Macroscopic Modeling of Road Traffic by Using Hydrodynamic Flow Models

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***Abstract*— In order to propose a model for describing the behavior of road traffic a parallel with the hydrodynamic theory is made. The road traffic is decomposed into basic elements, which are modeled by similarity with hydrodynamic systems, by assuming that the flow of vehicles is similar to the movement of a fluid through a pipe. The different elements that compose a traffic region are analyzed and a dynamic model is proposed for each such element. The models are validated in simulation.**

I. Introduction

Traffic congestion is one of the most worrying problems of the modern world, in order to meet the need for unhindered mobility. It is the price paid for the concentration of population and the increasing economic activities. Since land supply is limited and the development of road infrastructure is costly, it would not be advisable to increase the road capacity in order to provide continuous circulation patterns near the free regime. Even if developing the infrastructure this regime would be achieved, as traffic demand depends on the total cost (social), at the same time the effect of stimulating demand will inevitably lead to a new congested regime.

Furthermore, delays caused by traffic congestion have a significant impact on the quality of human life. The driver stuck in traffic is confronted with stress, noise and other problems, which increases the risk of accident. This can be seen as an indirect cost (health problems). Congestion has implications for fuel consumption, generating massive losses. Consequently, for a fuel price becoming higher and higher, there is a decrease in purchasing power. From the perspective of the environment, congestion has a negative effect by increasing pollution caused by exhaust gases. All these mentioned aspects have implications for the economic and environmental costs of road traffic.

Manuscript received January 30, 2012.

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Thus, problems due to traffic congestion are socio-economic problems that need a solution to be implemented as soon as possible. Therefore seems appropriate to seek solutions to address them. The fastest and easiest would be to build new infrastructure, but they are very expensive and reduce the natural space available. Another solution to reduce traffic congestion is to encourage people to use public transport and, with respect to trade, to transport goods by rail or water. In any case, the direct consequences imply significant costs.

As a compromise between the presented solutions, there is another which implies to use the existing road infrastructure and to find control algorithms to fluidize the traffic and prevent the apparition of congestion. In this context, the objective of this work is to propose a set of models that can be used to analyze the behavior of a road traffic network. Section 2 presents the main variables that will be used to describe the behavior of vehicles on a road. In section 3 the road is considered divided into smaller segments; the model of a segment is obtained taking into account different possible situations. The behavior of the entire road can be considered as the interconnection of its segments. Following, section 4 analyses a particular type of interconnections – junctions. Section 5 shows the results obtained by simulating the behavior of the presented models for real traffic data.

II. Traffic Variables

When modeling road traffic, there are two main approaches that are frequently found, depending on the level of detail used to describe the road sector: macroscopic [1] and microscopic [2]. Recent studies concerning the difficulties encountered when modeling traffic flow have also tried to make a hybrid model by combining the macroscopic and microscopic approaches, in order to adapt the traffic flow models to small scale problems [3].

However, in order to control a road traffic sector, the macroscopic point of view seems the most adequate choice. It doesn't require a large set of variables compared to the other type of models, and thus it leads to a shorter decision time. Furthermore, it permits a general representation of a traffic network, offering access to a global view of the entire system.

978-1-4673-2529-5/12/$31.00

The macroscopic models are usually derived from hydrodynamic models, resulting in describing the traffic as a flow of vehicles. The main variables used for describing its behavior are traffic flow, road density and average speed of vehicles.

Traffic flow expresses the number of vehicles passing a location in a unit time. Depending on the method of measurement the flow can be approximated either on a point of the road or on a section of the road. If the number of vehicles measured by a sensor in an interval Δt is $N(t)$, the flow of traffic is expressed as:

$$q(t) = \frac{N(t)}{\Delta t} \quad (1)$$

Road density expresses the number of vehicles that are on a section of the road. Using, for example, a pair of inductive loops it can be measured on a section determined by the location of the loops. If the section of road on which the density is measured is given by Δx and the number of vehicles that are on this section is given by $N(t)$, road density is the ratio between:

$$\rho(x,t) = \frac{N(t)}{\Delta x} \quad (2)$$

The average speed of vehicles expresses the average road speeds of vehicles that are on a section of the road. In practice, the value of the average speed is obtained by averaging the speeds of vehicles passing over a sensor for a fixed period of time (considering a number K of measurements):

$$v_m = \frac{\sum_{i=1}^{K} v(t_i)}{K} \quad (3)$$

In addition to the above mentioned parameters we added a new one, the road pressure, which is influenced by the number of vehicles on the road. The road pressure is defined as the ratio between the number of vehicles that are on a road section $N(t)$ and the maximum capacity of that section C:

$$p(t) = \frac{N(t)}{C} \quad (4)$$

The fundamental traffic diagram gives a relationship between traffic flow and road density. It can be used to predict the behavior of a road section, as seen from acquired data [4]. The general form is shown in Fig.1.

