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Instrumentation for electrical vehicle model on road slope using structural analysis

Zine-eddine Megatta, Blaise Conrard and Mireille Bayart

Abstract— **This paper focuses on the design of instrumentation of the control system for electrical vehicle model. The method is based on a structural model that describes qualitatively the different relations of the physical variables. The motivation for the choice of structural analysis approach is that this analysis uses a poor knowledge of the system; it uses only the relation between constraints and variables. By analyzing this model, we obtain the different ways to control the unknown variables in function of the sensors measurements and thanks to the available actuators. The main contributions of this paper combines the merits of: i) the modified structural analysis model in order to take into account different operating cases slope of the road constant and variable and their specific features; ii) to study reject disturbance using graph techniques; iii) to obtain the optimal instrumentation for the system of electrical vehicle which insure controllability in despite of the disturbance.**

I. INTRODUCTION

The cruise control system of a vehicle is one of the most common control systems encountered in everyday life. The control system attempts to keep the speed of the vehicle constant in spite of disturbances caused by changes in the slope of a road and variations in the wind and road surface. The controller compensates the disturbances by measuring the speed of the vehicle and adjusting the throttle appropriately. Modern light duty vehicles employ several electronic control systems which utilize vehicle and environment state information to increase efficiency, safety and comfort.

Many different methods for estimating the road slope can be found in the literature. One approach is to use a sensor directly related to the slope. This is used for instance in [1] where the grade is determined using a global positioning system(GPS) receiver which gives both a vertical and horizontal velocity. The road slope can then be found through the ratio of the velocities. Such a method relies heavily on the existence of a high quality GPS signal, something which is not always available. The idea of using vehicle sensor information to find the road slope has been explored in [2] where a Kalman filter is used to process a measured or estimated propulsion force or estimated retardation force and a measured speed. These methods is required an accurate mathematical model of the vehicle.

Associated with an increasing demand for high performance control as well as for more quality and control instrumentation of dynamics systems, and a natural trend toward system automation, structural modelling is becoming a strategic necessity as a result of increasing economic and environmental demands. Over the past few years, there has been significant research effort in the analysis of dynamics systems, namely structural analysis [3]-[6]. References [3] and [4] present how the structural analysis allows to determine all the possible ways to estimate an unknown quantity from the known ones such as measurements or control signals. References [5] and [6] give an extension of the structural model in order to take into account different behavioural modes and their specific features for a tank process. Structural analysis is an important tool, which is of interest in the early stage of the control and supervision system design, when detailed models are not available [3].

The objective of designing a good control system is to determine the best control instrumentation scheme, that is to say, a set of sensors and actuators that allows the system to perform its mission despite the disturbance. Disturbance rejection approach is studied by many researchers [8],[10]. Reference [8] has presented illustration of the disturbance decoupling problem by measurment feedback and solve this problem using geometric and graph techniques. In [9] the disturbance rejection problem for structured systems has presented. A graph is associated with such a structured system. Reference [10] shows new solutions to the disturbance decoupling problems by state feedback and by output measurment feedback and the solvability conditions are easily checked on the associated graphe of the system; In these approaches, the state space model of the system must be available.

Based on the aforementioned studies, the contribution of this paper is to develop a method which quickly allows to have an idea of the behavior for systems of complicated control, and so to work in model based on little information. Also, we develop a new method based on structural graph. In this paper, we extend the structural model [11]-[13] in order to take into account different behavioral in the different cases and their specific features for electrical vehicle dynamic model, and the using graphic techniques, we interest on reject disturbance problem, also determining the controllability of the electrical vehicle according to the disturbance sources.

This paper is organized as follows: Section 2 provides structural model and analysis. Section 3 describes modeling of the vehicle in both cases of road slope. section 4 explains the structural analysis used to determine the control signals U_t. Finally, concluding remarks are made in section 5 followed by the list of references.

