# Workspace tracking trajectory for 7-DOF ANAT robot using a hierarchical control strategy

Raouf Fareh, Mohamad Saad, and Maarouf Saad

***Abstract*—This paper presents a new hierarchical control strategy for a hyper redundant articulate nimble adaptable trunk (ANAT) robot. The objective is to track a desired trajectory in the robot's workspace. The pseudo-inverse of the Jacobian is used to transform the desired trajectory from the workspace to the joint space. The control strategy consists in controlling the last joint by assuming that the remaining joints are stable and follow their desired trajectories. Then going backward to the (n-1)-th joint, the same strategy is applied and so on until the first joint. In each step, the sliding mode technique is used to develop the control law. This control algorithm was experimented on a 7 DOF ANAT manipulator and gave effective results and good trajectory tracking in the workspace.**

## I. Introduction

THE workspace control of robot manipulators is very important in robotics control area. Workspace control is more interesting than joint space control because the tasks, such as painting, assembly etc., are performed by the end effectors in the robots workspace. Several types of manipulators such as redundant manipulators have been considered for the workspace tracking control. A manipulator is said redundant when the dimension of the joint space position vector is more than the dimension of its workspace vector. For redundant manipulators, the Jacobian matrix has a minimum dimension *n-m*, where *n* and *m* are the dimensions of the joint space and the workspace, respectively. Therefore, there is an infinite number of solutions [1-3] for the inverse kinematics problem.

Several techniques were used in the literature to solve manipulators tracking control problem. In general, two strategies were considered in the literature. The first one considers the robot as one system. One controller that depends on the overall joint space variables is then developed. The second strategy considers the robot manipulator as interconnected subsystems. Several controllers are then considered for each of them.

In the first case, several control techniques were used. Backstepping approach was applied in [4] to an n-DOF non redundant robot manipulator using passivity for trajectory tracking of the robot. Feedback linearization approach was used in [5-9] to solve the tracking control problem of robot manipulators. Adaptive control was used when the system's parameters are unknown. The adaptive control with stability analysis was widely studied in [10-12]. Sliding mode approach was applied in [12] to control robot manipulators using classical sliding surface. To reduce the input control chattering, a novel approach was proposed in [13] where the reaching law has an exponential term that is a function of the sliding surface.

In the second case, decentralized control is used when the robot dynamics can be viewed as interconnected subsystems (joints) [14-16]. A hierarchical control was developed and experimented on a 5 DOF redundant robot in [9, 17]. The control strategy consists in controlling the last joint by assuming that the remaining joints are stable and follow their desired trajectories. Then the same strategy was applied to all remaining joints by going backward until the first joint. Feedback linearization technique was used to develop the control law of each joint.

As stated previously, in the first case, robots are viewed as one system. All joints are then controlled as one MIMO system which makes their industrial implementation difficult. To overcome this problem, we propose, in this work, a hierarchical control strategy to solve the tracking control problem in the workspace for a 7-DOF hyper redundant articulated nimble adaptable trunks (ANAT) robot. For the inverse kinematics problem of the redundant robot, the pseudo-inverse of the Jacobian is used. The hierarchical control strategy consists in controlling the last joint while assuming that the remaining joints are stable and follow their desired trajectories. Then going backward to the (n-1)-th joint, we apply the same strategy, i.e. controlling the (n-1)-th joint while assuming that the remaining joints are stable and follow their desired trajectories, and so on until the first joint. The asymptotical stability is proved using Lyapunov theory. The hierarchical control strategy is tested on the 7-DOF ANAT robot.

The paper is organized as follows: Section 2 presents the hierarchical control strategy. Problem formulation, inverse kinematics and hierarchical control law are also presented in this section. Section 3 presents the experimental results. Finally, conclusions are given in Section 4.

