

# Nonlinear Approach For The Identification of Discrete Time Delay Systems

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**Abstract**—A new approach for simultaneous recursive identification of an unknown time delay of discrete-time delay systems is proposed in this paper. The proposed Levenberg-Marquardt's algorithm is used to deal with the identification problem. The effectiveness of this method has been illustrated through simulation.

## I. INTRODUCTION

Time delay system identification has received great attention in the last years. Since, a board class of industrial processes included time delay phenomena in their dynamics. Furthermore, actuators, sensors and field networks which are involved in feedback loops introduce the time delay. The neglect of its presence may lead to a source of complex behavior (ignored data, oscillations, destabilization of the closed-loop, etc.) [10], [18]. For the identification of time-delay systems, two main problems must be considered: first is the identification of time delay and second is the identification of dynamics parameters.

Several approaches have been proposed in the literature for the identification of time delay system identification [2], [3], [12], [14], [15], [17], [20], [21]. These approaches address the identification of continuous-time systems [1], [8], [9], [11], [16], [19] as well as the identification of discrete-time systems [4], [5], [6], [7], [22], [23].

Two strategies of identification are manly used by the existing methods. The first one estimates only the unknown time delay assuming that the dynamics parameters are known a priori. While the second one consists in estimating separately the time delay and the dynamics parameters. The two strategies favorize the sake of simplicity. However, successful implementations of online control requires simultaneously on line identification unknown time delay and unknown parameters. This paper proposes a new method for simultaneously online identification of time delay and dynamics parameters of time delay systems. Our methodology proposes a new formulation allowing to define the time delay and the dynamic parameters in the same vector.

The outline of this paper is as follows. Section II presents the identification problem and its assumptions. A new approach for the simultaneous identification of the time delay and the parameters of discrete-time delay systems is developed in Section III. Results of simulations are illustrated in Section IV.

## II. PROBLEM STATEMENT

In the following, we address the problem of estimating the time delay and the parameters of the following system:

$$A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k) + v(k) \quad (1)$$

where  $u(k)$  and  $y(k)$  are the system input and output, respectively,  $v(k)$  is a white noise,  $d$  is the time delay,  $A(q^{-1})$  and  $B(q^{-1})$  are two polynomials in the unit backward shift operator  $q^{-1}$ , [i.e.  $q^{-1}y(k) = y(k-1)$ ], defined by:

$$A(q^{-1}) = 1 + a_1q^{-1} + \dots + a_{n_a}q^{-n_a} =$$

$$B(q^{-1}) = b_1q^{-1} + \dots + b_{n_b}q^{-n_b} =$$

The following assumptions are made:

- A1. The polynomials  $A(q^{-1})$  and  $B(q^{-1})$  are coprime.
- A2. The orders  $n_a$  and  $n_b$  of the model are known.
- A3. The input sequence  $\{u(k)\}$  is a stationary ergodic process, independent of  $v(k)$  and is persistently exciting.
- A4. The disturbance  $v(k)$  is a sequence of independent, identically distributed random variable with zero mean and finite variance  $\sigma^2$ .
- A5. The input, the output and the noise are causal, i.e.  $u(k) = 0$ ,  $y(k) = 0$  and  $v(k) = 0$  for  $k \leq 0$ .

**Problem statement:** The goal is to develop an algorithm to estimate, simultaneously, the time delay  $d$  and the parameters  $\{a_i, b_i\}$  using the input/output measurement data  $\{u(k), y(k)\}$ .

## III. THE PROPOSED APPROACH

This paragraph proposes an alternative solution for the purpose of simultaneous identification of unknown time delay and the system parameters.

Equation (1) can be rewritten as:

$$y(k) = \varphi(k, d)\theta + v(k) \quad (2)$$

where  $\theta$  is the parameter vector and  $\varphi(k, d(k))$  is the observation vector which are defined as:

$$\varphi(k, d) = [-y(k-1), -y(k-2), \dots, -y(k-n_a), u(k-d-1), \dots, u(k-d-n_b)] \quad (3)$$

$$\theta = [a_1, a_2, \dots, a_{n_a}, b_1, b_2, \dots, b_{n_b}]^T$$

The estimated output is described by the following relation:

$$\hat{y}(k) = \hat{\varphi}(k, \hat{d})\hat{\theta} \quad (4)$$

where  $\hat{\theta}$  and  $\hat{d}$  represent the estimated parameter vector and the estimated time delay.

