Reflection Segments Based Stability Domain Approximation of the Robust PID Controller Parameters

Sergei Avanessov, Ülo Nurges

Abstract— **An innovative method for robust stabilizing PI,PD controller design and its stabilizing area approximation is presented for discrete-time SISO plants. It starts from the random generation of Schur stable polynomials using reflection coefficients and reflection segments of polynomials. Stable reflection segments are projected onto affine set of closed-loop characteristic polynomials which is defined by the controller parameters and the stable line segments in the controller parameter space are then determined. The segments of two systems with different parameters are used to generate a rectangular and polygonal approximations of the stabilizing area of robust controller parameters that stabilize both systems.**

Index Terms— **stability area, stabilizing control, robust control, polynomials, discrete-time systemsstability area, stabilizing control, robust control, polynomials, discrete-time systems.**

I. INTRODUCTION

Stabilization of SISO plants by low-order controllers is one of the challenging problems in control theory [1]-[3]. The most of the real-world controllers are PI- or PID-controllers having only two or three free parameters. Such controllers are easy to adjust and implement. However, few control parameters may be insufficient to obtain a stabilizing control law for unstable higher order plants.

In general, fixed-order controller design is a hard problem, since it can be transformed into the problem of finding a stable polynomial in an affine family, which is known to be NP-hard [2],[4]. No method can guarantee a success in finding a stabilizing PID controller for a given plant even if a solution exists. In case of controllers with only two adjustable parameters the classical method of D-decomposition can be applied [5]. Linear programming approach can be used for fixed-order controller design with some strong restrictions [4],[6]. For PID- and higher order controller design some random search methods have been recently proposed [7],[8]. The domain of stability in the controller parameter space is typically very small. That is why the straightforward randomized methods in the coefficient space have weak performance. Some promising results have been obtained via random generation of stable polynomials in the coefficient space of closed-loop characteristic polynomials [9]. To achieve better probability of finding a stabilizing solution a method of combining random stable polynomials and reflection segments approach was studied and analyzed [16].

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This work concentrates on finding robust controllers, stabilizing two different systems by using reflection segments approach. The idea is based on generating a number of stable random polynomials to create two pencils of reflection segments for each of the given unstable systems in the controller coefficients space. Using reflection segments a pencil of stabilizing line segments is obtained for each system, The segments of each pencil are then used to find the number of intersection points with segments from the other pencil. The point of intersection of those two segments, where the total sum of all the intersections in both segments is greatest, defines two best segment candidates, one from each system. Two subsegments of the candidate segments are chosen such that they are stable for both given plants and also intersect with each other. Once two stable subsegments are found, the rectangle consisting of 4 vertices of these subsegments is tested for stability for both given systems using Edge theorem [17, p.264]. Then the vertices of unstable edges of the rectangle are gradually shifted towards the point of intersection using interval halving algorithm to obtain the stability of the rectangle. The idea is then further developed by using the segments that intersect with the rectangle forming segments in a similar way. That allows to create a convex polygonal approximation of the common stability area for two systems.

The paper is organized as follows. In section 2 the problem of PID control is stated. The third section is devoted to the reflection coefficients of polynomials and to the generating of stable line segments in the polynomial coefficient space. In the fourth section the procedure of robust stabilizing fixed-order controller design for two different systems is developed and the stabilizing area approximations are found.

II. STABILIZATION BY CONTROLLERS OF FIXED ORDER

Let a linear SISO plant be specified by a transfer function of the form

$$G(z) = \frac{g(z)}{f(z)}, \qquad deg(g(z)) \leq deg(f(z))$$

and we are looking for a controller of the form

$$C(z) = \frac{q(z,c)}{p(z,c)}$$

where $c = (c_1, ...c_l)^T \in \mathcal{R}^l$ is the vector of unknown coefficients of the controller transfer function which enter linearly the numerator and denominator and let the degrees of the polynomials $q(z)$ and $p(z)$ be fixed.

Mathematically, the problem is as follows: determine if there exists polynomials $q(z, c)$ and $p(z, c)$ of given degrees such that the characteristic polynomial

$$a(z, c) = f(z)p(z, c) + g(z)q(z, c) \quad (1)$$

of the closed-loop system is Schur stable. In other words, the problem lies in determining the values of the parameters c^*, if any, for which $a(z, c^*)$ is stable.

