Preprints of the 2012 20th Mediterranean Conference on Control & Automation (MED), Barcelona, Spain, July 3-6, 2012

Rack Force Feedback For An Electrical Power Steering Simulator

L. Nehaoua, M. Djemaï and P. Pudlo

***Abstract*— This paper presents a force feedback control approach to enhance the realism and the fidelity of an electric power steering simulator. This simulator includes a real steering system with its electric assistance and a DC motor intended for the load rack force restitution. This motor allows to compensate for the lack of real tires and to simulate the tire/road contact as in a real driving situation. Besides, two different scenarios case study are introduced to prove the effectiveness of the various algorithms: virtual sensor based on sliding mode observer for the unknown input estimation and a sliding mode control for the force feedback.**

***Index Terms*— force feedback, driving simulators, sliding mode control, sliding mode observer.**

I. Introduction

The power steering has emerged as a design standard of personnel and professional vehicles. Indeed, in order to satisfy the expectations on driver comfort, the use of power steering system was widespread. This assessment is the fruit of a constantly rising complexity of car vehicles. Today, many driving assistive devices make the vehicle more and more intelligent but also heavy. In addition, the increased traffic in urban areas requiring maneuvers at low speeds were, among others, encourage the introduction of power steering system.

According to the actuation technology, we enumerate mainly the hydraulic power steering (HPS) and the electric power steering (EPS). The EPS assistance torque is usually defined from the torque boost-map implemented in the electrical control unit (ECU). Nevertheless, EPS system boost-map does not have well defined feedback design method. Among others, reversibility, maneuverability and vibrations are some of design criteria which are generally considered in the design of reference model used for the torque boost-map tuning [1]–[3]. However, driver physical capabilities are not taken into account and, consequently, the EPS optimality is not satisfied for a large population whose muscular activities are variable. The presence of the human-in-the-loop must be studied in-depth and a new approaches of assistance should be proposed and adapted to driver capabilities [4].

The validation of any developed control law is an essential step to assess its effectiveness and acceptability. Moreover, real circuit experimentations are often expensive and difficult to implement especially if they involve drivers with disabilities. Therefore, tests on an electrical power steering test-bench, in addition to real experimentation, seems to be an interesting alternative. In the field of driving simulation, multiple low-cost simulators have been emerged in the last decade [5], [6] while for EPS simulators, only reduced versions are designed [7], [8] and most of them were built by industrial institutions [9]–[11]. In this perspective, the VolHand project aims to build a EPS test-bench by integrating a real EPS and by ensuring a high simulation fidelity to be as close as possible to a real vehicle steering system.

To enhance the simulator realism and fidelity, a force feedback must be account for. In a classical way, the torque/force feedback methods are based on PID control on the Direct Current (DC) actuator current. These approaches require a dedicated torque/force sensor or the design of an additional virtual sensor (named observer) to estimate the torque/force to be reproduced. Instead, the torque/force feedback can be implemented as a robust reference position tracking computed from a reference model [12].

In the following, a force feedback control method is described. The remainder of this paper is organized as: section II exposes the problematic context. The EPS reference model and the simulator dynamics modeling are introduced in section III. In section IV, the driver torque estimation is illustrated and section V deals with the load force feedback control.

II. Problem Statement

Within the framework of the ANR project VolHand, an EPS test-bench is expected to be constructed. This simulator includes a real EPS system embedded into a modified car vehicle chassis [13].

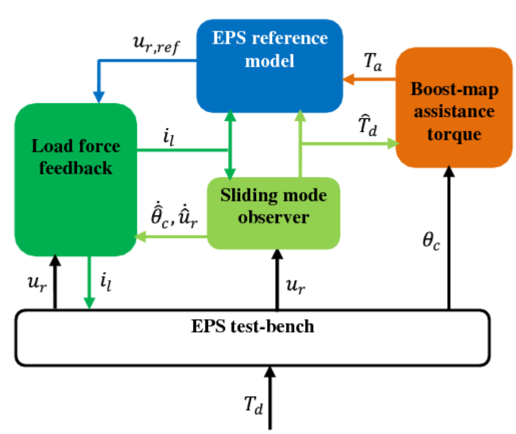

Fig. 1. Overall force feedback loop diagram

L. Nehaoua is with IBISC laboratory, EA 4526, CE 1455, 40 Rue du Pelvoux, 91020 Courcouronnes, Evry-Val-d'Essonne University, Evry, France, e-mail: lamri.nehaoua@ibisc.univ-evry.fr.