Now, let consider the prediction error:

$$e(k) = y(k) - \hat{y}(k) = y(k) - \hat{\varphi}(k, \hat{d})\hat{\theta} \quad (5)$$

This formulation does not admit the unknown time delay in the parameter vector and consequently it is not directly

applicable to achieve our objective which is simultaneous identification of the time delay and the parameters of time-varying delay systems. To overcome this problem, we suggest to consider the time delay in the vector of parameters to be estimated. Indeed, the new vector, called generalized parameter vector is given by:

$$\theta_G = [\theta^T, d] = [a_i, b_j, d]$$

where  $i \in \{1, \dots, n_a\}$  and  $j \in \{1, \dots, n_b\}$ .

Then, the equation (6) can be expressed as:

$$y(k) = f(k, \theta_G) \quad (6)$$

where  $f$  is nonlinear function depending on  $\theta_G$ . Now, the use of the negative gradient of the error, to obtain an appropriate observation vector, is proposed which is given by:

$$\phi(k, \hat{\theta}_G) = -\frac{\partial e}{\partial \hat{\theta}_G} \quad (7)$$

Then,

$$\phi(k, \hat{\theta}_G) = \left[ \varphi^T(\hat{d}), -\frac{\partial e}{\partial \hat{d}} \right]^T \quad (8)$$

Using the first order derivative approximation, we have:

$$\phi(k, \hat{\theta}_G) = \left[ \varphi^T(k, \hat{d}), -\sum_{i=1}^{n_b} \hat{b}_i q^{-\hat{d}} u(k-i)(1-q^{-1}) \right]^T \quad (9)$$

Replacing  $\varphi^T(k, \hat{d})$  by its expression in (9), we obtain the generalized parameter vector :

$$\phi(k, \hat{\theta}_G) = \begin{bmatrix} -y(k-1) \\ \vdots \\ -y(k-n_a) \\ q^{-\hat{d}} u(k-1) \\ \vdots \\ q^{-\hat{d}} u(k-n_b) \\ -\sum_{i=1}^{n_b} \hat{b}_i q^{-\hat{d}} \Delta u(k-i) \end{bmatrix} \quad (10)$$

where  $\Delta u(k) = u(k) - u(k-1)$ .

An estimation  $\hat{\theta}_G$  of  $\theta_G$  is given by the minimization of the following criterion:

$$J(\theta_G) = \frac{1}{2} e^T e \quad (11)$$

The used approach to identify the generalized parameter vector  $\theta_G$  is based on the minimization of the criterion  $J(k, \hat{\theta}_G)$  using the gradient algorithm. This method relies on an expansion of the criterion  $J$  limited to the first order.

Updating the generalized parameter vector is obtained from the following recurrence formula [13]:

$$\hat{\theta}_G(k) = \hat{\theta}_G(k-1) - \mu \frac{\partial J(k, \hat{\theta}_G)}{\partial \hat{\theta}_G} \quad (12)$$

where  $\mu$  is the step size or the convergence factor.

Then,

$$\hat{\theta}_G(k) = \hat{\theta}_G(k-1) - \mu \frac{\partial e(k, \hat{\theta}_G)}{\partial \hat{\theta}_G} e(k) \quad (13)$$

where  $\hat{\theta}_G(k-1)$  represents the value of the generalized parameter vector at iteration  $k-1$ ,  $\hat{\theta}_G(k)$  is the same vector to the next iteration.

It follows from (7):

$$\hat{\theta}_G(k) = \hat{\theta}_G(k-1) + \mu \phi(k, \hat{\theta}_G) e(k) \quad (14)$$

The technique used to identify the generalized parameter vector  $\theta_G$  is based on the minimization of a criterion  $J$  implementation using the Gauss-Newton algorithm. This method relies on an expansion of the criterion  $J$  limited to the second order.

