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Nonlinear vehicle lateral dynamics estimation with unmeasurable premise variable Takagi-Sugeno approach

Z. Yacine, D. Ichalal, N. Ait Oufroukh, S. Mammar and S. Djennoune

Abstract— **This paper deals with the problem of observer design for vehicle lateral dynamics. The nonlinear model of this last is transformed into Takagi-Sugeno (T-S) formulation by using the sector nonlinearity transformation. The main contribution of this paper is the representation of the vehicle nonlinear model by a T-S model with minimal loss of information (almost exact T-S model). This inevitably leads to a model with unmeasurable premise variables which is more difficult to study compared to the classical T-S models where premise variables are assumed to be measurable even if this is not really true as their are often estimated. The second contribution of this paper is the observer design for estimating the lateral dynamics of the vehicle. Stability conditions are established using a Lyapunov method and the concept of Input-To-State Stability (ISS). These conditions are then expressed in terms of optimization problem subject to LMI constraints. Simulation results are provided to illustrate the proposed approach, where the observer is synthesized with a T-S model and then applied directly to the nonlinear model of the vehicle. Some aspects of the robustness of the observer, with respect to time varying longitudinal velocity and measurement noise, are discussed.**

Index Terms— **Lateral vehicle dynamics, nonlinear systems, Takagi-Sugeno modeling, Lyapunov theory, Input-To-State stability (ISS), LMIs**

I. INTRODUCTION

The parameters of the vehicle's lateral dynamics are very important to the development of driver assistance, road safety and warnings technologies, especially for the lane keeping, the limitation of the yaw motion and lateral velocity. The knowledge of the side slip angle and the lateral velocity is then necessary. However the lateral dynamics are very difficult to measure accurately, or if they are, the sensors used are very expensive. To overcome the problem of the vehicle lateral dynamics measurements, many works have been undertaken in order to develop observers to such systems. See for example [15] where the authors proposed a Luenberger-like observer based on linear model of the vehicle. In [9], an unknown input and proportional observers are provided for estimating the lateral dynamics and the road attributes. A particular attention has been drawn for improving the estimation quality and accuracy since in the cited works, a linear model is used and it is known that a linear model is valid only around a specific operating points. The most known approach is that using a nominal linear model with taking into account some time-variations in the vehicle parameters, this leads to a model evolving in a large domain around the nominal operating point [7]. However, even if this approach can improve the state estimation, it remains unsatisfactory in the case of extreme driving situations corresponding to nonlinear behaviors. The major provided results for estimating the lateral dynamics are based on the use of the well-known Extended Kalman Filter [16], [6]. In the last years, new approaches have emerged, exploiting the Fuzzy modeling.

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In [18], a new structure for nonlinear representation has been introduced. It allows to represent nonlinear behaviors by in intuitive way by decomposing the operating space of the system in several regions and each region is modeled by a linear model valid on an associated operating point (region). The global nonlinear behavior of the system is then obtained by a set of nonlinear functions satisfying the convex sum property associated to each linear sub-model which leads to the following so-called T-S model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector and $y(t) \in \mathbb{R}^p$ represents the output vector. $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ are known matrices. The functions $\mu_i(\xi(t))$ are the weighting functions depending on the variables $\xi(t)$ which can be measurable (as the input or the output of the system) or unmeasurable variables (as the state of the system). These functions verify the following properties

$$\sum_{i=1}^{r} \mu_i(\xi(t)) = 1,\ 0 \le \mu_i(\xi(t)) \le 1 \quad \forall i \in \{1,...,r\} \tag{2}$$

Three approaches are commonly used to obtain a T-S model: linearization of an available nonlinear model around some operating points and using adequate weighting functions, black-box approaches (parameter identification from input-output data) [5] and the sector nonlinearity transformation [19], [12]. This last is an interesting approach since it allows to obtain an exact T-S representation of a general nonlinear model with no information loss, in a compact set of the state space.

Effectiveness of such a model is proven, for nonlinear systems, in many fields as stability and stabilization [19], [8], observation and fault diagnosis [3], [10].