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II. STRUCTURAL MODEL AND ANALYSIS

A controlled system can be modeled by a set of physical variables linked by a set of relations. The structural analysis is used to qualitatively represent the interaction between these variables without explicitly knowing the constraints. Despite the few information contained in the model, the structural analysis allows some properties to be determined such as observability, controllability and monitorability properties [3],[7]. In our method, structural modeling is used to determine all the ways to obtain a physical quantity or to act on it with a minimal disturbance sources.

A. General principle

The structural analysis provides a representation of the links between the physical variables. These links correspond to the constraints imposed by the physical relations between the variables. A structural model can be represented by a bipartite graph [12]. The corresponding incidence matrix is defined by the following relation :

$$S : \{F\} x \{Z\} \to \{0, 1, -1\}$$

- $S(f_i, z_i) = 1$ if and only if the variable z_i appears in the constraint f_i and if its value can be deduced from the others variables appearing in f_i.
- $S(fi, zi) = -1$ if and only if the variable z_i appears in the constraint f_i but its value cannot be deduced from the others variables appearing in f_i.
- $S(fi, zi) = 0$ if the variablez_i does not appears in the constraint f_i.

This modeling can be represented by an incidence matrix where each row is associated with a constraint and each column with a variable. For example, the matrix on table 1 corresponds to a tank equipped with a drain valve. Three variables are required to model this process: the restriction coefficient of the valve (C_v), the output flow(Q) and the tank level(H). With the assumption of a laminar flow, these three variables are linked by the two following constraints:

$$Q(t) = K_1 C_v \sqrt{H(t)} \tag{1}$$

$$\frac{dH(t)}{d(t)} = K_2 Q(t) \tag{2}$$

where K_1 and K_2 are two constant parameters. The second constraint is not an invertible function if the initial state is unknown. Consequently, the value of H cannot be deduced knowing only the value of Q, while the contrary is true. Finally, a brief note on how time differentiation in dynamic systems is handled here. There are at least three different ways to represent time differentiated variables:

- To extend the model with equations describing how, for example, $x(t)$ is related to $\dot{x}(t)$through the differentiation operator. which is added for each variable that appear differentiated in the original model[16].
- To consider x and $\dot{x}$ are separate variables and perform structural differentiation of the model [15].
- To consider x and dx/dt are the variables and to treat dynamic equations in the same way as static equations.

All three are possible choices, but for the structural analysis used here we select the second approach.

The studied model is represented by an incidence matrix, in which each row corresponds to an equation and each column to a variable. An x in position (i, j) indicates that variable j appears in equation i

TABLE I

INCIDENCE MATRIX

constraint	Aperture of the valve C_V	Output flow Q	tank level H
1	1	1	1
2		1	-1

B. Structural Modeling

Structural analysis is concerned with properties of the system structure. Structural information here means which variables appear in which equations and constraints [14]. Now it will be briefly outlined how an analysis of the structural model can provide a means to represent the different constraints that link physical quantities to know if a physical quantity may be evaluated according to the other variables. Another interest of this modeling is that it does not need the exact establishment of physical equations. Consequently, this model is easily constructed and the design phase is accelerated faster. The types of variables in a structural model can in a diagnosis context be divided into [13]:

- known variables, from the sensors.
- physical variables are supposed unknown.
- modes of operation of certain components.

The types of relation of constraints in a structural model can in a diagnosis context be divided into:

- those related to physical constraints verified whatever the operating mode.
- those linking physical quantities and measurement capabilities.
- those specific to particular modes of operation.

III. STRUCTURAL MOLDING OF ELECTRICAL VEHICLE DYNAMIC

A. Description of the process

The considered process is an electrical vehicle, as shown in figure 1.

The instruments that are available for a vehicle are:

- Electrical motor, for the vehicle propulsion.
- Wheel encoder, for speed measurement.
- GPS sensor, to measure the position.
- Inclinometer, to measure the angle of slope, elevation or depression of an object with respect to gravity, it is also measure both inclines (positive slopes, as seen by an observer looking upwards) and declines (negative slopes, as seen by an observer looking down ward).
- Chronometer, to measure time precisely, with high reliability.