If the controller order is not bounded the solution can be obtained using Yuola parametrization [2]. If the controller has only two adjustable parameters $c = (c_1, c_2)^T$ then the problem of designing a stabilizing controller has a graphic solution by the D-decomposition [5]. But that is not easy to use for analytical purposes.

The problem of stabilizing fixed-structure controller design is one of the hard problems in linear control theory [4]. It means, efficient methods for the solution of the problem are not known and cannot be devised in principle. As a result, the solution methods are either based on sufficient conditions or lean upon numerical procedures. One of the mixed methods, which combines the random search of stable polynomials and the analytical design of a controller, was proposed in [8]. The proposed method consists of the following steps: first, generate randomly a stable polynomial $a(z)$ of degree n; second, project this polynomial on the set $\mathcal{C}$, i.e. obtain a polynomial $a(z, c)$ which minimizes the distance to its prototype $a(z)$; if this projection denoted by $a(z, c^r)$ is stable the point c^r provides a stabilizing controller; third, local shift of the unstable projections to the stability domain in the space of controller parameters.

In principle, we follow the same procedure. However, instead of stabilizing points in $\mathcal{R}^l$ we generate stabilizing line segments in $\mathcal{R}^l$ by the use of reflection segments of polynomials. Because stable line segments are much powerful tools than single stable points the local iterative shift is not needed.

III. GENERATION OF STABLE POLYNOMIALS BY REFLECTION COEFFICIENTS

Polynomials can be determined in terms of their coefficients, roots or reflection coefficients. For controller design the coefficients are the most suitable definition of polynomials because the relation between closed-loop polynomials and controller parameters (1) are affine. The roots of the characteristic polynomial of a closed-loop system (poles of the system) are useful for stability and performance analysis. However, roots are not suited for random generation of stable polynomials in coefficients space [9]. For random generation of stable polynomials the reflection coefficients are most suitable because the relation between reflection coefficients and polynomial coefficients are multilinear.

The reflection coefficients are also known in the literature as FM parameters [8], Schur-Szegö parameters [11] , partial correlation (PARCOR) coefficients [12] or k-parameters [13]. They have been used efficiently in many applications in signal processing [13], system identification [12] and control [8], [14].

Let $a_n(z)$ be a monic polynomial of degree n with real coefficients $a_i \in \mathcal{R}$, $i = 0, ..., n$

$$a(z) = z^n + ... + a_1 z + a_0.$$

The reciprocal polynomial $a_n^*(z)$ of $a_n(z)$ is defined by [11]

$$a_n^*(z) = a_0 z^n + ... + a_{n-1} z + 1.$$

The reflection coefficients k_i , $i = 1, ..., n$ can be obtained from $a_n(z)$ by using backward Levinson's recursion [15]

$$za_{i-1}(z) = \frac{1}{1 - |k_i^2|}[a_i(z) - k_i a *_i (z)] \quad (2)$$

where $k_i = -a_{i,0}$ and $a_{i,0}$ denotes the last coefficient of an i-degree polynomial $a_i(z)$. From (2) the forward recursion can be obtained

$$a_i(z) = za_{i-1}(z) + k_i a_{i-1}^*(z). \quad (3)$$

The main advantage of using reflection coefficients instead of roots is the fact that the transformation from reflection coefficients to polynomial coefficients is very simple. Indeed, according to (3) polynomial coefficients a_i depend multilinearly from reflection coefficients k_i.

The next lemma follows immediately from the multilinear transformation (3) between reflection coefficients and polynomial coefficients [10].

Lemma. Through an arbitrary stable point $a = (a_0, a_1, ..., a_{n-1})$ with reflection coefficients $k_i(a) \in (-1, 1)$, $i = 1, ..., n$ one can draw n stable line segments

$$S^i(a) = conv\{a | k_i(a) = \pm 1\}$$

where $conv\{a | k_i^a = \pm 1\}$ denotes the linear cover of points a obtained by varying the reflection coefficient $k_i(a)$ between -1 and 1.

The endpoints $v_i^{\pm}(a)$ of the line segments $S^i(a)$ are called reflection vectors of the polynomial $a(z)$ [10].

Let us call the stable line segments defined by Lemma 2 *reflection segments* $S_i(a)$, $i = 1, ..., n$ of the Schur stable polynomial $a(z)$. Reflection segments are useful for design of stabilizing controllers, especially for random generation of controller candidates. Indeed, instead of a single stable point $a \in \mathcal{R}^n$ (polynomial) we can easily generate a couple of n stable reflection segments $S_i(a)$, $i = 1, ..., n$.

