Preprints of the 2012 20th Mediterranean Conference on Control & Automation (MED), Barcelona, Spain, July 3-6, 2012

Simulation of generic dynamics flight equations of a parafoil/payload system

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***Abstract*— This paper aims at obtaining a dynamic model as generic as possible of a parafoil/payload system. Modeling steps including the consideration of the added masses are detailed such that the reader can easily meet the result. Flight simulation and analysis tools are also presented with a user-friendly 3D visualization.**

I. INTRODUCTION

Self-guided airdrop systems equipped with parafoils have seen recent development thanks to the entry into market of affordable inertial systems using GPS. The applications for cargo air-delivery on badly reachable areas can be either military or civil/humanitarian. In any case, a very stringent landing precision for a wide range of weather conditions - both in terms of position and of impact speed- is a much desired requirement, which involves to develop new control and guidance algorithms efficiently exploiting the system dynamics. This objective has motivated the launch of a common pluri-annual research program between Onera and DGA. The first step concerns the development of models and tools apt to represent the behaviour of payload/parafoil systems with the desired accuracy. Flight dynamics of parafoil systems was the subject of numerous papers [7], [9], [10], [8], [5] and many models of different numbers of d.o.f. have been already published without step by step model building. This is the main contribution of this paper.

The paper is organized as follows. The first section reviews all notations -used in this paper- that highly depend on the reference frames. The second section explains step by step how to find a complete 9 d.o.f. modeling of a parafoil/payload system. Eulerian approach has been chosen for this multi-body problem. Special attention is paid to the definition of apparent masses/inertia. The last section concerns the model simulation in Matlab/Simulink with a possible 3D visualization.

II. NOTATIONS

A. Frame definitions

As shown in Fig. 1, the full system is composed of a payload attached to a parafoil (term designing the set {canopy+riser lines}). The payload reference frame is called $\mathcal{R}_b = \{B, x_b, y_b, z_b\}$, where B is the center of gravity of the payload. Similarly $\mathcal{R}_p = \{P, x_p, y_p, z_p\}$ defines the reference frame rigidly attached to the parafoil at its center of gravity P. z_p is conventionally a vertical axis drawn from top to bottom and x_p is directed forward in the symmetry plane of the parafoil. The relative movement of the parafoil w.r.t. the payload is defined around the hinge point C. Finally the inertial frame is denoted $\mathcal{R}_o = \{O, x_o, y_o, z_o\}$.

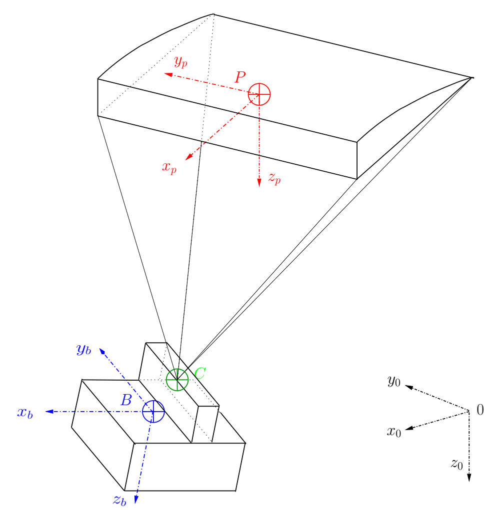

Fig. 1. Definition of the frames.

The transformation matrix from $\mathcal{R}_o$ to $\mathcal{R}_b$ (resp. $\mathcal{R}_p$) is called T_{bo} (resp. T_{po}) which is classically expressed in terms of EULER's angles $\varphi_b, \theta_b, \psi_b$ (resp. $\varphi_p, \theta_p, \psi_p$) :

$$T_{bo} = \begin{pmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi & \sin\varphi\sin\theta\sin\psi + \cos\varphi\cos\psi & \sin\varphi\cos\theta \\ \cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi & \cos\varphi\sin\theta\sin\psi - \sin\varphi\cos\psi & \cos\varphi\cos\theta \end{pmatrix}$$

Moreover a canopy rigging angle μ precises how the manufacturer has chosen the nominal length of each riser line. This angle is between the z_p axis and the line perpendicular to the chord of the canopy (see Fig. 2) and $\mu = 0$ is the particular case where front and back riser lines have the same lengths. The canopy has thus its own reference frame $\mathcal{R}_v = \{V, x_v, y_v, z_v\}$: V is the centroid of the canopy, x_v the axis running along the chord of the canopy from the trailing edge to the attack edge and y_v the axis normal to the plane of canopy profile.

