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Quasi-LPV Modelling and Non-Linear Identification of a Twin Rotor System

Fatiha Nejjari, Damiano Rotondo, Vicenç Puig and Mario Innocenti

Abstract— **This paper describes the experimental identification of the parameters of the Twin Rotor MIMO System (TRMS) non-linear model using data collected from the real lab set-up. From this non-linear model, a quasi-linear parameter varying (quasi-LPV) model has also been derived using a state transformation. This quasi-LPV model is approximated with a polytopic model using the bounding box approach. Such a model can later be used for control design. The model parameters have been calibrated by means of non-linear least-squares identification approach. Once the calibrated non-linear model has been obtained, a simulator has been built and validated against real data showing satisfactory results when compared to real data.**

I. INTRODUCTION

The TRMS is an aero-dynamical system similar to a helicopter (see Fig. 1). At both ends of its beam there are two propellers driven by DC motors, each perpendicular to the other one. The beam can rotate freely in the horizontal and vertical planes, in such a way that its ends move on spherical surfaces. The joined beam can be moved by changing the motors supply voltages, thus controlling the rotational speed of the propellers. There is a counter-weight fixed to the beam which is used for balancing the angular momentum in a stable equilibrium position. The rotor generating the vertical movement is called the main rotor. It enables the TRMS to pitch, which is a rotation in the vertical plane around the horizontal axis. The rotor generating the horizontal movement is called the tail rotor. It enables the model to yaw, which is a rotation in the horizontal plane around the vertical axis. The system is perceived as a challenging control engineering problem owing to its high non-linearity, cross-coupling between its two axes, frictions, saturations and inaccessibility of some of its states and outputs for measurements. Because of all these features, obtaining a non-linear model calibrated from data collected from the real set-up is a hard task.

The modelling of such a system has been addressed in several papers. In particular, in [1] two physical models based on Newtonian and Lagrangian approaches are presented and compared. Further modelling improvements can be found in [2], [3] and [4]. Looking at those models the reader can realize about the high non-linearity and cross-coupling of such a system. Regarding the experimental identification of the parameters of the TRMS model some preliminary work can be found in [5], [6], [7], [8] and [9].

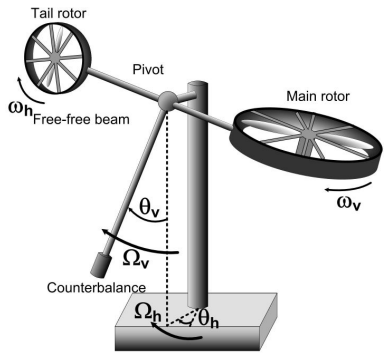

Fig. 1. Components of the Twin Rotor MIMO System

In this work, the non-linear model of the TRMS system proposed in [4] is identified using real data taken from the lab set-up. Moreover, from this non-linear model, a quasi-LPV model is derived using the approach proposed by [10]. The quasi-LPV model is then approximated in a polytopic fashion using the *bounding box* approach. The polytopic quasi-LPV model can later be used for designing controllers for the TRMS system.

At first, standard linear identification techniques [11] were applied to estimate the parameters of the TRMS non-linear model. Good models have been obtained around some given operating point, but the overall model failed in reproducing the system behavior in operating points different from the ones used during the identification. This is mainly due to the high non-linearity of the model. For this reason, a non-linear system identification approach has finally been used. This paper presents the results obtained solving several non-linear least square problems.

Once the calibrated non-linear model has been obtained, a simulator has been built and validated showing satisfactory results when compared to real data.

The structure of the paper is the following: In Section II, a quasi-LPV model is derived from the non-linear model of the TRMS system proposed in [4]. The non-linear identification approach used to estimate the parameters of the model is described in Section III and the results are presented in Section IV. Section V presents the validation of the calibrated model using real data. Finally, the main conclusions are summarized in Section VI.

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II. Twin Rotor MIMO System Modelling

A. Non-linear modelling

The strategy to model the TRMS is to split the system into simpler subsystems: the DC-Motors, the propellers and the beam. The first two have independent dynamics, that is, the main motor does not affect the behaviour of the tail motor, and vice versa. The same is true for the propellers. On the other hand, the dynamics of the beam is strongly non-linear with the presence of interaction phenomena among the horizontal and the vertical dynamics. Using such a modelling approach, the following non-linear model has been proposed in [4] introducing some improvement to the existing models already proposed in the literature ([1], [2], [3]):

$$\frac{d\omega_h}{dt} = \frac{k_a k_1}{J_{tr} R_a} u_h - \left(\frac{B_{tr}}{J_{tr}} + \frac{k_a^2}{J_{tr} R_a}\right)\omega_h - \frac{f_1(\omega_h)}{J_{tr}} \quad (1)$$

