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Fault Tolerant Control of Wind Energy System Subject To Actuator Faults and Time Varying Parameters

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Abstract--**In this paper we propose the Robust Fault Tolerant Control (RFTC) for Wind Energy System (WES) subject to actuator faults and time-varying bounded parameter uncertainties. The algorithm utilizes fuzzy systems based on "Takagi-Sugeno" (TS) fuzzy models to represent nonlinear systems. Sufficient conditions are derived for robust stabilization in the sense of Lyapunov asymptotic stability and are formulated in the format of Linear Matrix Inequalities (LMIs). The proposed algorithm is able to maintain the system stable during the actuator faults and parameter uncertainties. The design technique is applied to a dynamic model of wind energy system to illustrate the effectiveness of the proposed RFTC**

Index Terms—**LMIs, Parameters Uncertainties, Robust control, actuator Faults, TS Fuzzy Model, WES.**

I. Introduction

FAULTS in many control systems may occur at any uncertain time and in any locations such as sensors, actuators, or the plant. They will lead to performance deterioration or even instability of the systems [1]. Since actuators affect the system behaviors directly, faults in the actuators can cause more serious damaging results. So developing the reliability of the control systems with actuator faults is very significant in the control engineering. In the past two decades, Fault Tolerant Control (FTC) problems have been extensively studied by many researchers for linear systems, and several approaches have been developed [2], [3] and other methods have been extended to the nonlinear systems with parameter uncertainties, sensor and/or actuator faults [4]-[6].

Recently, we have witnessed rapidly growing interest in fuzzy logic control of uncertain nonlinear systems. Though there have been many successful applications, the heuristics based approach to fuzzy control lacks the formal and systematic design methodology which guarantees the basic requirement such as, stability and acceptable performance [7]. To solve these problems, the idea that a linear system is adopted as the consequent part of a fuzzy rule has evolved into the innovative Takagi-Sugeno (TS) fuzzy model, which becomes quite popular in recent years. Once a uncertain nonlinear system is represented by a TS fuzzy model, a fuzzy controller can be designed by using modern control theory on linear systems. As a common belief, the control technique based on the TS fuzzy model is conceptually simple and effective for the control of complex uncertain systems with nonlinearity [8], [9].

This paper considers the robust fault tolerant fuzzy control problem for uncertain nonlinear systems with actuator faults. The TS fuzzy model is employed to approximate a nonlinear uncertain system. Based on actuator fault model, an observer-based fuzzy controller is developed. Sufficient conditions are derived for robust stabilization in the sense of Lyapunov asymptotic stability in the presence of some parameters are uncertain. Wind Energy System (WES) example is given to illustrate the application of the proposed design method

This paper is organized as follows: Section II describes TS fuzzy plant model. Section III presents wind energy conversion system model and the TS fuzzy model for the WES. The proposed Robust Fault Tolerant Controller (RFTC) and the stability condition are presented in section IV. Section V presents the simulations results and discussion. Finally section VI presents the conclusions.

II. TS Fuzzy model with Parameter Uncertainties and Actuator Faults

Consider an uncertain nonlinear system represented by a TS fuzzy model described by plant rule i ($i=1,2,\ldots p$) [10].

IF $\xi_1(t)$ is N_{1i} AND … AND $\xi_g(t)$ is N_{gi}

Then $\dot{x}(t) = (A_i + \Delta A_i)x(t) + B_i u(t)$,

$$y(t) = C_i x(t) \tag{1}$$

where $x(t)\epsilon\kappa^{nx1}$ is the state vector, $u(t)\epsilon\kappa^{mx1}$control vector, $y(t)\epsilon\kappa^{sx1}$ measurable output vector, A_i, B_i, C_i are known constant matrices with appropriate dimensions, and $\Delta A_i\epsilon\kappa^{nxn}$ is the parametric uncertainties of A_i, $\xi(t)=[\xi_1(t),\ldots, \xi_g(t)]$ are the measurable premise variables, $N_{1i,}\ldots N_{gi}$ are the fuzzy sets and p is the number of rules. The plant dynamics is then described by,