Updating the generalized parameter vector is obtained from the following recurrence formula:

$$\hat{\theta}_G(k) = \hat{\theta}_G(k-1) - H(k, \hat{\theta}_G)^{-1} \frac{\partial J(k, \hat{\theta}_G)}{\partial \hat{\theta}_G}$$

The second partial derivative of the criterion with respect to the generalized vector parameter gives the Hessian matrix  $H(k, \hat{\theta}_G)$ :

$$H(k, \hat{\theta}_G) = \frac{\partial^2 J(k, \hat{\theta}_G)}{\partial \hat{\theta}_G \partial \hat{\theta}_G^T}$$

Then,

$$H(k, \hat{\theta}_G) = e(k) \frac{\partial^2 e(k)}{\partial^2 \hat{\theta}_G} + \phi(k, \hat{\theta}_G) \phi^T(k, \hat{\theta}_G)$$

Hence, an approached Hessian is obtained by:

$$H(k, \hat{\theta}_G) \simeq \phi(k, \hat{\theta}_G) \phi^T(k, \hat{\theta}_G)$$

So, we leads finally to the formula:

$$\hat{\theta}_G(k) = \hat{\theta}_G(k-1) + [\phi(k, \hat{\theta}_G) \phi^T(k, \hat{\theta}_G)]^{-1} \phi(k, \hat{\theta}_G) e(k) \quad (15)$$

Often, Hessian may not be invertible (singular matrix) or the inverse may not be definite positive, then the parameter update is not possible. To avoid these problems, the Levenberg-Marquardt's algorithm is used for ill-conditioned problems:

$$\hat{\theta}_G(k) = \hat{\theta}_G(k-1) - \mu [H(k, \hat{\theta}_G) + \lambda I]^{-1} \frac{\partial J(k, \hat{\theta}_G)}{\partial \hat{\theta}_G}$$

$$\hat{\theta}_G(k) = \hat{\theta}_G(k-1) + \mu [\phi(k, \hat{\theta}_G) \phi^T(k, \hat{\theta}_G) + \lambda I]^{-1} \phi(k, \hat{\theta}_G) e(k) \quad (16)$$

where  $\mu$  is the step size that minimize the criterion in the direction of vector  $H(k, \hat{\theta}_G)^{-1} \frac{\partial J(k, \hat{\theta}_G)}{\partial \hat{\theta}_G}$ ,  $\lambda$  is a scalar and  $I$  the identity matrix of appropriate dimension.

If  $\lambda$  is small then the Gauss-Newton's algorithm is used. On the other hand, if  $\lambda$  is great then the method of gradient descent is used.

The above approach can be summarized by the following step by step procedure:

Step1: set  $\hat{\theta}_G = \theta_{G_0}$ ,  $\mu = \mu_0$ ,  $\lambda = \lambda_0$  and  $k = 1$ .

Step2: Increment  $k$  and construct the observation vector  $\varphi(k, \hat{\theta})$ , the generalized observation vector  $\phi(k, \hat{\theta}_G)$  using (3) and (10).

Step3: Perform the following:

$$\hat{\theta}_G(k) = \hat{\theta}_G(k-1) + \mu [\phi(k, \hat{\theta}_G)\phi^T(k, \hat{\theta}_G) + \lambda I]^{-1} \phi(k, \hat{\theta}_G)e(k)$$

Step3: Return to step 2 until  $k = N$  where  $N$  is the number of input/output data.

#### Covariance matrix

With the assumption A4, the covariance matrix of  $\hat{\theta}_G$  is given by:

$$E[(\hat{\theta}_G - \theta_G)(\hat{\theta}_G - \theta_G)^T] = (\phi(k, \theta_G)\phi(k, \theta_G)^T)^{-1} \sigma^2 \quad (17)$$

#### Proof

Consider the following first order Taylor series expansion around the real parameter of  $\theta_G$

$$\frac{\partial J(k, \hat{\theta}_G)}{\partial \hat{\theta}_G} = \frac{\partial J(k, \theta_G)}{\partial \theta_G} + \frac{\partial^2 J(k, \theta_G)}{\partial \theta_G^2} (\hat{\theta}_G - \theta_G) \quad (18)$$