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## II. HIERARCHICAL CONTROL STRATEGY

### A. *Problem Formulation and Preliminaries*

The ANAT robot is shown in Figure 3. The equation of motion of an n-DOF manipulator can be written as [18]:

$$B(q)\ddot{q} + H(q,\dot{q})\dot{q} + F\dot{q} + G(q) = \tau \quad (1)$$

where $q \in \mathcal{R}^n$ denotes the vector of the generalized coordinates in the joint space, $B(q) \in \mathcal{R}^{n\times n}$ is a symmetric positive definite mass and inertia matrix, $H(q,\dot{q})\dot{q}$ is the Coriolis and centrifugal forces vector, $F\dot{q} \in \mathcal{R}^n$ is the friction vector, $G(q) \in \mathcal{R}^n$ is a vector of gravity terms, $\dot{q}$ and $\ddot{q}$ are the joints velocity and acceleration vectors respectively, and finally $\tau \in \mathcal{R}^n$ is the joints input torque vector. The model has the following properties:

- Since the inertia and mass matrix $B(q)$ is a symmetric positive definite matrix, the diagonal elements are positive:

$$B_{ii}(q) > 0; \text{for i} = 1 \ldots \text{n} \quad (2)$$

- $\dot{B}(q) - 2H(q,\dot{q})$ is an anti-symmetric matrix. Then:

$$\dot{B}_{ii}(q) - 2H_{ii}(q,\dot{q}) = 0; \text{for i} = 1 \ldots \text{n} \quad (3)$$

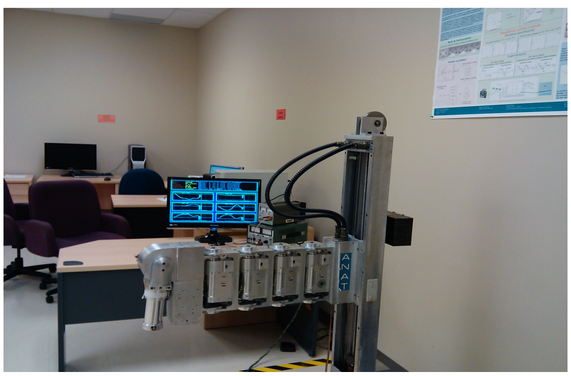

Fig. 1. ANAT robot.

The objective of this works is to track a desired trajectory defined in the ANAT workspace. To achieve this objective, three steps are considered. In the first step, the desired trajectory is transformed from the workspace to the joint space. The generalized inverse Jacobian matrix is then used [18]:

$$\dot{q} = J^{+}\dot{x} \quad (4)$$

where $J^{+} = J^T(J\,J^T)^{-1}$ is the generalized inverse Jacobian matrix, and $\dot{q}$ and $\dot{x}$ are the velocities in the joint space and the workspace respectively.

The second step consists to generate a hierarchical control law to ensure the tracking in the joint space. The direct kinematics is used, in the third step, to transform the joint space trajectory given by the ANAT robot to the workspace. The position of the tool relative to the base reference is given by [18]:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} L_4c_{2345}s_6 + L_3c_{2345} + L[c_{234} + c_{23} + c_2] + L_1 \\ L_4s_{2345}s_6 + L_3s_{2345} + L[s_{234} + s_{23} + s_2] \\ -L_4c_6 + L_2 + q_1 \end{bmatrix} \quad (5)$$

where $s_i = sin(q_i)$ , $c_i = cos(q_i)$ , $s_{ij} = sin(q_i + q_j)$ and $c_{ij} = cos(q_i + q_j)$.

The procedure of the tracking trajectory is given in Figure 2.

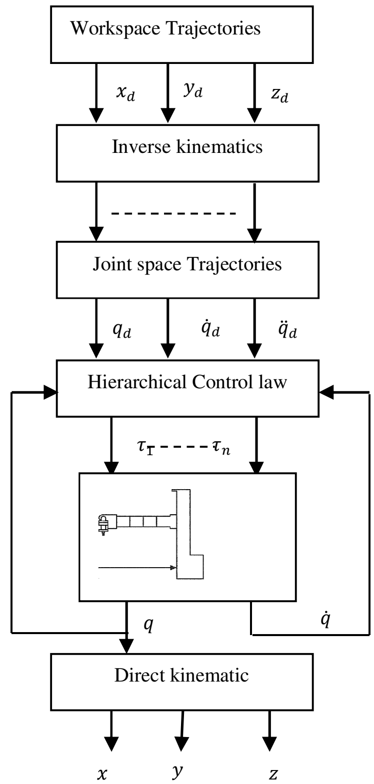


Fig. 2. Tracking control algorithm

### B. *Hierarchical Control Law*

This section presents the hierarchical control strategy. It consists to start by controlling the last joint, then going backward to the (n-1)-th joint and so on until the first joint. At each step, the control law is developed using sliding mode technique and assuming that the remaining joints are stable and follow their desired trajectories. The asymptotical stability is proved, at each step, using Lyapunov theory.