As explained later, $\mathcal{R}_v$ is useful to define added masses/inertia, and the aerodynamic forces/moments applied to the canopy. The rotation matrix T_{vp} from $\mathcal{R}_p$ to $\mathcal{R}_v$ is :

$$T_{vp} = \begin{bmatrix} \cos\mu & 0 & -\sin\mu \\ 0 & 1 & 0 \\ \sin\mu & 0 & \cos\mu \end{bmatrix}$$

This work was supported by the DGA.
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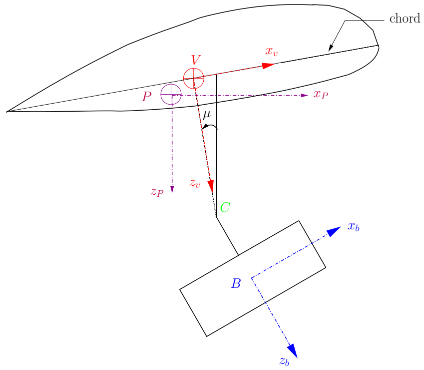

Fig. 2. Rigging angle μ (with $\theta_p = 0$).

B. Parafoil and payload data

Concerning the parafoil :

m_p	mass of the parafoil ($M_p = m_p \text{diag}([1\,1\,1])$)
$\overrightarrow{\Omega}_p$	absolute angular velocity vector of $\mathcal{R}_p$ w.r.t. $\mathcal{R}_o$; its components[1] in $\mathcal{B}_p$ are $[p_p\, q_p\, r_p]^T = \omega_p$
$\overrightarrow{v}_P$	inertial velocity at point P
$\overrightarrow{V}_{ae,p}$	aerodynamic speed of the parafoil $\overrightarrow{v}_P = \overrightarrow{V}_{ae,p} + \overrightarrow{V}_{wind}$
$\varphi_r, \theta_r, \psi_r$	EULER's angles representing three composed rotations that move $\mathcal{B}_p$ to $\mathcal{B}_b$
$\mathbb{I}_{p,P}$	moment of inertia tensor of the parafoil at P ; its matrix expression in $\mathcal{B}_p$ is I_p
CP	projection of $\overrightarrow{CP}$ in $\mathcal{B}_p$
$\overrightarrow{F}_{\text{ext.,p}}$	external forces applied to the parafoil its projection in $\mathcal{B}_p$ is noted F_p
$\overrightarrow{\mathcal{M}}_{P,\text{ext.,p}}$	external torques applied to the parafoil at P ; its projection in $\mathcal{B}_p$ is noted $\mathcal{M}_P$

Concerning the canopy, it is supposed to be symmetric with respect to the (x_v, z_v) and (y_v, z_v) planes for simplicity reasons.

m_a apparent mass ; its expression

in $\mathcal{B}_v$ is $M_{a|v} = \begin{bmatrix} m_{a1} & 0 & 0 \\ 0 & m_{a2} & 0 \\ 0 & 0 & m_{a3} \end{bmatrix}$

in $\mathcal{B}_p$ is $M_a = T_{vp}^T M_{a|v} T_{vp}$

$\mathbb{I}_a$ moment of apparent inertia tensor[2] ; its expression

in $\mathcal{B}_v$ is $J_{a|v} = \begin{bmatrix} J_{a1} & 0 & 0 \\ 0 & J_{a2} & 0 \\ 0 & 0 & J_{a3} \end{bmatrix}$

in $\mathcal{B}_p$ is $J_a = T_{vp}^T J_{a|v} T_{vp}$

[1] The EULER angle kinematics is defined as follows :

$$\begin{bmatrix} \dot{\varphi}_b \\ \dot{\theta}_b \\ \dot{\psi}_b \end{bmatrix} = \begin{bmatrix} 1 & \sin\varphi_b \tan\theta_b & \cos\varphi_b \tan\theta_b \\ 0 & \cos\varphi_b & -\sin\varphi_b \\ 0 & \sin\varphi_b / \cos\theta_b & \cos\varphi_b / \cos\theta_b \end{bmatrix} \begin{bmatrix} p_b \\ q_b \\ r_b \end{bmatrix}$$

[2] The point about which $\mathbb{I}_a$ is calculated is defined in subsection III-B.

Concerning the payload :

m_b	mass of the payload
$\overrightarrow{\Omega}_b$	absolute angular velocity vector of $\mathcal{R}_b$ w.r.t. $\mathcal{R}_o$; its components in $\mathcal{B}_b$ are $[p_b\, q_b\, r_b]^T = \omega_b$
$\overrightarrow{\Omega}_r$	relative angular velocity vector of $\mathcal{R}_b$ w.r.t $\mathcal{R}_p$; $\overrightarrow{\Omega}_r = \overrightarrow{\Omega}_b - \overrightarrow{\Omega}_p$ its components in $\mathcal{B}_b$ are $[p_r\, q_r\, r_r]^T$
$\mathbb{I}_{b,B}$	moment of inertia tensor of the body at B its expression in $\mathcal{B}_b$ is I_b
CB	the projection of $\overrightarrow{CB}$ in $\mathcal{B}_b$
$\overrightarrow{F}_{\text{ext.,b}}$	external forces applied to the payload its expression in $\mathcal{B}_b$ is F_b
$\overrightarrow{\mathcal{M}}_{B,\text{ext.,b}}$	external torques applied to the payload at B ; its projection in $\mathcal{B}_b$ is noted $\mathcal{M}_B$

Concerning the hinge point :