Since  $\frac{\partial J(k, \hat{\theta}_G)}{\partial \hat{\theta}_G} = 0$ , it derives from (18)

$$\begin{aligned} (\hat{\theta}_G - \theta_G)(\hat{\theta}_G - \theta_G)^T &= \left[ \frac{\partial^2 J(k, \theta_G)}{\partial \theta_G^2} \right]^{-1} \frac{\partial J(k, \theta_G)}{\partial \theta_G} \\ \left[ \frac{\partial J(k, \theta_G)}{\partial \theta_G} \right]^T &\left[ \left( \frac{\partial^2 J(k, \theta_G)}{\partial \theta_G^2} \right)^{-1} \right]^T \end{aligned} \quad (19)$$

The second partial derivative of the criterion with respect to the generalized vector parameter is

$$\frac{\partial^2 J(k, \theta_G)}{\partial \theta_G^2} = e(k) \frac{\partial^2 e(k)}{\partial \theta_G^2} - \frac{\partial e(k)}{\partial \theta_G} \phi(k, \theta_G)$$

So,

$$\frac{\partial^2 J(k, \theta_G)}{\partial \theta_G^2} = e(k) \frac{\partial^2 e(k)}{\partial \theta_G^2} + \phi(k, \theta_G)\phi^T(k, \theta_G)$$

Hence, an approached of  $\frac{\partial^2 J(k, \theta_G)}{\partial \theta_G^2}$  is obtained:

$$\frac{\partial^2 J(k, \theta_G)}{\partial \theta_G^2} \simeq (\phi(k, \theta_G)\phi^T(k, \theta_G))$$

Applying the mean value of  $\frac{\partial J(k, \theta_G)}{\partial \theta_G} \frac{\partial J(k, \theta_G)^T}{\partial \theta_G}$ , we get:

$$\begin{aligned} E \left[ \frac{\partial J(k, \theta_G)}{\partial \theta_G} \frac{\partial J(k, \theta_G)^T}{\partial \theta_G} \right] &= \\ \phi(k, \theta_G)\phi(k, \theta_G)^T E(e(k)e(k)^T) \end{aligned}$$

So,

$$E \left[ \frac{\partial J(k, \theta_G)}{\partial \theta_G} \frac{\partial J(k, \theta_G)^T}{\partial \theta_G} \right] = \phi(k, \theta_G)\phi(k, \theta_G)^T \sigma^2$$

Then, we have:

$$\begin{aligned} E[(\hat{\theta}_G - \theta_G)(\hat{\theta}_G - \theta_G)^T] &= \\ E \left[ \left( \frac{\partial^2 J}{\partial \theta_G^2} \right)^{-1} \phi(k, \theta_G)\phi(k, \theta_G)^T \sigma^2 \left[ \left( \frac{\partial^2 J}{\partial \theta_G^2} \right)^{-1} \right]^T \right] \end{aligned} \quad (20)$$

Finally, we obtain:

$$E[(\hat{\theta}_G - \theta_G)(\hat{\theta}_G - \theta_G)^T] = (\phi(k, \theta_G)\phi(k, \theta_G)^T)^{-1} \sigma^2 \quad (21)$$

#### Remarks

- It is well known that the time delay  $d$  is an integer. However, the proposed algorithm returns a real value. Thus, we retain as time delay value the nearest integer.
- The proposed algorithm can identify simultaneously and recursively the time delay and the parameters which are constants or slowly varying ones (such as single step changes in the parameters and the time delay) but not a higher frequency variations.

## IV. SIMULATION RESULTS

Now, we present two simulation examples to illustrate the performance of the proposed approach for simultaneous identification of the unknown time delay and the parameters of time delay systems. This simulation study considers two types of models which are first order plus time delay and second order plus time delay. This choice is justified by the fact that these models are mainly used for the purpose of industrial modeling.

The simulations are performed under the following conditions:

- The input  $\{u(k)\}$  is a persistent excitation signal sequence with zero mean and unit variance.
- The additive noise  $\{v(k)\}$  is a white noise sequence with zero mean and constant variance  $\sigma^2$  computing to obtain the desired Signal-to-Noise Ratio:

$$SNR(db) = 10 \text{Log} \left[ \frac{\sigma_x^2}{\sigma^2} \right] \quad (22)$$

where  $\sigma_x^2$  is the variance of the noise free output sequence  $\{x(k)\}$ .