Consider the following variables: a is the acceleration of the vehicle, v is the speed of the vehicle, x is the position of

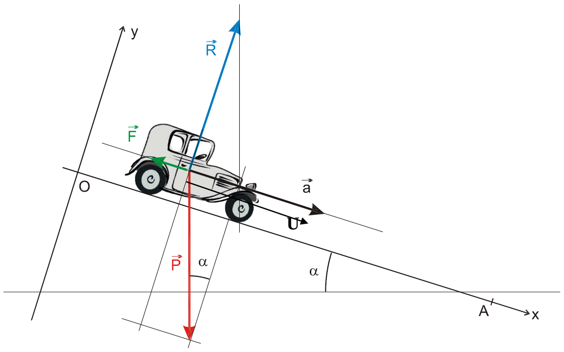

Fig. 1. Dynamic model of vehicle

the vehicle,U is the control signal of the motor, and α is the angle of slope, I is the measure given by the inclinometer, Cv is the measure given by the encoder placed on the wheel, gps is the measure given by the GPS sensor, CH the measure given by the chronometer.

B. Structural analysis of the vehicle

The slope can be a natural constraint for the driving of the vehicle; it depends on road profile, in this section. We will explain both the slope of the road (constant and variable) and the structural analysis outlined in section 2 will be applied on the electrical vehicle model.

C. The road slope constant

Applying the principle basis of mechanics of solids, for the balance of forces acting on the vehicle exterior and according to figure 1, a vehicle mass is $1t$, it is initially at rest, on top of a road of slope 5% ($5m$ of descent for $100m$ of path real), long of $200m$, the friction of the road $\overrightarrow{F}$ is $100N$, the air resistance is $50N$.
To model the electrical vehicle dynamic in the case where the slope is constant, we consider the following points:

- system study: vehicle
- referential: road (earth)
- reference: shown in figure1
- external forces: $\overrightarrow{P}$, $\overrightarrow{R}$, $\overrightarrow{F_r}$, $\overrightarrow{U}$ are the weight, reaction of the road, the total force of friction and the force induced by the motor respectively.
- The fundamental principle is given by:

$$\sum \overrightarrow{F} = m \cdot \overrightarrow{a}$$

- The projection on the axes is given by:

$$\sum \overrightarrow{F_x} = m\overrightarrow{a_x} \Leftrightarrow Psin\alpha - F_r + U = ma_x$$

$$\sum \overrightarrow{F_y} = m\overrightarrow{a_y} \Leftrightarrow R - Pcos\alpha = ma_y$$

The movement is rectilinear along the axis Ox, so $ay = 0$. The reaction of the road given as follow:

$$R = mgcos(\alpha) \tag{3}$$

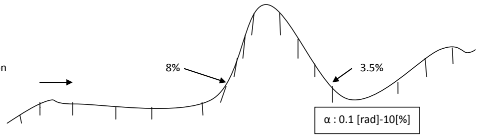

Fig. 2. The road undulation

The acceleration is the parallel to OX and oriented in the direction Ox (otherwise the car will not start moving), The a_x is given by:

$$\begin{aligned} a_x &= \frac{Psin(\alpha) - F_r + U}{m} \\ &= gsin(\alpha) - \frac{F_r}{m} + \frac{U}{m} \end{aligned} \tag{4}$$

