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Biased Sinusoidal Disturbance Rejection with Plant Uncertainty via an Adaptive Third-Order Generalized Integrator

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***Abstract*— In this paper a method to reject periodic biased disturbances of unknown frequency is presented. The entire set of unknown disturbance parameters is estimated via a third-order generalized integrator (TOGI). An interesting property of the method is that internal signals of TOGI are combined linearly to produce the control signal, resulting in a very simple algorithm. Only a single adaptive parameter permits to govern the system. It is shown that, within the assumptions of an averaging analysis, the adaptive system is stable and completely rejects the disturbance, even if a rough estimation of the plant is available. Bounds on the uncertainty of the plant are given in terms of the bounds on the input disturbance. Simulations demonstrate the properties of the algorithm in a variety of conditions.**

I. Introduction

The problem of complete rejection of external inaccessible disturbances plays an important role in the modern control theory. Possible applications can be found in control noise and vibrations in helicopters due to the rotation of the blades [1] or vibrations reduction in space structures due to control moment gyroscopes and cryogenic coolers [2], just to name a few. Methods proposed in the literature for disturbance rejection can be classified depending on *a priori* required information about the disturbance. The known frequency case has been extensively investigated and several techniques are available, including internal model control, adaptive feed-forward cancellation, and repetitive control. Among these techniques, the internal model principle approach, going back to [3], is one of the most common techniques. The basis idea is that perfect disturbance rejection can be performed in a linear feedback system if and only if the controller gain is infinite at that frequency. If the plant is completely known, as reported in [4], the frequency known-case problem has a classical solution by modeling the disturbance as a linear exosystem and by using an observer which provides an asymptotic estimate of the disturbance so that it can be cancelled. If the frequency is unknown, the rejection problem has been investigated in the case of an unbiased sinusoidal disturbance. In [4], [5] a possible extension to the case of biased sinusoid signal at unknown frequency is presented. Unlike known-frequency methods, techniques to handle unknown and time-varying disturbance frequencies have only recently emerged. Interesting feedback control systems have been proposed that do not assume independent measurement of the disturbance, so that the signals driving the control actuator are obtained directly, and solely, from the error sensors. Generally, the most intuitive approach in this direction consists in combining a frequency estimator with a disturbance cancellation algorithm for known frequency; such techniques were labeled *indirect* in [6]. On the other hand, *direct* approaches have been proposed to design a stable adaptive controller for disturbance rejection without a clear separation between frequency estimation and disturbance cancellation. While the direct scheme is used for local initial conditions of the frequency estimate, the indirect one [7] can be used for larger initial conditions; however only the direct scheme guarantees exact disturbance compensation. Moreover, the case of not-completely known plant have received growing interest in the last years. In the signal processing literature, gradient algorithms (*i.e.*, adaptive least-mean-squares or LMS algorithm) have been coupled with an on-line identifier of the plant's impulse response [8], [9], [10]. Even if such methods seem to be theoretically attractive, they suffer for additional excitation for convergence requirement. A different approach results in harmonic steady-state (HSS) methods that simplify the problem by approximating the plant by its steady-state sinusoidal response. In [11], higher harmonic control (HHC) was proposed for the cancellation of periodic noise in an acoustic drum. This approach requires a separate tuning procedure for the plant estimate, which was to be repeated whenever performance degraded. In [12] an algorithm combining two gradient-type adaptation steps guaranteeing stability properties and without additional excitation has been proposed. [13] proposes an implementation of adaptive harmonic steady-state that provides continuous adaptation to unknown plant parameters. However, the frequency of the disturbance was assumed to be known.