In this work the considered premise variable $\xi(t)$ depends on the state of the system which is not totally measurable. The problem of state estimation of nonlinear systems using

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T-S model approach have been addressed with different methods, but most of the published works have considered T-S models with measurable premise variables [2], [14], [10], [3]. It is clear that the choice of measurable premise variables offers a good simplicity to generalize the methods already developed for linear systems. Contrarily, the problem becomes harder when the premise variables are not measurable. However, this formalism is very important in both the exact representation of nonlinear behaviors by T-S model and in observer based diagnosis. Furthermore, the T-S models with unmeasurable premise variables may represent a larger class of nonlinear systems compared to the T-S model with measurable premise variables [20]. Only a few works are devoted to the case of unmeasurable premise variables, nevertheless, we can cite [4], [13] where the authors proposed the fuzzy Thau-Luenberger observer which is an extension of the classical Luenberger observer and, in [20], a filter estimating the state and minimizing the effect of disturbances is proposed. Finally, a similar problem is dealt with for LPV discrete-time systems with uncertain scheduled parameters in [11].

In this paper, an alternative approach is proposed to construct a nonlinear observer for estimating the vehicle lateral dynamics in a large range of functioning which includes some extreme situations as tires lateral forces saturation. The main objective is to provide an observer that estimates accurately the lateral dynamics of a vehicle. In particular, the estimation of the sideslip angle of the vehicle and its lateral velocity for which a sensor measuring it is very expensive (around 15000€), the accuracy of the estimated lateral velocity is related to the used vehicle lateral dynamics model. In fact the model should be able to describe the whole operating domain, including the linear and the saturated behavior of the lateral tire-road forces occurring for higher tire sideslip angles. In addition, LMI conditions are provided, which are easily solved by dedicated numerical tools (Matlab LMI toolbox, YALMIP,...).

The paper begins by a brief description of the nonlinear vehicle lateral dynamics. Thereafter, a T-S model is derived from sector nonlinearity transformation which constitutes the heart of the proposed work since it allows to describe exactly nonlinear behaviors with no loss of information. Obviously, as discussed in the last paragraph, this modeling approach leads generally to T-S model with state dependent weighting functions which are not measurable. This represents the second particularity of the proposed work in theoretical point of view. An observer is than proposed and its convergence is studied with the Lyapunov theory and the ISS concept to ensure a bounded state estimation error which is optimized for more accuracy. The established stability conditions are then expressed in LMI formulation. Finally, simulation results and discussions on some robustness aspects as time-varying longitudinal velocity v, uncertainties and measurement noise of the observer are given to illustrate the performances of the proposed observer.

A. Vehicle lateral dynamics nonlinear model

In many studies of controllers and observers design for vehicle lateral dynamics, a single track model is used. The nonlinear model is reported in [1] given by the following differential equations

$$\begin{cases} mv\dot{\beta} = F_f + F_r - mv\dot{\psi} \\ I_z\ddot{\psi} = a_f F_f - a_r F_r \end{cases} \tag{3}$$

where β represents the sideslip angle, $\dot{\psi}$ is the yaw rate, F_f and F_r are the lateral forces at the front and rear tires respectively, v is the longitudinal velocity. Several works consider a constant longitudinal velocity which is not very realistic. In this work we assume that the velocity is constant and measured variable but the observer is applied to the nonlinear system with time varying longitudinal velocity (robust observer with respect to time varying velocity v). I_z is the yaw moment of inertia, m is the mass of the vehicle, a_f and a_r are respectively the distance from the front axle to the center of gravity and the distance of the rear axle to the center of gravity. The tire forces are modeled by the well-known Pacejka's magic formula which is largely used in the study of the interactions between the tires and the road. They are thus given by the following formula

$$F_i = D_i \sin(C_i \tan^{-1}((B_i(1-E_i)\alpha_i + E_i \tan^{-1}(B_i\alpha_i)))) \tag{4}$$

The parameters B_i, C_i, D_i, E_i $i = \{f, r\}$ depend on the tire characteristics and vertical load. α_f and α_r are respectively the sideslip angle of the front and rear tires which are given by

$$\begin{cases} \alpha_f = \delta_f - \beta - \tan^{-1}\left(\frac{a_f}{v}\dot{\psi}\cos(\beta)\right) \\ \alpha_r = -\beta - \tan^{-1}\left(\frac{a_r}{v}\dot{\psi}\cos(\beta)\right) \end{cases} \tag{5}$$

where δ_f is the front tires steering angle.