There are two areas that correspond to two types of traffic:

- free flow traffic, where vehicles move without constraints with a speed equal to the free speed (defined as the maximum recommended speed for the road section);
- congested flow traffic, where vehicles travel at a speed below the free speed.

In the free flow zone the density varies between zero and the critical density. Zero density corresponds to the case where there is no vehicle on the road. The critical density corresponds to the case where there are a number of vehicles on the road moving with a free speed, but after this value if the number of vehicles increases the speed begins to descend and vehicles cannot travel anymore at free speed. Congested area of flow is given by a density between the critical value and the maximum value. The maximum density corresponds to the case where the road is filled to maximum capacity and there are no moving vehicles.

For maintaining a constant speed in the area of free flow we considered the following expression for the speed:

$$v(\rho) = v_l \left[1 - f\left(\frac{\rho}{2\alpha\rho_{\max}} \right) \cdot \frac{\rho - \alpha\rho_{\max}}{(1-\alpha)\rho_{\max}} \right] \quad (5)$$

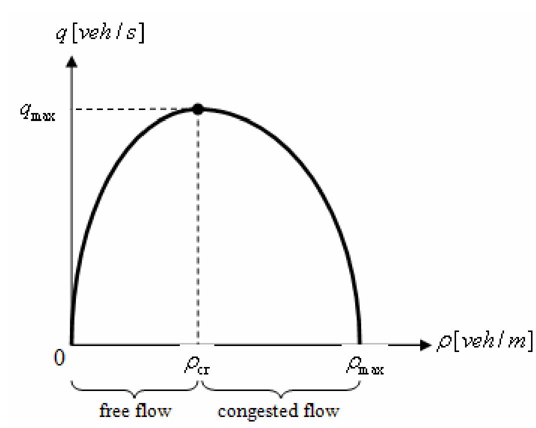


Fig.1. Fundamental traffic diagram.

III. Basic Traffic Models

In order to illustrate the variables discussed in section 2, we consider the road segment from Fig.2. The parameters used are the following:

- C, segment capacity;
- ρ, segment density;
- p, internal pressure;
- p_{ext}, external pressure;
- q_e, input flow ;
- q_s, output flow.

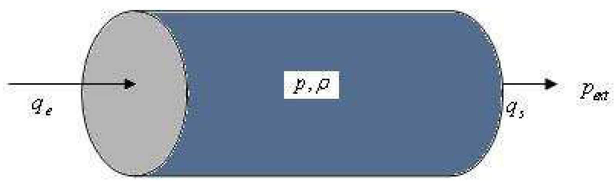


Fig. 2. Road traffic segment (pipe representation).

As the number of vehicles that are on the road section at any given time is limited:

$$0 \le N(t) \le C \tag{6}$$

interior pressure is also limited:

$$0 \le p(t) \le 1 \tag{7}$$

The internal and external pressures of the section determine the dynamics of movement for the road section. Similarly with the hydraulic theory, the output flow is given by the expression:

$$q_s(t) = k\sqrt{p(t)\left(1 - p_{ext}(t)\right)} \tag{8}$$

where k is a proportionality constant that depends on the number of output lanes.

A. *Simple Road Segment*

For the road segment in Fig. 2 we will associate a dynamic representation, from which the model will be obtained. The input of the process is represented by the input flow $q_e(t)$ and the output is represented by the number of vehicles inside the segment $N(t)$.

The internal pressure can be easily replaced by the number of vehicles in the segment (see relation (4)) and the dynamic model from relation (8) takes the form:

$$\begin{aligned} \dot{N}(t) &= q_e(t) - q_s(t) \\ q_s(t) &= k\sqrt{\frac{N(t)}{C}\left(1 - p_{ext}(t)\right)} \end{aligned} \tag{9}$$

It can be noticed that it is a nonlinear model. Therefore, in order to determine an equivalent transfer function, the model had been linearized.