M. Djemaï and P. Pudlo are with University Lille Nord, F59000 Lille and UVHC, LAMIH, CNRS, FRE 3304, Campus du Mont Houy F59313 Valenciennes, France, e-mail: {Mohamed.Djemai;Philippe.Pudlo}@univ-valenciennes.fr.

978-1-4673-2529-5/12/$31.00 ©2012 IEEE

However, on the simulator, the load force on the steering system rack, resulting from the tire/road interaction, is absent. One must to replace the vehicle tire with a DC motor to simulate the corresponding rack load force. As shown in figure (1), when the driver applies a given torque T_d on the vehicle steering-wheel, a corresponding rack displacement u_r is measured. Consequently, the simulator fidelity is achieved if one can makes the simulator rack displacement to be as close as possible to the rack displacement on a real vehicle steering system. In other words, the DC motor assembly, intended to reproduce a given load rack force, should be versatile with respect to a real EPS system. This issue can be realized by controlling the rack displacement u_r towards a reference rack displacement $u_{r,ref}$ computed from an EPS reference model. In such way, one can also ensures that the reproduced load force on the simulator's rack is exactly the same as one developed in the presence of a real tire/road interaction.

Nevertheless, the driver torque T_d is an unknown variable which should be estimated in order to drive the reference model as shown in figure (1). For this, an estimation procedure is described in section IV by using the well known sliding mode unknown input observer (SMUIO). In the other side, the tracking of the reference rack displacement $u_{r,ref}$ should also be robust against model uncertainties and also against the sum of the driver and assistance torque $T_d + T_a$ which may be large for some driving maneuvers.

III. Dynamics modeling

A. EPS reference model

In this section, a reduced order model of the EPS system is considered [14]. Figure (2) schematized the interconnection between the EPS system and the load motor where the bilateral effort interaction is propagate over the rack force F_r.

At first, the steering column dynamics is given by the following equation:

$$J_c\ddot{\theta}_c + \beta_c\dot{\theta}_c = T_d - T_{cc} \tag{1}$$

where θ_c is the steering-wheel angle, T_d is the steering-wheel torque applied by the driver and T_{cc} is the transmitted torque via the column stiffness modeled as:

$$T_{cc} = K_c\left(\theta_c - \frac{u_r}{r_p}\right) \tag{2}$$

Here u_r is the rack displacement and r_p is the pinion radius. Next, the EPS electric motor is located between pinion and rack. Its mechanical equation is expressed as following:

$$J_a\ddot{\theta}_a + \beta_a\dot{\theta}_a = T_a - T_{cm} \tag{3}$$

where θ_a is the motor axis rotation, $T_a = k_{t,a}i_a$ is the assistance motor delivered torque and T_{cm} is the transmitted torque to the rack via the EPS gears. Finally, the rack dynamics is given as:

$$m_r\ddot{u}_r + \beta_r\dot{u}_r = \frac{1}{r_p}T_{cc} + \frac{N_a}{r_p}T_{cm} - F_r \tag{4}$$

in which F_r is the load rack force computed from the vehicle dynamics. The rack displacement is related to the EPS motor rotation by $u_r = \frac{r_p}{N_a}\theta_a$. From (3,4), the motor/rack equivalent dynamics is:

$$m_{eq}\ddot{u}_r + \beta_{eq}\dot{u}_r = \frac{K_c}{r_p}\left(\theta_c - \frac{u_r}{r_p}\right) + \frac{N_a k_{t,a}}{r_p}i_a - F_r \tag{5}$$

By combining (1,2,5), the EPS steering system dynamics is written as a linear state space formulation, as follow:

$$\dot{\mathbf{x}}_r = \mathcal{A}_r\mathbf{x}_r + \mathcal{B}_r\begin{bmatrix} T_d \\ i_a \\ F_r \end{bmatrix} \tag{6}$$

where $\mathbf{x}_r = [\dot{\theta}_c, \theta_c, \dot{u}_r, u_r]^T$ is the state vector and the system output is the rack reference displacement $u_{r,ref}$.