B. T-S modeling of the vehicle nonlinear lateral dynamics model

Let us consider the nonlinear model given in (3). The aim of this sub-section is to re-write this model in T-S formulation with no loss of information and without simplification. For that purpose, let us start with the Pacejka's model of the tires. The general form is given by

$$F = D\sin(C\tan^{-1}((B(1-E)\alpha + E\tan^{-1}(B\alpha)) \tag{6}$$

with simple mathematical manipulations, it is easy to write

$$\begin{aligned} F &= (\mathscr{A}\frac{\sin(S_3)}{S_3}\frac{\tan^{-1}(S_2)}{S_2} \\ &+ \mathscr{B}\frac{\sin(S_3)}{S_3}\frac{\tan^{-1}(S_2)}{S_2}\frac{\tan^{-1}(S_1)}{S_1})\alpha \end{aligned} \tag{7}$$

where

$$S_1 = B\alpha \tag{8}$$

$$S_2 = B(1-E) + E\tan^{-1}(S_1) \tag{9}$$

$$S_3 = C\tan^{-1}(S_2) \tag{10}$$

$$\mathscr{A} = BCD(1-E) \tag{11}$$

$$\mathscr{B} = BCDE \tag{12}$$

It is known that $\frac{sin(x)}{x}$ is defined on $\mathbb{R}$ and when $x \to 0$ $\frac{sin(x)}{x} \to 1$. It is the case also for the function $\frac{tan^{-1}(x)}{x}$. Indeed, by using the following formula

$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - .... \quad (13)$$

it follows

$$\frac{\tan^{-1}(x)}{x} = 1 - \frac{x^2}{3} + \frac{x^4}{5} - \frac{x^6}{7} + \frac{x^8}{9} - .... \quad (14)$$

It is then obvious that $\lim_{x\to 0}\left(\frac{\tan^{-1}(x)}{x}\right) = 1$. Knowing that α is a bounded angle the function $\frac{\tan^{-1}(x)}{x}$ is also bounded.

Now, consider the nonlinearity $f(\alpha)$ given by

$$\begin{aligned} f(\alpha) &= \mathscr{A}\frac{\sin(S_3)}{S_3}\frac{\tan^{-1}(S_2)}{S_2} \\ &+ \mathscr{B}\frac{\sin(S_3)}{S_3}\frac{\tan^{-1}(S_2)}{S_2}\frac{\tan^{-1}(S_1)}{S_1} \end{aligned} \quad (15)$$

The function $f(\alpha)$ is bounded $\forall \alpha$ as follows

$$f_{\min} \le f(\alpha) \le f_{\max} \quad (16)$$

Let us define the $f(\alpha)$ as a premise variable, it follows

$$\mu_1(\alpha) = \frac{f(\alpha) - f_{\min}}{f_{\max} - f_{\min}},\ \mu_2(\alpha) = \frac{f_{\max} - f(\alpha)}{f_{\max} - f_{\min}} \quad (17)$$

Thus, the exact T-S model is obtained as follows

$$F = \sum_{i=1}^{2} \mu_i(\alpha) M_i \alpha \quad (18)$$

where the parameters $M_i,\ \ i = 1,2$ are defined by $M_1 = f_{max}$ and $M_2 = f_{min}$. Next, both F_f and F_r will be expressed in exact T-S formulations.