Where $a_y = 0$ and $a_x \succ 0$, $a = a_x$; the norm of $\overrightarrow{a}$ is coordinated along the axis Ox. The acceleration $\overrightarrow{a}$ is constant over time, so the movement is varied and uniformly rectilinear(MRUV).
Suppose that at $t = 0$, the vehicle is at the origin of the axis Ox, and starts to go down the road, on arrival $x = 200m$. We can determine the length of descent, the relation between x and t for MRUV is defined as follows(only valid for acceleration is constant):

$$x(t) = \frac{1}{2}a_x t^2 + v_{0x}t + x_0 \tag{5}$$

With the initial conditions at $t = 0$, the vehicle starts to go down the road, with initial speed zero, the time of descent $t_{descent}$ as follow:

$$v_{0x}(t = 0) = 0 : x(t) = \frac{1}{2}a_x t^2 \Leftrightarrow t_{descent} = \sqrt{\frac{2x}{a_x}} \tag{6}$$

If the slope is constant, the profile of the road is right. The incidence matrix which represents the relations between the variables for vehicle in road with slope constant is given in Table 2

D. The road slope not constant

Concerning the vehicle in the road undulation as shown in figure 2. The following constraints are added which express the movement of vehicle in the road with slope not constant; the slope in road undulation is function evolves in time. It will be changed at each point as follows:

$$\alpha_t = \alpha_{t-\delta t} + \Delta\alpha_t \tag{7}$$

where: $\Delta\alpha_t$ is the variation of the slope. In case the road profile is a curve at each point of the road, the slope is different. In general, the movement of the center of inertia of a moving object is stored in a sampling that is to say that we have discrete points corresponding to positions at instants separated by a period of time shorter. Therefore, the change in velocity vector gives the acceleration vector, by approximation, the tangential component is given by:

$$\begin{aligned} a_x &= \frac{v_x(t+\delta t) - v_x(t)}{\delta t} \\ &= \frac{v(t+\delta t) - v(t)}{\delta t} \end{aligned} \tag{8}$$

TABLE II

INCIDENCE MATRIX FOR VEHICLE IN ROAD WITH SLOPE CONSTANT

	disturbance	physical quantities						actuator	sensors			
	α	a_x	R	v_t	x	$t_{descent}$	a_y	U_t	C_v^t	gps	CH	I_t
f_1	1	1						1				
f_2				1					1			
f_3					1					1		
f_4	1											1
f_5	1		1				1					
f_6		1			1	1						
f_7				1	-1							
f_8		1		-1								
f_9						1					1	

TABLE III

INCIDENCE MATRIX FOR VEHICLE IN ROAD WITH SLOPE NOT CONSTANT

	disturbance		physical quantities					actuator	sensors		
	α_t	$\alpha_{t-\delta t}$	a_t	$v_{t+\delta t}$	v_t	δt	$\Delta\alpha_t$	U_t	C_v^t	CH	I_t
f_1			1	1	1	1					
f_2	1	1					1				
f_3	1		1					1			
f_4					1				1		
f_5	1										1
f_6						1				1	

Indeed, in the Frenet curves, we have $v_x(t) = v(t)$, and we made approximation $v_x(t+\delta t) = v(t+\delta t)$. where δt is the sampling period.

Therefore, any change in slope will result in a change in acceleration and speed. The incidence matrix which represents the relations between the variables for vehicle in road with slope not constant is given in Table 3.

E. Structural analysis

A structural analysis is used to describe the system and the global incidence matrix for the whole system is given in Table 4.

According to this matrix the different paths to access the variables to be controlled or estimated can be found. More especially, the speed of the vehicle can be control using the relation between the measures of α_t, U_t, δ_t and v_t valid in case the slope is not constant. Consequently, we obtain:

$$(\alpha_t \wedge U_t \wedge v_t \wedge \delta_t) \Rightarrow v_{t+\delta t} \tag{9}$$

In the same way, the acceleration of the vehicle can be estimated by three ways: thanks to a relation of U_t and α_t, or thanks to a relation between the measured position(x) and time of descent($t_{descent}$) valid in case the slope is constant and, also relation between the speed of the vehicle v_t and short time δ_t.

$$(U_t \wedge \alpha_t) \vee (x_t \wedge t_{descent}) \vee (v_{t+\delta t} \wedge v_t \wedge \delta_t) \Rightarrow a_t \tag{10}$$

However, these ways to estimate a quantity are only valid when all constraints are available and satisfied the operating case associated to some of them. Consequently, the number of ways is reduced according to the state of cases here, these cases are the road slope (constant or variable).