In this paper, the case of unknown biased sinusoid disturbance and not-completely known plant is treated. Only an estimate of the plant is known, although with some degree of uncertainty. The indirect approach is revisited by considering a frequency estimator based on a third-order generalized integrator (**TOGI**) [14]. An averaging analysis was used to prove the stability of the TOGI method. Averaging theory approximates a set of non-linear time-varying differential equations by a simpler time-invariant system. The theory is reviewed in [15], where it was found to provide useful information on the dynamic properties of adaptive control systems. It is possible to consider the method here discussed as an indirect implementation of adaptive internal model principle, as opposed to the direct implementations mentioned earlier. An original feature is that the disturbance cancellation scheme uses the internal signals produced by the TOGI. Therefore, an important property associated with the proposed approach is that no additional dynamics are required into cancellation algorithm: the correction signal is simply a linear combination of the TOGI signals. This specific choice results in an extremely simple algorithm where a single adaptive parameter permits to estimate the frequency of the disturbance signal and to determine the dynamics of the system. An interesting by-product of the approach is that it makes possible to prove asymptotic disturbance rejection even in the presence of error in the plant estimate. Conditions on the plant estimate error can be checked for a range of disturbance frequencies. The paper is organized as follows. In Section II the rejection scheme is presented and the TOGI filter properties are discussed. In Section III the stability of the scheme and the rejection of the input disturbance is proved. In Section IV some severe simulations are conducted to put in evidence the strength of the method and Section V is devoted to conclusions.

II. Proposed algorithm

The scheme that permits to estimate the frequency of the sinusoidal disturbance and cancel its effect on the plant output is shown in Fig. 1. The signal $d(t)$ is the sinusoidal

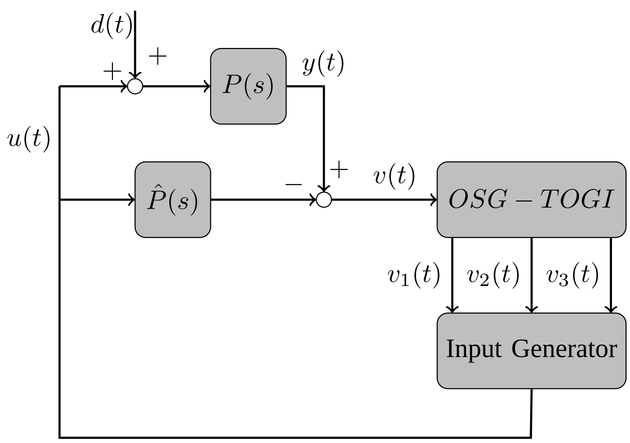

Fig. 1. Disturbance rejection scheme.

disturbance acting at the input channel of the plant. In the definition of the algorithm, the disturbance is assumed to have fixed frequency. However, in its application, the frequency is allowed to vary.

The output plant $y(t)$ is described by the following

$$y(t) = P(s)[u(t) + d(t)], \tag{1}$$

where $P(s)$ is the plant transfer function and $u(t)$ is the control input. The notation $P(s)[(\cdot)]$ stands for the time-domain output of the system $P(s)$ with input $(\cdot)$. $P(s)$ is assumed to be stable and the disturbance is a biased sinusoid

$$d(t) = A_0 + A_c \sin(\omega_c t + \phi_c). \tag{2}$$

The disturbance frequency is estimated via an orthogonal signals generator based on a third-order generalized integrator, namely **OSG-TOGI** (see Fig. 2). The OSG-TOGI input is defined as

$$v(t) = y(t) - \hat{P}(s)[u(t)] \tag{3}$$

where $\hat{P}(s)$ is an estimate of the plant $P(s)$. The OSG-TOGI is characterized by a resonant frequency ω_s and a gain K_s. For ω_s constant, the output signals ($v_1(t)$, $v_2(t)$ and $v_3(t)$) are the outputs of linear time-invariant systems with input $v(t)$ and transfer functions

$$F_1(s) = \frac{V_1(s)}{V(s)} = \frac{K_s\omega_s s}{s^2 + K_s\omega_s s + \omega_s^2}, \tag{4}$$

$$F_2(s) = \frac{V_2(s)}{V(s)} = \frac{K_s\omega_s^2}{s^2 + K_s\omega_s s + \omega_s^2} \tag{5}$$

and

$$F_3(s) = \frac{V_3(s)}{V(s)} = \frac{K_s\omega_s(s^2 + \omega_s^2)}{(s + \omega_s)(s^2 + K_s\omega_s s + \omega_s^2)} \tag{6}$$

where $V(s)$ and $V_i(s)$ are the Laplace transforms of the input $v(t)$ and the signal $v_i(t)$ respectively.