$$\dot{x}(t) = \sum_{i=1}^{p} \mu_i(\xi(t))[(A_i + \Delta A_i)x(t) + B_i u(t)] \quad ,$$

$$y(t) = \sum_{i=1}^{p} \mu_i(\xi(t)) C_i x(t) \tag{2}$$

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where

$$h_i(\xi(t)) = \prod_{j=1}^{g} N_{ij}(\xi(t)), \quad \mu_i(\xi(t)) = \frac{h_i(\xi(t))}{\sum_{i=1}^{p} h_i(\xi(t))} \tag{3}$$

In this work, we consider actuator faults occur. Then the TS fuzzy system (2) becomes:

$$\dot{x}_f(t) = \sum_{i=1}^{p} \mu_i(\xi(t))[(A_i + \Delta A_i)\, x_f(t) + B_i u(t) + E_i f(t)] \quad ,$$

$$y_f(t) = \sum_{i=1}^{p} \mu_i(\xi(t)) C_i x_f(t) \tag{4}$$

Where $f(t)$ is the actuator fault vectors which is assumed to be bounded, E_i is constant matrix with appropriate dimensions assumed full column rank, $\xi_i(t)$ assumed not effect by the actuator faults [11].

III. System Model and TS Fuzzy Description

The state equation and output equation of the system is given by[12], [13]:

$$\dot{x}(t) = A(x)x(t) + B(x)u(t) \qquad y = Cx(t) \tag{5}$$

Where $x(t) = \begin{bmatrix} V_b \\ \omega_s \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, u(t) = \begin{bmatrix} E_{fd} \\ I_{ref} \end{bmatrix} = \begin{bmatrix} u_1(t) \\ u_1(t) \end{bmatrix}$

$$A(x) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{L_f}{\tau L_{md}} q_1(x) & \frac{L_f}{\tau L_{md}} q_1(x)(L_d i_{sd} - i_{sq} r_a q_1(x)) \\ \frac{P_{ind} - P_{load}}{J_s} q_2(x) & -\frac{D_s}{J_s} \end{bmatrix}$$

$$B(x) = \begin{bmatrix} 1 & -\frac{V_c}{J_s} q_1(x) \\ 0 & -\frac{V_c}{J_s} q_1(x) \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$q_1(x)=1/\omega_s$ and $q_2(x)=1/V_b\omega_s$, ω_s is angular speed, J_s, D_s are the inertia and frictional damping, i_{sd}, i_{sq} are the direct and quadrature current component, L_d, L_f are the stator and rotor inductance, L_{md} is the field mutual inductance, τ is the transient open circuit time constant and r_a is the rotor resistance of SG, P_{ind} is the power of the induction generator, P_{load} is the power of the load. The control inputs are the excitation field voltage (E_{fd}) of the synchronous machine and the direct-current set point of the converter (I_{ref}) to the diesel engine. The measurements are the voltage amplitude (y_1) of the common ac bus-bar and the angular speed (y_2) of the synchronous generator.

To design the fuzzy controller, we first represent the wind-turbine system (5) with a TS fuzzy model. The sector nonlinearity as follow,

$$q_1(t)=q_{1max}N_1(q_1(t))+q_{1min}N_2(q_1(t))$$

$$q_2(t)=q_{2max}M_1(q_2(t))+q_{2min}M_2(q_2(t))$$

where $N_1(q_1(t))+N_2(q_1(t))=1$, $M_1(q_2(t))+M_2(q_2(t))=1$, N_1^i and M_1^i are a fuzzy term of rule i. The membership function for $q_1(t)$ and $q_2(t)$ are shown in Fig.1. Then, the nonlinear WES system (5) is represented by the following fuzzy model.

