A New Look at Observers for Systems with Measurement Uncertainty

Monty J. Smith

Abstract -The work that follows is a continuation of the investigation of metric based nonlinear state transformations, but with applications to observers for plants with measurement uncertainty. In particular, these transformations are shown to be useful for linear systems, and appear to be highly effective when bounded sensor measurement uncertainty is present. It turns out that a nonlinear observer can be constructed based upon the existing Luenberger linear structure, i.e., Kalman filter. This nonlinear structure enables the estimated state reconstruction to become less sensitive to sensor uncertainty due to the transformation itself.

1. Introduction

The idea of state estimation with measurement uncertainty is a current topic of rigorous study, and the topic evolved from modern control theory and the need for estimation of states that weren't available via direct measurements (for example, [1-3]). During the evolution of these algorithms, various strategies have been proposed for robust state reconstruction, even as sensor measurements become contaminated with biases superimposed onto the measured signals (see [6],[7],[9],[14-16] for rigorous studies). Numerous investigations with hardware specifically targeting sensor bias and estimation have been published (see [8],[10-13], for a partial listing).

In this note, a different approach will be presented for the successful construction of the state of linear systems with bounded sensor measurement uncertainties. This approach uses the same isomorphic metric based transformation of state that has been previously considered for control algorithm development [5]. The state estimation formulation under investigation will be based on the linear detectable system of the form

$$(\Sigma_1) \quad \dot{x} = Ax + Bu\,, \quad z = Mx + v \tag{1}$$

where $x \in \Re^n$ are coordinates in state space with initial states at time zero designated by $x(0) = x_0$, the known state space realization $\{A, B, M, 0\}$ and the matrix A Hurwitz, the measurement vector $z \in \Re^z$, and measurement uncertainties denoted by $\{v\}$, and the notation $\dot{x} = dx/dt$ denoting the time derivative. In particular, the measurement uncertainty is assumed to be any bounded time varying signal, i.e., $\|v\| \le \gamma_v$, where the notation $\|\cdot\|$ denotes the standard Euclidean norm and γ_v is an assumed known constant. In addition, the realization is assumed detectable.

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The presented material that follows is organized by topics. A very brief review of the pertinent structure of classical state estimation is provided first, followed by the metric based state transformation that is applicable to state estimation with sensor measurement uncertainty. The mathematical derivations will be provided for the construction of the estimation algorithm, and the advantages and limitations will be discussed. The final section concludes with simulations and a brief discussion on future research.

2. Linear Estimation / Brief Review

Similar to the format given in (1), assume that the following system replicates the system (Σ_1) with the exception that the measurement uncertainty is assumed zero mean without the bias, and there exists an exogenous system disturbance,

$$(\Sigma_2) \quad \dot{x} = Ax + Bu + Dw, \quad z = Mx + v \tag{2}$$

From consulting the literature on estimation theory (for example, [1-3]) and denoting the estimated state and measurement x_2 and z_2, respectively, the state x can be reconstructed by forming the following realization,

$$\begin{aligned}(\Sigma_3)\ \dot{x}_2 &= Ax_2 + Bu + F(z - z_2) \quad z = Mx + v \\ &= Ax_2 + Bu + FMx_2 + Fv - FMx_2,\ z_2 = Mx_2\end{aligned} \tag{3}$$

where F is constructed to assure the matrix $(A - FM)$ is Hurwitz. By defining $\tilde{x}_2 = x_2 - x$ as the error between the state x and the estimated state x_2, the first derivative is given by

$$\begin{aligned}\dot{\tilde{x}}_2 &= (A - FM)\tilde{x}_2 + Fv - Dw \\ &= A\tilde{x}_2 - FMx_2 + Fz - Dw.\end{aligned} \tag{4}$$

This particular realization is assumed time invariant for the sake of presentation. However,

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the results being presented are not limited to time invariance, and can be easily formatted to the time varying realization (omitted due to brevity). The transformation of interest will be discussed in the next section along with some comments pertaining to the current application.