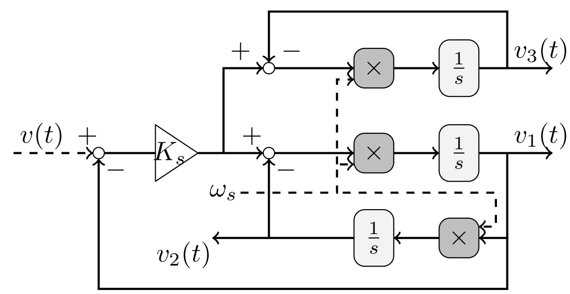

Fig. 2. OSG-TOGI block diagram.

As discussed in [14], for a signal

$$v(t) = A_{v0} + A_v \sin(\omega_c t + \phi_v), \tag{7}$$

the output signals converge exponentially to

$$v_{1\infty}(t) = mA_v \sin(\omega_c t + \phi_v + \phi), \tag{8}$$

$$v_{2\infty}(t) = K_s A_{v0} - mA_v \frac{\omega_s}{\omega_c} \cos(\omega_c t + \phi_v + \phi) \tag{9}$$

and

$$\begin{aligned} v_{3\infty}(t) \quad = \quad & K_s A_{v0} - K_s A_v \omega_s \sqrt{\frac{1-m^2}{\omega_s^2 + \omega_c^2}} sign[\omega_s - \omega_c] \\ & \times \cos\left(\omega_c t + \phi_v + \phi - \arctan\left(\frac{\omega_c}{\omega_s}\right)\right) \end{aligned} \tag{10}$$

where

$$m = \frac{K_s\omega_s - \omega_c}{\sqrt{(\omega_s^2 - \omega_c^2)^2 + K_s^2\omega_s^2\omega_c^2}}, \tag{11}$$

$$\phi = sign[\omega_s - \omega_c]\frac{\pi}{2} - \arctan\left(\frac{K_s\omega_s\omega_c}{\omega_s^2 - \omega_c^2}\right), \tag{12}$$

and the $sign[\cdot]$ function is defined as follows

$$sign[x] : -1 \ x < 0 \text{ and } +1 \ x \geq 0. \tag{13}$$

In Figs. 3-5 the Bode diagrams of the TOGI filters have been reported for constant ω_s equal to 100 rad/s. An immediate result is that, in steady-state conditions, the signal

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$v_1(t)$ converges to an unbiased sinusoid, due to the derivative term in $F_1(s)$, in contrast with the outputs $v_2(t)$ and $v_3(t)$ that are both affected by the bias term $K_s A_{v0}$. The transfer functions $F_1(s)$ and $F_2(s)$ represent second order filters with a bandwidth depending on the gain K_s and a resonant frequency equal to ω_s. In particular, $F_2(s)$ presents second order low-pass filtering characteristics with static gain K_s and F_1 behaves as a second order band-pass filter with no attenuation and no phase shift at the resonant frequency. If K_s decreases, the bandwidth of the filter $F_1(s)$ becomes narrower resulting a heavy filtering, nevertheless this entails a slowdown on the dynamic response of the system increasing oscillations and the stabilization time. Lastly, the transfer function $F_3(s)$ represents a proper notch filter with a band stop centered at ω_s. Eq. (6) reveals that the filter gain at the frequency ω_s is equal to zero, i.e.

$$F_3(j\omega_s) = 0. \tag{14}$$

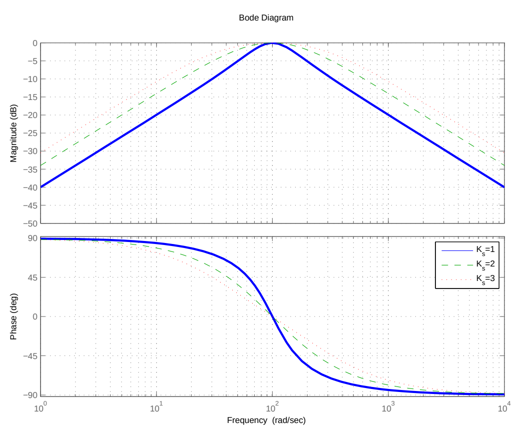

Fig. 3. Bode diagrams of $F_1(s)$ filter for constant $\omega_s = 100$ rad/s.