Generally, we work around an operating point corresponding to the static regime. If we make the linearization of the relation (9) around the operating point characterized by the internal pressure p_0 and it is assumed that the external pressure is constant (zero), keeping only the first order term, the following dynamic model was obtained:

$$\tau_{P1}\frac{dy_1(t)}{dt} + y_1(t) = k_{P1}u_1(t) \tag{10}$$

with

$$\begin{aligned} \tau_{P1} &= \frac{2N_0}{q_0} \\ k_{P1} &= \frac{2N_0}{q_0} \end{aligned} \tag{11}$$

where p_0 and q_0 are the steady-state values of the internal pressure and respectively of the input/output flow. The input and output variables are as stated:

$$\begin{aligned} u_1(t) &= \Delta q_e(t) \\ y_1(t) &= \Delta N(t) \end{aligned} \tag{12}$$

B. *Road Segment with External Variations*

If the external pressure is variable in time, the operating point considered for the linearization of relation (9) is also characterized by the value of this pressure p_{ext0}. The number of vehicles is still the output variable, but to the input variable previously represented by the input flow is added a second one, the external pressure:

$$\begin{aligned} u_{21}(t) &= \Delta q_e(t) \\ u_{22}(t) &= \Delta p_{ext}(t) \\ y_2(t) &= \Delta N(t) \end{aligned} \tag{13}$$

The new dynamic model is rewritten as follows:

$$\tau_{P2}\frac{dy_2(t)}{dt}+y_2(t)=k_{P21}u_{21}(t)+k_{P22}u_{22}(t) \quad (14)$$

with

$$\begin{aligned}\tau_{P2}&=\frac{2N_0}{q_0}\\ k_{P21}&=\frac{2N_0}{q_0}\\ k_{P22}&=\frac{N_0}{1-p_{ext,0}}\end{aligned} \quad (15)$$

The model assumes the external pressure varies in time. If it is zero then the dynamic model from (14) becomes the dynamic model from (10).

C. *Road Segment with Vehicle Queue*

If we add the accumulation of vehicles at the end of the road section, in addition to the variables in Fig.2, we will consider a new variable: the length of the queue of vehicles (l_q). The model dynamics will be given only by the change in queue length of vehicles, by the variation of the number of vehicles that make up the queue. Vehicles entering the segment will decelerate and stop adding themselves to the queue. For the purpose of this case, the dynamics of deceleration is neglected. Similarity with the hydraulic theory, we propose to model the dependence of the output flow upon the length of the queue of vehicles with a relationship such as:

$$q_s(t)=a\sqrt{l_q(t)\left(1-p_{ext}(t)\right)} \quad (16)$$

For this model, the input variable remains as in the previous cases the input flow, while the output variable is given by the queue length:

$$\begin{aligned}u_3(t)&=\Delta q_e(t)\\ y_3(t)&=\Delta l_q(t)\end{aligned} \quad (17)$$

The dynamic model of a road segment with vehicles queue is of the form:

$$\tau_{P3}\frac{dy_3(t)}{dt}+y_3(t)=k_{P3}u_3(t) \quad (18)$$

with

$$\begin{aligned}\tau_{P3}&=\frac{2l_{q0}}{l_{veh}q_0}\\ k_{P3}&=\frac{2l_{q0}}{q_0}\end{aligned} \quad (19)$$

There are some differences that should be highlighted. In the case of the first two models (without accumulation of vehicles), all the vehicles are considered vehicles in motion, moving with a free speed. In the last case (with vehicle queue) all the vehicles are considered stopped vehicles. In both cases the variable of interest is the same - the number of vehicles, either running on the section or forming the queue associated with the section.

By connecting the basic models described in this section, the model of an entire road can be obtained. Thus the complexity of the model of a road section is given by the required level of description. The higher the level, the more subsections the road will be divided into. In the following section, as a particular case of interconnecting the presented models, different junction models will be considered.

IV. Junction Models

We have considered the behavior of basic road sections that can be directly connected one to another. But, in reality, the sections are connected by junctions. Among the different types of junctions, the following can be listed as the most often encountered:
- Y-junction;
- T-junction;
- Cross-junction.

A. *Y-junction*

For the behavior of a Y-junction, we considered the connection of three road sections, one being input and two being outputs. For the two output sections the percentage of vehicles that intend to choose the respective section is a priori known. Thus their input flows are expressed in terms of output flow of the first section by the relations:

$$\begin{aligned}q_{e2}(t)&=rq_{s1}(t)\\ q_{e3}(t)&=\left(1-r\right)q_{s1}(t)\end{aligned} \quad (20)$$

with r the ratio of vehicles going for one of the two exit sections. The output flow of the first section is also dependent of this ratio:

$$q_{s1}(t) = k_1\sqrt{p_1(t)\left[1-\left(rp_2(t)+(1-r)\,p_3(t)\right)\right]} \quad (21)$$

with $p_1(t)$, $p_2(t)$ and $p_3(t)$ the pressures inside the first, second and third section respectively.