IV. PROCEDURE OF ROBUST STABILIZING CONTROLLER DESIGN VIA REFLECTION SEGMENTS

A. Generation of random polynomials

According to Lemma 1 by sweeping the unit cube of reflection coefficients $k_i(a) \in (-1, 1)$, $i = 1, ..., n$ and by using the transformation (3) all the stable polynomials $a(z)$ of degree n will be obtained. In this paper the reflection coefficients $k_i(a)$ are generated randomly uniformly on [-1,1]. Neither the coefficients, nor the roots of polynomials generated in such a way are distributed uniformly in the respective domains [9]. However, the method of generation is quite simple, fast and numerically stable and gives quite a representative description of the (bounded) set of all stable polynomials in the coefficients space [8].

B. Projecting on the set of closed-loop polynomials

We identify the nth-degree polynomial $a(z)$ with the n-dimensional vector $a = (a_0, ..., a_{n-1})^T$ of its coefficients. Let a_c denote the corresponding vector of closed-loop polynomial $a_c(z) \in \mathcal{A}_c \subset \mathcal{R}^n$. According to (1) the closed-loop polynomials $a_c(z)$ can be presented via certain constituent polynomials $p_j(z)$, $j = 1, ...l$ (having different degree)

$$a_c(z) = p_0(z) + \sum_{j=1}^{l} c_j p_j(z). \quad (4)$$

This relation can be re-written in the form

$$a_c = Pc + p_0$$

where the rectangular matrix $P \in \mathcal{R}^{n \times l}$ is composed from the coefficient vectors p_j with the properly added zero components - to have the same length. The projection problem of finding a controller which minimizes the distance

$$min_c ||a_c - a|| = min_c ||Pc + p_0 - a||$$

can be solved as the least squares problem

$$c = (P^T P)^{-1} P^T (a - p_0). \quad (5)$$

The projection (5) is linear in respect of polynomials $a(z)$. So, the projections C^i of the reflection segments $S^i(a)$ generated by reflection vectors $v_i^{\pm}(a)$ of a polynomial $a(z)$ are obtained as the line segments between the points

$$c_i^{\pm}(a) = (P^T P)^{-1} P^T (v_i^{\pm}(a) - p_0),$$

$$C^i = conv\{c_i^+(a), c_i^-(a)\}.$$

C. Reflection segments and segments of stabilizing controllers

Reflection segments $S^i(a)$ of a stable polynomial $a(z)$ are stable by Lemma. However, the projection segments C^i of reflection segments $S^i(a)$, $i = 1, ..., n$ may contain a stabilizing controller or not. In order to solve this problem we have to transform all the projection segments $C^i \in \mathcal{R}^l$ by linear mapping (4) into the space of closed-loop parameters $\mathcal{A}_c \subset \mathcal{R}^n$ and check the stability of the resulting line segments. The problem of checking the Schur stability of the line joining two real polynomials $a_1(z)$ and $a_2(z)$ can be solved by Schur Segment Lemma if the leading coefficients of the extreme polynomials are of same sign and the degree remains constant along the segment [1, p.85].

It is possible that a projection segment C^i is fully stable and has one or both endpoints transformations located inside the stability boundary. In this case the segment is extended by moving each stable endpoint c_i^+ or c_i^- along the segment line by one full length of the segment away from the original position. So, for c_i^+ located inside the stability domain, the new location is identified as $c_{i,new}^+ = c_i^+ + (c_i^+ - c_i^-)$ and thus $C^{i,new} = conv\{c_{i,new}^+, c_i^-\}$ is found. If the transformation of the point $c_{i,new}^+$ in the space of closed loop parameters is still stable, the point can be moved by the same length again until it is outside of the stability domain.

If there are some stability boundary polynomials $a_{ki}^{s*}(z)$, $ki = 1, ..., N_i$ on the line segment $conv\{a_{c_i^+}, a_{c_i^-}\}$ then the number of them N_i is even as both endpoints of the segment are located outside of the stability domain. Then segments of stabilizing controllers are determined by line segments A_{li}^{s*} and the linear transformation (4). It is reasonable to increase the collection of stabilizing controllers by addition of some stable polynomials $a^s(z) \in A_{li}^{s*}$ with stable sets of reflection segments $S^i(a^s)$.