$$\frac{d\Omega_h}{dt} = \frac{l_t f_2(\omega_h)\cos\theta_v - k_{oh}\Omega_h - f_3(\theta_h) + f_6(\theta_v)}{D\cos^2\theta_v + E\sin^2\theta_v + F} + \frac{k_m \omega_v \sin\theta_v \Omega_v \left(D\cos^2\theta_v - E\sin^2\theta_v - F - 2E\cos^2\theta_v\right)}{\left(D\cos^2\theta_v + E\sin^2\theta_v + F\right)^2} + \frac{k_m \cos\theta_v \left(\frac{k_a k_2}{R_a} u_v - \left(B_{mr} + \frac{k_a^2}{R_a}\right)\omega_v - f_4(\omega_v)\right)}{J_{mr}\left(D\cos^2\theta_v + E\sin^2\theta_v + F\right)} \quad (2)$$

$$\frac{d\theta_h}{dt} = \Omega_h \quad (3)$$

$$\frac{d\omega_v}{dt} = \frac{k_a k_2}{J_{mr} R_a} u_v - \left(\frac{B_{mr}}{J_{mr}} + \frac{k_a^2}{J_{mr} R_a}\right)\omega_v - \frac{f_4(\omega_v)}{J_{mr}} \quad (4)$$

$$\frac{d\Omega_v}{dt} = \frac{l_m f_5(\omega_v) + k_g \Omega_h f_5(\omega_v)\cos\theta_v - k_{ov}\Omega_v}{J_v} + \frac{g\left((A-B)\cos\theta_v - C\sin\theta_v\right) - {\Omega_h}^2 H \sin\theta_v \cos\theta_v}{J_v} + \frac{k_t \left(\frac{k_a k_1}{R_a} u_h - \left(B_{tr} + \frac{k_a^2}{R_a}\right)\omega_h - f_1(\omega_h)\right)}{J_v J_{tr}} \quad (5)$$

$$\frac{d\theta_v}{dt} = \Omega_v \quad (6)$$

The non-linear functions f_i, that take into account the frictions and coupling effects between horizontal/vertical dynamics, are defined as follows:

$$f_1(\omega_h) = \begin{cases} k_{thp}\omega_h^2 & \text{if} \quad \omega_h \geq 0 \\ -k_{thn}\omega_h^2 & \text{if} \quad \omega_h < 0 \end{cases} \quad (7)$$

$$f_2(\omega_h) = \begin{cases} k_{fhp}\omega_h^2 & \text{if} \quad \omega_h \geq 0 \\ -k_{fhn}\omega_h^2 & \text{if} \quad \omega_h < 0 \end{cases} \quad (8)$$

$$f_3(\theta_h) = \begin{cases} k_{chp}\theta_h & \text{if} \quad \theta_h \geq 0 \\ k_{chn}\theta_h & \text{if} \quad \theta_h < 0 \end{cases} \quad (9)$$

$$f_4(\omega_v) = \begin{cases} k_{tvp}\omega_v^2 & \text{if} \quad \omega_v \geq 0 \\ -k_{tvn}\omega_v^2 & \text{if} \quad \omega_v < 0 \end{cases} \quad (10)$$

$$f_5(\omega_v) = \begin{cases} k_{fvp}\omega_v^2 & \text{if} \quad \omega_v \geq 0 \\ -k_{fvn}\omega_v^2 & \text{if} \quad \omega_v < 0 \end{cases} \quad (11)$$

$$f_6(\theta_v) = \begin{cases} k_{cvp}(\theta_v - \theta_v^0)^2 & \text{if } \theta_v \geqslant \theta_v^0 \\ k_{cvn}(\theta_v - \theta_v^0)^2 & \text{if } \theta_v < \theta_v^0 \end{cases} \quad (12)$$

The system input vector is $u = [u_h,\ u_v]^T$ where u_h is the input voltage of the tail motor and u_v is the input voltage of the main motor. On the other hand, the system states are $x = [\omega_h,\ \Omega_h,\ \theta_h,\ \omega_v,\ \omega_v,\ \theta_v]^T$ where Ω_h is the angular velocity around the vertical axis, θ_h is the yaw angle of the beam (horizontal plane), ω_h is the rotational velocity of the tail rotor, Ω_v is the angular velocity around the horizontal axis, θ_v is the pitch angle of the beam (vertical plane) and ω_v is the rotational velocity of the main rotor. $J_{tr/mr}$ are the moments of inertia in DC-motor tail/main propeller subsystem, respectively. θ_v^0 is the equilibrium value for the vertical angle when no input voltage acts on the main propeller. The constants of the non-linear model (1)-(6) are defined as:

$$A = \left(m_t/2 + m_{tr} + m_{ts}\right) l_t \qquad B = \left(m_m/2 + m_{mr} + m_{ms}\right) l_m$$
$$C = m_{cb} l_{cb} + m_b l_b/2 \qquad H = A l_t + B l_m + m_b l_b^2/2 + m_{cb} l_{cb}^2$$
$$D = m_b l_b^2/3 + m_{cb} l_{cb}^2 \qquad F = m_{ms} r_{ms}^2 + m_{ts} r_{ts}^2/2$$
$$E = \left(m_{mr} + m_{ms} + m_m/3\right) l_m^2 + \left(m_{tr} + m_{ts} + m_t/3\right) l_t^2$$