Rule i: IF $q_1(t)$ is N_{1i} and $q_2(t)$ is M_{1i}

Then $\dot{x}(t) = (A_i + \Delta A_i)x(t) + B_i u(t) + E_i f(t)$,

$$y(t) = C_i\, x(t) \qquad \text{For} \quad i=1,2,3,4 \tag{6}$$

From (4), the inferred system is given by,

$$\dot{x}_f(t) = \sum_{i=1}^{4} \mu_i(q(t))[(A_i + \Delta A_i)\, x_f(t) + B_i u(t) + E_i f(t)]$$

$$y_f(t) = \sum_{i=1}^{p} \mu_i(q(t)) C_i\, x_f(t) \tag{7}$$

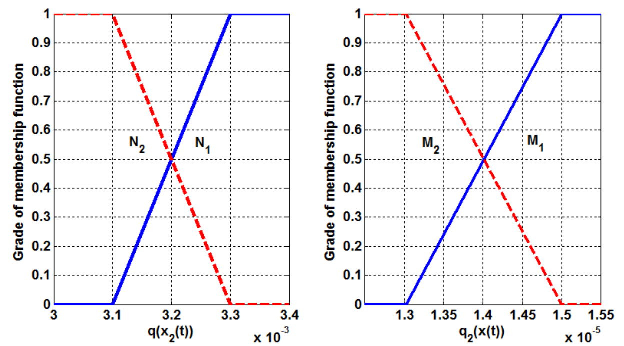

Fig.1. Membership functions for $q_1(x(t))$ and $q_2(x(t))$

If ΔL_d, ΔL_f , Δr_a and ΔD_s are denoted as the uncertainties introduced by system parameters L_f, L_d, r_a and D_s, respectively, then from (7), the uncertain matrices can be expressed as

$$A_1 + \Delta A_1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{L_f + \Delta L_f}{\tau L_{md}} q_{1\max} & \frac{L_f + \Delta L_f}{\tau L_{md}} q_{1\max}((L_d + \Delta L_d) i_{sd} - (r_a + \Delta r_a) i_{sq} q_{1\max}) \\ \frac{P_{ind} - P_{load}}{J_s} q_{2\min} & -\frac{D_s + \Delta D_s}{J_s} \end{bmatrix}$$

$$A_2 + \Delta A_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{L_f + \Delta L_f}{\tau L_{md}} q_{1\max} & \frac{L_f + \Delta L_f}{\tau L_{md}} q_{1\max}((L_d + \Delta L_d) i_{sd} - (r_a + \Delta r_a) i_{sq} q_{1\max}) \\ \frac{P_{ind} - P_{load}}{J_s} q_{2\max} & -\frac{D_s + \Delta D_s}{J_s} \end{bmatrix}$$

$$A_3 + \Delta A_3 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{L_f + \Delta L_f}{\tau L_{md}} q_{1\min} & \frac{L_f + \Delta L_f}{\tau L_{md}} q_{1\min}((L_d + \Delta L_d) i_{sd} - (r_a + \Delta r_a) i_{sq} q_{1\min}) \\ \frac{P_{ind} - P_{load}}{J_s} q_{2\min} & -\frac{D_s + \Delta D_s}{J_s} \end{bmatrix}$$

$$A_4 + \Delta A_4 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{L_f + \Delta L_f}{\tau L_{md}} q_{1\min} & \frac{L_f + \Delta L_f}{\tau L_{md}} q_{1\min}((L_d + \Delta L_d) i_{sd} - (r_a + \Delta r_a) i_{sq} q_{1\min}) \\ \frac{P_{ind} - P_{load}}{J_s} q_{2\max} & -\frac{D_s + \Delta D_s}{J_s} \end{bmatrix}$$

$$B_1 = B_2 = \begin{bmatrix} 1 & -\frac{V_c}{J_s} q_{1\max} \\ 0 & -\frac{V_c}{J_s} q_{1\max} \end{bmatrix}, B_3 = B_4 = \begin{bmatrix} 1 & -\frac{V_c}{J_s} q_{1\min} \\ 0 & -\frac{V_c}{J_s} q_{1\min} \end{bmatrix}$$

We assume $E_i = \begin{bmatrix} 10 & 1 \\ 0.1 & 0.01 \end{bmatrix}$

The choice of E depends on the two inputs of the system.