D. Stable rectangle for two unstable systems

We consider two different systems G_1, G_2 represented by two characteristic polynomials $a_1(z), a_2(z)$. Using the reflection segments method we may find bunches of stable segments $A_{li}^{s1*}, A_{li}^{s2*}$ for each individual system based on the same row of initial random polynomials. Their respective stabilizing segments C_1^{i*}, $i = 1...N_1$ and C_2^{j*}, $j = 1...N_2$ in the space of controller parameters are then analyzed.

In order to form a stable rectangle inside the common stability area, the following steps are taken:

1) find all intersection points between the stabilizing segments of the two systems such that $c_k^x \in \{C_1^{i*} \cap C_2^{j*}\}, k = 1, .., N_k$. These points represent robust stabilizing controllers for both of the given systems.
2) choose a combination i_x and j_x of two segments C_1^{ix*} and C_2^{jx*} such that the total number N_{cx} of intersection points on the two segments $c_k^x \subset [C_1^{ix*} \cup C_2^{jx*}], k = 1, ..., N_{cx}$ is maximal.
3) find a part on each segment C_1^{ix*} and C_2^{jx*}, which includes their point of intersection and is stable for both systems described by $a_1(z), a_2(z)$. We denote those segment parts as $C_1^{ix**} = [c_1^{ix**}; c_2^{ix**}]$ and $C_2^{jx**} = [c_1^{jx**}; c_2^{jx**}]$, so that their transformations in the space of closed-loop coefficients $A_{ix}^{s1*}, A_{ix}^{s2*}$ and $A_{jx}^{s1*}, A_{jx}^{s2*}$ are all stable thus meaning that these subsegments are stabilizing for both systems.
4) create a rectangle $R^x = [c_1^{ix**}, c_1^{jx**}, c_2^{ix**}, c_2^{jx**}]$ and check it for stability for both systems via Edge Theorem [17].
5) if R^x has an unstable edge for one or both systems, then the vertices of the unstable edge are shifted towards the point of intersection c_{ij}^{x**} of C_1^{ix*} and C_2^{jx*} and then using interval halving algorithm the most remote location from c_{ij}^{x**} of the two vertices is found such that the edge between them remains stable for both systems. Continue again from the previous step.
6) if R^x is fully stable for G_1 and G_2 then a convex stability area approximation os found.

E. Repetitive stable polygon generation

Once a pair of segments, forming a stable rectangle, is found, the idea can be further developed by extending the existing rectangle into a convex polygon. The following steps are completed:

7) select those segments from the initially created stable pencils C_1^{i*}, C_2^{j*}, $i = 1...N_1$, $j = 1...N_2$ that they

have a point of intersection with one of the rectangle forming segments $c_{ki}^{x*} \in \{C_1^{i*} \cap C_2^{jx**}\}$ or $c_{kj}^{x*} \in \{C_2^{j*} \cap C_1^{ix**}\}$. Let us denote such segments as sets C_{cx}^{i*}, $i = 1...N_{cx}^{i}$ and C_{cx}^{j*}, $j = 1...N_{cx}^{j}$ respectively. The intersection points c_{ki}^{x*} and c_{kj}^{x*} are denoted a a set $C_{ki,kj}^{x*} = \{c_{ki}^{x*}, c_{kj}^{x*}\}$, $ki = 1..Ki, kj = 1..Kj$.