where m_{ms} and m_{ts} are the masses of the main and tail shields, m_m and m_t are the masses of the main and the tail parts of the beam, m_{mr} and m_{tr} are the masses of the main and the tail DC-motor with main and tail rotor, m_b and l_b are the mass and the length of the counter-weight beam, m_{cb} and l_{cb} represent the mass of the counter-weight and the distance between the counter-weight and the joint and r_{ms} and r_{ts} are the radius of the main and tail shield.

B. Quasi-LPV modelling of TRMS

To take into account the TRMS high non-linearity and cross-coupling between its two axes, an accurate dynamic model of the system is thus required to achieve control objectives satisfactorily. A quasi-LPV system is defined as a linear time-varying plant whose state-space matrices are fixed functions of some vector of varying parameters ψ_t that depend on the state variables. Then, it is described by state-space equations of the form:

$$\dot{x}(t) = A(\psi_t)\, x(t) + B(\psi_t)\, u(t) \quad (13)$$

where $x(t) \in R^{n_x}$ represents the state vector and $u(t) \in R^{n_u}$ denotes the control inputs. Here $A(\psi_t) \in R^{n_x \times n_x}$ and $B(\psi_t) \in R^{n_x \times n_u}$ are parameter-varying matrices.

Following the non-linear embedding in the parameters approach proposed in [10], the non-linear model (1)-(6) can be expressed in quasi-LPV form by:

$$\begin{bmatrix} \dot{\omega}_h(t) \\ \dot{\Omega}_h(t) \\ \dot{\theta}_h(t) \\ \dot{\omega}_v(t) \\ \dot{\Omega}_v(t) \\ \dot{\theta}_v(t) \end{bmatrix} = A(\psi_t) \begin{bmatrix} \omega_h(t) \\ \Omega_h(t) \\ \theta_h(t) \\ \omega_v(t) \\ \Omega_v(t) \\ \theta_v(t) - \theta_v^0 \end{bmatrix} + B(\psi_t) \begin{bmatrix} u_h(t) \\ u_v(t) \end{bmatrix} \quad (14)$$

$$A(\psi_t) = \begin{bmatrix} \psi_t^1(p_t) & 0 & 0 & 0 & 0 & 0 \\ \psi_t^2(p_t) & \psi_t^3(p_t) & \psi_t^4(p_t) & \psi_t^5(p_t) & \psi_t^6(p_t) & \psi_t^7(p_t) \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \psi_t^8(p_t) & 0 & 0 \\ \psi_t^9(p_t) & \psi_t^{10}(p_t) & 0 & \psi_t^{11}(p_t) & a_{55} & \psi_t^{12}(p_t) \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (15)$$

$$B(\psi_t) = \begin{bmatrix} b_{11} & 0 & 0 & 0 & b_{51} & 0 \\ 0 & \psi_t^{13}(p_t) & 0 & b_{42} & 0 & 0 \end{bmatrix}^T$$

with:

$$\psi_t^1(p_t) = -\frac{k_a^2/R_a + B_{tr} + f_1(\omega_h)/\omega_h}{J_{tr}}, \quad \psi_t^2(p_t) = \frac{l_t \cos\theta_v f_2(\omega_h)/\omega_h}{D\cos^2\theta_v + E\sin^2\theta_v + F}$$
$$\psi_t^3(p_t) = -\frac{k_{oh}}{D\cos^2\theta_v + E\sin^2\theta_v + F}, \quad \psi_t^4(p_t) = -\frac{f_3(\theta_h)/\theta_h}{D\cos^2\theta_v + E\sin^2\theta_v + F}$$
$$\psi_t^5(p_t) = -\frac{k_m\left(\frac{k_a^2}{R_a} + B_{mr} + f_4(\omega_v)/\omega_v\right)\cos\theta_v}{\left(D\cos^2\theta_v + E\sin^2\theta_v + F\right)J_{mr}}$$
$$\psi_t^6(p_t) = \frac{k_m\omega_v \sin\theta_v\left(D\cos^2\theta_v - E\sin^2\theta_v - F - 2E\cos^2\theta_v\right)}{\left(D\cos^2\theta_v + E\sin^2\theta_v + F\right)^2}$$
$$\psi_t^7(p_t) = \frac{f_6(\theta_v)/(\theta_v - \theta_v^0)}{D\cos^2\theta_v + E\sin^2\theta_v + F}$$
$$\psi_t^8(p_t) = -\frac{\frac{k_a^2}{R_a} + B_{mr} + \frac{f_4(\omega_v)}{\omega_v}}{J_{mr}}, \quad \psi_t^9(p_t) = \frac{-k_t\left(B_{tr} + f_1(\omega_h)/\omega_h + k_a^2/R_a\right)}{J_v J_{tr}}$$
$$\psi_t^{10}(p_t) = \frac{k_g\cos\theta_v f_5(\omega_v)/\omega_v - \Omega_h H\sin\theta_v\cos\theta_v}{J_v}, \quad \psi_t^{11}(p_t) = \frac{l_m f_5(\omega_v)/\omega_v}{J_v}$$
$$a_{55} = -\frac{k_{ov}}{J_v}, \quad \psi_t^{12}(p_t) = \frac{g((A-B)\cos\theta_v - C\sin\theta_v)}{J_v(\theta_v - \theta_v^0)}, \quad b_{11} = \frac{k_a k_1}{J_{tr}R_a}$$
$$\psi_t^{13}(p_t) = \frac{k_m\cos(\theta_v)k_a k_2}{(D\cos^2\theta_v + E\sin^2\theta_v + F)R_a J_{mr}}, \quad b_{42} = \frac{k_a k_2}{J_{mr}R_a}, \quad b_{51} = \frac{k_a k_1 k_t}{J_v J_{tr} R_a}$$