The dynamical model given in (1) can be written as follows:

$$\begin{bmatrix} B_1^T(q) \\ \vdots \\ B_n^T(q) \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \vdots \\ \ddot{q}_n \end{bmatrix} + \begin{bmatrix} H_1^T(q,\dot{q}) \\ \vdots \\ H_n^T(q,\dot{q}) \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \vdots \\ \dot{q}_n \end{bmatrix} + \begin{bmatrix} F_1\dot{q}_1 \\ \vdots \\ F_n\dot{q}_n \end{bmatrix} + \begin{bmatrix} G_1(q) \\ \vdots \\ G_n(q) \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_n \end{bmatrix} \quad (6)$$

where: $B_i^T(q) = [B_{i1}(q) \quad B_{i2}(q) \quad \ldots \quad B_{in}(q)]$ and $H_i^T(q,\dot{q}) = [H_{i1}(q,\dot{q}) \quad H_{i2}(q,\dot{q}) \quad \ldots \quad H_{in}(q,\dot{q})]$; *i=1..n.*

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Let $\overline{q}_n$ the new generalized coordinates for the last joint and $\dot{\overline{q}}_n$ and $\ddot{\overline{q}}_n$ their first and second time derivatives:

$$\overline{q}_n = [q_{1d} \quad \cdots \quad q_{(n-1)d} \quad q_n]^T \tag{7}$$

$$\dot{\overline{q}}_n = [\dot{q}_{1d} \quad \cdots \quad \dot{q}_{(n-1)d} \quad \dot{q}_n]^T \tag{8}$$

$$\ddot{\overline{q}}_n = [\ddot{q}_{1d} \quad \cdots \quad \ddot{q}_{(n-1)d} \quad \ddot{q}_n]^T \tag{9}$$

Then, the equation of motion of the n-th joint becomes:

$$B_n^T(\overline{q}_n)\ddot{\overline{q}}_n + H_n^T(\overline{q}_n, \dot{\overline{q}}_n)\dot{\overline{q}}_n + F_n\dot{q}_n + G_n(\overline{q}_n) = \tau_n \tag{10}$$

To achieve tracking of the last joint, we propose the following control law:

$$\tau_n = B_n^T(\overline{q}_n)\ddot{\overline{q}}_n^* + H_n^T(\overline{q}_n, \dot{\overline{q}}_n)\dot{\overline{q}}_n^* + F_n\dot{q}_n + G_n(\overline{q}_n) + K_{pn}\tilde{q}_n + K_{dn}\dot{\tilde{q}}_n \tag{11}$$

where: $K_{pn}$ and $K_{dn}$ are positive gains, $\dot{\overline{q}}_n^* = \dot{\overline{q}}_{nd} + \gamma_n\tilde{\overline{q}}_n$; $\tilde{\overline{q}}_n = \overline{q}_{nd} - \overline{q}_n$; and $\gamma_n = \frac{K_{pn}}{K_{dn}}$.
Suppose that the sliding surface [12] is as follows:

$$S_n = \dot{\tilde{q}}_n + \gamma_n\tilde{q}_n \tag{12}$$

where: $\tilde{q}_n = q_{nd} - q_n$ is the error of the n-th joint angle. Then we can write:

$$K_{pn}\tilde{q}_n + K_{dn}\dot{\tilde{q}}_n = K_{dn}S_n \tag{13}$$

Inserting the control law (11) in the equation of motion (10), the error dynamics becomes:

$$B_n(\overline{q}_n)\left[\ddot{\overline{q}}_n^* - \ddot{\overline{q}}_n\right] + H_n(\overline{q}_n, \dot{\overline{q}}_n)\left[\dot{\overline{q}}_n^* - \dot{\overline{q}}_n\right] + K_{pn}\tilde{q}_n + K_{dn}\dot{\tilde{q}}_n = 0 \tag{14}$$

In addition, we can write:

$$\left[\dot{\overline{q}}_n^* - \dot{\overline{q}}_n\right] = [0 \ \ldots 0 \ \ S_n]^T \tag{15}$$

$$\left[\ddot{\overline{q}}_n^* - \ddot{\overline{q}}_n\right] = \left[0 \ \ldots 0 \ \ \dot{S}_n\right]^T \tag{16}$$

Using (13), (15) and (16), the error dynamics is given by:

$$B_{nn}(\overline{q}_n)\dot{S}_n + H_{nn}(\overline{q}_n, \dot{\overline{q}}_n)S_n + K_{dn}S_n = 0 \tag{17}$$