3. Transformation

In this section, we will briefly review the transformation used in [5] and discuss the implications regarding the systems being considered in this paper.

Recall that the functional $\tilde{x} = \phi(x) = \|x\|^{2/p-1} x$ for $x \in \Re^n$ and any $p \in (0, \infty)$ is isomorphic on $\Re^n$ and dilates (or contracts, depending on the size of $\|x\|$) the original state and provides a means for scaling the state variables defined by $\phi(x)$, $\|\phi(x)\|^p = \left\| \|x\|^{2/p-1} x \right\|^p = \|x\|^2$, and it's inverse, $x = \phi^{-1}(\phi(x)) = \|\phi(x)\|^{p/2-1} \phi(x)$. It will be of interest in the latter part of this paper to use this functional, but introduce a positive weight $q \in \Re^+$ on the state $x \in \Re^n$ to assure that the norm $\|q\, x\| < 1$, and hence $\|q\, x\|^{2/p} < 1 \;\forall\; p \le 2$, i.e., $\tilde{x} = \phi(q\, x) = \|q\, x\|^{2/p-1} q\, x$. In fact,

$$\lim_{p \to 0} \|q\, x\|^{2/p} = 0 \;\; \forall \|q\, x\| < 1$$

In the current estimation problem, this transformation will be useful for setting up a new realization with applications to the state estimation error $\tilde{x}_2 = x_2 - x$ as presented in the next section. Finally, recall that this transformation applied to a standard first order state space realization results in the transformed state (recited from [5]).

<u>Lemma</u> *If the system ($\sum$) is linear, i.e., defined by the following realization,*

$$\dot{x} = Ax + B(u + w), \qquad z = Mx + v \tag{5}$$

then the resulting dilated state transformed system (has the following structure;

$$\dot{\tilde{x}} = \tilde{A}(\tilde{x})\tilde{x} + \tilde{B}(\tilde{x})(u + w), \quad \tilde{z} = M\phi^{-1}(\tilde{x}) + v \tag{6}$$

where the state dependent matrices are given by

$$\tilde{A}(\tilde{x}) = \left\{ I + (2/p - 1)\frac{\tilde{x}\tilde{x}'}{\|\tilde{x}\|^2} \right\} A \;\; and$$
$$\tilde{B}(\tilde{x}) = \left\{ I + (2/p - 1)\frac{\tilde{x}\tilde{x}'}{\|\tilde{x}\|^2} \right\} \|\tilde{x}\|^{1-p/2} B. \tag{7}$$

<u>Comment</u> A function defined by individually normalizing and multiplying each element of the state $x \in \Re^n$ with a weight $Q(x) \in R^{nxn}$ could be used to prioritize certain elements of the state, i.e., for an example $x \in \Re^3$,

$$\tilde{x} = Q(x)\, x = \begin{bmatrix} \|q_1\, x_1\|^{2/p_1 - 1} & 0 & 0 \\ 0 & \|q_2\, x_2\|^{2/p_2 - 1} & 0 \\ 0 & 0 & \|q_3\, x_3\|^{2/p_3 - 1} \end{bmatrix} x \tag{8}$$

where the positive scalar pairs satisfy $\|q_i x_i\| < 1 \;\; i = 1, 2, 3$ and hence $\|q_i x_i\|^{2/p} < 1 \;\forall\; 0 < p_i \le 2$ and can be individually chosen.

The following section details exclusively how this transformation can be applied to systems with measurement bias (sensor uncertainty), and the potential assets with applications to estimation and state recovery when measurement error is present.

4. Estimator in Transformed Coordinates

In this section, the nonlinear structure of the estimator being proposed is derived, and a careful discussion is presented on the potential assets to state estimation when sensor bias exists.