B. *T-junction*

In the case of a Y-junction there was assumed that there is no possibility of vehicles between the three sections. To account for this case too, we have the case of a T-junction. The vehicles coming from the first section go through an additional section that links with the other two sections. The output flows of the intermediate section are influenced by the estimated percentage of vehicles going in the proper direction:

$$\begin{aligned} q_{s12}(t+\tau) &= rq_{s1}(t) \\ q_{s13}(t+\tau) &= (1-r)\,q_{s1}(t) \end{aligned} \quad (22)$$

with τ the time interval required for vehicles crossing the T-junction to enter the desired exit section.

C. *Cross-junction*

Both the Y-junction and the T-junction allowed the vehicles coming out of the first section to go either forward (on the second section) or to the right (on the third section). In the case of a cross-junction there is also the possibility of going to the left. Like in the case of the T-junction, there is an intermediate section that connects the first section with the three possible output sections. These output flows are expressed in terms of output flow of the first section by the relations:

$$\begin{aligned} q_{s12}(t+\tau) &= r_{12}q_{s1}(t) \\ q_{s13}(t+\tau) &= r_{13}q_{s1}(t) \\ q_{s14}(t+\tau) &= r_{14}q_{s1}(t) \end{aligned} \quad (23)$$

with τ the time interval required for vehicles crossing the cross-junction to enter the desired exit section.

V. Simulation Results

The simulations were carried using Matlab/Simulink program for the basic traffic model and 20-sim program for the junction models. In 20-sim bond graph models implementing the proposed relations were designed [5-7].

We considered the simple road segment with the non linear and linearized models. In Fig.3 it can be seen the error for the stationary number of vehicles on the segment, obtained by using the linearized and the non linear models. We can conclude that the proposed linearized model approximates with an acceptable accuracy the behavior of the nonlinear model.

Considering the basic road segments models valid, we analyzed their connections. For the Y-junction we have the following situation:

- the three sections are the same length ($L = L_1 = L_2 = L_3 = 200\,m$), therefore the same capacity ($C = 40\,veh$);
- the inflow is equal to $q_e = 1\,veh/s$ up to time $t = 500\,s$, after which it becomes void;
- the output of the second section is disturbed from time $t = 200\,s$ until time $t = 300\,s$ ($p_{ext2} = 0.5$);
- the output of the third section is disturbed from time $t = 100\,s$ until time $t = 400\,s$ ($p_{ext3} = 0.5$).

The resulting variations of the number of vehicles and the output flows for each section are shown in Fig.4.

The same situation was considered for the T-junction, with the additional information that the junction has the length $L_{\text{int}} = 25\,m$, therefore a capacity of $C_{\text{int}} = 5\,veh$. The resulting variations for the number of vehicles and the output flows are shown in Fig.5.

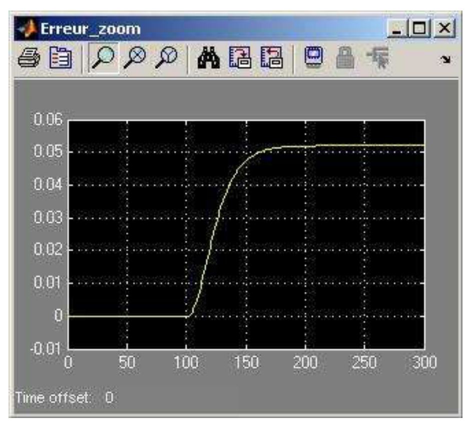
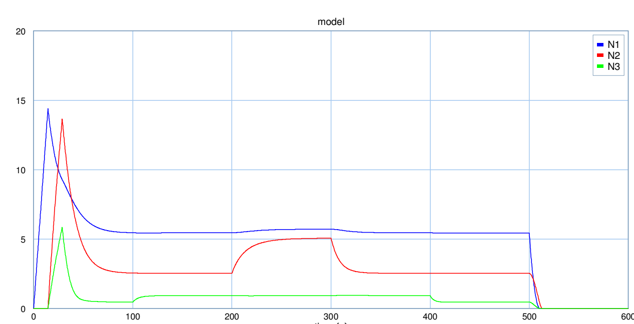


Fig. 3. Vehicle number – output error between linearized and non linear models.