$\overrightarrow{v}_C$	velocity of point C its projection in $\mathcal{B}_o$ is $v_C = [v_{C1}\, v_{C2}\, v_{C3}]^T$
$\overrightarrow{F}_C$	action/reaction forces applied to the payload at the link its expression in $\mathcal{B}_o$ is F_C
$\overrightarrow{\mathcal{M}}_C$	action/reaction torques applied to the payload at the link ; its expression in $\mathcal{B}_b$ is $\mathcal{M}_C$

C. Matrix expressions of vectorial formula

Let us consider two vectors $\overrightarrow{u}$ and $\overrightarrow{v}$. If $u = [u_1\, u_2\, u_3]^T$ and $v = [v_1\, v_2\, v_3]^T$ are respectively the coordinate vectors of $\overrightarrow{u}$ and $\overrightarrow{v}$ in a specified frame $\mathcal{R}$, then the dot product $\overrightarrow{u}.\overrightarrow{v}$ is equal to $u^T v$ in $\mathcal{R}$ and the cross product $\overrightarrow{u} \times \overrightarrow{v}$ is written in $\mathcal{R}$ as a matrix-vector product $u^\times v$ where

$$u^\times = \begin{bmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{bmatrix}$$

This notation leads to the following relations (T_{12} is the transformation matrix from a frame $\mathcal{B}_2$ to another $\mathcal{B}_1$) :

$$(T_{12}u)^\times = T_{12} u^\times T_{21} \tag{1}$$

$$\frac{\mathrm{d}T_{po}}{\mathrm{d}t} = -\omega_p^\times T_{po} \qquad \frac{\mathrm{d}T_{bo}}{\mathrm{d}t} = -\omega_b^\times T_{bo} \tag{2}$$

III. RIGID MULTIBODY DYNAMICS EQUATIONS

Newtonian mechanics equations are used for each rigid subsystem (parafoil and payload, both supposed undeformable) in order to get a dynamical model as rich as possible for analyses and simulations. The payload dynamics is first studied because of its simplicity and is then a basis for the harder parafoil dynamics, which has to account for added masses. Added masses consist :

- of the included mass/inertia of the air inside the canopy
- and of apparent masses/inertia describing the unsteady aerodynamic effects on the canopy.

Only the more complex contribution of the apparent masses/inertia is explicitly detailed here.

A. Dynamics of the payload

The NEWTON's second law and the EULER's equations (at point B) applied to the payload give :

$$\frac{\mathrm{d}}{\mathrm{d}t_{|\mathcal{B}_o}}(\vec{p}_b) = \frac{\mathrm{d}}{\mathrm{d}t_{|\mathcal{B}_b}}(\vec{p}_b) + \vec{\Omega}_b \times \vec{p}_b = \vec{F}_{\text{ext., b}} \quad (3a)$$

$$\frac{d}{dt_{|\mathcal{B}_o}}\overrightarrow{\sigma_{b/B}} = \frac{d}{dt_{|\mathcal{B}_b}}\overrightarrow{\sigma_{b/B}} + \vec{\Omega}_b \times \overrightarrow{\sigma_{b/B}} = \vec{\mathcal{M}}_{B,\text{ext.,b}} \quad (3b)$$

where

$$\vec{p}_b = m_b \vec{v}_B = m_b \left[\vec{v}_C + \vec{\Omega}_b \times \overrightarrow{CB}\right] \quad (3c)$$

and

$$\overrightarrow{\sigma_{b/B}} = \mathbb{I}_{b,B}\left(\vec{\Omega}_b\right) \quad (3d)$$

are respectively the payload linear momentum and the payload angular momentum.

In taking (2) into account, the projection of (3a) and (3b) into $\mathcal{B}_b$ can be written as :

$$m_b T_{bo} \dot{v}_C - m_b CB^\times \dot{\omega}_b = F_b + m_b\, \omega_b^\times CB^\times \omega_b \quad (4)$$

$$I_b \dot{\omega}_b = \mathcal{M}_B - \omega_b^\times I_b\, \omega_b \quad (5)$$

B. Dynamics of the parafoil

Similar equations can be obtained for the parafoil in $\mathcal{B}_p$, but they must be completed by additional terms characterizing the apparent masses and inertia. These effects are significant but they are hardly identifiable [1].

Here the NEWTON's second law and the EULER's equations (at point P) applied to the parafoil state :

$$\frac{\mathrm{d}}{\mathrm{d}t_{|\mathcal{B}_p}}(\vec{p}_p + \vec{p}_a) + \vec{\Omega}_p \times (\vec{p}_p + \vec{p}_a) = \vec{F}_{\text{ext.,p}} \quad (6a)$$

$$\frac{d}{dt_{|\mathcal{B}_o}}\left[\overrightarrow{\sigma_{p/P}} + \overrightarrow{\sigma_{a/P}}\right] = \vec{\mathcal{M}}_{P,\text{ext.,p}} + \vec{p}_a \times \vec{v}_P \quad (6b)$$

where the apparent linear and angular momentums $\vec{p}_a$ and $\overrightarrow{\sigma_{a/P}}$ depend on the location of the apparent mass center. In fact two apparent mass centers are commonly distinguished : the first one, G_{pitch}, for all canopy rotation vector along (V, y_v) and the second one, G_{roll}, for all canopy rotation vector in the plane (x_v, z_v). Due to the symmetry properties of the canopy, G_{pitch} and G_{roll} are located on the (V, z_v) axis.