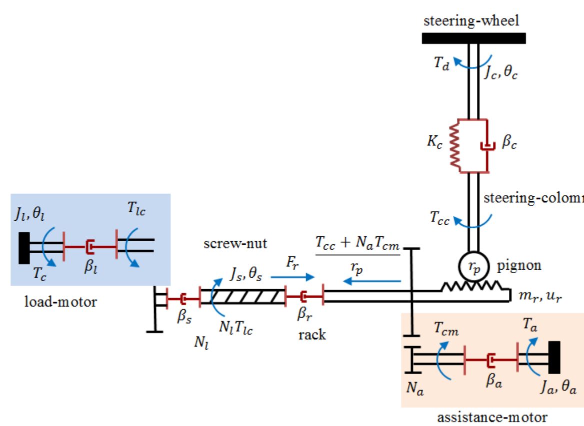

Fig. 2. Scheme of the EPS system and the load motor dynamics

B. EPS simulator dynamics

To drive the reference model of the previous section, an information about the driver torque applied on the simulator's steering-wheel is required. In that case, it is necessary to write the simulator dynamics.

As stated in section II, the simulator includes a real EPS system, so, equations (1-5) hold, and, it remains only to integrate the dynamics of the load motor part where the mechanical equation of the load motor and the screw-nut are described by:

$$\begin{aligned} J_l\ddot{\theta}_l + \beta_l\dot{\theta}_l &= T_l - T_{lc} \\ J_s\ddot{\theta}_s + \beta_s\dot{\theta}_s &= N_l T_{lc} + \frac{p}{2\pi\eta}F_r \end{aligned} \tag{7}$$

where θ_l, θ_s are respectively the load motor and the screw-nut angle positions and $T_l = k_{t,l}i_l$ is the motor delivered torque. In the other hand, θ_l, θ_s and rack displacement u_r are related to the EPS motor angle θ_a by the following relations:

$$\theta_s = \frac{2\pi}{p} u_r = \frac{1}{N_l}\theta_l \qquad \theta_a = \frac{N_a}{r_p} u_r$$

By reporting these relations in (7) and combining the resulting expression with (5), we get the equivalent dynamics of the assistance motor/rack/screw-nut/load motor assembly:

$$m'_{eq}\ddot{u}_r + \beta'_{eq}\dot{u}_r = N_l k_{t,l} i_l + \frac{p}{2\pi\eta r_p}\left(T_{cc} + N_a k_{t,a} i_a\right) \quad (8)$$

Finally, with (1) the overall EPS simulator dynamics is written by the following linear state space formulation:

$$\begin{aligned} \dot{\mathbf{x}}_s &= \mathcal{A}_s\mathbf{x} + \mathcal{B}_s \begin{bmatrix} i_a \\ i_l \end{bmatrix} + \mathcal{D}_s T_d \\ y_s &= u_r = \mathcal{C}_s \mathbf{x}_s \end{aligned} \quad (9)$$

where $\mathbf{x}_s = [\dot{\theta}_c, \theta_c, \dot{u}_r, u_r]^T$ is the state vector, y_s is the measured rack displacement signal and T_d is the unknown input to be estimated.

The next section concerns the development of virtual sensor to estimate the driver torque by introducing SMO theory.

Remark 3.1: In this paper, it is supposed that the assistance and the load motors are driven by the current signal i_a and its current dynamics is not considered.

IV. SMO DESIGN FOR STATES AND UNKNOWN INPUTS ESTIMATION

In this section, an application of a class of SMO developed in [15] is applied by considering the linear time invariant (LTI) system (9) with $n = 4$ states, one measurable output and one unknown input. Consider the following definitions [15], [16]:

Definition 4.1: The relative degree of the system output y_s with respect to the unknown input T_d is r if $\mathcal{C}_s\mathcal{A}^j\mathcal{D}_s = 0$, where, $j = 0, \cdots, r-2$ and $\mathcal{C}_s\mathcal{A}^{r-1}\mathcal{D}_s \neq 0$.

Definition 4.2: System (9) is strongly observable if and only if the relative degree r satisfies $r = \text{rank}(\mathcal{P})$ where $\mathcal{P}$ is the observability matrix $\mathcal{P} = \left[\mathcal{C}_s^T, (\mathcal{C}_s\mathcal{A}_s)^T, \cdots, (\mathcal{C}_s\mathcal{A}_s^{n-1})^T\right]^T$.

From these definitions, the relative degree of the system output y_s with respect to the unknown input T_d is 4 which is equal to the system state order n (or equal to the rank of the observability matrix), hence, system (9) is strongly observable. In addition, as reported in [15], if system (9) satisfies the following two assumptions:

1) The unknown input T_d is a bounded function where $|T_d| \leq T^+_{max}$ (the applied driver's torque is obviously bounded),
2) The k successive derivatives of T_d are bounded by the same constant $T^+_{1,max}$ and the $(k+1)$-th derivative is a Lipschitzian function.