8) find the parts of these segments C_{cx}^{i*} and C_{cx}^{j*}, which contain a point of intersection from the set $C_{ki,kj}^{x*}$ and are stable for both plants represented by $a_1(z), a_2(z)$. These subsegments of pencils C_{cx}^{i*} and C_{cx}^{j*} are denoted as sets of segments C_{cx}^{i**}, $i = 1...N_i^{**}$ and C_{cx}^{j**}, $j = 1...N_j^{**}$.
9) The vertices of the previously found segments polygon R^x and newly created stable pencils C_{cx}^{i**} and C_{cx}^{j**} are then used to create a set of vertices $C_1^{**,1} = \{R^x, C_{cx}^{i**}, C_{cx}^{j**}\}$. The vertices in $C_1^{**,1}$ are stable for both systems. Therefore, a polygon $P_1^{x,1} = conv\{C_1^{**,1}\}$ can be defined and found. Let the vertices of the polygon be defined as $P_1^{x,1} = p_1, p_2, ..., p_{M1}$. Then the edges of the polygon $P_1^{x,1}$ are checked for stability using Edge Theorem [17].
10) if $P_1^{x,1}$ has an edge unstable for one or both plants, then the pair of vertices $[p_i, p_j], i, j \in [1, ..., M1]$ of the unstable edge is shifted along their segments towards their respective points of intersection $c_{pi}^{x*}, c_{pj}^{x*} \subset C_{ki,kj}^{x*}$ and then interval halving algorithm is used to find such locations of $[p_i^*, p_j^*]$ that are the most remote from c_{pi}^{x*}, c_{pj}^{x*} respectively and so that the edge $[p_i^*, p_j^*]$ remains stable for both plants. Then the vertices are substituted into the modified set of points $C_1^{**,2}$ and polygon $P_1^{x,2} = conv\{C_1^{**,2}\} = \{p_1, p_2, ..., p_{M2}\}$ is considered again. The amount of vertices $M2$ can be different from $M1$ as the vertices of the edge that was stabilized in the previous step can now be located inside the polygon. If $P_1^{x,2}$ still has an edge that is unstable for one of the systems, than the procedure of stabilizing that edge is repeated again and the set of points $C_1^{**,2}$ is modified. As the result all possibly unstable edges in the polygon are modified and stabilized. Hence after L cycles of this step a stable polygon $P_1^{x,L}$ is obtained. We denote this final stable polygon simply P_1^x and its final set of points C_1^{**}.
11) In order to get a polygon that includes more segment vertices the whole cycle can be repeated from step 7. This time, however, instead of the rectangle forming segments C_1^{ix*} and C_2^{jx*}, we can use the segments $C_{cx}^{i**}, C_{cx}^{j**}$ and look for the not yet added segments from C_1^{i*} and C_1^{j*} that have robustly stable intersection points c_k^x with the segments of the sets $C_{cx}^{i**}, C_{cx}^{j**}$. That will give a new polygon P_2^x, which includes more parts from different segments.

The cycle from step 7 to 11 is repeated N times until the number of points in C_N^{**} and C_{N-1}^{**} is the same, which potentially provides a best convex stability area approximation P_N^x for the two different systems. The procedure of stable polygon generation is more time consuming, but may give a better approximation of the common stability area for two systems.

Example. Let us consider two fifth-order closed loop systems of the fourth-order plants defined by the transfer functions

$$G_1(z) = \frac{z^2 - 0.2z + 0.57}{z^4 + 0.4z^3 - 1.57z^2 - 0.424z + 0.3696}$$

$$G_2(z) = \frac{z^2 - 0.2z + 0.57}{z^4 + 0.93z^3 - 1.57z^2 - 0.424z + 0.3696}$$

The first plant originates from [8, example 1]. And the second plant has one slightly modified coefficient different from the first system. The first plant have two unstable poles while the second plant has only one unstable pole.

For simplicity and illustrative reasons in both cases we are looking for a stabilizing PD controller of the form

$$C_1(z) = C_2(z) = \frac{c_1 + c_2 z}{z}.$$

First, 10 random initial stable points were generated and their reflection vectors were projected on each of the two systems stability areas in the space of controller parameters c. Each group of projected vectors for each plant consists of $N_{1,2}^{rv} = 10 \cdot 5 \cdot 2 = 100$ projected vectors forming 50 line segments by using pairs of projected vectors from the same level. Every line segment for each system was then independently analyzed using Schur Stability Lemma. In our test we obtained $N_1 = 31$ stabilizing segments C_1^{i*} for system described by G_1 and $N_2 = 32$ segments C_2^{j*} for G_2 (Figure 1.). The D-decomposition was used to show the entire stability domain in Figure 1. for illustrative purposes.

Then 16 points of intersection $c_k^x, k = 1, .., 16$ between the stable segments of the first and the second plant systems were found. We took one stable segment from each pencil of segments $C_1^{i*}, i = 1, .., N_1$ and $C_2^{j*}, j = 1, .., N_2$ such that they intersect and the total sum of points c^x located on both of the segments is greatest. In these two segments we found segment parts C_1^{ix**} and C_2^{jx**}, which are stable for both of the given plants. Then a rectangle R^x was built using 4 vertices of the two segment parts. The total amount of the intersection points on the two segments was 7. The rectangle created by the pair of subsegments C_1^{ix**} and C_2^{jx**} became fully stable for both of the systems (Figure 2.). The point of intersection c_{ij}^{x**} is also marked in (Figure 2.) as a circle inside the rectangle.