ψ_t is the vector of varying parameters scheduled by $p_t = [\ \theta_h \quad \theta_v \quad \omega_h \quad \omega_v \quad \Omega_h\ ]^T$. Notice that the vector p_t does not contain the state Ω_v as this state variable does not affect the values of the elements in the model (14).

C. Polytopic Quasi-LPV Representation

In [12] and [13], linear matrix inequality techniques are applied for gain-scheduled control of LPV systems. These techniques have been developed for LPV systems whose time-varying parameter vector ψ_t varies within a polytope Θ. More precisely, in this kind of LPV system, the state matrices range in a polytope of matrices defined as the convex hull of a finite number N of matrices. Each polytope vertex corresponds to a particular value of the scheduling variable ψ_t. In other words:

$$\begin{pmatrix} A(\psi_t) \\ B(\psi_t) \end{pmatrix} \in Co\left\{\begin{pmatrix} A_j \\ B_j \end{pmatrix}, j = 1,\ldots,N\right\} := \sum_{j=1}^{N} \alpha_t^j(\psi_t)\begin{pmatrix} A_j \\ B_j \end{pmatrix} \quad (16)$$

with $\alpha_t^j(\psi_t) \geq 0$ and $\sum_{j=1}^{N}\alpha_t^j(\psi_t) = 1$, where each j^{th} model is called a *vertex* system. Because of this property, this type of LPV systems is referred as polytopic.

A controller/observer can be designed for quasi-LPV system using these techniques by approximating the quasi-LPV system with a polytopic model. The simplest polytopic approximation relies on bounding each parameter by an interval. This approximation is known as *bounding box* approach. To obtain such an approximation, each of the element of the vector p_t is assumed to take values in an interval (e.g. $\theta_h \in \left[\underline{\theta}_h, \overline{\theta}_h\right]$). Then, for each of the scheduling parameters in the vector ψ_t their minimum and maximum values over the allowed values of the elements of p_t are calculated. For example, consider the parameter $\psi_t^1(p_t)$. In this case, the extreme values $\underline{\psi}_t^1$ and $\overline{\psi}_t^1$ are given by:

$$\underline{\psi}_t^1 = -\left(k_a^2/R_a + B_{tr} + \max\left\{k_{thp}\overline{\omega}_h, -k_{thn}\underline{\omega}_h\right\}\right)/J_{tr}$$
$$\overline{\psi}_t^1 = -\left(k_a^2/R_a + B_{tr}\right)/J_{tr}$$

Taking into account all possible combinations of extreme values of the parameter vector elements, vertex systems in (16) are generated. The values of the non-linear model parameters must be known in order to calculate the vertex systems for control design purposes. Some of these values are given by the manufacturer while others need to be estimated. This leads to the necessity of performing system identification.

TABLE I
KNOWN AND UNKNOWN PARAMETERS

Param.	Value	Param.	Value	Param.	Value	Param.	Value
r_{ts}	0.1	r_{ms}	0.155	J_{tr}	?	J_{mr}	?
l_t	0.282	l_m	0.246	B_{tr}	?	B_{mr}	?
l_b	0.290	l_{cb}	0.276	$k_{thp/n}$	?	$k_{tvp/n}$	?
m_b	0.022	m_{cb}	0.068	k_{fhp}	?	k_{fhn}	?
m_m	0.014	m_t	0.015	k_{fvp}	?	k_{fvn}	?
m_{mr}	0.236	m_{tr}	0.221	k_{chp}	?	k_{chn}	?
m_{ms}	0.219	m_{ts}	0.119	k_{cvp}	?	k_{cvn}	?
k_1	6.5	k_2	8.5	k_m	?	k_t	?
$R_{ah/av}$	8	$L_{ah/av}$	0.86e-3	k_{oh}	?	k_{ov}	?
k_g	0.2	$k_{ah/av}$	0.0202				

There are several ways for implementing (16) depending on how $\alpha_t^j(\psi_t)$ functions are defined [14]. Here, the function $\alpha_t^j(\psi_t)$ is calculated via barycentric combination of vertices as suggested by [12].