In order to compute $\vec{p}_a$ and $\overrightarrow{\sigma_{a/P}}$, let us consider a point A situated in the plane (V, x_v, z_v), such that

Vector	Expression	in the frame
$\overrightarrow{AG_{roll}}$	$d_{r\|v} = \begin{bmatrix} d_{r1} & 0 & d_{r3}\end{bmatrix}^T$	$\mathcal{B}_v$
$\overrightarrow{AG_{pitch}}$	$d_{t\|v} = \begin{bmatrix} d_{t1} & 0 & d_{t3}\end{bmatrix}^T$	$\mathcal{B}_v$
$\overrightarrow{G_{roll}G_{pitch}}$	$d_{rt\|v} = \begin{bmatrix} 0 & 0 & d_{rt3}\end{bmatrix}^T$	$\mathcal{B}_v$
$\vec{v}_A$	$v_{\|\mathcal{B}_v} = \begin{bmatrix} v_1 & v_2 & v_3\end{bmatrix}^T$ v	$\mathcal{B}_v$ $\mathcal{B}_p$

As the apparent mass center differs according to the component of the absolute angular velocity vector, the vectorial relation equivalent to (3c) for $\vec{p}_a$ can be written on each axis of $\mathcal{B}_v$ as :

$$\begin{cases} p_{a1,v} &= m_{a1}\left(v_1 + q_{p|v}\, d_{t3}\right) \\ p_{a2,v} &= m_{a2}\left(v_2 + r_{p|v}\, d_{r1} - p_{p|v}\, d_{r3}\right) \\ p_{a3,v} &= m_{a3}\left(v_3 - q_{p|v}\, d_{t1}\right) \end{cases}$$

The equivalent matrix equation is as follows :

$$p_{a,v} = M_{a|v}\left(v_{|\mathcal{B}_v} - D_{a,v}\, T_{vp}\, \omega_p\right)$$

where

$$D_{a,v} = \begin{bmatrix} 0 & -d_{t3} & 0 \\ d_{r3} & 0 & -d_{r1} \\ 0 & d_{t1} & 0 \end{bmatrix}.$$

The expression of the apparent linear momentum in $\mathcal{B}_p$ can also be easily deduced :

$$p_a = M_a\left(v - D_a\, \omega_p\right) \quad \text{with} \quad D_a = T_{pv} D_{a,v} T_{vp} \quad (7)$$

In a same manner, the apparent angular momentum at point A depends on different apparent mass centers according to the projection axis. The projection of $\vec{\sigma}_{a/A}$ into $\mathcal{B}_v$ can be easily deduced :

$$\sigma_{a/A,\mathcal{B}_v} = -D_{a,v}^T\, M_{a|v}\, v_{|\mathcal{B}_v} + \left[J_{a|v} - D_{a,v}^T\, M_{a|v}\, D_{a,v}\right] T_{vp}\, \omega_p$$

Its expression in $\mathcal{B}_p$ is then straightforward :

$$\sigma_{a/A,\mathcal{B}_p} = T_{pv}\, \sigma_{a/A,\mathcal{B}_v}$$

With the notation :

$$J_{Aa} = \left[J_a - D_a^T\, M_a\, D_a\right]\, \omega_p$$

the expressions of $\vec{p}_a$ and $\vec{\sigma}_{a/A}$ in $\mathcal{B}_p$ are gathered as follows :

$$\begin{bmatrix} p_a \\ \sigma_{a/A,\mathcal{B}_p} \end{bmatrix} = \begin{bmatrix} M_a & -M_a D_a \\ -D_a^T M_a & J_{Aa} \end{bmatrix} \begin{bmatrix} v \\ \omega_p \end{bmatrix} \quad (8)$$

This reminds the expressions of $\vec{p}_p$ and $\overrightarrow{\sigma_{p/P}}$ in $\mathcal{B}_p$:

$$\begin{bmatrix} p_p|_{\mathcal{B}_p} \\ \sigma_{p/C}|_{\mathcal{B}_p} \end{bmatrix} = \begin{bmatrix} m_p I_3 & -m_p CP^\times T_{po} \\ m_p CP^\times T_{po} & \mathbb{I}_{p,C} \end{bmatrix} \begin{bmatrix} v_C \\ \omega_p \end{bmatrix} \quad (9)$$

Afterwards $D_{a/P}$ (resp. $D_{a/C}$) is the value of D_a, when A is placed at point P (resp. C). These two values are linked through :

$$D_{a/C} = CP^\times + D_{a/P} \quad (10)$$

Using (8) and (9) the expression of (6a) in $\mathcal{B}_p$ is quite simple, if A is chosen at point C (this choice raises the most interesting velocity v_C) :

$$\begin{aligned} \left[M_p + M_a\right] T_{po}\, \dot{v}_C - \left[M_p\, CP^\times + M_a\left(CP^\times + D_{a/P}\right)\right] \dot{\omega}_p \\ = F_p + \omega_p^\times \left[M_p\, CP^\times + M_a\left(CP^\times + D_{a/P}\right)\right] \omega_p \\ + \left[M_a\, \omega_p^\times\, T_{po} - \omega_p^\times\, M_a\, T_{po}\right] v_C \end{aligned} \quad (11)$$

since $v = T_{po} v_C$ and $\dot{v} = T_{po} \dot{v}_C - \omega_p^\times\, T_{po}\, v_C$.