To begin, recall that our major assumption is that the exogenous process noise is negligible. This implies that the observer error structure from section 2 can be expressed in the following form,

$$\dot{\tilde{x}}_2 = x_2 - x = A\tilde{x}_2 - FMx_2 + Fz = (A - FM)\tilde{x}_2 + Fv \tag{9}$$

In addition, the solution to (9) is given by

$$\tilde{x}_2(t) = e^{(A-FM)t}\tilde{x}_2(0) + \int_0^t e^{(A-FM)(t-\tau)} v(\tau)\, d\tau. \tag{10}$$

Recall that the solution in (10) is unobtainable (in the format shown) due to the unknown measurement noise v(t) for any initial guess $x_2(t = 0)$. However, it will be shown that the first order format for $\dot{\tilde{x}}_2 = \dot{x}_2 - \dot{x}$ in (9) provides us sufficient information to build a new state estimate $x_p(t)$ that converges to the state $x(t)$ in accordance with the following limit,

$$\lim_{t \to \infty} x_p - x = \|x_2 - x\|^{2/p-1}(x_2 - x) = \|\tilde{x}_2\|^{2/p-1}\tilde{x}_2 \tag{11}$$

and hence $x_p \to x$. In addition, the error converges for any initial assigned state estimates $x_p(t = 0)$, $x_2(t = 0)$, and unknown state $x(t = 0)$ due to the Hurwitz criteria on the matrix

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A. The existence of a state vector x_p satisfying (11) provides a means for removing the effects of sensor bias provided the error $\|\tilde{x}_2\| < 1$. This bound can always be accomplished by introducing an appropriate normalizing weight q in the transformed system $\|q\tilde{x}_2\| < 1$, as previously discussed in section 3.

The following result proves that a state vector x_p exists that satisfies the equality in (11), and additionally provides an algorithm for computing the state x_p.

Theorem 1 *Consider the linear time invariant stable system with bounded sensor noise* $(\|v\| \le \gamma_v)$ *in accordance with the following realization*

$$\dot{x} = Ax + Bu, \quad z = Mx + v,$$

with a pre-constructed linear estimator

$$\dot{x}_2 = Ax_2 + Bu + F(z - z_2) \quad z_2 = Mx_2,$$

where the matrix $(A - FM)$ *is Hurwitz. Denote the state derivative* $\dot{\tilde{x}}_2$ *by the error between the state* $\dot{x}$ *and the estimated state* $\dot{x}_2$,

$$\dot{\tilde{x}}_2 = \dot{x}_2 - \dot{x} = A\tilde{x}_2 - FMx_2 + Fz$$

Then a new state x_p *can be constructed that satisfies*

$$\lim_{t\to\infty} x_p - x = \|x_2 - x\|^{2/p-1}(x_2 - x) = \|\tilde{x}_2\|^{2/p-1}\tilde{x}_2 \cdot$$

In addition, the resulting first order form of x_p *is given by*

$$\dot{x}_p = Ax_p + Bu - A\|\tilde{x}_2\|^{2/p-1}\tilde{x}_2 + \frac{d}{dt}\{\|\tilde{x}_2\|^{2/p-1}\tilde{x}_2\} \quad (12)$$

and the state $\tilde{x}_p = x_p - x$ *results in stable tracking.*

Proof Outline

Assume that $x_p - x \to \|\tilde{x}_2\|^{2/p-1}\tilde{x}_2$ holds. In the limit, $x_p - x = \|\tilde{x}_2\|^{2/p-1}\tilde{x}_2$ and taking derivatives we obtain