then, the reconstruction of the unknown input T_d is possible and the observer is built in the form:

$$\begin{aligned} \dot{\mathbf{z}} &= \mathcal{A}_s\mathbf{z} + \mathcal{B}_s \begin{bmatrix} i_a \\ i_l \end{bmatrix} + \mathcal{L}(y_s - \mathcal{C}_s\mathbf{z}) \\ \hat{\mathbf{x}}_s &= \mathbf{z} + \mathcal{P}^{-1}\omega \\ \dot{\mathbf{v}} &= \mathbf{W}\left(\mathbf{v}, y_s - \mathcal{C}_s\mathbf{z}\right) \\ \mathcal{C}_s &= \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (10)$$

where, $\omega = [v_1, \cdots, v_n]^T$ and $\mathbf{v} = [v_1, \cdots, v_{n+k+1}]^T$ is the nonlinear part of the observer computed by the following high-order sliding mode differentiator (recall that $n = 4$ and $k = 1$):

$$\begin{aligned} \dot{v}_1 &= -8M^{\frac{1}{6}}\left|v_1 - y_s + \mathcal{C}_s\mathbf{z}\right|^{\frac{5}{6}} \text{sign}(v_1 - y_s + \mathcal{C}_s\mathbf{z}) + v_2 \\ \dot{v}_2 &= -5M^{\frac{1}{5}}\left|v_2 - \dot{v}_1\right|^{\frac{4}{5}} \text{sign}(v_2 - \dot{v}_1) + v_3 \\ \dot{v}_3 &= -3M^{\frac{1}{4}}\left|v_3 - \dot{v}_2\right|^{\frac{3}{4}} \text{sign}(v_3 - \dot{v}_2) + v_4 \\ \dot{v}_4 &= -2M^{\frac{1}{3}}\left|v_4 - \dot{v}_3\right|^{\frac{2}{3}} \text{sign}(v_4 - \dot{v}_3) + v_5 \\ \dot{v}_5 &= -1.5M^{\frac{1}{2}}\left|v_5 - \dot{v}_4\right|^{\frac{1}{2}} \text{sign}(v_5 - \dot{v}_4) + v_6 \\ \dot{v}_6 &= -1.1M\text{sign}(v_6 - \dot{v}_5) \end{aligned} \quad (11)$$

where, M is chosen sufficiently large from definition (4.1) and assumption (1) as:

$$M \geq \left|\mathcal{C}_s\mathcal{A}^{r-1}\mathcal{D}_s\right| T^+_{max}$$

Finally, the unknown input T_d is estimated by using:

$$\hat{T}_d = \frac{1}{\left|\mathcal{C}_s\mathcal{A}^{r-1}\mathcal{D}_s\right|}\left(v_5 - \mathcal{P}(\mathcal{A}_s - \mathcal{L}\mathcal{C}_s)\mathcal{P}^{-1}\omega\right) \quad (12)$$

Remark 4.1: Matrix $\mathcal{A}_s$ in (9) has one eigenvalue at the origin ([-4.216±68.543j, -2.973, 0]) then system (9) is unstable. It goes that the use of $\mathcal{L}(y_s - \mathcal{C}_s\mathbf{z})$ is necessary to stabilize the system. Since (9) is strongly observable, an arbitrary eigenvalues assignment can be done.

Remark 4.2: Unlike Walcott-Zak observer [17], the present observer is well adapted for high-order system. However, its direct application requires the boundedness of the unknown input T_d and also its $k+1$ successive derivatives. Fortunately, in mechanical system applications, assumption can be made and generally, only the first derivative of the unknown input T_d is considered (so $k = 1$ in assumption (2)).

At this level, the virtual sensor is implemented and the EPS reference model can be simulated. The next section is dedicated to the development of a control strategy to reproduce a load force on the simulator's rack.

V. MOTOR CONTROL FOR THE LOAD RACK FORCE FEEDBACK

The main key is that the simulator's rack displacement should be as close as to the reference rack displacement, computed from the reference EPS model. For this, from (8), the rack/screw-nut/load motor assembly equation can be written as:

$$\ddot{u}_r = -\frac{\beta'_{eq}}{m'_{eq}}\dot{u}_r + u + w(x,t) \tag{13}$$

where the new control input u and the external perturbation w are given by:

$$u = \frac{N_l k_{t,l}}{m'_{eq}} i_l \tag{14}$$
$$w(x,t) = \frac{p}{2\pi\eta r_p m'_{eq}} \left(N_a k_{t,a} i_a + T_{cc}\right)$$