Finally the term $\vec{p}_a \times \vec{v}_P$ due to the added masses in (6b) calls for comments, since it is not classical (and overlooked in several publications). As mentionned by [1], a part of this term, $v_O^\times M_a v_O$ (where O is some origin point), accounts for the stationnary aerodynamic torque on the canopy: it is thus part of the aerodynamic model. So $v_O^{\times} M_a v_O$ has to be removed from $\overrightarrow{p}_a \times \overrightarrow{v}_P$. However, it is not clear in [1] which O should be considered. The natural choice for an aerodynamic contribution is P. Eventually, the remaining contribution of $\overrightarrow{p}_a \times \overrightarrow{v}_P$ projected in $\mathcal{B}_p$ is $-\left(M_a D_{a/P}\,\omega_p\right)^{\times} v_P$.

At last, in taking into account that

$$v_P = T_{po}\, v_C - CP^{\times}\omega_p$$

and

$$\dot{v}_P = T_{po}\,\dot{v}_C - \omega_p^{\times} T_{po} v_C - CP^{\times}\dot{\omega}_p$$

(6b) becomes, when written in $\mathcal{B}_p$:

$$\begin{aligned} -D_{a/P}^T M_a T_{po}\,\dot{v}_C + \left(I_p + J_{Pa} + D_{a/P}^T M_a CP^{\times}\right)\dot{\omega}_p = \\ \mathcal{M}_P - \left(M_a D_{a/P}\,\omega_p\right)^{\times}\left(T_{po}\, v_C - CP^{\times}\omega_p\right) \\ -\omega_p^{\times}\left(I_p + J_{Pa} + D_{a/P}^T M_a CP^{\times}\right)\omega_p \\ +\omega_p^{\times} D_{a/P}^T M_a T_{po}\, v_C - D_{a/P}^T M_a \omega_p^{\times} T_{po} v_C \end{aligned} \tag{12}$$

C. Forces and torques

The equation (4) needs F_b, which is composed of :

$$F_b \left| \begin{array}{lll} \star & F_b^A & : \text{ aerodynamic forces applied to the payload, written in } \mathcal{B}_b \\ \star & F_b^G & : \text{ weight of the payload written in } \mathcal{B}_b \\ & & \quad F_b^G = T_{bo}\begin{bmatrix} 0 \\ 0 \\ m_b\, g \end{bmatrix} \\ \star & T_{bo}F_C & : \ \overrightarrow{F}_C \text{ written in } \mathcal{B}_b \end{array}\right.$$

F_p appears in (11) ; it is composed of :

$$F_p \left| \begin{array}{lll} \star & F_p^A & : \text{ aerodynamic forces applied to the parafoil, written in } \mathcal{B}_p \\ \star & F_p^G & : \text{ weight of the parafoil written in } \mathcal{B}_p \\ & & \quad F_p^G = T_{po}\begin{bmatrix} 0 \\ 0 \\ m_p\, g \end{bmatrix} \\ \star & -T_{po}F_C & : \ -\overrightarrow{F}_C \text{ applied to the parafoil and written in } \mathcal{B}_p \end{array}\right.$$

The equation (5) uses $\mathcal{M}_B$, which is composed of :

$$\mathcal{M}_B \left| \begin{array}{lll} \star & \mathcal{M}_B^A & : \text{ aerodynamic torques applied to the payload and written in } \mathcal{B}_b \\ \star & \mathcal{M}_C & : \ \mathcal{M}_C = [\mathcal{M}_{C1}\,\mathcal{M}_{C2}\,\mathcal{M}_{C3}]^T \\ \star & -CB^{\times}T_{bo}F_C & : \text{ torques induced by } \overrightarrow{F}_C \text{ and written in } \mathcal{B}_b \end{array}\right.$$

Let us note that weight induces obviously no torque around the center of gravity.