1) T-S form of the nonlinear vehicle lateral dynamics : The state vector of the system is defined by $x^T(t) = [\beta \ \ \dot{\psi}]^T$. Considering only a small values of the slip angles of the front and rear tires, which corresponds to normal and pseudo-sliding regions. Indeed, if vehicle is in these regions the side slip angles do not exceed 8 deg. This assumption leads to simplify the expressions of α_f and α_r as follows

$$\alpha_f = -\beta - \frac{a_f}{v}\dot{\psi} + \delta_f \quad (19)$$

$$\alpha_r = -\beta + \frac{a_r}{v}\dot{\psi} \quad (20)$$

Using these above equations in the Pacejka's forces defined in (18) for each wheel (front and rear), the following representation is obtained

$$F_f = \sum_{i=1}^{2} \mu_i^f(x,\delta_f)\begin{pmatrix} -M_i^f & -\frac{M_i^f a_f}{v} \end{pmatrix} x + \delta_f \quad (21)$$

$$F_r = \sum_{i=1}^{2} \mu_i^r(x)\begin{pmatrix} -M_i^r & \frac{M_i^r a_r}{v} \end{pmatrix} x \quad (22)$$

Using (21) and (22), the vehicle lateral dynamics in T-S representation can be written as follows

$$\dot{x}(t) = \sum_{i=1}^{4} h_i(x(t))\left(A_i x(t) + B_i u(t)\right) \quad (23)$$

where

$$\begin{aligned} h_1(x) &= \mu_1^f(x,\delta_f)\mu_1^r(x) \\ h_2(x) &= \mu_2^f(x,\delta_f)\mu_1^r(x) \\ h_3(x) &= \mu_1^f(x,\delta_f)\mu_2^r(x) \\ h_4(x) &= \mu_2^f(x,\delta_f)\mu_2^r(x) \end{aligned}$$

$$\sum_{i=1}^{4} h_i(x) = 1$$

$$x(t) = \begin{pmatrix} \beta \\ \dot{\psi} \end{pmatrix}, \quad u(t) = \delta_f$$

$$A_1 = \begin{pmatrix} -\frac{M_1^f + M_1^r}{mv} & -\frac{a_f M_1^f + a_r M_1^r}{mv^2} - 1 \\ -\frac{a_f M_1^f - a_r M_1^r}{I_z} & -\frac{a_f^2 M_1^f + a_r^2 M_1^r}{vI_z} \end{pmatrix}$$

$$A_2 = \begin{pmatrix} -\frac{M_2^f + M_1^r}{mv} & -\frac{a_f M_2^f + a_r M_1^r}{mv^2} - 1 \\ -\frac{a_f M_2^f - a_r M_1^r}{I_z} & -\frac{a_f^2 M_2^f + a_r^2 M_1^r}{vI_z} \end{pmatrix}$$

$$A_3 = \begin{pmatrix} -\frac{M_1^f + M_2^r}{mv} & -\frac{a_f M_1^f + a_r M_2^r}{mv^2} - 1 \\ -\frac{a_f M_1^f - a_r M_2^r}{I_z} & -\frac{a_f^2 M_1^f + a_r^2 M_2^r}{vI_z} \end{pmatrix}$$

$$A_4 = \begin{pmatrix} -\frac{M_2^f + M_2^r}{mv} & -\frac{a_f M_2^f + a_r M_2^r}{mv^2} - 1 \\ -\frac{a_f M_2^f - a_r M_2^r}{I_z} & -\frac{a_f^2 M_2^f + a_r^2 M_2^r}{vI_z} \end{pmatrix}$$

$$B_1 = B_3 = \begin{pmatrix} \frac{M_1^f}{mv} \\ \frac{a_f M_1^f}{I_z} \end{pmatrix}, B_2 = B_4 = \begin{pmatrix} \frac{M_2^f}{mv} \\ \frac{a_f M_2^f}{I_z} \end{pmatrix}$$

C. Available measurements

In this work, only the yaw rate $\dot{\psi}$ and the front steering angle δ_f are measured, thus, the output equation is constructed as follows

$$y(t) = Cx(t) \quad (24)$$

where $C = \begin{pmatrix} 0 & 1 \end{pmatrix}$ Finally, the T-S model of the vehicle lateral dynamics is given by

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{4} h_i(x(t))\left(A_i x(t) + Bu(t)\right) \\ y(t) = Cx(t) \end{cases} \quad (25)$$

A simulation is performed with the nonlinear model, the T-S model obtained by simplifying the side slip angles α_f and α_r and a linear model $\dot{x} = Ax + Bu$ obtained by linearizing the nonlinear one around the operating point $(\beta, \dot{\psi}, \delta_f) = (0,0,0)$. The steering angle used in this simulation is given in the figure 1, where large values are used in order to reveal the nonlinear dynamics of the system. The obtained results are depicted in the figure 2 with fixed longitudianl velocity $v = 30m/s$.