IV. THE CONTROL SIGNAL DETERMINED BY STRUCTURAL ANALYSIS

This section presents, on one hand, the way of the structural analysis can be used to determined the control of physical quantities of a system, and, on the other hand, a way to determine whether a disturbance can be rejected or not.

A. A first case with a single disturbance

The main objective is to determine the control signal u_t , that is sent to the propulsion, in order to obtain $V_{t+\delta t}$ equal or near to a desired speed given by the operator. Here, we consider that the sampling interval δt is known and constant. A structural model is used to describe the system and its corresponding incidence matrix is given in table 4. With this matrix, the different paths that allow the value of u_t to be evaluated according to desired the speed $V_{t+\delta t}$ can be found.

The controllability is an important property of a control system, and the controllability property plays a crucial role in many control problems, such as optimal control. A system is controllable if for every state at the initial instant t, and every state at the final instant $t+\delta t$, there exists a control U_t, applied a finite time interval $[t, t+\delta t]$, which allows to rejoin the final state starting from the initial state.

More especially, in case of the vehicle where the only considered perturbation is the road slope, the control signal U_t can be easily evaluated if an inclinometer sensor is implemented on the vehicle. The relations to evaluate can be represented by a tree as shown in figure 3. In this tree, each node represents an algebraic relation between the variables placed at the end of the arcs (or arrows). This tree is built from the incidence matrix; according to the constraints defined in it, a path that leads to the variable to evaluate, can be deduced. Among all possible paths, the one that

TABLE IV

THE GLOBAL INCIDENCE MATRIX FOR THE WHOLE SYSTEM

	α_t	$\alpha_{t-\delta t}$	R	a_t	$v_{t+\delta t}$	v_t	x_t	δt	$t_{descent}$	a_y	$\Delta\alpha_t$	U_t	C_v^t	gps	CH	I_t	*case*
f_1				1	1	1		1									variable slope
f_2	1			1								1					
f_3						1							1				
f_4							1							1			
f_5	1															1	
f_6	1		1							1							constant slope
f_7				1			1		1								constant slope
f_8	1	1									1						variable slope
f_9						1	-1										constant slope
f_{10}									1						1		constant slope
f_{11}								1							1		variable slope

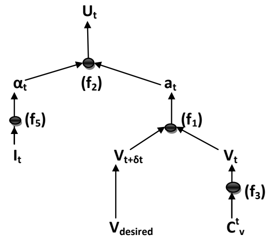

Fig. 3. The control u_t with a single disturbance

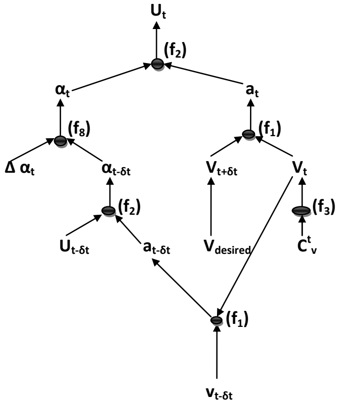

Fig. 4. The control U_t with a single disturbance without inclinometer

is retained, is the one that uses known variables (measure, desired physical quantities, and control signal) and a minimal number of unknown disturbances. Indeed, the main aim of this analysis is to find a control that rejected a maximum number of disturbances.

Consequently, with the inclinometer sensor, the speed of the vehicle is controllable, since the control signal U_t can be evaluated without disturbance as shown in figure 3. Of course, this is valid in the case where the disturbance is the slope.