#### A. Example 1

Consider a first-order plus time delay system with the following transfer function:

$$T_p \dot{y}(t) + y(t) = K_p u(t - \tau(t)) \quad (23)$$

where  $\tau(t) = 0.5 + |\sin(\frac{t}{250})|$ ,  $K_p = 1$  and  $T_p = 2$ .

The evolution of the time delay is given by Fig. 1:

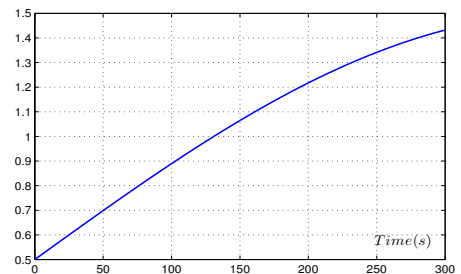


Fig. 1. Variation law of the time delay  $\tau$ .

Since  $\tau = dT_e + \varepsilon$ ,  $0 < \varepsilon < T_e$ ,  $T_e$  is the sampling period and  $d$  is a non-negative integer, it can be easily, using Zero Order Holder (ZOH), that (23 is given, in discrete-time by:

$$A(q^{-1})y(k) = q^{-d(k)}B(q^{-1})u(k) + v(k) \quad (24)$$

where

$$A(q^{-1}) = 1 + a_1q^{-1}$$

$$B(q^{-1}) = b_1q^{-1} + b_2q^{-2}$$

: Thus, the fractional delay  $\varepsilon$  gives rise in the  $z$ -domain to a pole at the origin and to a real negative zero. Suppose the sampling frequency is  $2Hz$ , we can obtain the plant model in the discrete time domain, The system's output is subject to additive zero mean white noise. The Signal-to-Noise Ratio ( $SNR$ ) is equal to 15dB on system's output.

The estimation starts with zero initial values for the parameters and the time delay.

Fig. 2– 5 show the evolution of the real and the estimated parameters and the time delay:

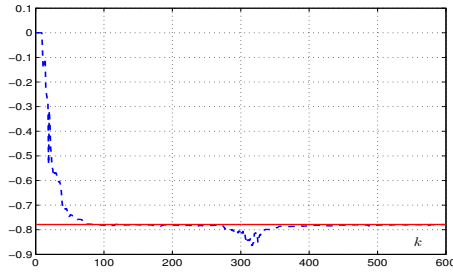


Fig. 2. The evolution of the real(–) and the estimated (– –) parameters  $a(k)$ .

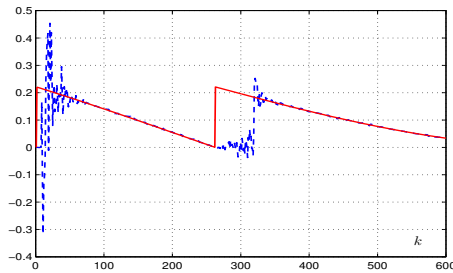


Fig. 3. The evolution of the real(–) and the estimated (– –) parameters  $b_1(k)$ .

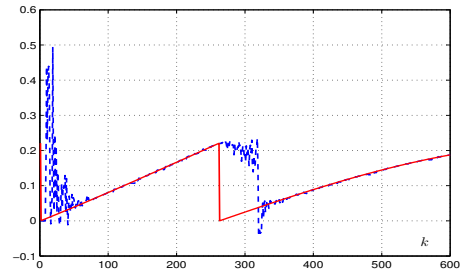


Fig. 4. The evolution of the real(–) and the estimated (– –) parameters  $b_2(k)$ .

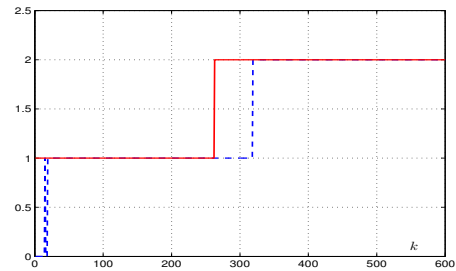


Fig. 5. The evolution of the real(–) and the estimated (– –) time delay  $d(k)$ .