It appears that the T-S model represents the nonlinear behavior of the vehicle with a good accuracy compared to the linear model. For small values of δ_f (time between 6 and 10 sec), both the linear model and the T-S model represent the system accurately but if the values of δ_f increase, only the T-S model provides an accurate results.

In the next section, the obtained T-S model is used in order to construct an observer estimating the slide slip angle β and

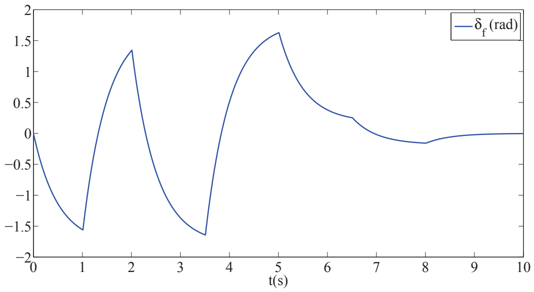

Fig. 1. Steering angle of the front wheel

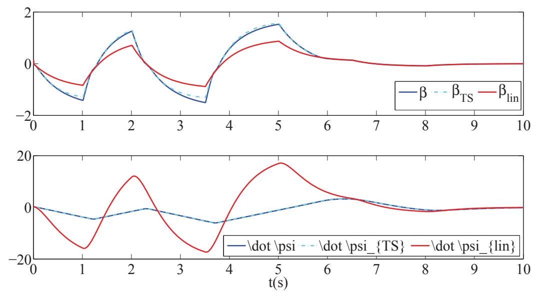

Fig. 2. Comparison between the nonlinear model, the T-S model and the linear one

the lateral velocity v_y obtained from $v_y = v\beta$ where v is the longitudinal velocity.

II. Observer for Sideslip Angle and Lateral Velocity Estimation

Let us consider the T-S model of the vehicle lateral dynamics given in (25). The proposed observer is in the form

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{4} h_i(\hat{x}(t))\left(A_i\hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))\right) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{26}$$

where L_i $(i = 1,\ldots 4)$ is the observer gain related to each sub-model. Define the state estimation error $e(t) = x(t) - \hat{x}(t)$, its dynamics obeys to the following differential equation

$$\dot{e}(t) = \sum_{i=1}^{4} h_i(\hat{x}(t))\left(A_i - L_i C\right)e(t) + \Delta(t) \tag{27}$$

where

$$\Delta(t) = \sum_{i=1}^{4}\left(h_i(x(t)) - h_i(\hat{x}(t))\right)\left(A_i x(t) + B_i u\right) \tag{28}$$

is a vanishing perturbation term, i.e. if $\hat{x} \to x$ then $\Delta(t) \to 0$.

Assumption 1: In order to prove the convergence of the observer, assume that

- the state x and the control input u are bounded.

Definition 1: [17] The system (27) is said to be ISS if there exist a $\mathcal{KL}$ function $\beta : \mathbb{R}\times\mathbb{R} \to \mathbb{R}$ and a $\mathcal{K}$ function $\alpha : \mathbb{R} \to \mathbb{R}$ such that, for each input $\Delta(t)$ satisfying $\|\Delta(t)\|_\infty < \infty$ and each initial condition $e(0) \in \mathbb{R}^n$, the trajectory of (27) associated with $e(0)$ and $\Delta(t)$ satisfies

$$\|e(t)\| \le \beta\left(\|e(0)\|, t\right) + \alpha\left(\|\Delta(t)\|_\infty\right), \forall t \tag{29}$$

From assumption 1 and the fact that h_i are bounded, the term $\Delta(t)$ is bounded. The assumption 1 is not restrictive for our framework. Indeed, the system is stable which provides a bounded states for bounded input δ_f. Under bounded vanishing perturbation term $\Delta(t)$, the observer (26) is synthesized by solving the optimization problem under LMI constraints given in the theorem 1.