IV. Proposed Fuzzy Fault Tolerant Algorithm and Stability Conditions

In this section, we design RFTC based on Linear Matrix Inequalities (LMIs) for nonlinear systems with actuator faults and time varying parameter uncertainties described by the TS fuzzy model, such that the control law must be able to maintain the stability of the system during the faults and parameter uncertainties.

A. TS Fuzzy Observer

To detect and estimate the faults, assume that the fuzzy system (4) is locally observable [14]. The TS Fuzzy observer with actuator faults and parameter uncertainties is formulated as follows [11]:

IF $\xi_1(t)$ is N_{1i} AND … AND $\xi_g(t)$ is N_{gi}

Then $\dot{\hat{x}}_f(t) = A_i\hat{x}_f(t) + B_i u(t) + E_i\hat{f}(t) + K_i\tilde{y}_f(t)$,

$$\dot{\hat{f}}(t) = L_i\tilde{y}_f$$

$$\hat{y}_f(t) = C_i\hat{x}_f(t) \qquad i=1, 2,\ldots,p \tag{8}$$

where $\hat{x}_f$ is the estimated state, $\hat{y}_f(t)$ is the fuzzy observer output, K_i is the proportional gain and L_i is the integral gain for the i-th observer rule, $\tilde{y}_f = (y_f - \hat{y}_f)$. The structure of the proportional observer is chosen as follows

$$\dot{\hat{x}}_f(t) = \sum_{i=1}^{p}\mu_i(\xi(t))[A_i\hat{x}_f(t) + B_i u(t) + E_i\hat{f}(t) + K_i\tilde{y}_f(t)]$$

$$\dot{\hat{f}}(t) = \sum_{i=1}^{p}\mu_i(\xi(t))L_i\tilde{y}_f$$

$$\hat{y}_f(t) = \sum_{i=1}^{p}\mu_i(\xi(t))C_i\hat{x}_f(t) \tag{9}$$

B. Proposed RFTC Controller

To design the fault tolerant control, assume that the fuzzy system (4) is locally controllable [14]. Using Parallel Distributed Compensation (PDC) [15] the i-th rule of the fuzzy controller is given by

IF $\xi_1(t)$ is N_{1i} AND … AND $\xi_g(t)$ is N_{gi}

Then $Bu(t)=B_i u_i(t)$ (10)

Where $u_i(t)\epsilon\kappa^{mx1}$ is the output of the i-th rule controller. The global output of the fuzzy controller is given by

$$Bu(t) = \sum_{i=1}^{p}\mu_i(\xi(t))B_i u_i(t) \tag{11}$$

From (3), (4) and (11), writing $\mu_i(\xi(t))$ as μ_i, we have,

$$\dot{x}_f(t) = \sum_{i=1}^{p}\mu_i[(A_i + \Delta A_i)\, x_f(t) + Bu(t) + Ef(t)]$$

$$y_f(t) = \sum_{i=1}^{p}\mu_i C_i\, x_f(t) \tag{12}$$

We use the property

$$\sum_{i=1}^{p}\mu_i = \sum_{i=1}^{p}\sum_{j=1}^{p}\mu_i\mu_j = 1,\ B = \sum_{i=1}^{p}\mu_i B_i,\ E = \sum_{i=1}^{p}\mu_i E_i \tag{13}$$

Note that B and E are known.

C. Reference Model

Consider the stable reference model without faults described as follows:

$$\dot{\bar{x}}(t) = A_r\bar{x}(t) + B_r r(t)$$

$$\bar{y}(t) = C_r\bar{x}(t) \tag{14}$$

where $\bar{x}(t)\in\kappa^{nx1}$ is the system state, $\bar{y}(t)\in\kappa^{sx1}$ is measured output, $r(t)\in\kappa^{mx1}$ is the system input, A_r, B_r and C_r are known constant matrices with appropriate dimensions.