To prove the asymptotical stability of the last joint, we define the following positive definite Lyapunov function:

$$V_n(t) = \frac{1}{2}B_{nn}(\overline{q}_n)S_n^2 \tag{18}$$

Taking the time derivative of $V_n(t)$, we get:

$$\begin{aligned}\dot{V}_n(t) &= B_{nn}\dot{S}_n s_n + \frac{1}{2}\dot{B}_{nn}S_n^2 \\ &= (-H_{nn}S_n - K_{dn}S_n)S_n + \frac{1}{2}\dot{B}_{nn}S_n^2 \\ &= -K_{dn}S_n^2 + (\frac{1}{2}\dot{B}_{nn} - H_{nn})S_n^2\end{aligned}$$

Using the proprieties given in Section 2, the time derivative of $V_n(t)$ becomes:

$$\dot{V}_n(t) = -K_{dn}S_n^2 \leq 0 \tag{19}$$

Since $K_{dn}$ is positive and using LaSalle theorem, the error dynamics (17) resulting from the above control law (11) are asymptotically stable.
The same strategy is applied backward to the (n-1)-th joint and so on until the first joint. Taking for example the i-th joint, the equation of motion is given by the following expression:

$$B_i^T(\overline{q}_i)\ddot{\overline{q}}_i + H_i^T(\overline{q}_i, \dot{\overline{q}}_i)\dot{\overline{q}}_i + F_i\dot{q}_i + G_i(\overline{q}_i) = \tau_i \tag{20}$$

where: $B_i^T(\overline{q}_i) = [B_{i1} \ldots \quad B_{ii} \quad \ldots \quad B_{in}]$ and $H_i^T(\overline{q}_i, \dot{\overline{q}}_i) = [H_{i1} \ldots \quad H_{ii} \quad \ldots \quad H_{in}]$
The new generalized coordinates associated with the i-th joint are given by:

$$\overline{q}_i = [q_{1d} \quad \cdots q_{(i-1)d} \ q_i \ q_{(i+1)d} \ \cdots \quad q_{nd}]^T \tag{21}$$

$$\dot{\overline{q}}_i = [\dot{q}_{1d} \quad \cdots \dot{q}_{(i-1)d} \ \dot{q}_i \ \dot{q}_{(i+1)d} \cdots \quad \dot{q}_n]^T \tag{22}$$

$$\ddot{\overline{q}}_i = [\ddot{q}_{1d} \quad \cdots \quad \ddot{q}_{(i-1)d} \ \ddot{q}_i \ \ddot{q}_{(i+1)d} \cdots \quad \ddot{q}_n]^T \tag{23}$$

The proposed control law for the i-th joint is:

$$\tau_i = B_i^T(\overline{q}_i)\ddot{\overline{q}}_i^* + H_i^T(\overline{q}_i, \dot{\overline{q}}_i)\dot{\overline{q}}_i^* + F_i\dot{q}_i + G_i(\overline{q}_i) + K_{pi}\tilde{q}_i + K_{di}\dot{\tilde{q}}_i \tag{24}$$

where $K_{pi}$ and $K_{pi}$ are positive gains, $\dot{\overline{q}}_i^* = \dot{\overline{q}}_{di} + \gamma_i\tilde{\overline{q}}_i$, and $\gamma_i = \frac{K_{pi}}{K_{di}}$.
Inserting the control law (24) in the equation of motion (20), the error dynamics is given as follows:

$$B_i(\overline{q}_n)\left[\ddot{\overline{q}}_i^* - \ddot{\overline{q}}_i\right] + H_i(\overline{q}_i, \dot{\overline{q}}_i)\left[\dot{\overline{q}}_i^* - \dot{\overline{q}}_i\right] + K_{pi}\tilde{q}_i + K_{di}\dot{\tilde{q}}_i = 0 \tag{25}$$

or in the following form:

$$B_{ii}(\overline{q}_i)\dot{S}_i + H_{ii}(\overline{q}_i, \dot{\overline{q}}_i)S_i + K_{di}S_i = 0 \tag{26}$$

where $S_i = \dot{\tilde{q}}_i + \gamma_i\tilde{q}_i$ is the sliding surface for the i-th joint.
With the following positive Lyapunov function:

$$V_i(t) = \frac{1}{2}B_{ii}(\overline{q}_i)S_i^2 \tag{27}$$

The time derivative of $V_i(t)$ is:

$$\dot{V}_i(t) = -K_{di}S_i^2 \leq 0 \tag{28}$$

Using LaSalle theorem, the error dynamics is then asymptotically stable.