And $\mathcal{M}_P$ emerges in (12) ; it is composed of :

$$\mathcal{M}_P \left| \begin{array}{lll} \star & \mathcal{M}_P^A & : \text{ aerodynamic torques applied to the parafoil and written in } \mathcal{B}_p \\ \star & -T_{po}T_{bo}^T\mathcal{M}_C & : \ -\overrightarrow{\mathcal{M}}_C \text{ applied to the parafoil and written in } \mathcal{B}_p \\ \star & CP^{\times}T_{po}F_C & : \text{ torques induced by } -\overrightarrow{F}_C \text{ and written in } \mathcal{B}_p \end{array}\right.$$

D. Aerodynamic model

Aerodynamic models are needed to determine the resulting aerodynamic efforts on the parafoil and on the payload as a function of the system states. Parafoil aerodynamic models are similar to airplane's, i.e. make use of dimensionless coefficients which are the summation of the effects of flow angles, control surface deflections and angular velocity rates. Formulas can be found in the literature to construct longitudinal coefficient models from the main geometric features of the canopy [2] with reasonnable accuracy. Such information does not seem to be available for lateral coefficients. Only a few papers [3], [4], [5], [6] provide sets of coefficients, but these are not supported by physical or geometrical reasons.

The parafoil aerodynamic coefficients $C_{\cdot}$ are determined in the canopy frame $\mathcal{B}_v$. Hence the forces and torques in $\mathcal{B}_p$ are :

$$F_p^A = \frac{1}{2}\rho S V_{ae,p}^2\, T_{pv}\begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix} \quad M_p^A = \frac{1}{2}\rho S l V_{ae,p}^2\, T_{pv}\begin{bmatrix} C_l \\ C_m \\ C_n \end{bmatrix} \tag{13}$$

The only significant aerodynamic efforts on the payload is drag.

$$F_b^A = -\frac{1}{2}\rho S_b\, C_{X\,b}\,||V_{ae,b}||V_{ae,b} \qquad M_b^A = 0 \tag{14}$$

E. Reference dynamical model

Finally in gathering (4), (11), (5) and (12) and in exploiting the previous expressions of forces and torques, the most 9 d.o.f. complete dynamical model can be obtained (see (15)). This system determines jointly the derivatives $\dot{v}_C$, $\dot{\omega}_b$, $\dot{\omega}_p$ and F_C by solving the 12 linear equations.

Another version uses the relative acceleration $\dot{\omega}_r$ of the payload w.r.t. the parafoil instead of $\dot{\omega}_p$. This new formulation, easily obtained thanks to

$$\begin{array}{ll} \omega_r = \omega_b - T_{bp}\,\omega_p & (\text{written in } \mathcal{B}_b) \\ \dot{\omega}_p = -\omega_r^{\times} T_{pb}\left(\omega_b - \omega_r\right) + T_{pb}\left(\dot{\omega}_b - \dot{\omega}_r\right) & (\text{written in } \mathcal{B}_p) \end{array}$$

is able to take into account constraints on the relative displacement (freeze of degrees of freedom...).

As 12 equations describe this 9 d.o.f. model, 3 equations are used to implicitly compute the force transmitted at the joint F_C. *A contrario* as $\mathcal{M}_C$ is a term in the right side of (15), it can not be deduced ; it must be defined *a priori* by the user, who intrinsically chooses a joint model. The most common option for $\mathcal{M}_C$ is a yaw-wise return moment. If $\mathcal{M}_C$ is fixed infinite according to a direction, the associated degree of freedom is then deleted ; it is a way of obtaining a 8 d.o.f. dynamical model.

By the way simpler dynamical equations can be derived from (15).

1) Simplified 9 d.o.f. models: The first kind of simplification concerns the location of G_{roll} and G_{pitch}. Indeed the 9 d.o.f. model suggested in [7] can be recovered with the assumption G_{roll} is placed exactly at G_{pitch} ; i.e.:

$$d_{rt} = 0 \qquad \text{et} \qquad D_{a/P} = PG_{roll}^{\times}$$

$$\begin{bmatrix} m_b T_{bo} & -m_b CB^\times & 0_{3\times3} & -T_{bo} \\ [M_p + M_a] T_{po} & 0_{3\times3} & -\left[M_p CP^\times + M_a \left(CP^\times + D_{a/P}\right)\right] & T_{po} \\ 0_{3\times3} & I_b & 0_{3\times3} & CB^\times T_{bo} \\ -D_{a/P}^T M_a T_{po} & 0_{3\times3} & I_p + J_{Pa} + D_{a/P}^T M_a CP^\times & -CP^\times T_{po} \end{bmatrix} \begin{bmatrix} \dot{v}_C \\ \dot{\omega}_b \\ \dot{\omega}_p \\ F_C \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} \tag{15}$$

with

$$\begin{aligned} B_1 &= F_b^A + F_b^G + m_b\, \omega_b^\times CB^\times \omega_b \\ B_2 &= F_p^A + F_p^G + \omega_p^\times \left[M_p CP^\times + M_a \left(CP^\times + D_{a/P}\right)\right] \omega_p + \left[M_a \omega_p^\times T_{po} - \omega_p^\times M_a T_{po}\right] v_C \\ B_3 &= \mathcal{M}_B^A + \mathcal{M}_C - \omega_b^\times I_b\, \omega_b \\ B_4 &= \mathcal{M}_P^A - T_{po} T_{bo}^T \mathcal{M}_C - \omega_p^\times \left(I_p + J_{Pa} + D_{a/P}^T M_a CP^\times\right) \omega_p + \omega_p^\times D_{a/P}^T M_a T_{po} v_C - D_{a/P}^T M_a \omega_p^\times T_{po} v_C \\ &\quad - \left(M_a D_{a/P}\, \omega_p\right)^\times \left(T_{po}\, v_C - CP^\times \omega_p\right) \end{aligned}$$

Let us note that the definition of B_4 in [7] has a sign error. Another hypothesis consists in placing G_{roll} and G_{pitch} at point P. Here $D_{a/P} = 0_{3\times3}$ and the simplification of (15) leads to the model proposed in [8].