$$\begin{aligned}\dot{x}_p &= \dot{x} + \frac{d}{dt}\{\|\tilde{x}_2\|^{2/p-1}\tilde{x}_2\} = Ax + Bu + \frac{d}{dt}\{\|\tilde{x}_2\|^{2/p-1}\tilde{x}_2\} \\ &= A(x_p - \|\tilde{x}_2\|^{2/p-1}\tilde{x}_2) + Bu + \frac{d}{dt}\{\|\tilde{x}_2\|^{2/p-1}\tilde{x}_2\} \\ &= Ax_p + Bu + \{-A\|\tilde{x}_2\|^{2/p-1}\tilde{x}_2 + \frac{d}{dt}\{\|\tilde{x}_2\|^{2/p-1}\tilde{x}_2\}\end{aligned}$$

which is identically (12). In addition, this term can be expressed in the following format,

$$\dot{x}_p = Ax_p + Bu + e^{At}\frac{d}{dt}\{e^{-At}(\|\tilde{x}_2\|^{2/p-1}\tilde{x}_2)\} \cdot$$

Following integration of the last equation we obtain

$$\begin{aligned}x_p(t) &= e^{At}\{x_p(0) - \|\tilde{x}_2(0)\|^{2/p-1}\tilde{x}_2(0)\} \\ &+ \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau + \|\tilde{x}_2(t)\|^{2/p-1}\tilde{x}_2(t)\end{aligned} \quad (13)$$

Subtracting the deterministic state x(t),

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau$$

from the previous term, we obtain

$$\begin{aligned}x_p(t) - x(t) &= e^{At}\{x_p(0) - \|\tilde{x}_2(0)\|^{2/p-1}\tilde{x}_2(0) - x(0)\} \\ &+ \|\tilde{x}_2(t)\|^{2/p-1}\tilde{x}_2(t)\end{aligned} \quad (14)$$

and hence $x_p - x \to \|\tilde{x}_2\|^{2/p-1}\tilde{x}_2$ for any stable system and any set of initial conditions $\{x(0), x_p(0), \tilde{x}_2(0)\}$, and this completes the proof.

Comment 1 The first order state space format expressed in (7) is well defined for $0 < p \le 2$ (non-smooth otherwise [4]), and explicitly given by

$$\dot{x}_p = Ax_p + Bu - A\|\tilde{x}_2\|^{2/p-1}\tilde{x}_2 + \frac{d}{dt}\{\|\tilde{x}_2\|^{2/p-1}\tilde{x}_2\}$$

where the last derivative was explicitly given in [1], and is given by the following expression

$$\frac{d}{dt}\{\|\tilde{x}_2\|^{2/p-1}\tilde{x}_2\} = \|\tilde{x}_2\|^{2/p-1}\left\{I + (2/p-1)\frac{\tilde{x}_2\,\tilde{x}_2'}{\|\tilde{x}_2\|^2}\right\}\dot{\tilde{x}}_2$$

with $\dot{\tilde{x}}_2$ as previously given in (9).

Comment 2 If the initial state is deterministic and known, and the appropriate initial conditions are satisfied, i.e., $x_p(0) - \|\tilde{x}_2(0)\|^{2/p-1}\tilde{x}_2(0) - x(0) = 0$, then $x_p - x = \|\tilde{x}_2\|^{2/p-1}\tilde{x}_2$ holds for any matrix A and any initial state estimate $x_2(t=0)$ as clearly seen in (14)

The main contribution of this note is summarized in the next section, which outlines the application of Theorem 1 to systems with sensor bias. A resulting nonlinear first order realization is constructed for implementation and simulation purposes.

5. Applications to Sensor Bias

In this section, applications of Theorem 1 will be applied to systems with sensor bias. It is presumed that an existing estimation filter has been constructed (a gain F exists based on some pre-existing performance criteria), and the

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transformation under consideration is being applied to reduce the effects of sensor bias and sensor measurement uncertainty. The main result is presented in the following Theorem.