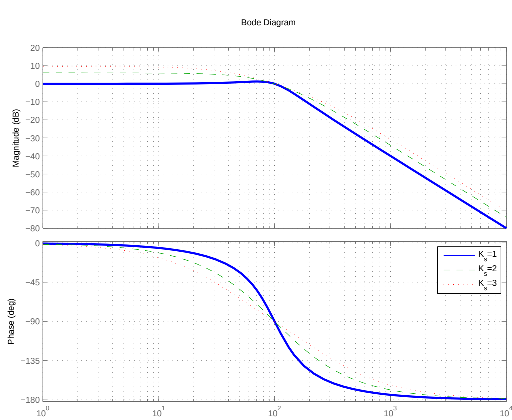

Fig. 4. Bode diagrams of $F_2(s)$ filter for constant $\omega_s = 100$ rad/s.

This fact implies that every sinusoidal input term with frequency equal to the resonant one is completely filtered. As a consequence, an adaptation of the resonant frequency to the unknown one ω_c means a complete filtering of the signal

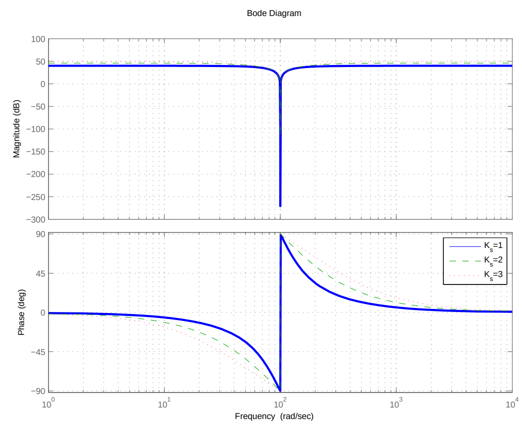

Fig. 5. Bode diagrams of $F_3(s)$ filter for constant $\omega_s = 100$ rad/s.

$v(t)$ that results in a constant $v_3(t)$ signal depending on the bias term. Therefore, in order to converge to the unknown frequency ω_c, the resonant frequency ω_s is adapted according to the following differential equation

$$\dot{\omega}_s = \gamma_s \left(v(t) - v_1(t) - v_3(t)/K_s\right)\left(v_3(t) - v_2(t)\right)\omega_s \tag{15}$$

where $\gamma_s > 0$ is the adaptation gain [14]. In the next section the convergence of the adaptation law (15) is revisited and it is proved that every sinusoidal term at the same frequency of the OSG-SOGI does not affect the estimation process. Based on the frequency estimate, a number of approaches can be obtained by combining the frequency estimator with a control law for a sinusoidal disturbance of known frequency. Interestingly, the signals of the OSG-TOGI can be shared with the disturbance rejection scheme, yielding a very simple control algorithm. Specifically, the control signal $u(t)$ is generated as linear combination of the OSG-TOGI signals

$$u = -\frac{Re(\hat{P}(j\omega_s))}{\left|\hat{P}(j\omega_s)\right|^2} v_1 + \frac{Im(\hat{P}(j\omega_s))}{\left|\hat{P}(j\omega_s)\right|^2}(v_3 - v_2) - \frac{1}{K_s \hat{P}(0)} v_3. \tag{16}$$

The motivation of such a choice is that, for fixed ω_s,

$$u(t) = -C(s)[v] \tag{17}$$

where the expression of the controller $C(s)$ is reported in Eq. (18). Interestingly , for $\hat{P} = P$ and $\omega_s = \omega_c$

$$u(t) = -C(s)P(s)[d] \tag{19}$$

with $C(0) = 1/P(0)$ and $C(j\omega_c) = 1/P(j\omega_c)$, so that in steady-state, $u = -d$ and $y = 0$.