The system under control is controllable and the disturbance $w(x,t)$ and its gradient are bounded. Consider the sliding surface $s = \lambda e + \dot{e}$, consequently, the system error dynamics is given by:

$$\ddot{e} = -\frac{\beta'_{eq}}{m'_{eq}}(s - \lambda e) + u + w \tag{15}$$

where $e = u_r - u_{r,ref}$ is the position error tracking. When the motion reaches the sliding surface $s = 0$, the error dynamics $\dot{e} = -\lambda e$ can be tuned to reach a prescribed performance. Next, to compensate for the disturbance $w(x,t)$, an injection term v is added to the equivalent control in order to form the global control variable u which can be expressed by:

$$u = u_{eq} + v = -\left(\lambda - \frac{\beta'_{eq}}{m'_{eq}}\right)(s - \lambda e) + v \tag{16}$$

When the perturbation $w(x,t)$ is bounded by a known function $\rho(x)$, the first-order sliding mode control (SMC) can be used at the expense of a discontinuous control. Otherwise, since system (13) has a relative degree 2, the super-twisting algorithm (STA) can be employed to avoid control chattering, however, the STA was designed as an absolutely continuous control law allowing to compensate Lipschitz unbounded perturbations but with a bounded time derivative [18], [19]. Then, we need VGSTA because the uncertainties are state dependent and bounded with known functions [20]. In that case, v is taken to be as:

$$v = -k_1(t,x)\phi_1(s) - \int_0^t k_2(t,x)\phi_2(s)dt \tag{17}$$
$$\phi_1(s) = |s|^{1/2}\operatorname{sign}(s) + k_3 s$$
$$\phi_2(s) = \frac{1}{2}\operatorname{sign}(s) + \frac{3}{2}k_3 |s|^{1/2}\operatorname{sign}(s) + k_3^2 s$$

where $k_1(t,x)$ and $k_2(t,x)$ are the variable gains which make the sliding surface insensitive to perturbations growing with system states and k_3 allows to deal with perturbations growing linearly in s. For this the perturbation $w(x,t)$ is split into to parts: $g_1(x,s,t)$ is growing linearly in s and $g_2(x,t)$ is independent from the sliding surface s like following:

$$g_1(x,s,t) = 0$$

$$g_2(x,t) = \frac{p}{2\pi\eta r_p m_{eq}}\left(K_c\theta_c - \frac{K_c}{r_p}u_r + N_a k_{t,a} i_a\right)$$

So, according to assumption (3), VGSTA is insensitive to perturbations $w(x,t)$ satisfying:

$$|g_1(x,s,t)| \le \rho_1(t,x)\,|\phi_1(s)| \to \rho_1(t,x) = 0$$

and:

$$\left|\frac{d}{dt}g_2(x,t)\right| \le \rho_2(t,x)\,|\phi_2(s)|$$
$$\to \rho_2(t,x) \ge \frac{pK_c}{2\pi\eta r_p m'_{eq}}\max\left\{\frac{1}{r_p k_3^2}, 2\left|\dot{\theta}_c + \frac{\lambda}{r_p}e\right|\right\}$$

In these two expressions, the bound functions $\rho_i(t,x)$ are used to compute the variable gains $k_i(t,x)$ as shown in [20].

VI. Simulations results

A. Simulation configuration

In the following, simulations are carried-out using the system model (9) as sketched in Fig.1 where the EPS assistance is deactivated ($i_a = 0$). The measured variables are the steering-wheel angle θ_c and the load motor position angle θ_l. The simulator's rack displacement u_r is is obtained from the load motor angle position θ_l where $u_r = p/(2\pi N_l)\theta_l$.

B. Case 1: parking maneuver

The first simulation aims to test the platform behavior for a parking maneuver. In that case, the steering-wheel rotation consists on a $\pm 4\pi$ (rad) which corresponds to 2 steering-wheel tours at right and left.

Fig.3.a presents the estimated driver's torque with respect to the steering-wheel angle where, an exact estimation in a finite time is obtained, in the case of noiseless signals and by using the nominal system parameters. Next, to check the robustness of the present observer against parameters uncertainties, a simulation is carried-out by considering 20% of parameters variation with respect to the nominal values (Fig.3.b). Despite the parameters uncertainty, one can conclude that the observer ensures a good asymptotic convergence.

Figures (3.c-d) point-out the effectiveness of the VGSTA control for the load rack force feedback. The proposed control is able to compensate for high level perturbations and provides an exact tracking of the reference rack displacement. This was possible thanks to the variable gains which allow to compensate for bounded perturbations growing with the system states (Fig.3.e-f).