Theorem 1: Assume that the assumption 1 is hold. Given a scalar $\sigma \in [0\ 1]$. If there exist a symmetric and positive definite matrix P, gain matrices K_i and a positive scalars $\bar{\gamma}$ and $\bar{\alpha}$ solution to the following optimization problem

$$\min_{P,K_i,\bar{\gamma},\bar{\alpha}} \quad \sigma\bar{\alpha} + (1-\sigma)\bar{\gamma}$$

$$\begin{pmatrix} A_i^T P + P A_i - K_i C - C^T K_i^T + I & P \\ P & -\bar{\gamma} I \end{pmatrix} < 0 \tag{30}$$

$$P \ge I \tag{31}$$

$$i = 1,\ldots,4$$

then the error dynamics (27) is ISS with respect to $\Delta(t)$ and satisfy the following inequality

$$\|e(t)\|_2 \le \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\|e(0)\|_2 e^{-\frac{1}{2\lambda_{\max}(P)}t} + \gamma\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\|\Delta(t)\|_\infty \tag{32}$$

The gains of the observer are computed by $L_i = P^{-1}K_i$ and the attenuation level of the transfer from $\Delta(t)$ to $e(t)$ is $\rho = \gamma\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}$.

Proof: Assume that the LMIs (30) are feasible. Let us consider the vector

$$T = \begin{pmatrix} e^T(t) & \Delta^T(t) \end{pmatrix} \tag{33}$$

By multiplying (30) on the left by T and by T^T on the right, the following is obtained

$$\begin{aligned} & e^T(t)\left(\Phi_i^T P + P\Phi_i\right)e(t) + e^T(t)P\Delta(t) + \Delta^T(t)Pe(t) \\ + \quad & e^T(t)e(t) - \gamma^2\Delta^T(t)\Delta(t) < 0 \end{aligned} \tag{34}$$

By multiplying (34) by $\sum_{i=1}^{4} h_i(\hat{x})$, one obtains

$$\begin{aligned} & \sum_{i=1}^{4} h_i(\hat{x}(t))(e^T(t)\left(\Phi_i^T P + P\Phi_i\right)e(t) + e^T(t)P\Delta(t) \\ + \quad & \Delta^T(t)Pe(t)) < -e^T(t)e(t) + \gamma^2\Delta^T(t)\Delta(t) \end{aligned} \tag{35}$$

which is equivalent to

$$\dot{V}(t) < -e^T(t)e(t) + \gamma^2\Delta^T(t)\Delta(t) \tag{36}$$

where

$$V(t) = e^T(t)Pe(t), \quad P = P^T > 0 \tag{37}$$

Since (30) is feasible, $\gamma^2 > 0$, then,

$$\lambda_{\min}(P)\|e(t)\|_2^2 \le V(t) \le \lambda_{\max}(P)\|e(t)\|_2^2, \quad \forall e(t) \in \mathbb{R}^2 \tag{38}$$

Consequently, (36) can be bounded as follows

$$\dot{V}(t) \leq -\frac{1}{\lambda_{\max}(P)} V(t) + \gamma^2 \|\Delta(t)\|_2^2 \tag{39}$$

which leads to

$$\begin{aligned} V(t) &\leq V(0) e^{-\frac{1}{\lambda_{\max}(P)}t} + \gamma^2 \int_0^t e^{-\frac{1}{\lambda_{\max}(P)}(t-s)} \|\Delta(s)\| ds \\ &\leq V(0) e^{-\frac{1}{\lambda_{\max}(P)}t} + \gamma^2 \lambda_{\max}(P) \|\Delta(t)\|_\infty^2 \end{aligned} \tag{40}$$