III. Averaging Analysis

An averaging analysis (see [15]) permits to demonstrate that the frequency ω_s in Eq. (15) converges to the unknown input frequency ω_c. The steady-state responses of the components of the system are computed assuming that ω_s is constant, and the right-hand side of the differential equation for ω_s is averaged, yielding the averaged system. The dynamics of the averaged system can then be analyzed to predict properties of the original system under a small

$$C(s)=\frac{\omega_s}{\hat{P}(0)}\frac{s^2+\omega_s^2}{(s+\omega_s)(s^2+K_s\omega_s s+\omega_s^2)}+\frac{K_s\omega_s s}{\left|\hat{P}(\jmath\omega_s)\right|^2}\frac{Re(\hat{P}(\jmath\omega_s))(s+\omega_s)-Im(\hat{P}(\jmath\omega_s))(s-\omega_s)}{(s+\omega_s)(s^2+K_s\omega_s s+\omega_s^2)}. \quad (18)$$

gain assumption. The motivation for the update law (15) is that, in sinusoidal steady-state, given an input of the form $v(t)=A_{v0}+A_v\sin(\omega t)$, the average of the right-hand side of (15) is equal to

$$AVG[(v(t)-v_1(t)-v_3(t)/K_s)(v_3(t)-v_2(t))] = \frac{1}{2}A_v^2(\xi_1(\omega)-\xi_2(\omega)). \quad (20)$$

The terms $\xi_1(\omega)$, $\xi_2(\omega)$ are defined as

$$\xi_1(\omega)=(1-Re[F_1(\jmath\omega)]-Re[F_3(\jmath\omega)]/K_s)\times \times(Re[F_3(\jmath\omega)]-Re[F_2(\jmath\omega)])$$

and

$$\xi_2(\omega)=(Im[F_3(\jmath\omega)]-Im[F_2(\jmath\omega)])\times \times(Im[F_1(\jmath\omega)]+Im[F_3(\jmath\omega)]/K_s)$$

where ω is the frequency of the considered input. The OSG-TOGI input signal in Eq. (3) is now rewritten as

$$v(t)=\overline{A}_0+\overline{A}_c\sin(\omega_c t+\overline{\phi}_c)+m_0+m_y\sin(\omega_s t+\phi_y), \quad (21)$$

where the term $m_0+m_y\sin(\omega_s t+\phi_y)$ is due to the control input $u(t)$ while the term $\overline{A}_0+\overline{A}_c\sin(\omega_c t+\overline{\phi}_c)$ represent the output of the plant at the disturbance $d(t)$, i.e., $\overline{A}_0 = A_0P(0)$, $\overline{A}_c = A_c|P(\jmath\omega_c)|$ and $\overline{\phi}_c=\phi_c+\angle P(\jmath\omega_c)$. A question arises as to the effect of the control input on the signal that is used for frequency estimation. As stated in the following proposition, the averaging analysis predicts that a sinusoidal term at the same frequency as the OSG-TOGI resonant frequency does not influence the estimation process.

Proposition 1: Consider the averaged system of the differential equation (15). Then, a biased sinusoidal term at the resonant frequency ω_s does not affect the convergence of the averaged system.

Proof: The update law for the frequency estimate is approximated by averaging the right-hand side of (15), using signals computed while assuming that the frequency estimate is constant. By considering Eqs. (4)-(6) and the averaged system (20), any term depending on an input sinusoid at the frequency ω_s drops out of the differential equation because $\xi_1(\omega_s)=0$ and $\xi_2(\omega_s)=0$ due to the fact that $F_1(\jmath\omega_s)=1$, and $F_3(\jmath\omega_s)=0$. A similar approach can be provided for the constant term and therefore $\dot{\omega}_s=0$. ■

From Proposition 1, it follows that, with a control input at the frequency ω_s, the estimator behaves as the input $v(t)$ contains only the component depending on the disturbance at the frequency ω_c. The expressions for the transfer functions $F_1(s)$, $F_2(s)$ and $F_3(s)$ yield the following adaptive law for ω_s:

$$\dot{\omega}_s=-\frac{\gamma_s}{2}K_s\overline{A}_c^2\frac{w_s^3w_c^2(\omega_s^2-\omega_c^2)}{(\omega_c^2+\omega_s^2)\left((\omega_c^2-\omega_s^2)^2+K_s^2\omega_s^2\omega_c^2\right)}. \quad (22)$$

Considering the frequency estimator error

$$\omega_e \triangleq \omega_s-\omega_c, \quad (23)$$

the candidate Lyapunov function $V=\frac{1}{2}\omega_e^2$ has been proposed. Its derivative with respect to time is equal to

$$\dot{V}=-\frac{\gamma_s}{2}K_s\overline{A}_c^2\frac{\omega_s^3\omega_c^2(\omega_s+\omega_c)(\omega_s-\omega_c)^2}{(\omega_c^2+\omega_s^2)\left((\omega_c^2-\omega_s^2)^2+K_s^2\omega_s^2\omega_c^2\right)}. \quad (24)$$