III. TWIN ROTOR MIMO SYSTEM IDENTIFICATION

In this section, the identification procedure to estimate the parameters of TRMS is presented (see Table I for the list of known and unknown parameters). At first, the unknown parameters of motor-propellers are identified. Then, those of the aerodynamical part are estimated. The spirit of the method is to compare, at each step, the data obtained from the real set-up with the data obtained by simulating part of the TRMS continuous-time non-linear model. This is included in an optimization problem so as to find the parameter values that give simulation results that better approximate the real system in a least squares fashion.

A. Tail and Main Propeller

The identification procedure is based on the knowledge of the non-linear model of the tail ((1) and (7)) and main propellers ((4) and (10)) and should identify some values for the unknown parameters in such a manner that the model behaviour resembles the real behaviour of the tail and main propellers. It is assumed to have at disposal N_ω sets of data $u_{h/v}^i(k), \omega_{h/v}^i(k)$ for the propellers where $i = 1,\ldots,N_\omega$ and $k = 1,\ldots,K_i$ where K_i is the number of samples of the i-th set of data. The identification procedure finds the minimum of the following objective function over the unknown parameters $J_{tr/mr}$, $B_{tr/mr}$, $k_{th/vp}$ and $k_{th/vn}$:

$$J_\omega = \sum_{i=1}^{N_\omega}\sum_{k=1}^{K_i}\left(\omega_{h/v}^i(k) - \hat{\omega}_{h/v}^i(k)\right)^2 \quad (17)$$

where $\hat{\omega}_{h/v}^i(k)$ is the simulation provided by Eqs. (1) and (4). Notice that, while all the sets of data can be used to identify $J_{tr/mr}$ and $B_{tr/mr}$, a numerical value for $k_{thp/vp}$ can be obtained only from those sets of data where the input voltage $u_{h/v}$ is positive. Analogously, a numerical value for $k_{thn/vn}$ can be obtained only from those sets of data where the input voltage $u_{h/v}$ is negative.

B. Tail Propeller to Horizontal Aeromechanics

The identification procedure is based on the knowledge of the non-linear model (2), (3), (8), (9), (12). In this step, the horizontal dynamics due to an input voltage acting on the tail propeller are identified (unknown parameters: k_{fhp}, k_{fhn}, k_{oh}, k_{chp}, k_{chn}). When no input voltage acts on the main propeller, (2) and (3) simplify to:

$$\frac{d\Omega_h}{dt} = \frac{l_t f_2(\omega_h)\cos\theta_v^0 - k_{oh}\Omega_h - f_3(\theta_h)}{D\cos^2\theta_v^0 + E\sin^2\theta_v^0 + F} \tag{18}$$

$$\frac{d\theta_h}{dt} = \Omega_h \tag{19}$$

Replacing (19) in (18) and using Laplace transform leads to:

$$\Theta_h(s) = \frac{K}{1 + 2\xi T_w s + T_w^2 s^2}\Psi_h(s) \tag{20}$$

where $\Psi_h(s)$ is the Laplace transform of $\omega_h^2(t)$ and:

$$K = \begin{cases} \frac{l_t k_{fhp}\cos\theta_v^0}{k_{chp}} if \omega_h \geq 0 & 2\xi T_w = \frac{k_{oh}}{k_{chp/n}} \\ -\frac{l_t k_{fhn}\cos\theta_v^0}{k_{chn}} if \omega_h < 0 & T_w^2 = \frac{D\cos^2\theta_v^0 + E\sin^2\theta_v^0 + F}{k_{chp/n}} \end{cases} \tag{21}$$

Hence, the knowledge of K, ξ and T_w can be used for estimation of the unknown parameters:

$$\begin{array}{ll} k_{chp/n} = \frac{K_v^0}{\left(T_w^{(p/n)}\right)^2} k_{oh}^{(p/n)} & k_{fhp} = \frac{K^{(p)} k_{chp}}{l_t \cos\theta_v^0} \\ k_{oh}^{(p/n)} = 2\xi^{(p/n)} T_w^{(p/n)} k_{chp/n} & k_{fhn} = \frac{-K^{(n)} k_{chn}}{l_t \cos\theta_v^0} \end{array} \tag{22}$$