In the next step the segments that intersect with C_1^{ix**} or C_2^{jx**} were considered. There were 6 more such segments, obtained from 6 points of intersection on C_1^{ix**} and C_2^{jx**}. Using those parts of the 6 segments, which are Schur stable for both G_1 and G_2 systems and contain the points of intersection from the sets C_{cx}^{i**} or C_{cx}^{j**}, a robustly stable polygon P_1^x was created, which included 8 segment parts that were stable for both systems. After subsequent cycle repetitions P_2^x was found having 21 segments, P_3^x included 33 segments and finally P_4^x was created using 42 segments and consisted of 9 vertices. Figure 3. demonstrates the vertices of P_4^x as stars, and the initial stable rectangle R^x

is shown by solid lines, while P_4^x edges have dotted lines. It can be noticed that the gain in the stability area by using polygonal object was quite substantial with P_4^x the major part of the common stability area of two systems.

The calculation tests of this example were performed using Intel Core 2 Duo processor with 4GB RAM in Windows Vista environment. The generation of stable segments for plant described by G_1 took $0.0936s$ and $0.0780s$ for G_2 system. The subsequent search and determining of robustly stable segments and the rectangle and polygon in the space of controller coefficients took $0.1092s$ and $0.1872s$ of CPU time respectively.

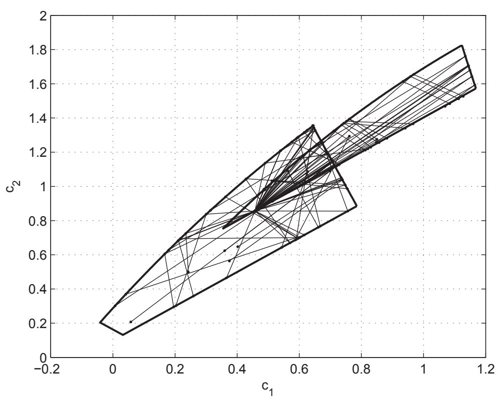

Fig. 1. Stabilizing line segments for each of the given systems

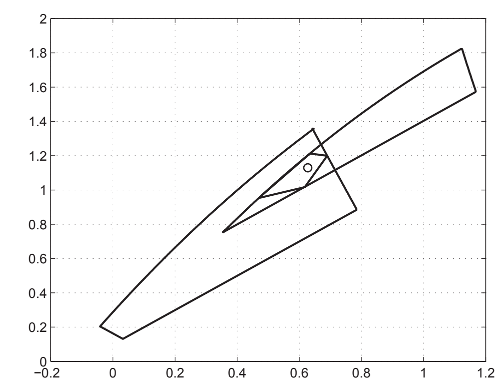

Fig. 2. The rectangular approximation R^x of the common stability area

V. CONCLUSIONS

A straightforward and efficient approach to stabilizing robust controller design with two adjustable parameters for two discrete-time SISO plants is proposed. It is based on the random generation of Schur stable polynomials using reflection coefficients and reflection segments of polynomials. Stabilizing line segments for every given system are determined by projecting of the reflection segments onto affine set of closed-loop characteristic polynomials. A novel approach is proposed for finding a rectangular and polygonal

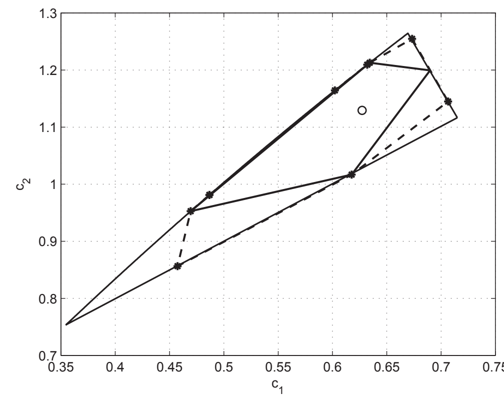

Fig. 3. The polygonal P_4^x (dashed lines) and rectangular R^x (solid lines) approximation inside the common stability area

approximation of the stability area of the two systems by analyzing the points of intersection between the stabilizing segments of the first and the second systems. The stabilizing objects can be used for system parameters optimization.

This approach can also be utilized for robust stabilizing of three or more systems. The convex approximation of the common stability area provides opportunity for future parametric optimization. The idea of using stabilizing segments can also be further developed for PID or other controllers with three adjustable parameters.

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