Finally, by using (38) with the square root, one obtains

$$\|e(t)\|_2 \leq \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \|e(0)\|_2 e^{-\frac{1}{2\lambda_{\max}(P)}t} + \gamma \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \|\Delta(t)\|_\infty \tag{41}$$

From this equation, we conclude that if $\|\Delta(t)\|_\infty = 0$ then $\|e(t)\| \to 0$ when $t \to \infty$. Moreover, in the presence of the perturbation $\Delta(t)$, the error $\|e(t)\|$ is bounded by $\gamma\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \|\Delta(t)\|_\infty$. As a conclusion, the Input-To-State (ISS) stability is proven with the inequality (41). Now, in order to enhance the state estimation accuracy, the quantity $\gamma\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}$ should be minimized. For that purpose, the objective function should be linear under LMI constraints. Let us consider the following inequality

$$\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \leq \sqrt{\alpha} \tag{42}$$

where α is a positive scalar to minimize. Since $\lambda_{\max}(P) > \lambda_{\min}(P)$, the minimal value of $\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}$ which can be obtained is equal to 1. From this, one can impose $P \geq I$ which leads to $\lambda_{\min}(P) \geq 1$. It follows

$$\lambda_{\max}(P) \leq \alpha \Leftrightarrow P^T P - \alpha^2 I \leq 0 \tag{43}$$

Using Schur's complement lemma

$$\begin{pmatrix} -\alpha^2 I & P \\ P & -I \end{pmatrix} \leq 0 \tag{44}$$

defining $\bar{\alpha} = \alpha^2$ and considering the objective function

$$\min \quad \sigma\bar{\alpha} + (1-\sigma)\bar{\gamma} \tag{45}$$

where $\sigma \in [0,1]$, the optimization problem (45) under LMI constraints (30)-(31) is then obtained which ends the proof. ■

III. Simulation results and discussions

An observer for vehicle lateral dynamics estimation is constructed by following the presented approach and the gains of the observer are computed after solving the optimization problem given in the theorem 1, with longitudinal velocity $v = 30m/s$, by YALMIP toolbox. The obtained gains are

$$L_1 = \begin{pmatrix} 29.57 \\ 113.01 \end{pmatrix}, L_2 = \begin{pmatrix} 114.07 \\ 114.90 \end{pmatrix},$$

$$L_3 = \begin{pmatrix} -94.63 \\ 114.50 \end{pmatrix}, L_4 = \begin{pmatrix} -0.57 \\ 115.36 \end{pmatrix}$$

The obtained attenuation level of the transfer of the perturbation $\Delta(t)$ to the state error $e(t)$ is $\rho = 0.0267$. The computation of $\|\Delta(t)\|_2$ at the steady state, gives values less than 10^{-3}, so the obtained bound of the error $\|e(t)\|_2$ is $\rho\|\Delta(t)\|_2 = 2.67.10^{-5}$. In order to test the convergence behavior of the observer, initial conditions are set different for the system and the observer. The initial conditions of the system are $x(0) = [0.15 \quad 0.2]^T$ and those of the observer are $\hat{x}(0) = [0 \quad 0]^T$.

A. Observer for the nonlinear model

The developed observer is applied directly to the nonlinear model of the vehicle lateral dynamics. In this case, the simplifications on α_f and α_r are only in the observer while the nonlinear model accounts with all nonlinearities. The term Δ has the bound 10^{-2} in the steady state which corresponds to $\|e(t)\|_2 \leq \rho\|\Delta(t)\|_2 = 2.67 \times 10^{-4}$. The simulation results are shown in figure 3.

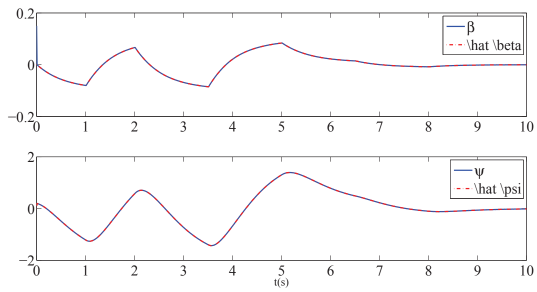

Fig. 3. States of the nonlinear model and the estimated states

Note that, even if the used T-S model is not exact regarding to the original model due to the simplifications on α_f and α_r, the obtained estimations are acceptable since the state estimation error norm $\|e(t)\|_2$ is less than 1.5×10^{-4} which is less than the bound 2.67×10^{-4}.