<u>*Theorem 2*</u> *Consider the linear time invariant stable system with sensor noise, i.e., defined by the following realization*

$$\dot{x} = Ax + Bu\,, \quad z = Mx + v$$

and a pre-constructed linear estimator (designed without consideration of sensor bias,

$$\dot{x}_2 = Ax_2 + Bu + F(z - z_2) \quad z_2 = Mx_2\,,$$

where the matrix $(A - FM)$ *is Hurwitz. Then assuming that a predetermined positive scalar q has been chosen such that the bound holds*

$$\| q\,\tilde{x}_2(t) \| = \| q\{ \int_0^t e^{(A-FM)(t-\tau)} (v_1(\tau) + v(\tau)) d\tau \} \| < 1 \; \forall\, t \ge t_1$$

for the sensor bias and zero mean noise source $\bar{v} = v_1 + v$*, then there exists a nonlinear state estimator* x_p *satisfying the limit as* $t \to \infty$*,* $x_p - x \to 0$ *Furthermore, recalling that* $\dot{\tilde{x}}_2 = x_2 - x = A\tilde{x}_2 - FMx_2 + Fz$*, then the structure of this particular state estimator is given by*

$$\dot{x}_p = Ax_p + Bu - A\| q\,\tilde{x}_2 \|^{2/p-1} \tilde{x}_2 + \frac{d}{dt}\{ \| q\,\tilde{x}_2 \|^{2/p-1} q\,\tilde{x}_2 \}$$

and converges for any given initial condition pair $\{x_p(0), \tilde{x}_2(0)\}$.

<u>Proof Outline</u>

Replacing $x_2 - x$ with $q(x_2 - x)$ in (13) we obtain

$$x_p(t) - x(t) = e^{At}\{x_p(0) - \| q\,\tilde{x}_2(0) \|^{2/p-1} q\,\tilde{x}_2(0) - x(0)\} + \| q\,\tilde{x}_2(t) \|^{2/p-1} q\,\tilde{x}_2(t)$$

and since A is Hurwitz and a q can be chosen such that $\| q\,\tilde{x}_2 \| < 1$, the following limits hold,

$$\lim_{p \to 0} \| q\tilde{x}_2 \|^{2/p-1} = 0 \qquad \lim_{t \to \infty} e^{At}\eta = 0 \;\; \forall \eta$$

Thus, for any epsilon $\varepsilon > 0$ there exists positive constants $\{t_1, p_1, \lambda, \gamma_0\}$ such that for all time $t \ge t_1 > 0$ and all p satisfying $0 < p \le p_1 < 2$

$$\begin{aligned}
\| x_p(t) - x(t) \| &= \| e^{At}\{x_p(0) - \| q\,\tilde{x}_2(0) \|^{2/p-1} q\,\tilde{x}_2(0) - x(0)\} + \| q\,\tilde{x}_2(t) \|^{2/p-1} q\,\tilde{x}_2(t) \| \\
&\le \| e^{At}\{x_p(0) - \| q\,\tilde{x}_2(0) \|^{2/p-1} q\,\tilde{x}_2(0) - x(0)\} \| + \| \| q\,\tilde{x}_2(t) \|^{2/p-1} q\,\tilde{x}_2(t) \| \\
&\le e^{-\lambda t}\gamma_0 + \| \| q\,\tilde{x}_2(t) \|^{2/p-1} q\,\tilde{x}_2(t) \| \\
&\le \varepsilon/2 + \| \| q\,\tilde{x}_2(t) \|^{2/p-1} q\,\tilde{x}_2(t) \| \;\; \forall\, t \ge t_1 \\
&\le \varepsilon/2 + \varepsilon/2 \quad \forall\, t \ge t_1 \textit{ and } p \le p_1
\end{aligned}$$

and hence $x_p - x \to 0$.

Although extremely restrictive, if the state initial condition is assumed known ($x(t=0)$), then Theorem 2 is good for any given realization (unstable inclusive) as stated in the following Corollary.