2) Reduced d.o.f. models: The second kind of simplification is relative to the characterization of the link between parafoil and payload as explained above. If the set {parafoil + payload} is considered as one rigid body, i.e.:

$$\omega_b = \omega_p = \omega \quad \text{and} \quad T_{bo} = T_{po} = T$$

(15) can be rewritten with only 6 equations. F_C is no more significative and disappears. 6 d.o.f. models like those used in [9], [5] can be thus easily restored from (15).
Some papers [10] proposed to lock the roll d.o.f. of the parafoil w.r.t. the payload by means of :

$$\varphi_r = 0$$

By successive time derivatives, this last expression gives :

$$\dot{\varphi}_r = p_r + \tan\theta_r\, r_r = 0 \tag{16}$$

$$\ddot{\varphi}_r = \dot{p}_r + \tan\theta_r\, \dot{r}_r + \dot{\theta}_r(1 + \tan^2\theta_r)\, r_r = 0 \tag{17}$$

As $\dot{\theta}_r = q_r$, (17) leads to :

$$\dot{p}_r + \tan\theta_r\, \dot{r}_r = -(1 + \tan^2\theta_r)\, q_r r_r$$

When expressed in terms of $\dot{\omega}_p$ and $\dot{\omega}_b$, this last relationship has to be added to (15) to block the d.o.f.. This additional equation allows to compute the first component of $\overrightarrow{\mathcal{M}}_C$ in $\mathcal{B}_p$. Finally such 8 d.o.f. model is characterized by 13 equations making its resolution paradoxically most complex.

IV. Tool development for HQ assessment

Handling quality assessment is a prerequisite before the development of control laws, which is a subsequent objective of this work. The requirements are typically :

- to calculate trimmed points for given brake settings, both in longitudinal flight and in turn;
- to perform time simulations of the parafoil motion, either interactive or with preset commands;
- to get linearized models, so as to access the dynamic modes and to allow controller design.

These requirements have been answered by developing suitable tools in MATLAB environment. The various dynamic models (from 9 to 6 d.o.f.) have been augmented by adding the EULER angles to the state vector (plus the position for the simulation) to yield a numerical system $\dot{X} = f(X, u)$, where X is the augmented state vector (see Table I), and u the brake positions. Typically, the trim computation comes with the resolution of $f(X, u) = 0$, the simulator with the time integration of $f(X, u)$, and the model linearization with the numerical differentiation.

9 d.o.f.	8 d.o.f.	6 d.o.f.
$X = \begin{bmatrix} v_{C1} & v_{C2} & v_{C3} & p_p & q_p & r_p & \varphi_b & \theta_b & \psi_b & p_r & q_r & r_r & \varphi_r & \theta_r & \psi_r \end{bmatrix}^T$	$X = \begin{bmatrix} v_{C1} & v_{C2} & v_{C3} & p_p & q_p & r_p & \varphi_b & \theta_b & \psi_b & q_r & r_r & \theta_r & \psi_r \end{bmatrix}^T$	$X = \begin{bmatrix} v_{C1} & v_{C2} & v_{C3} & p_p & q_p & r_p & \varphi_b & \theta_b & \psi_b \end{bmatrix}^T$

TABLE I
STATE VARIABLES OF 9,8 AND 6 D.O.F. MODELS.

A. Trim computation

The trim computation determines the parafoil equilibrium for given entries u. There are two entries corresponding to brake deflections : the symmetric brake deflection δs and the asymmetric brake deflection δa. More precisely if br_l (resp. br_r) denotes the left (resp. right) brake deflection,

$$u = \begin{bmatrix} \delta a \\ \delta s \end{bmatrix} \quad \text{with} \quad \begin{cases} \delta a = br_r - br_l \\ \delta s = \dfrac{br_r + br_l}{2} \end{cases}$$

Remark : Brakes can only be deflected down. Mathematically this leads to the constraints :

$$0 \le br_r \le 1 \quad \text{and} \quad 0 \le br_l \le 1 \tag{18}$$

The trim computation is done after taking out $\dot{\psi}_b$ equation and imposing ψ_b: this suppresses the indetermination due to the fact that ψ_b has no interaction with the dynamics.