B. Robustness with respect to longitudinal velocity variation

As explained previously, the longitudinal velocity v is considered constant, but in the real life, it is not the case. In this section, the proposed observer is tested in the case where v is time-varying. The observer is constructed by choosing a nominal value $v = 30m/s$ but in the nonlinear system, it is assumed to be time-varying as $v \in [3 \quad 57]$ (m/s), the longitudinal velocity is $v = 30 + 27sin(t)$. The states of the nonlinear system with time-varying longitudinal speed and their estimations are depicted in the figure 4.

C. Robustness with respect to measurement noise

In this last simulation, the proposed observer is applied on the original nonlinear system with time-varying longitudinal velocity v and measurement noise. The obtained results are shown in the figure 5. The estimated lateral velocity v_y is obtained from the estimated sideslip angle β and the measured longitudinal velocity v (figure 6).

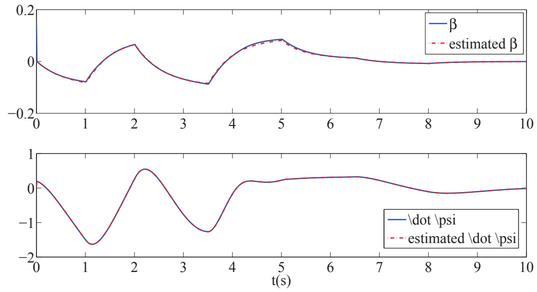

Fig. 4. State estimation in time-varying longitudinal velocity

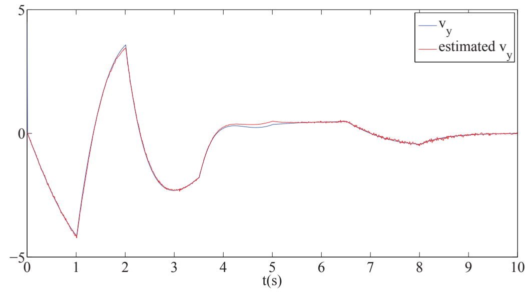

Fig. 6. Estimated lateral velocity v_y

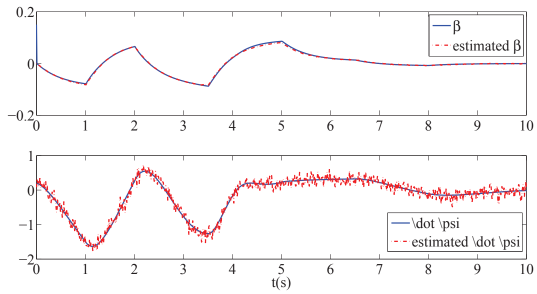

Fig. 5. States of the system and their estimations in measurement noisy scenario

IV. CONCLUSIONS AND FUTURE WORKS

In this paper, an observer for estimating the lateral velocity of a vehicle is proposed. First, the nonlinear model of the vehicle is transformed into a Takagi-Sugeno model with the well-known sector nonlinearity transformation. The obtained model takes into account several nonlinear behaviors of the original model. Secondly, a T-S observer is synthesized. Since the weighting functions of the T-S model are partially unmeasured, the observer weighting functions depend inevitably on the estimated states. The ISS stability and the Lyapunov theory are then used to prove the convergence of the proposed observer. The obtained conditions are expressed as an optimization problem subject to LMI constraints. Finally, the proposed observer is applied on the original nonlinear system to estimate the state vector and the lateral velocity. Some robustness aspects such as time-varying longitudinal velocity and noisy measurements are discussed. We can conclude that the observer is robust to the measurement noise and it can work in large range longitudinal time-varying velocities. As future work, an application to observer-based controller design for driving assistance will be considered.

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