<u>Corollary</u> *If the initial condition is known* ($x(t=0)$)*, then a nonlinear filter* x_p *described in Theorem 2 can be constructed that converges to the original state* x *(i.e.,* $x_p - x \to 0$) *for any observable system (unstable inclusive) with the sensor bias described by the format in (1).*

<u>Proof</u> Since the following equality holds,

$$x_p(t) - x(t) = e^{At}\{x_p(0) - \| \tilde{x}_2(0) \|^{2/p-1} \tilde{x}_2(0) - x(0)\} + \| \tilde{x}_2(t) \|^{2/p-1} \tilde{x}_2(t)$$

Setting $x_p(0) - \| \tilde{x}_2(0) \|^{2/p-1} \tilde{x}_2(0) = x(0)$ implies that $x_p(t) - x(t) = \| \tilde{x}_2(t) \|^{2/p-1} \tilde{x}_2(t) \;\forall\, t \ge 0$ and hence $x_p - x \to 0$ from the proof of Theorem 2.

It's interesting to note that if there is a disturbance w (as in equation (2)), then the loop feed through from the disturbance w to the state x exists in the error. It's easily shown that this error is the open loop input to output map (equivalent to setting the gain F to zero in the standard existing linear estimation filter). That is,

$$\lim_{p \to 0} x_p - x = -\int_0^t e^{A(t-\tau)} Dw(\tau) d\tau\,.$$

In order to implement this nonlinear state estimation filter, the first order states $\{\dot{x}_2, \dot{\tilde{x}}_2, \dot{x}_p\}$ are integrated simultaneously for any given set of initial conditions. This can be summarized as follows.

<u>*Numerical Strategy*</u> A nonlinear state x_p that converges to the original state x (for the intent of increased robustness to sensor bias) exists as described in (1). Furthermore, this state x_p can be constructed by simultaneous integration of the following three nonlinear equations,

$$\dot{x}_p = Ax_p + Bu - A\| q\tilde{x}_2 \|^{2/p-1} q\tilde{x}_2 + \frac{d}{dt}\{ \| q\tilde{x}_2 \|^{2/p-1} q\tilde{x}_2 \}$$

$$\dot{\tilde{x}}_2 = x_2 - x = A\tilde{x}_2 - FMx_2 + Fz$$

$$\dot{x}_2 = Ax_2 + Bu + F(z - z_2) \quad z_2 = Mx_2\,,$$

where the initial conditions can be independently selected. It's interesting that this set of equations requires the integration of three states of n'th order for the construction of x_p. Additionally,

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(as noted in the Corollary), if the state initial condition is known, then an appropriate nonlinear filter starting point results in applications for all linear systems, unstable inclusive. The next section discusses the results from numerical simulation of these nonlinear first order differential equations.

6. Examples / Numerical Simulations

The results of an example simulation are presented. Two scenarios were simulated for comparison, one with an external disturbance, and one with the disturbance set to zero. All state initial conditions were randomly selected with relatively small values.

Example (2nd Order Stable System)

In this example, a randomly selected second order system was chosen for estimating the state using the approach described in this note, with sensor bias inclusive. The sensor noise intensity covariance R and the exogenous disturbance intensity covariance Q used to compute a stabilizing gain matrix F are listed, along with the dc bias and input driver signal.

$$A = \begin{bmatrix} 0 & 1 \\ -0.395 & -1.005 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} \quad M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 9.03 & -0.18 \\ -0.18 & 9.26 \end{bmatrix} \quad Q = 0.0049 \quad v_1 = 1e^3$$

$$F = 1e^{-5}\begin{bmatrix} 0.508 & 0.010 \\ 0.004 & 0.195 \end{bmatrix} \quad u(t) = 0.06 \cdot \sin(2\pi\, 0.01 t)$$

All state initial conditions were randomly selected during simulation, and had relatively little effect on the outcome due to stability, and thus the rapid dissipation due to the initial condition response.