As a consequence, if $\omega_s(0)>0$, then $\dot{V}<0$ unless $\omega_e\equiv 0$, so that the system is globally stable (within the subspace where initial conditions are allowed). It has proved earlier that, if $\hat{P}=P$, the disturbance signal is completely removed in steady-state. In the presence of plant estimation error, the transfer functions from the disturbance $d(t)$ to the signals $u(t)$ and $y(t)$ are

$$u(t) = -\frac{C(s)P(s)}{1+C(s)(P(s)-\hat{P}(s))}[d(t)] \quad (25)$$

$$y(t) = \frac{P(s)(1-C(s)\hat{P}(s))}{1+C(s)(P(s)-\hat{P}(s))}[d(t)]. \quad (26)$$

Therefore, stability of the inner loop is determined by the zeros of $1+C(P-\hat{P})$. Note that $C(s)$ is a function of the estimate ω_s as well, and the averaging analysis requires that the roots of $1+C(P-\hat{P})$ are in the open left-half plane for all ω_s along the trajectories of the averaged system. The Nyquist criterion implies the following condition for stability

$$\left|P(j\omega)-\hat{P}(j\omega)\right|<\frac{1}{|C(j\omega)|} \quad (27)$$

for all $\omega>0$ and for ω_s along the trajectories of the adaptive system. Given that ω_s converges monotonically to the disturbance frequency, this condition can be checked *a priori* if the disturbance frequency is known to lie within a certain range and the frequency estimate is initialized within that range. Since

$$C(j\omega_s)=\frac{1}{\hat{P}(j\omega_s)} \text{ and } C(0)=\frac{1}{\hat{P}(0)} \quad (28)$$

then $1-C(j\omega_c)\hat{P}(j\omega_c)=0$ if $\omega_s=\omega_c$ and $1-C(0)\hat{P}(0)=0$ for the bias term. This implies that the disturbance is rejected asymptotically even if the plant estimate is inaccurate, as long as the uncertainty is within the specified bound.

Remark 1: The conditions to be satisfied are that the adaptation be sufficiently slow and that the system be closed-loop stable for all values of the frequency estimate. Given bounds on the disturbance frequency and estimates of the plant and associated uncertainty, the stability condition can be checked. Even in the presence of rough estimates, the disturbance is rejected if the conditions are satisfied.

IV. SIMULATIONS

In this section some simulations have been conducted to highlight the performances of the proposed method in a wide range of working scenarios. The cases of both frequency step and frequency sweep are discussed. The case of offset steps is also covered. For the three simulations, the observation time is $T_{obs} = 15s$ and the number of samples is equal to $n = 6.4 \cdot 10^6$. The plant to be controlled remains the same for all the experiments

$$P(s) = \frac{8000}{(s+100)(s+80)} \tag{29}$$

and the estimated one is

$$\hat{P}(s) = \frac{60}{s+60}. \tag{30}$$

To guarantee the stability of the closed-loop system, the condition in Eq. (27) has been checked in the working range $\omega_s \in [20, 60]$.

Example 1**. Frequency steps**. The case of a frequency steps is here analyzed. The input disturbance is equal to

$$d(t) = 1 + \sin(\omega_c(t)t) \tag{31}$$

where

$$\omega_c(t) = \begin{cases} 50, & \text{for } 0 < t \leq T_{obs}/4, \\ 60, & \text{for } T_{obs}/4 < t \leq T_{obs}/2, \\ 50, & \text{for } T_{obs}/2 < t \leq 3T_{obs}/4, \\ 40, & \text{for } 3T_{obs}/4 < t \leq T_{obs}. \end{cases} \tag{32}$$

The OSG-TOGI parameters are chosen equal to $K_s = 1, \gamma = 50$ and the initial resonant frequency is equal to $\omega_s(0) = 0.5\omega_c(0)$. In Fig. 6 the estimation process is reported while in Fig. 7 the rejection action of the proposed method is depicted.