The same strategy is applied to all joints. The error dynamics are then asymptotically stable. This strategy was tested on the 7-DOF ANAT robot.

### III. APPLICATION ON THE ANAT ROBOT

In this section, experimental results are presented by applying the previous strategy to the ANAT robot. The robot has seven degrees of freedom: the first joint is prismatic, followed by three redundant rotary joints (joints 2, 3, and 4), and an end effector that consists of three rotary joints (joints 5, 6, and 7). In this work, we only consider the prismatic joint and the following four joints and we assume that the sixth and seventh joints are locked.

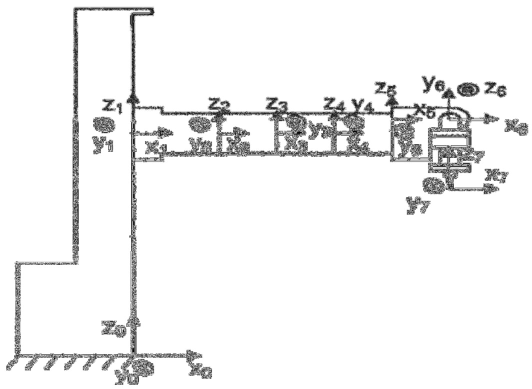


Fig. 3. ANAT robot axes.

Simulink with Real-Time Workshop (RTW) of Mathworks® is used for the implementation of the proposed controller. The national Instruments PCI 6024E digital card is chosen for the real-time target. The ATMEGA 16 microcontrollers are used to translate signals to serial peripheral interface.

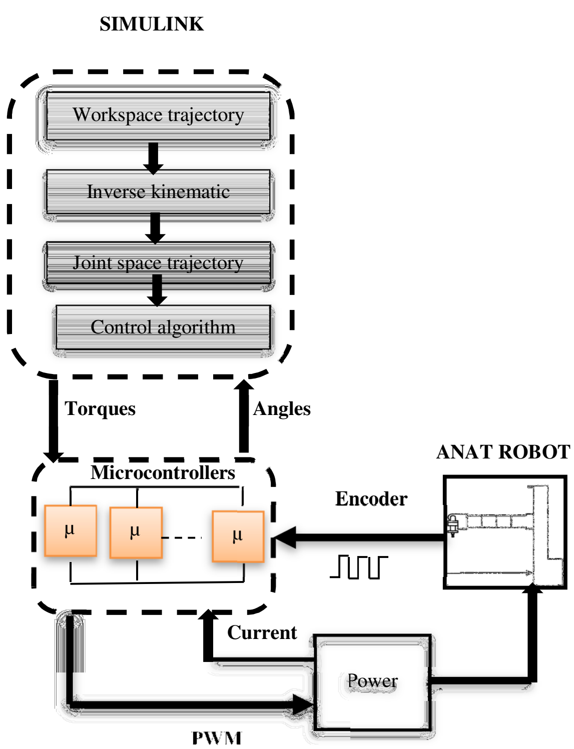
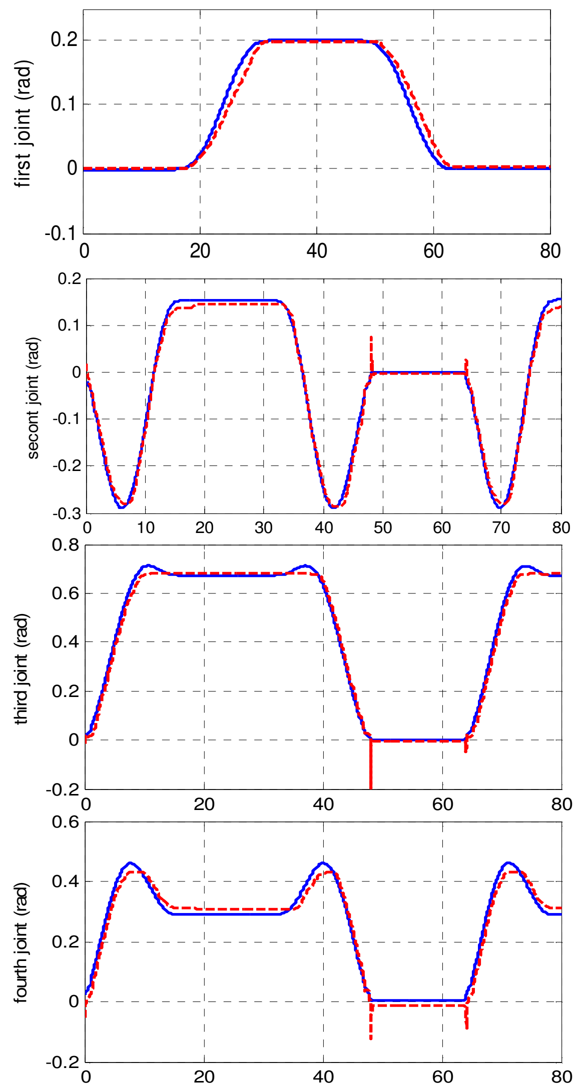