From Fig. 2– 5, we can remark that the proposed approaches gives acceptable precision. In fact, the estimates parameters converge fastly to the true values and it's notice that the prposed approach is robust when the order  $n_b$  is over estimated. A validation of the model is realized. Fig. 6 gives the evolution of real output  $y(k)$  and estimated  $\hat{y}(k)$ .

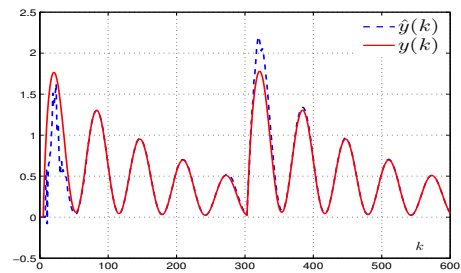


Fig. 6. The evolution of the real(–) and the estimated (– –) outputs signals.

This figure shows that the estimated output  $\hat{y}(k)$ , which is generated by the proposed approach, tracks fastly and accurately the real output.

### B. Exemple 2

In order to test the proposed identification algorithm, we consider a second-order plus time delay process with the

following transfer function:

$$G_c = \frac{K_p w_n^2}{s^2 + 2\xi w_n s + w_n^2} e^{-\tau s} \quad (25)$$

where  $\tau = dT_e + \varepsilon$ ,  $0 < \varepsilon < T_e$ ,  $T_e$  is the sampling period and  $d$  is a non-negative integer.

Suppose that the parameters and the time delay of the system are time varying, and the variation law is given in table I:

Time (sec)	$K_p$	$\xi$	$w_n$	$\tau$
$0 < t \leq 60$	1	0.1	3	0.5
$60 < t \leq 120$	2	0.3	4	0.6

TABLE I

VARIATION LAW OF THE PARAMETERS AND THE TIME DELAY.

Suppose that the sampling frequency is  $5Hz$ . A signal of noise is added to the system's output. The Signal-to-Noise Ratio ( $SNR$ ) is equal to  $15dB$ .

In same way, we discretize the system using Zero-Order Hold at each sample period  $T_e$ , we can obtain the plant model in the discrete time domain.

The evolution of the parameters and time delay of the discrete system is given by the two equations (26) and (27):

$$G_{d1}(q^{-1}) = q^{-2} \frac{0.04378q^{-1} + 0.245q^{-2} + 0.04042q^{-3}}{1 - 1.558q^{-1} + 0.8869q^{-2}} \quad (26)$$

$$G_{d2}(q^{-1}) = q^{-3} \frac{0.5211q^{-1} + 0.4426q^{-2}}{1 - 1.137q^{-1} + 0.6188q^{-2}} \quad (27)$$

We apply again the proposed algorithm to identify the time delay and the parameters of discrete-time systems.

The estimation starts with zero initial values for the parameters and the time delay.

Fig. 7–12 show the evolution of the real and the estimated parameters and the time delay:

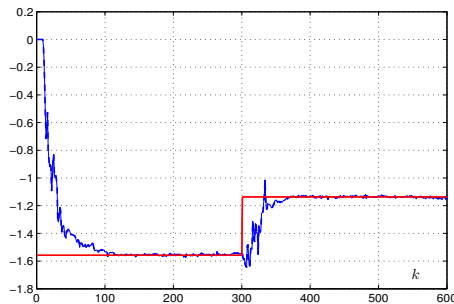


Fig. 7. The evolution of the real(–) and the estimated (– –) parameters  $a_1(k)$ .

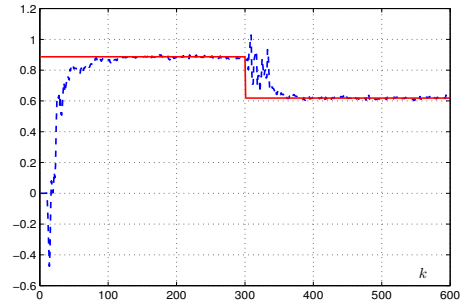


Fig. 8. The evolution of the real(–) and the estimated (– –) parameters  $a_2(k)$ .