D. Stability and Robustness Analyses for The Proposed Algorithm

A proof of the stability and robustness conditions for the plant dynamics described by (12) is shown in the appendix. The main result is summarized in the following lemma and theorem.

Lemma: The closed loop system of (12) with bounded plant actuator faults and parameter uncertainties is guaranteed to be asymptotically stable, and its states will follow those of a stable model of (14), if the following condition satisfy; the robust fuzzy fault tolerant control (11) are designed as

$$B_i u_i(t) = (B_i^T B_i)^{-1} B_i^T B_i Z_{ui} = B_i^{+} B_i Z_{ui} \tag{15}$$

Where B_i^+ is the pseudo inverse matrix of B_i. With $n \geq m$, the inverse of $(B_i^T B_i)$ exists, when B_i is full rank matrix, the rank of B_i and B_i^+ is m, Z_{ui} is given by

$$Z_{ui}(t) = \Big\{[He_1(t) + A_r\,\bar{x}(t) + B_r r(t) - A_i\, x(t) - S\hat{f}(t) - \frac{e_1(t)\|e_1(t)\|\|P_1\|\|\Delta A_i\|_{\max}\|x(t)\|}{e_1(t)^T P_1 e_1(t)} - \frac{e_1(t)\|x(t)\|\|D\|_{\max}\|x(t)\|}{e_1(t)^T P_1 e_1(t)} - \frac{e_1(t)\|e_1(t)\|\|P_1\|\|E\|_{\max}\|\tilde{f}(t)\|}{e_1(t)^T P_1 e_1(t)}]\Big\} \tag{16}$$

$\|\cdot\|$ denotes the l_2 norm for vectors and l_2 induced norm for matrices, $\|\Delta A_i\| \leq \|\Delta A_i\|_{\max}$, $\|E\| \leq \|E\|_{\max}$, $\|D\| \leq \|D\|_{\max}$,

$H\in\kappa^{nxn}$ is a stable matrix and choosing S so that $S=E$, $D = B_{oi}^T B_{oi}$.

Theorem: System (12) with output-feedback controller (11) is asymptotically stable if there exist symmetric definite positive matrices P_{11}, some matrices K_i and L_i ,and matrices X_i, Y_i, satisfying the following Linear Matrix Equations (LMEs) for $i=1,2,\ldots,p$

$$A_i^T P_{11} + P_{11}A_i + (X_i C_i)^T + (X_i C_i) = -\sigma I \tag{17}$$

$$(Y_i C_i)^T + Y_i C_i = -\sigma I \tag{18}$$

Where $X_i=-P_{11}K_i$ and $Y_i=-P_{11}L_i$, σ is robustness index. Solving (17) and (18) related to the wind turbine system, it is clear that the uncertainty is affected by σ , since at certain value of σ the control system is stable in spite of large value of uncertainty. We choose $H = \begin{bmatrix} -4 & -4 \\ 0 & -1 \end{bmatrix}$ which is a stable matrix. The stable linear model is chosen as follows,

$$A_r = \begin{bmatrix} -4 & -4 \\ 0 & -4 \end{bmatrix}, B_r = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, C_r = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Solve LMEs (17) and (18) give the computation of the matrices K_i and L_i. Based on the lemma, construct the fuzzy fault tolerant controller (11) and based on the theorem construct the fuzzy observer (8).

V. Simulation Studies

The simulations are performed on a model wind-turbine system (5). The proposed RFTC for the wind-turbine system is tested for two cases, case 1; compares the results of the proposed algorithm and the previous algorithm [13] subject to parameter uncertainty, case 2; studying responses for wind-turbine system model given in (5) subject to parameter uncertainty and actuator faults. The reference input ($r(t)=\omega_{t(opt)}=v\lambda_{opt}/R$) is applied to the reference model and the controller, v is the wind speed, λ is the Tip Speed Ratio (TSR) and R is the turbine radius (m). The proposed controller is tested for random profiles of wind speed signal to prove the effectiveness of the proposed algorithm as shown in Fig.2, and change of parameter uncertainties L_f, L_d, r_a and D_s within 35 % of their nominal values are shown in Fig.3.