Values for $K^{(p)}$ and $K^{(n)}$ can be found in a simple way through the knowledge of the steady-state values of ω_h and θ_h:

$$K^{(p/n)} = \theta_h^\infty / (\omega_h^\infty)^2 \tag{23}$$

Values for $T_w^{(p)}$, $T_w^{(n)}$, $\xi^{(p)}$ and $\xi^{(n)}$ can be estimated by means of some optimization method. It is assumed to have at disposal N_h sets of data $\omega_h^i(k)$ and $\theta_h^i(k)$ where $i = 1,\ldots,N_h$, $k = 1,\ldots,K_i$ and K_i is the number of samples of the i-th set of data. By applying the described procedure to each set of data, different values for the unknown parameters can be obtained. Such values can be used to define lower bounds and upper bounds for the values of the unknown parameters. Then, another identification stage can be performed. In this stage the minimum of the objective function over the unknown parameters (k_{fhp}, k_{fhn}, k_{oh}, k_{chp}, k_{chn}):

$$J_h = \sum_{i=1}^{N_h}\sum_{k=1}^{K_i}\left(\theta_h^i(k) - \hat{\theta}_h^i(k)\right)^2 \tag{24}$$

should be found, where $\hat{\theta}_h^i(k)$ is the solution of (18)-(19). $\hat{\theta}_h$ can be calculated by considering that both ω_h and θ_v are data provided from the sensors.

C. Main Propeller to Horizontal Aeromechanics

In this step, the horizontal dynamics due to an input voltage acting on the main propeller are identified (unknown parameters: k_m, k_{cvp}, k_{cvn}). When no input voltage acts on the tail propeller, (2) and (3) simplify to:

$$\begin{aligned} \frac{d\Omega_h}{dt} &= \frac{k_m\omega_v\sin\theta_v\Omega_v\left(D\cos^2\theta_v - E\sin^2\theta_v - F - 2E\cos^2\theta_v\right)}{\left(D\cos^2\theta_v + E\sin^2\theta_v + F\right)^2} \\ &+ \frac{-k_{oh}\Omega_h - f_3(\theta_h) + f_6(\theta_v)}{D\cos^2\theta_v + E\sin^2\theta_v + F} \\ &+ \frac{k_m\cos\theta_v\left(\frac{k_a k_2}{R_a}u_v - \left(B_{mr} + \frac{k_a^2}{R_a}\right)\omega_v - f_4(\omega_v)\right)}{J_{mr}\left(D\cos^2\theta_v + E\sin^2\theta_v + F\right)} \end{aligned} \tag{25}$$

$$\frac{d\theta_h}{dt} = \Omega_h \tag{26}$$

It is assumed to have at disposal M_h sets of data $\omega_v^i(k)$,$\theta_v^i(k)$ and $\theta_h^i(k)$ where $i = 1,\ldots,M_h$, and K_i is the number of samples of the i-th set of data. The identification procedure should identify values for k_m, k_{cvp} and k_{cvn}, as all other parameter values are given by the manufacturer or have been identified previously. If the input voltage in the main propeller is a step signal, at steady-state:

$$\Omega_v^\infty = \Omega_h^\infty = \frac{k_a k_2}{R_a}u_v - \left(B_{mr} + \frac{k_a^2}{R_a}\right)\omega_v - f_4(\omega_v) = 0 \tag{27}$$

and considering (25) it reduces to:

$$\begin{cases} -k_{chp}\theta_h^\infty + k_{cvp}\left(\theta_v^\infty - \theta_v^0\right)^2 = 0 & if \theta_v \geq \theta_v^0 \\ -k_{chp}\theta_h^\infty + k_{cvn}\left(\theta_v^\infty - \theta_v^0\right)^2 = 0 & if \theta_v < \theta_v^0 \end{cases} \tag{28}$$

A first estimation of k_{cvp} and k_{cvn} is obtained from (28) as:

$$k_{cvp/n} = \frac{k_{chp}\theta_h^\infty}{(\theta_v^\infty - \theta_v^0)^2} \tag{29}$$

Later, k_m, k_{cvp} and k_{cvn} can be refined by solving an optimization problem with the same objective function (24) that has been previously used, optimizing over the parameters k_m, k_{cvp} and k_{cvn}.

D. Main Propeller to Vertical Aeromechanics

In this case, the identification procedure should identify the values of k_{fvp} , k_{fvn} and k_{ov}. Eq. (5) and (6), when no input voltage acts on the tail propeller, simplify to:

$$\begin{aligned} \frac{d\Omega_v}{dt} &= \frac{l_m f_5(\omega_v) + k_g\Omega_h f_5(\omega_v)\cos\theta_v - k_{ov}\Omega_v}{J_v} \\ &\frac{-0.5\Omega_h^2 H\sin 2\theta_v + g\left((A-B)\cos\theta_v - C\sin\theta_v\right)}{J_v} \end{aligned} \tag{30}$$

$$\frac{d\theta_v}{dt} = \Omega_v \tag{31}$$

At steady-state, as $\Omega_h^\infty = 0$ and $\Omega_v^\infty = 0$:

$$\frac{l_m f_5(\omega_v^\infty) + g\left((A-B)\cos\theta_v^\infty - C\sin\theta_v^\infty\right)}{J_v} = 0 \tag{32}$$