To compare simulations, three transformation scalars were used, $p = \{1.9,\ 1.6,\ 0.5\}$, and the simulations are presented in Figure 1 with a sensor bias superimposed onto the measurement vector at time t = 180.0 seconds. The graphs pertaining to the green, red, and black lines represent the results of the simulation due to the decreasing values for p. That is, the top graph is the results pertaining to $p = \{1.9\}$,the middle plot is the resulting simulation for $p = \{1.6\}$, and finally the bottom plot of Figure 1 is the resulting simulation for $p = \{0.5\}$. The reduction in estimated state position error occurs as $p \to 0$ as the theoretical results predict.

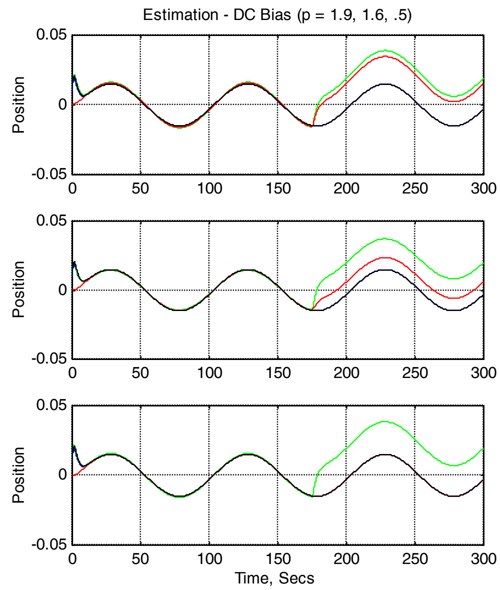

Figure 1 Simulation (Zero External Disturbance)

As mentioned, the dc bias in this simulation was imposed onto the sensor at 180 seconds and zero exogenous disturbances were imposed (i.e., w(t) = 0). From these simulations, it's apparent that the nonlinear estimation algorithm has shown improvement over the existing linear approach when the dc bias has been superimposed onto the sensor signal.

Since in real physical applications it's likely that there would be an external disturbance, a signal was generated with a covariance matching the signal (i.e., Q) and superimposed onto the control signal and simulated. It's important to recall that the theoretical results for the estimation algorithm used were developed on the assumption that any external driving disturbances would pass through as an open loop throughput. In fact, this would be the simulation obtained in the standard Kalman filter if the estimator gain F were set to zero. However, in real hardware it's usually a presumption that the sensors are being processed and hence the current algorithm suggested for dc bias removal would have its advantages. These results are

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displayed in Figure 2 for the same values of p.

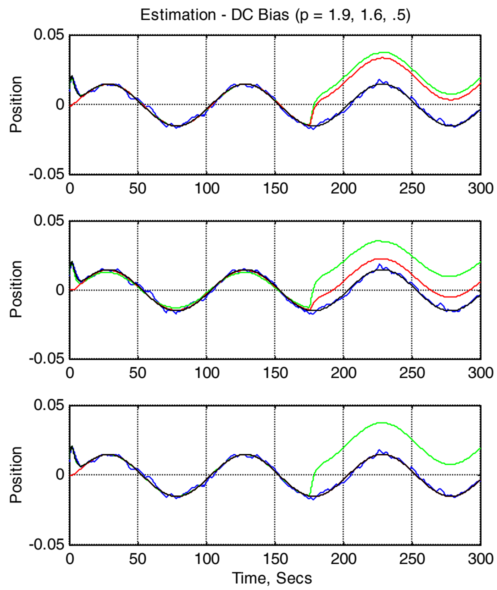

Figure 2 Simulation (External Disturbance Inclusive)

Although the example is simple, the simulations suggest that these results are effective for systems with external disturbances as well.

7. Conclusions

The results presented in this note appear promising for the implementation of nonlinear state estimation algorithms, with particular emphasis on increasing robustness to sensor bias. Numerical bounds pertaining to actual limits on sensor bias and effective performance, and the criteria for stability have to be investigated thoroughly.

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