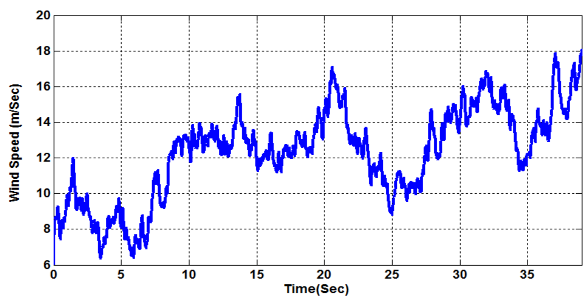

Fig. 2. Wind speed

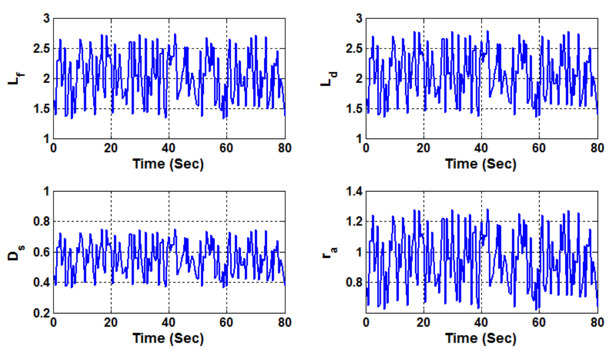

Fig. 3. The change of the parameter uncertainties L_f, L_d, r_a and D_s within *35%*

A. Case 1: System States of the Fuzzy Control System With Parameter Uncertainties

To compare the results of the proposed algorithm, and the previous algorithm [13], the system responses with the reference input (solid line) $r(t)=[V_{b(ref)} \quad \omega_{s(ref)}]^T$ for the proposed algorithm (dashed line) and the previous algorithm (dotted line) are shown in Fig. 4.

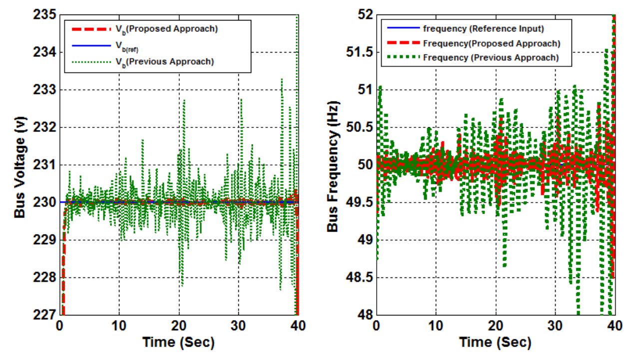

Fig. 4. Responses of bus voltage (V_b) (left) and rotor speed (ω_s) (right) of the proposed algorithm (dashed line), and the previous algorithm [13] (dotted line)

B. Case 2: System States of the Fuzzy Control System With Actuator Faults and With Parameter Uncertainties

The random profiles of wind speed signal as shown in Fig.2. We consider respectively the excitation field voltage actuator fault of the SG $f_1(t)$ and the direct-current set point actuator fault of the converter $f_2(t)$ are modeled as follow:

$$f_1(t) = \begin{cases} 0 & t < 22.5\,\text{sec} \\ 3 & t \geq 22.5\,\text{sec} \end{cases} \quad , \quad f_2(t) = \begin{cases} 0 & t \leq 22.5\,\text{sec} \\ 2 & t \geq 22.5\,\text{sec} \end{cases} \tag{19}$$

Fig. 5 shows the faults and their estimations based on (19) and parameter uncertainties L_f, L_d, r_a and D_s are shown in Fig.3. The system states (dashed line), observer states (dotted line) and stable linear states (solid line) are shown in Fig.6. It is shown that, there is only spike when the fault is detected at 22.5 sec.