C. Case 2: cornering maneuver

The second simulation aims to test the simulator behavior for a cornering maneuver in which, and according to the visual environment, the driver applies a torque to follow a defined trajectory. System (9) is considered with an uncertainty of 20% on the nominal parameters. Once again,

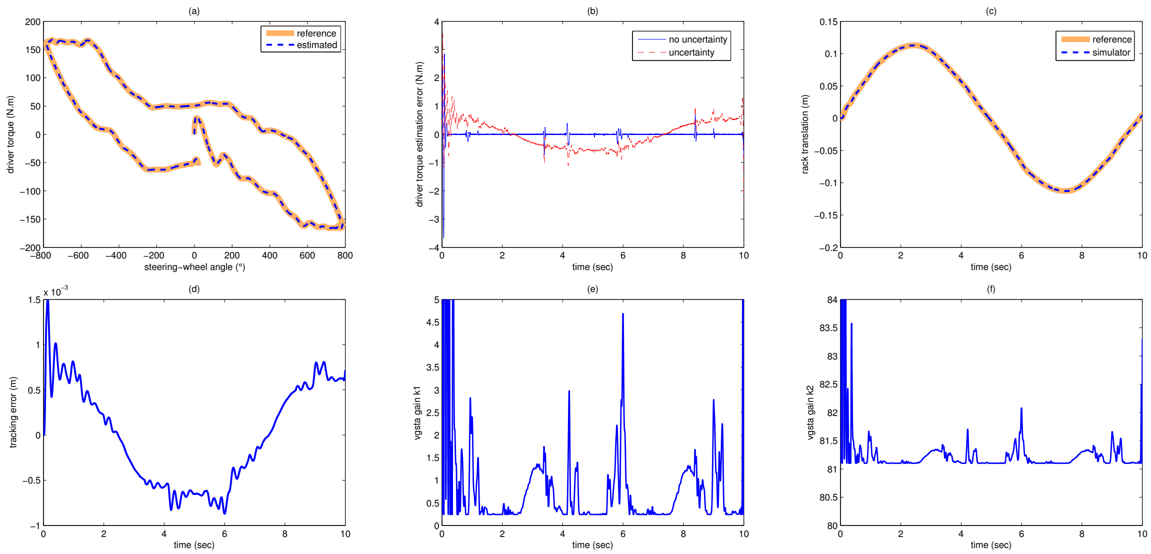

Fig. 3. Simulations: parking maneuver case

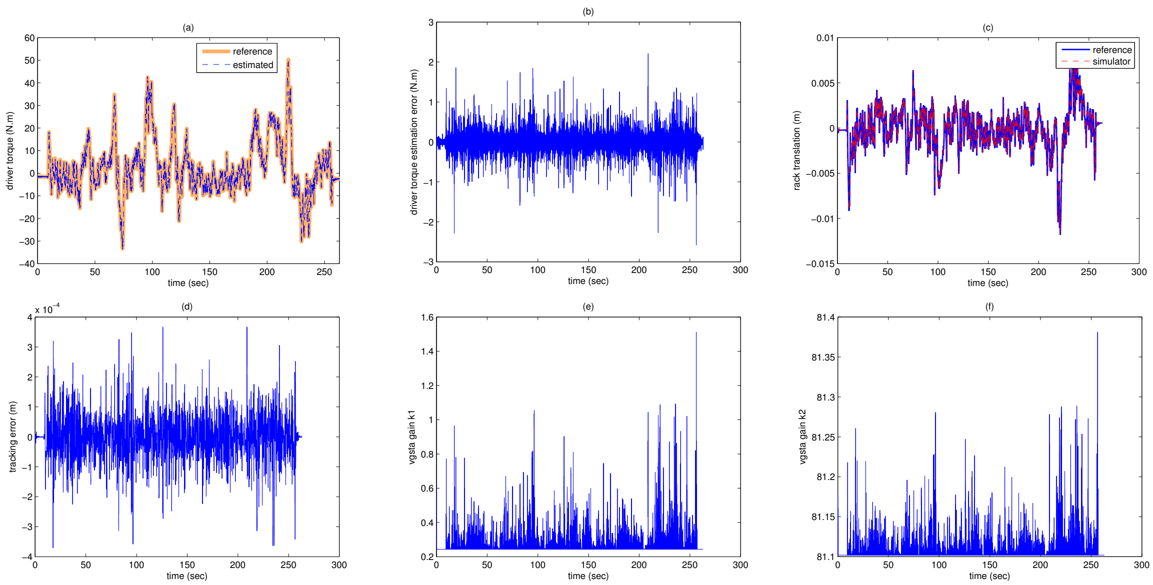

Fig. 4. Simulations: cornering maneuver case

it can be seen in Fig.4.a-b, that the unknown input (driver torque) is accurately reconstructed by the proposed observer where an asymptotic convergence is ensured. Finally, as in the parking maneuver, VGSTA control allows to achieve an exact tracking of the reference rack displacement (Fig.4.c-d) and to compensate for external perturbations and parameters uncertainties where the control gains are shown in Fig.4.e-f.

