontrolled vehicle the driver is not able to compensate for either the understeering or the oversteering motion, therefore the tracking error of the yaw-rate significantly increases. In the controlled vehicles



(a) Course of vehicle
(b) Yaw-rate error
(c) Half track change
(d) Height of roll center
(e) Roll of chassis
(f) a_y input of suspension

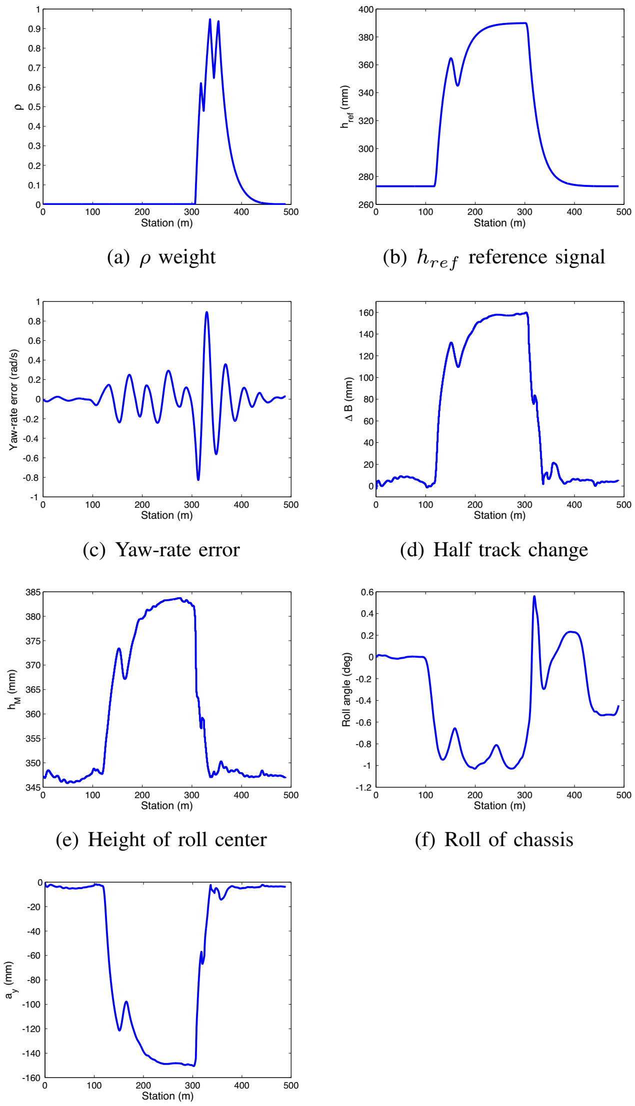
Fig. 7. Performances in the driver assistance system
(solid: $\rho = 1$, dashed: $\rho = 0, h_{ref} = h_M$), dashed-dotted: $\rho = 0, h_{ref} = h_{ref,max}$)

the tracking of the reference yaw-rate with an acceptable threshold is guaranteed.

When the suspension system focuses on the tracking task, i.e., $\rho = 1$, the yaw-rate error is significantly reduced, which is shown in Figure 7(c). In the default case, when the suspension system focuses on the half-track change, i.e., $\rho = 0$, $h_{ref} = h_M$, the half-track change is minimized, see Figure 7(c). However, in this case the roll angle increases. When the suspension system focuses on the roll angle minimization, i.e., $\rho = 0$, $h_{ref} = h_{ref,max}$, the half-track change significantly increases since lifting up the height of the roll center requires significant control input a_y, see Figure 7(d) and Figure 7(f). Figure 7(d) and Figure(e) illustrate the relationship between the increased roll center and the reduced roll angle. Figure 7(f) shows the control inputs of the variable-geometry suspension a_y.

In the second example the efficiency of the driver assistance system during the same cornering maneuver is presented. Figure 8(a) and Figure 8(b) show scheduling variable ρ and reference signal h_{ref}, respectively. The reconfigurable control of the variable-geometry suspension system is based on these variables.

In Figure 8(c) the yaw-rate error of the vehicle is shown. The scheduling variable ρ shows the relationship between the



(a) ρ weight

(b) h_{ref} reference signal

(c) Yaw-rate error

(d) Half track change

(e) Height of roll center

(f) Roll of chassis

(g) a_y input of suspension

Fig. 8. Operation of the driver assistance system

variable ρ and the yaw-rate error. When the control focuses on the half-track change and it decreases, the control action requires significant control input and at the same time the yaw-rate error increases, see Figure 8(d) and Figure 8(g). Figure 8(e) and Figure 8(f) show the height of roll center and roll angle of the chassis. The change of h_{ref}, which is also shown in Figure 8(b), induces the modification of h_M. The change of the reference height modifies the roll angle of the chassis ϕ, thus the roll dynamics.

VII. Conclusion

The paper has proposed the design of the robust control of a variable-geometry suspension system for the enhancement of vehicle stability. The control system guarantees various performances which are related to the yaw rate tracking, the reduction of the chassis roll angle and the reduction of the half-track change. Since the performances are in conflict, a trade-off must be guaranteed between them by the reconfiguration of the camber angles. The paper has proposed the variable-geometry system as a driver assistance system. The control design is based on robust LPV methods, in which both performance specifications and model uncertainties are taken into consideration.

The simulation results show that the variable-geometry suspension system is able to modify both lateral and roll dynamics effectively. The different settings of the variable-geometry suspension system make it possible to improve vehicle dynamic properties. Using the reconfiguration structure, the variable-geometry suspension system is able to focus on trajectory tracking in an emergency. Thus, the driver is able to drive the vehicle with self-confidence.

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