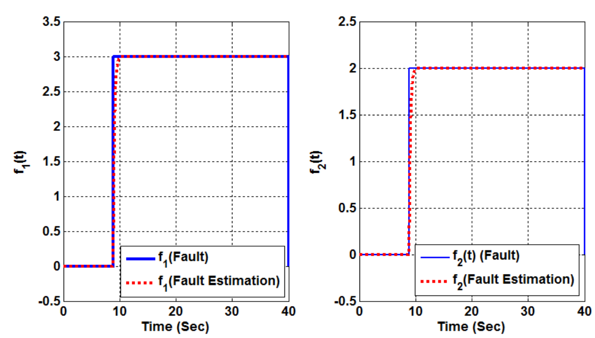

Fig. 5. Faults and their estimations(the excitation field voltage actuator fault of the SG $f_1(t)$ and its estimate and is the direct-current set point actuator fault of the converter $f_2(t)$ and its estimate)

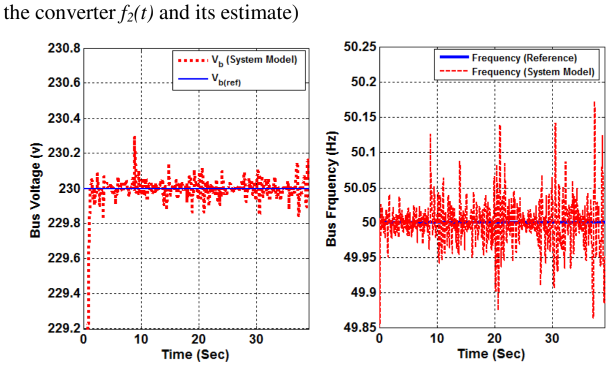

Fig. 6. The bus voltage and the bus frequency of the system (dashed and reference input (solid)

Fig. 4 and Fig. 6 show the simulation results of applying the proposed control scheme to the system (5) for tracking the reference model. We can see that both states of the system are bounded and good tracking performance can be obtained though the nonlinearities of the system, actuator faults and parameter uncertainties.

In summary results, in case 1; the response of the system subject to parameter uncertainties (L_f, L_d, r_a and D_s within 35% of their nominal values) is studying. It can be seen that the proposed robust fuzzy controller can control the plant well in the whole range of wind velocities, while the previous algorithm [13] cannot give good responses. In case 2; we can be seen that the system trajectory follows the trajectory of the reference model. Thus, the TS fuzzy model based controller is robust against norm-bounded parametric uncertainties and actuator faults.

VI. Conclusion

This paper addresses the design of robust fuzzy fault tolerant controllers to control a variable-speed wind turbine, while taking into account actuator fault(s) and parametric uncertainties as well as enabling the system to capture as much wind power as possible. A fuzzy observer design is proposed to achieve fault estimation of TS models with time-varying actuator faults and parameter uncertainties. Furthermore, based on the information of online fault estimation, an observer-based fuzzy-fault tolerant controller is designed to compensate the effect of faults. The proposed control scheme can guarantee the stability of the closed-loop system and the convergence of the output tracking error. The simulation results proved the effectiveness, robustness and better tracking performance of the proposed controller. In future work, it could be interesting to develop the RFTC law by taking into the premise variables affected by faults based on the neural fuzzy algorithm.

Appendix

Proof. let $e_1(t) = x_f(t)\text{-}\bar{x}(t)$ (A.1)

$$e_2(t) = x_f(t)\text{-}\hat{x}_f(t) \quad , \quad \tilde{f}(t) = f(t)\text{-}\hat{f}(t) \tag{A.2}$$

The dynamic of $e_1(t)$ is given by $\dot{e}_1(t) = \dot{x}_f(t)\text{-}\dot{\bar{x}}(t)$

$$\dot{e}_1(t) = \sum_{i=1}^{p} \mu_i [(A_i + \Delta A_i)\, x_f(t) + B\, u_i(t) + Ef(t)\text{-}A_r\, \bar{x}(t)\text{-}B_r r(t)] \tag{A.3}$$