Fig. 4. Real-Time setup.

Using Matlab/Simulink, the trajectory in the workspace is first defined. The generalized inverse Jacobian matrix is used to transform the desired trajectory from the workspace to the joint space. Using the desired trajectory of the joints and their real values from the microcontrollers, the control algorithm executes and sends computed torques to the microcontrollers. The microcontrollers translate the control signals into a pulse width modulation (PWM) signal. The latter is applied to the H-bridge drive of the actuators of the ANAT robot. The current of each actuator is measured by a current sensor located in the H-bridge drive. The microcontrollers process the digital information of the actuators encoders and send the angles positions to Simulink. The real-time setup is given in Figure 4.

The control strategy presented in the previous section was tested on the ANAT manipulator to track a desired trajectory in rectangular form. The controller gains are chosen as follows: $K_{p1} = 100, K_{p2} = 35, K_{p3} = 20,\ K_{p4} = 25,\ K_{p5} = 20;\ K_{d1} = 100, K_{d2} = 1.85, K_{d3} = 2, K_{d4} = 4, K_{d5} = 4$ ;



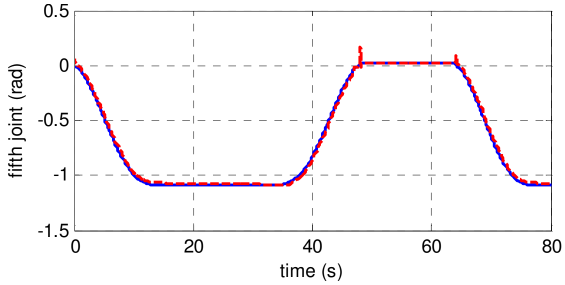



Fig. 5. Joint space tracking.