Remark 6.1: To prove the observer convergence, it is a custom to use different initial conditions to integrate the system states and observer states. Nevertheless, in this paper the same initial conditions are considered since the simulator will be calibrated at the beginning of each simulation.

VII. Conclusion

In this paper, a force feedback control approach is presented. It allows to enhance the EPS simulator fidelity and to provide a convenient load rack force feedback. An estimation procedure is illustrated for the sensorless measurement of the driver torque by using a step by step high-order sliding mode observer (HOSMO). The described algorithm allows to give an accurate estimation even in the presence of external perturbations and/or parameters uncertainties. Otherwise, the force feedback on the simulator's rack is carried-out by using a position tracking problem. This issue is easiest and more suitable then a force tracking as in the traditional torque/force feedback methods. However, since external per-

TABLE I

ABBREVIATIONS, NOTATION AND NUMERICAL VALUES

J_c (Kg.m^2)	column inertia (0.04)
J_a	EPS motor inertia (4.52E-4)
J_l, J_s	load motor and screw-nut inertia (98E-5, 17E-5)
β_c (N.m.s/rad)	column damping (0.361)
β_a	EPS motor damping (3.339E-3)
β_l, β_s	load motor and screw-nut damping (0, 0.0405)
m_r (Kg)	rack mass (2.5)
β_r (N.s)	rack damping (0.361)
K_c (N.m/rad)	column stiffness 172
r_p (m)	pinion radius (0.0077)
$k_{t,a}$, $k_{t,l}$ (N.m/A)	torque constant (1.345, 1.07)
N_a,N_l	EPS gear ratio (16, 10)
p (m), η	screw-nut thread and efficiency (0.025, 80%)

turbations applied on the simulator's rack are of high level, a VGSTA control is applied to achieve an accurate position tracking and compensate for growing perturbations. Finally, estimation and force feedback are validated by considering two maneuver cases, at parking and at cornering. There effectiveness is proven.

VIII. ACKNOWLEDGMENTS

This research was supported by the French National Research Agency (ANR) (VOLHAND project "ANR-09-VTT-14-01/06"), the International Campus on Transportation Safety and Intermodality, the European Community, the Regional Delegation for Research and Technology, the Ministry of Higher Education and Research, the Nord/Pas-de-Calais Region and the CNRS.

APPENDIX

$$m_{eq} = m_r + \frac{N_a^2}{r_p^2} J_a \quad \beta_{eq} = \beta_r + \frac{N_a^2}{r_p^2}\beta_a$$

$$m'_{eq} = \frac{p}{2\pi} m_{eq} + \frac{2\pi}{p}\left(J_s + N_l^2 J_s\right)$$

$$\beta'_{eq} = \frac{p}{2\pi}\beta_{eq} + \frac{2\pi}{p}\left(\beta_s + N_l^2 \beta_s\right)$$

$$\mathcal{A}_r = \begin{bmatrix} -\frac{\beta_c}{J_c} & -\frac{K_c}{J_c} & 0 & \frac{K_c}{r_p J_c} \\ 1 & 0 & 0 & 0 \\ 0 & \frac{K_c}{r_p m_{eq}} & -\frac{\beta_{eq}}{m_{eq}} & -\frac{K_c}{r_p^2 m_{eq}} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathcal{B}_r = \begin{bmatrix} \frac{1}{J_c} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\frac{N_a}{r_p m_{eq}} & -\frac{1}{m_{eq}} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{A}_s = \begin{bmatrix} -\frac{\beta_c}{J_c} & -\frac{K_c}{J_c} & 0 & \frac{K_c}{r_p J_c} \\ 1 & 0 & 0 & 0 \\ 0 & \frac{pK_c}{2\pi\eta r_p m'_{eq}} & -\frac{\beta'_{eq}}{m'_{eq}} & -\frac{pK_c}{2\pi\eta r_p^2 m'_{eq}} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathcal{B}_s = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{pN_a k_{t,a}}{2\pi\eta r_p m'_{eq}} & \frac{N_l k_{t,l}}{m'_{eq}} \\ 0 & 0 \end{bmatrix} \quad \mathcal{D}_s = \begin{bmatrix} \frac{1}{J_c} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

REFERENCES

[1] A. Marouf, C. Sentouh, M. Djemaï, and P. Pudlo, "Control of Electric Power Assisted Steering System Using Sliding Mode Control", IEEE Conf. on Intelligent Transportation Systems, 2011, pp. 107-112, Washington, DC , USA.