Thus, a preliminary estimation of k_{fvp} and k_{fvn} can be obtained as:

$$k_{fvp} = \frac{g\left((B-A)\cos\theta_v^\infty + C\sin\theta_v^\infty\right)}{l_m(\omega_v^\infty)^2} \tag{33}$$

TABLE II

IDENTIFIED PARAMETERS VALUES

Par.	Value	Par.	Value	Par.	Value	Par.	Value
B_{tr}	0.0097	B_{mr}	0.0032	k_{thp}	0.0032	k_{thn}	4.8951e-4
J_{tr}	0.0101	J_{mr}	0.0562	k_t	0	k_m	0.0122
k_{tvp}	0.0177	k_{tvn}	0.0154	k_{oh}	0.0154	k_{ov}	0.0326
k_{fhp}	0.0187	k_{fhn}	0.0271	k_{fvp}	0.4311	k_{fvn}	0.2850
k_{chp}	0.0085	k_{chn}	0.0043	k_{cvp}	0.0207	k_{cvn}	0.0094

$$k_{fvn} = -\frac{g\left((B-A)\cos\theta_v^{\infty} + C\sin\theta_v^{\infty}\right)}{l_m\left(\omega_v^{\infty}\right)^2} \tag{34}$$

Later, the minimum over k_{fvp}, k_{fvn} and k_{ov} of the objective function:

$$J_v = \sum_{i=1}^{N_v}\sum_{k=1}^{K_i}\left(\theta_v^i(k) - \hat{\theta}_v^i(k)\right)^2 \tag{35}$$

where $\hat{\theta}_v^i(k)$ is the solution of (30)-(31), should be found. $\hat{\theta}_v$ can be calculated by considering that both ω_v and Ω_h are data provided from the sensors.

E. Tail Propeller to Vertical Aeromechanics

It has been noticed experimentally that when the main propeller is off, the TRMS remains in its equilibrium vertical angle θ_v^0 for each value of u_h. This suggests that $k_t = 0$ is a good approximation of the real value of this constant.

IV. RESULTS

A. Non-linear Identification Results

The identification procedure has been applied to sets of data that correspond to experiments that lasts 60 s, with a sample period of 0.01 s. Each set is the response of the system to a step input voltage signal with a pseudo-random binary sequence overlapped from 30 s to 60 s. The minima of (17), (24) and (35) are found by means of *fmincon* MATLAB function of the Optimization Toolbox. This function attempts to find a constrained minimum of a scalar function of several variables (in this case, the unknown parameters). This is generally referred to as constrained non-linear optimization or non-linear programming. Even though such a function does not give guarantees of being achieved the global optimum, it has been noticed that performing the same optima search through the more robust *genetic algorithms* (GA) led to similar results as those obtained through *fmincon* but needing more computing time. The values returned by the *fmincon* function are listed in Table II.

B. Polytopic Bounding Box Results

Once the parameters of the non-linear model have been identified, a polytopic quasi-LPV model can be obtained by applying the procedure described in Section II-C. The motor inputs u_h and u_v and each of the state variables that influence the values of the elements of the state-space matrix (15) is assumed to take values in an interval, as resumed in Table III. In order to design a digital controller, a discrete-time polytopic model is required. Here, this has been obtained using Euler approximation with sampling time $T_s = 0.01s$[1]. Minimum and maximum values for each of the elements of the scheduling vector ψ_k have been calculated, and such values are listed in Table IV.

TABLE III

STATE VARIABLES LIMITS

Var.	Min.	Max.	Var.	Min.	Max.
u_h, u_v	-2.5 V	2.5 V	Ω_h	-2 rad/s	2 rad/s
ω_h	-3.5 V	2.5 V	ω_v	-1.8 V	1.8 V
θ_h	-3.4 rad	2.4 rad	θ_v	0.7 rad	1.7 rad

TABLE IV

SCHEDULING PARAMETERS LIMITS

ψ_k^j	$\underline{\psi}_k^j$	$\overline{\psi}_k^j$	ψ_k^j	$\underline{\psi}_k^j$	$\overline{\psi}_k^j$
ψ_k^1	0.9840	0.9903	ψ_k^2	2.59e-4	0.0048
ψ_k^3	0.9895	0.9975	ψ_k^4	-0.0058	-7.04e-4
ψ_k^5	-0.0013	-1.15e-4	ψ_k^6	-0.0075	0.0070
ψ_k^7	-0.0045	0.0156	ψ_k^8	0.9947	0.9994
ψ_k^9	0	0	ψ_k^{10}	-0.0092	0.0254
ψ_k^{11}	0	0.0247	ψ_k^{12}	-0.0412	-0.0240

V. VALIDATION OF THE CALIBRATED TRMS MODEL

In order to validate the values obtained by means of identification, the model has been tested with various inputs. The simulation of the non-linear model has been compared to the data obtained from the real system. Two different validation scenarios have been performed. In these validation scenarios the input voltages for both the main and the tail propeller are sinusoidal with mean values different from zero, as resumed in Table V.