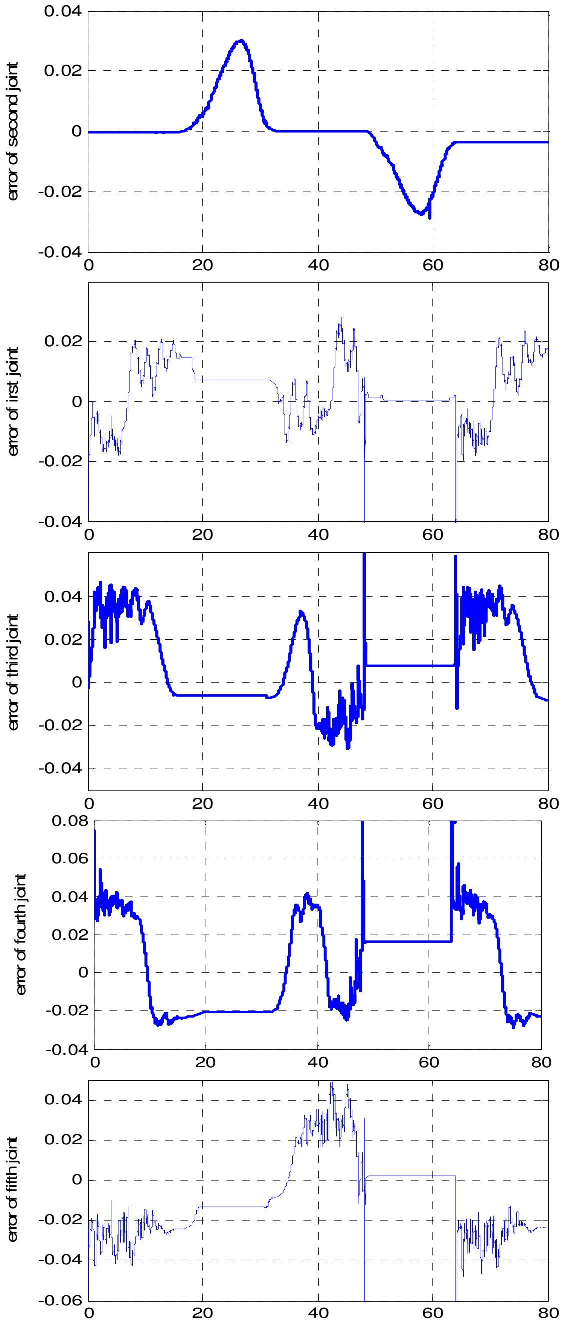
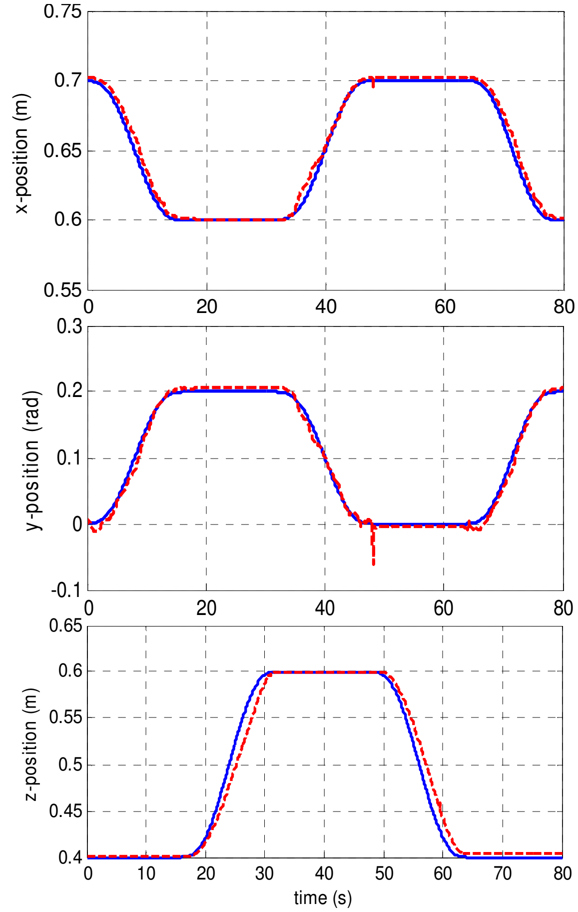



Fig. 6. Error tracking in joint space.



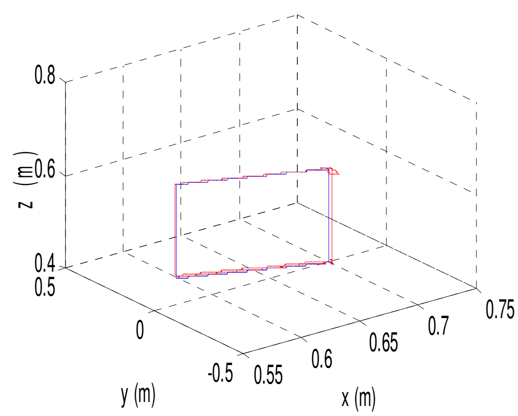




Fig. 7. Workspace tracking trajectories.

The experimental results given in Figure 5 show a good tracking of the desired trajectories in the joint space. The tracking errors given in Figure 6 confirm the result of figure 5. Using the direct kinematics, Figure 7 shows a good tracking in the workspace. According to the experimental results, for the hierarchical control, we can observe very small tracking errors for the five joints affected by the rectangular workspace trajectory. This illustrates the effectiveness of the proposal approach.

## 5. CONCLUSION

A nonlinear hierarchical control strategy is proposed in this paper for a hyper redundant articulated nimble adaptable trunk (ANAT) robot to track a desired trajectory, in the workspace, in rectangular form. The strategy of the hierarchical control consists to start by controlling the last joint while assuming that the remaining joints are stable. Then, we follow the same strategy backward until the first joint. The developed control laws are based on sliding mode technique. Lyapunov theory is used in this work to prove the (local) asymptotical stability of the errors dynamics. The effectiveness of the hierarchical control is confirmed by the experimental results that show a good tracking in the workspace. Analysis was limited to local stability in this paper. Further work is being developed to extend the results to prove global stability of the errors dynamics.

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