The choice of X vector has been dictated by the urge to simplify the model formulations. In particular, the variable v_C is projected in the absolute frame of reference. As a consequence, solving directly $f(X, u) = 0$ cannot yield steady turn equilibria. Two equations have to be modified to take into account the turn rate $\Omega = \dot{\psi}_b$:

$$\dot{v}_{C1} = \Omega\, v_{C2} \quad \text{and} \quad \dot{v}_{C2} = -\Omega\, v_{C1} \tag{19}$$

Solving the resulting set of equations is not straightforward since the problem is non linear, not only because of the mechanic formulation, but also (and mostly?) because of the aerodynamic model. An efficient algorithm is the MATLAB function `fsolve`. We have also implemented a GAUSS-NEWTON method which relies on a Jacobian matrix $\partial f / \partial X$ obtained by finite differences. Both methods require a starting point to converge to a solution, which does not ensure that we have all the possible solutions. (As a matter of fact, some parafoil models that we have tested do show multiple solutions for a unique brake setting, both in longitudinal flight and in steady turn. This behaviour has been encountered by other authors [8].)

B. *Interactive simulation*

SIMULINK is used to get time simulations. The dynamics flight equations $f(X, u)$ of the full system is described within a S-function, which is a convenient and compact way to integrate the state vector X with SIMULINK solvers (see Fig. 3).

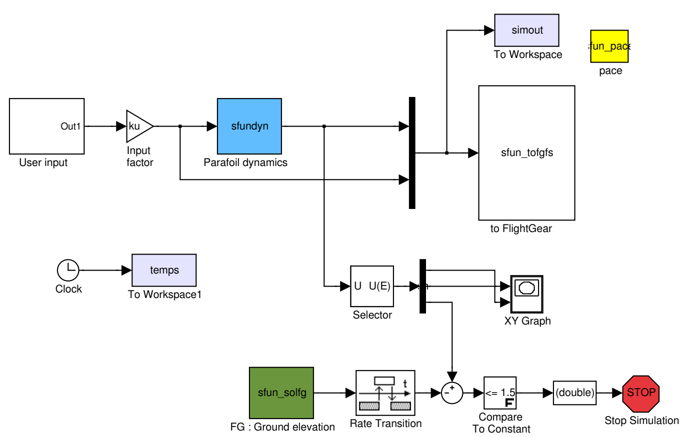

Fig. 3. Simulink block diagram.

An input called "mouse/keyboard" allows the user to deflect brakes in real-time. Fig. 4 shows the user interface. The red square points out brake deflections, which are changed through the mouse, the keyboard or a joystick. The shaded part corresponds to the unauthorized area of $(-\delta s, \delta a)$ due to (18).

C. *3D visualization using* FLIGHTGEAR

In addition to the MATLAB tools, a visualization software has been employed in order to facilitate the flight dynamics analysis for interactive simulations: FLIGHTGEAR[3]. FG is

[3] see at http://www.flightgear.org/

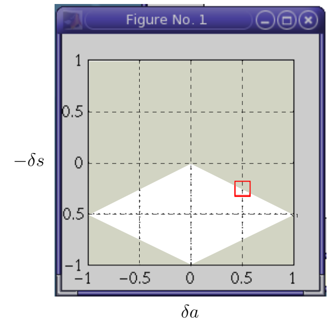

Fig. 4. User interface.

essentially an open-source flight simulation which allows the user to introduce his own aircraft model *via* xml files. For purposes like ours, FG can be conveniently reduced to its visual interface, displaying a flight computed externally.

The implementation of the MATLAB/FG coupling requires the development of an interface which transfers the needed variables during the simulation. This interface was written in C language, and integrated in MATLAB in a mex-function. For the needs of visualization a 3D model is also required. This was done in a schematic way (see fig. 5) which is quite enough to represent the interesting features of parafoil flight, such as the relative payload/canopy motion.

Finally, the MATLAB/FG simulation has proven a good pedagogic tool to perceive and illustrate the peculiarity of parafoil flight dynamics, in particular as regards turns.

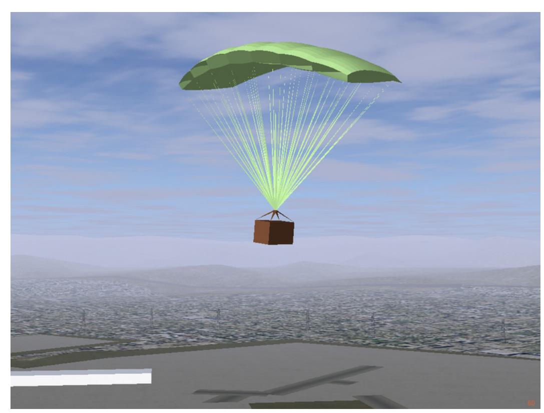

Fig. 5. 3D visualization (longitudinal flight).

V. CONCLUSION

A generalized 9 d.o.f. dynamics flight model of a parafoil/payload system has been developed. It takes into account not only the relative rotations of the canopy and the payload, but also added mass effects, considered essential in many papers. This significance must be further assessed ; indeed some of these additional terms have a similar effect as aerodynamic coefficients. Finally first tools for trim computations, model linearizations and simulations have been implemented with a view to future works in control and guidance framework.

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