[2] A. Marouf, C. Sentouh, M. Djemaï and P. Pudlo, "Control of an Electric Power Assisted Steering System Using Reference Model", IEEE Conf. Decision and Control , 2011, pp. 6684-6690, Orlando, USA.

[3] A. Marouf, M. Djemaï, C. Sentouh and P. Pudlo, "A New Control Strategy for an Electric Power Assisted Steering System", IEEE Trans. on Vehicular Technology, 2012, (To appear).

[4] F. Gabrielli, P. Pudlo and M. Djemaï, "Instrumented Steering Wheel for Biomechanical Measurements", International Journal of Mechatronics, 2012, (To appear).

[5] L. Nehaoua, H. Mohellebi, A. Amouri, H. Arioui, S. Espié and A. Kheddar, "Design and Control of a Small-Clearance Driving Simulator", IEEE Trans. on vehicular technology, 2008, vol. 57(2) pp. 736-746.

[6] L. Nehaoua, S. Hima, H. Arioui, N. Séguy and S. Espié, "Design and Modeling of a New Motorcycle Riding Simulator", IEEE American Control Conference, pp.176-181 , 2007, New York, USA.

[7] T. Park, C. Han and S. Lee, "Development of the electronic control unit for the rack-actuating steer-by-wire using the hardware-in-the-loop simulation system", Mechatronics, 2005, Vol. 15(8), pp. 899-918.

[8] W. Ren, H. Chen and J. Song, "Model-based development for an electric power steering system", Proc. of the Institution of Mechanical Engineers, Part C: J. of Mechanical Engineering Science, 2008, vol. 222(7), pp. 1265-1269.

[9] Y. Chih and H. Shih, "Development of a test bench for tuning and validating electrical power steering control method", IEEE Vehicle Power and Propulsion Conf., 2007, pp. 618-622.

[10] M. Segawa, M. Higashi and S. Nakano, "Development of simulator for evaluation of steering systems", Koyo Engineering J., 2005, vol. 168, pp. 29-34 .

[11] M. Segawa, M. Higashi and S. Nakano, "Development of steering hardware-in-the-loop (HIL) simulator", Proceedings of the FISITA world cong., 2006, vol. 10, pp. 22-27.

[12] L. Nehaoua and H. Arioui and L. Fridman, "Force Feedback Control based on VGSTA for Single Track Riding Simulator", IEEE CDC-ECC, pp. 8243-8248, 2011, Orlando, USA.

[13] L. Nehaoua, A. Marouf, J.J. Santin, P. Pudlo and M. Djemaï, "Towards An Electrical Power-Assisted Steering Simulator : Modeling Specifications", IFAC Symposium on Mechatronic Systems, 2010, pp. 571-576, Cambridge, MA, USA.

[14] A. Badawy, J. Zuraski, F. Bolourchi and A. Chandy, "Modeling and analysis of an electric power steering system", SAE paper, 1999.

[15] L. Fridman, A. Levant and J. Davila, "Observation of linear systems with unknown inputs via high-order sliding-modes", Int. J. of Systems Science, Vol. 38, N. 10, October 2007, pp. 773-791.

[16] A. Isidori, "Nonlinear Control Systems", London: Springer-Verlag, 1996.

[17] S. Hui and S.H. Zak, "Observer design for systems with unknown inputs", International Journal of Applied Mathematics and Computer Science 2005, vol. 15(14), pp 431-446.

[18] M. Djemaï, J.P. Barbot and K. Busawon, "Designing r-sliding mode control using smooth iterative manifolds", Med. J. of Measurement and Control, 2008, vol(4), pp. 86-93.

[19] J.A. Moreno, "A Linear Framework for the Robust Stability Analysis of a Generalized Super-Twisting Algorithm", Proc.IEEE Conf. on Electrical Engineering, Computing Science and Automatic Control, 2009, pp 1-6, Toluca, Mexico.

[20] A.Dávila, J.A. Moreno and L. Fridman, "Variable Gains Super-Twisting Algorithm: A Lyapunov Based Design", Proc. IEEE American Control Conf., 2010, pp. 241-246, Baltimore, USA.