The dynamic of $e_2(t)$ is expressed as follow:

$$\dot{e}_2(t) = \sum_{i=1}^{p} \mu_i [(A_i \text{-} K_i C_i) e_2(t) + \Delta A_i\, x_f(t) + E\tilde{f}(t)] \tag{A.4}$$

The dynamic of the fault error estimation can be written, $\dot{\tilde{f}}(t) = \dot{f}(t)\text{-}\dot{\hat{f}}(t)$. It is assumed to be the piecewise constant, that is, $\dot{f}(t) = 0$. Then the derivative of $\tilde{f}(t)$ can be written as.

$$\dot{\tilde{f}}(t) = \text{-}\dot{\hat{f}}(t) = -\sum_{i=1}^{p} \mu_i L_i C_i e_2(t) \tag{A.5}$$

From the (A.4) and (A.5), one can obtain:

$$\dot{\varphi} = A_o \varphi + B_o x(t) \tag{A.6}$$

With $\varphi = \begin{bmatrix} e_2(t) \\ \tilde{f}(t) \end{bmatrix}$, $A_o = \sum_{i=1}^{p} \mu_i A_{oi}$, $B_o = \sum_{i=1}^{p} \mu_i B_{oi}$

where $A_{oi} = \begin{bmatrix} A_i - K_i C_i & E \\ -L_i C_i & 0 \end{bmatrix}, B_{oi} = \begin{bmatrix} \Delta A_i \\ 0 \end{bmatrix}$

Consider the Lyapunov function as follows

$$V(e_1(t), \varphi(t)) = \frac{1}{2} e_1(t)^T P_1 e_1(t) + \varphi(t)^T P_2 \varphi(t) \tag{A.7}$$

where P_1 and P_2 are symmetric and positive definite matrices. By derivative (A.7) and $u_i(t)$ is designed as in Lemma, we obtain

$$\begin{aligned} \dot{V}(e_1(t), \varphi(t)) &\le -\frac{1}{2} e_1(t)^T Q_1 e_1(t) - \varphi(t)^T Q_2 \varphi(t) \\ &+ \|e_1(t)\| \|P_1\| (\|E\| \text{-} \|E\|_{\max}) \|\tilde{f}(t)\| + \|x(t)\| (\|D\| - \|D\|_{\max}) \|x(t)\| \\ &+ \sum_{i=1}^{p} \mu_i \|e_1(t)\| \|P_1\| (\|\Delta A_i\| \text{-} \|\Delta A_i\|_{\max}) \|x(t)\| \end{aligned} \tag{A.8}$$

where $Q_2 = -(A_{oi}^T P_2 + P_2 A_{oi} + P_2 P_2)$, $Q_1 = \text{-}(H^T P_1 + P_1 H)$ are a symmetric positive definite matrix. As $\|E\| \le \|E\|_{\max}$, $\|\Delta A_i\| \le \|\Delta A_i\|_{\max}$, $\|D\| \le \|D\|_{\max}$. From (A.8), we one obtain

$$\dot{V}(e_1(t), \varphi(t)) \le -\frac{1}{2} e_1(t)^T Q_1 e_1(t) - \varphi(t)^T Q_2 \varphi(t) \tag{A.9}$$

$\dot{V} < 0$ if there exists a common positive definite matrix P_1 and P_2 such that

$$H^T P_1 + P_1 H < 0 \tag{A.10}$$

$$A_{oi}^T P_2 + P_2 A_{oi} + P_2 P_2 < 0 \quad i=1,2,\ldots,p \tag{A.11}$$

Assuming $P_2 = diag(P_{11}, P_{22})$. By multiplying (A.11) from left and right by P_{11}^{-1} and apply the change of variables $X_i = \text{-}P_{11} K_i$ and $Y_i = \text{-}P_{11} L_i$. One obtains the LMEs (17) and (18) respectively in theorem.

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