The comparisons between the simulation data and the real data are shown in Fig.2, Fig.3, Fig.4, Fig.5, Fig.6 and Fig.7.

The validation scenarios show that the model approximates the behaviour of both the propellers in a quite perfect manner. Nevertheless, the model and the real system are not so close when comparing the yaw and pitch angle responses. Although many experiments have been done with different sets of data and input signals, it has been proved hard to improve the results without modifying the non-linear model of the system, due to unmodelled frictions (e.g. the torque of the friction force presented in [1]) that are very difficult to characterize because of their high non-linearity and change in time.

However, the results prove to be satisfactory, as the model has to be used for control purposes and the control loop will compensate the imperfections in the model.

[1]The discrete-time versions of ψ_t and p_t are referred to as ψ_k and p_k.

TABLE V

INPUT VOLTAGES FOR VALIDATION OF THE TRMS MODEL

Scenario	Input	Mean value	Sine amplitude	Sine frequency
1	u_h	0.7 V	0.3 V	0.1 Hz
1	u_v	0.7 V	0.3 V	0.05 Hz
2	u_h	0.6 V	0.3 V	0.1 Hz
2	u_v	0.6 V	0.1 V	0.05 Hz

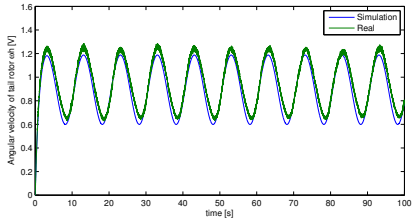

Fig. 2. Validation scenario 1: Angular velocity of tail rotor ω_h.

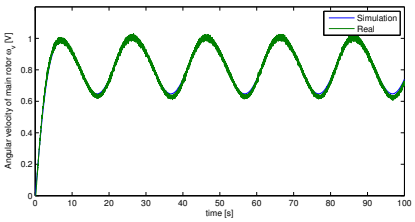

Fig. 3. Validation scenario 1: Angular velocity of main rotor ω_v.

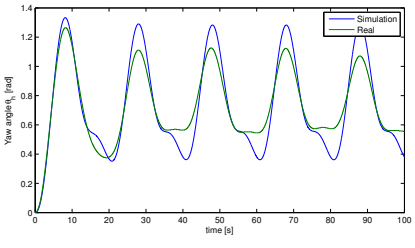

Fig. 4. Validation scenario 1: Yaw angle of the TRMS θ_h.

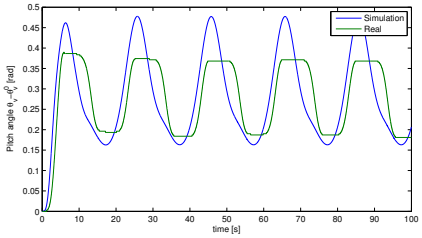

Fig. 5. Validation scenario 1: Pitch angle of the TRMS $\theta_v - \theta_v^0$.

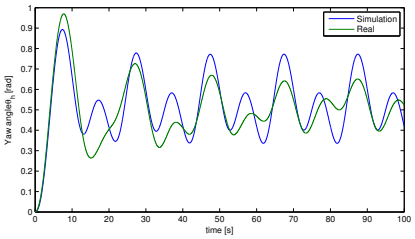

Fig. 6. Validation scenario 2: Yaw angle of the TRMS θ_h.

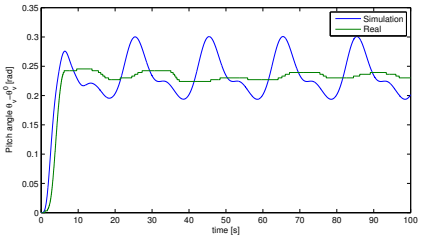

Fig. 7. Validation scenario 2: Pitch angle of the TRMS $\theta_v - \theta_v^0$.

VI. Conclusions and Future Work

This paper has presented the experimental identification of the parameters of the TRMS non-linear model using data collected from the real laboratory setup. From this non-linear model, a quasi-LPV model has also been obtained and, later, transformed into a polytopic one. The model parameters have been calibrated by means of nonlinear least-square identification approach. The results have been validated and the model has proved to show good performances in approximating the real behavior of the rotors. Results concerning the mechanical part were not so good, probably due to the non-linear and non-repetitive effect of frictions.

As a future work, the model obtained in this paper will be used for control purposes, so as to test some LPV techniques with a real set-up. Moreover, methods for deriving polytopic models that reduce conservativeness will be investigated (e.g. *bounding diamonds*, *convex hulls*). Finally, further investigation in the modelling of the TRMS will be done so as to include the non-linear time-varying effects due to frictions that have been neglected or approximated.

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