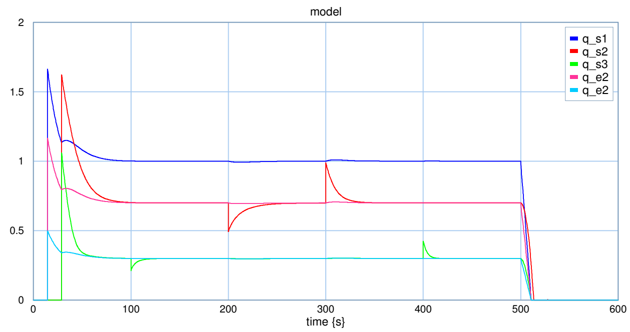


Fig. 4. Changes in the number of vehicles and output flows for a Y-junction.

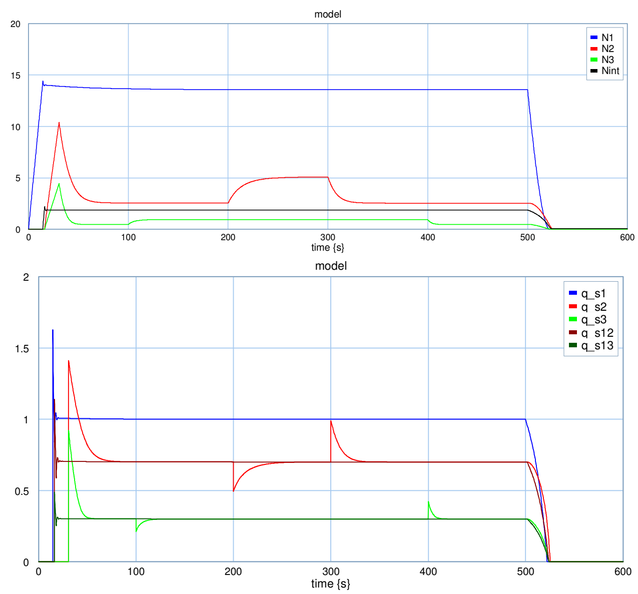


Fig. 5. Changes in the number of vehicles and output flows for a T-junction.

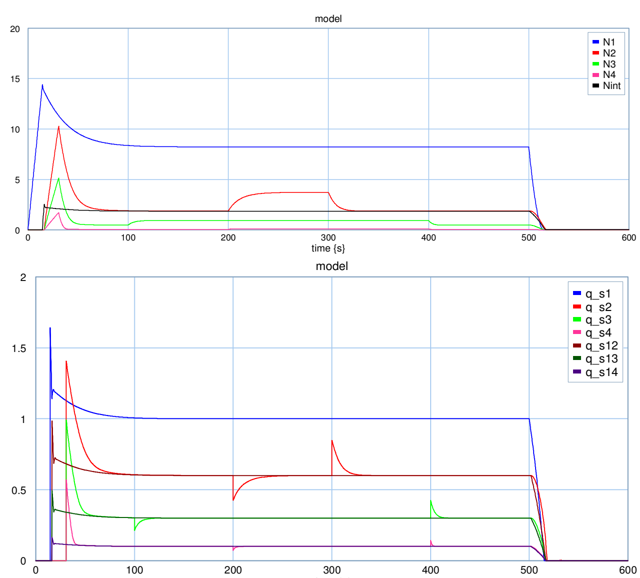


Fig. 6. Changes in the number of vehicles and output flows for a cross-junction.

For the cross-junction, compared to the T-junction, we have another section for going to the left. It was considered to be similar as the one for going to the right. The resulting variations of the number of vehicles and the output flows are shown in Fig.6.

For each of the three junctions the obtained results are similar. The number of vehicles on each section tends to arrive to a stationary value, which determines a constant output flow. Whenever an exterior disturbance appears, the number of vehicles starts to increase, determining a decrease of the output flow. Once the disturbance disappears it returns to the stationary value. We can conclude that the models reflect the same behavior encountered in real road traffic.

VI. Conclusion

By considering the traffic flow similar to the flow of a fluid a series of models are constructed. By making connections between these models more complex structures encountered in a traffic region can be modeled. The models take into account only one-way traffic; an extension to bidirectional traffic is to be further developed. Also, we study the implementation of a control algorithm using the presented models to account for the number of vehicles allowed on the road at any given time.

Acknowledgement

Thanks to the support of ERRIC: Empowering Romanian Research on Intelligent Information Technologies FP7-REGPOT-2010-1/264207

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