In a second case, without the implementation of the inclinometer sensor, the same study can be carried out. Here in this subpart, we consider $\Delta\alpha t$ is the variation of the slope over a undulation road which acts on the control. Where $\Delta\alpha t = \alpha_t - \alpha_{t-\delta t}$ in this case, the new graph to evaluate the control signal U_t is given in figure 4.

According to the found graph where the lack of inclinometer is compensate by an estimation of the slop, the system is controllable. Indeed, the change of the slope $\Delta\alpha t$ in the road undulation is unknown and cannot be rejected.

B. Vehicle with several disturbances

In this section, we consider the same system of electrical vehicle model but completed by several previously inconsiderate disturbances that are:

- the disturbance of the road slope $(\alpha t, \Delta\alpha t)$ are previously.
- the disturbance of the mass; we assume that the load of the vehicle can be changed at each trip. In this case, we have:

$$m_t = M + \Delta m_t \tag{11}$$

  where M is a fixed parameter used in the model and corresponding to the assumed mass of the vehicle. Δm_t is the variation of the mass due to the various transported loads, while m_t is a real mass quantity of the vehicle.
- the disturbance concerning the measurement errors of sensors; for any measure a physical quantity, a sensor can never get the exact value due to the imperfections of the measuring instrument. On the vehicle, two disturbances on the measurement errors are considered: The first one concerns the speed sensor with the following relation: $c_v^t = v_t + \varepsilon_v^t$. Where: c_v^t is the measured speed provided by speed sensor, ε_v^t is the uncertainty of the measurement error while v_t is the real speed. The second considered disturbance concerning the inclinometer sensor with the constraint: $I_t = \alpha_t + \varepsilon_I^t$. Where: I_t is the measured slope provided by inclinometer, ε_I^t is the uncertainty of the measurement error and α_t is the real slope.

Therefore, the graph for evaluating the control signal U_t with several all these disturbances is deduced and is given in figure5. This graph shows the most influential disturbance that is the slope which can be rejected, but the other can not. Thus, the evaluation of the control signal U_t is affected by 5 errors sources (ε_I^t, Δm_t, ε_v^t,$\varepsilon_v^{t-\delta t}$, $\Delta\alpha_t$).

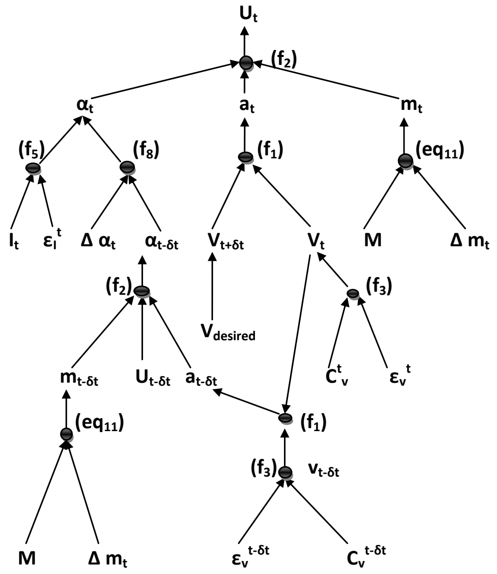

Fig. 5. The control u_t with several disturbance

V. CONCLUSION

In this paper, the speed control of the vehicle is presented in various cases according to the number of considered disturbances: a single one (the slope of a road) or several disturbances (the slope and the load transported and error of measurement of the sensors).

The used structural analysis shows that it is a fast tool for estimate the quality, according to the number of rejected disturbance and according to number of implemented and used sensors. It can help the designer to select the sensor has to be implemented according the ratio between instrumentation cost and quality of control. Moreover, the used method is a relatively easy to use and requires a reduced amount of data, that is to say, an incidence matrix built by a structural modeling.

In our future work, we foresee to take into account various aspects. The first one is to quantify qualitatively the effect of each disturbance in order to select the best graph of evaluation that rejects the most influent disturbances. Another one concerns the capacity of fault tolerance, with the aim to determine instrumentation architecture that can perform their mission despite one or several failures.

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