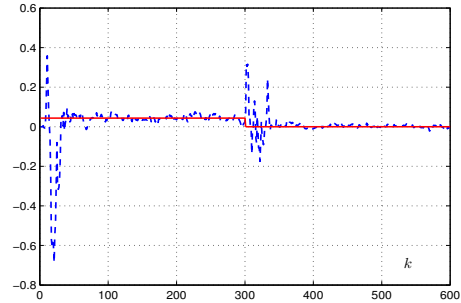


Fig. 9. The evolution of the real(–) and the estimated (– –) parameters  $b_1(k)$ .

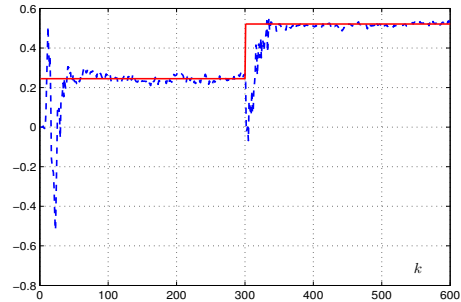


Fig. 10. The evolution of the real(–) and the estimated (– –) parameters  $b_2(k)$ .

In same way, we realized a validation test. Fig. 13 gives the evolution of real output  $y(k)$  and estimated  $\hat{y}(k)$ .

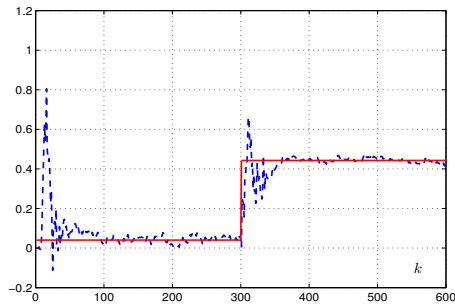


Fig. 11. The evolution of the real(—) and the estimated ( - - ) parameters  $b_3(k)$ .

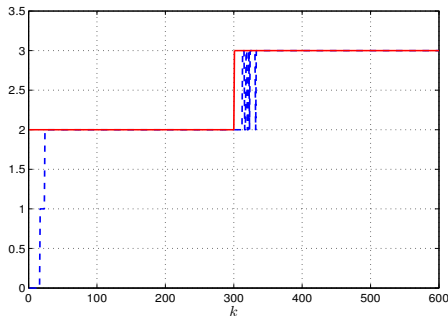


Fig. 12. The evolution of the real(—) and the estimated ( - - ) parameters  $b_2(k)$ .

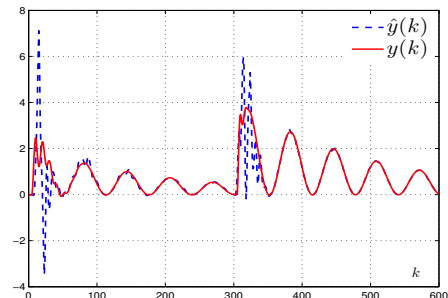


Fig. 13. The evolution of the real(—) and the estimated ( - - ) outputs signals.

## V. CONCLUSION

In the most publications, the problems of discrete time delay identification and discrete parameter identification require two different approaches to deal with the identification issue. So, in this paper, an identification algorithm based on Levenberg-Marquardt's method is developed to both estimate the system parameters and the time delay. To achieve our aim, we have proposed a new generalized regression vector and defined the time delay and the rational dynamic parameters in the same vector. The formulation problem is nonlinear. In fact, we have used the Levenberg-Marquardt' to solve the obtained system. A priori knowledge of the time delay and parameters are not required. This efficiency of the considered

algorithm is illustrated by some simulation examples of time delay systems.

## VI. ACKNOWLEDGMENTS

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## REFERENCES

- [1] S. Ahmed, B. Huang, and S.L. Shah. Parameter and delay estimation of continuous time models using a linear filter. *Journal of Process Control*, 16(4):323–331, 2006.
- [2] S. Bedoui, M. Ltaief, K. Abderrahim, and R. Ben Abdennour. Representation and control of time delay system: Multimodel approach. In *Proceedings of the 8<sup>th</sup> International Multi-Conference On Systems, Signals And Devices*, 2011.
- [3] S.V. Drakunova, W. Perruquettib, J.P. Richard, and L. Belkoura. Delay identification in time-delay systems using variable structure observers. *Annual Reviews in Control*, 30(2):143–158, 2006.
- [4] A. Elnaggar, G.A. Dumont, and A.L. Elshafei. New method for delay estimation. 3:1929–1930, 1990.
- [5] G. Ferretti, C. Maffezzoni, and R. Scattolini. Recursive estimation of time delay in sampled systems. *Automatica*, 27(4):653–661, 1991.
- [6] W. Gao, Y.C. Li, G.J. Liu, and T. Zhang. An adaptive fuzzy smith control of time-varying processes with dominant and variable delay. In *Proceedings of the American Control Conference*, volume 1, pages 220–224, 2003.
- [7] W. Gao, M. L. Zhou, Y. C. Li, and T. Zhang. An adaptive generalized predictive control of time varying delay system. In *Proceedings of the second World Conference on Machine Learning and Cybernetics, Xi'an, Shanghai*, pages 878–881, 2004.
- [8] P.J. Gawthrop and M.T. Nihtila. Identification of time delays using polynomial identification method. *Systems and Control Letters*, 5:276–271, 1985.
- [9] O. Gomez, Y. Orlov, and I. V. Kolmanovsky. Online identification of siso linear time-invariant delay systems from output measurements. *Automatica*, 43(12):2060–2069, 2007.
- [10] Kolmanovskii, V.B. Niculescu, S.I. Gu, and K. Delay effects on stability: A survey. In *Proceedings of the 38<sup>th</sup> IEEE Conference on Decision and Control*, volume 2, pages 1993–1998, 1999.
- [11] H. Kurz and W. Goedecke. Digital parameter adaptive control of process with unknown dead time. *Automatica*, 17(1):245–252, 1981.
- [12] M. De la Sen. Robust adaptive control of linear time-delay systems with point time-varying delays via multiestimation. *Applied mathematical modelling*, 33(2):959–977, 2009.
- [13] O. Nelles. *Nonlinear System Identification: From Classical Approaches to Neural Networks and Fuzzy Models*. Springer, 2001.
- [14] Y. Orlov, L. Belkoura, M. Dambrine, and J.P. Richard. On identifiability of linear time-delay systems. *IEEE Transactions on Automatic Control*, 47(8):1319–1324, 2002.
- [15] Y. Orlov, L. Belkoura, J.P. Richard, and M. Dambrine. Adaptive identification of linear time-delay systems. *International Journal on Robust and Nonlinear Control*, 13(9):857–872, 2003.
- [16] A.B. Rad, W.L. L, and K.M. Tsang. Simultaneous online identification of rational dynamics and time delay: A correlation based approach. *IEEE Transactions on Control Systems Technology*, 2003.
- [17] X.M. Ren, A.B. Rad, P.T. Chan, and W.L. Lo. Online identification of continuous-time systems with unknown time delay. *IEEE Transactions on Automatic Control*, 50(9), 2005.
- [18] J.P. Richard. Time delay systems: an overview of some recent advances and open problems. *Automatica*, 39:1667–1694, 2003.
- [19] S.W. Sung and I.B. Lee. Prediction error identification method for continuous time processes with time delay. In *Industrial and Engineering Chemistry Research*, volume 40, pages 5743–5751, 2001.
- [20] Q. G. Wang and Y. Zhang. Robust identification of continuous systems with dead time from step responses. *Automatica*, 37(3):377–390, 2001.
- [21] T. Zhang and Y. Cui. A bilateral control of teleoperators based on time delay identification. In *IEEE Conference on Robotics, Automation and Mechatronics*, 2008.
- [22] T. Zhang and Y. Li. A fuzzy smith control of time-varying delay systems based on time delay identification. In *Proceedings of Machine Learning and Cybernetics*, volume 1, pages 614–619, 2003.
- [23] W.X. Zheng and C.B. Feng. Identification of stochastic time lag systems in the presence of colored noise. *Automatica*, 26(4):769–779, 1990.