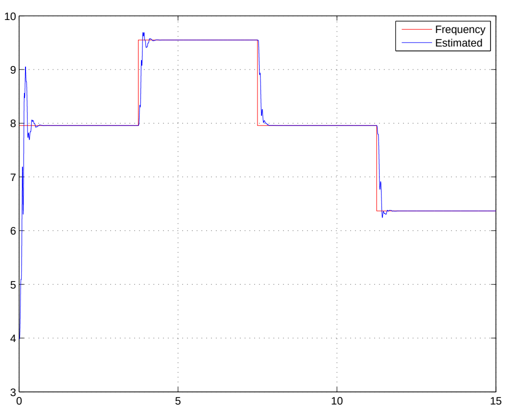

Fig. 6. **Example** 1. Frequency estimation.

In Fig. 7 the effectiveness of the method is shown. The controlled output reaches fast its zero steady-state condition and it remains constantly in its state except for the instants that correspond to frequency steps.

Example 2. Frequency sweep In such a case, an input disturbance is considered of the form

$$d(t) = 1 + \sin(\omega_c(t)t) \tag{33}$$

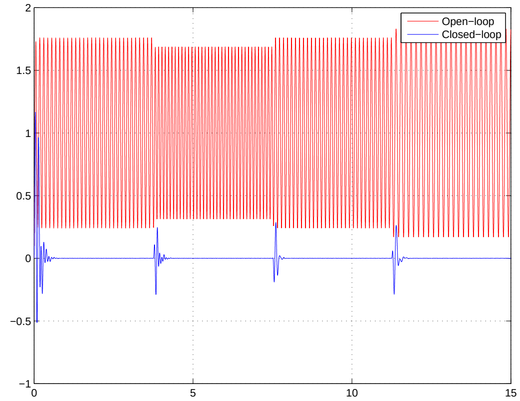

Fig. 7. **Example** 1. Plant outputs.

with

$$\omega_c(t) = \begin{cases} \frac{20}{T_{obs}}t + 50 & \text{for } 0 < t \leq T_{obs}/2, \\ -\frac{20}{T_{obs}}t + 70 & \text{for } T_{obs}/2 < t \leq T_{obs}. \end{cases} \tag{34}$$

The results are reported in Figs. 8-9.

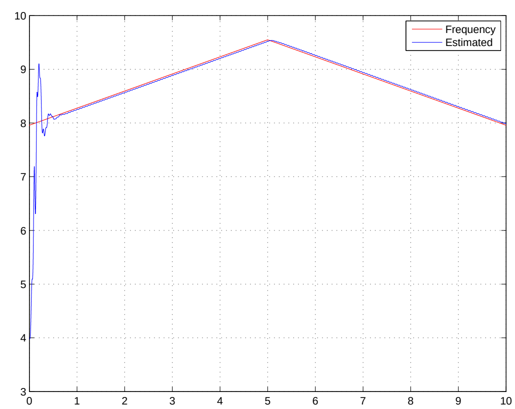

Fig. 8. **Example** 2. Frequency estimation.

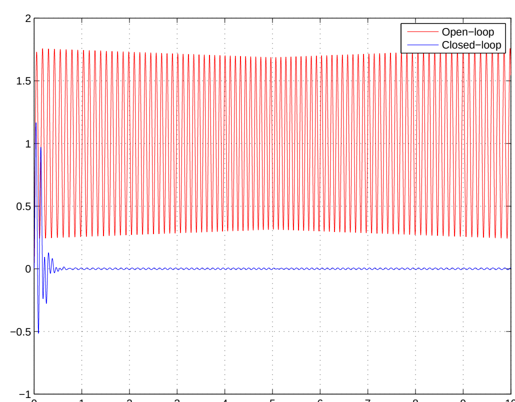

Fig. 9. **Example** 2. Plant outputs.

This is a more severe test due to the fact that the OSG-TOGI is tracking a not constant input frequency so that the steady-state condition can not be taken into account.

However, as it is shown, the proposed scheme is able to significantly reduce the disturbance effect on the output signal.

Example 3**. Offset steps.** In such an example, the bias term is allowed to vary as follows

$$d(t) = A_0(t) + \sin(50t) \tag{35}$$

where

$$A_0(t) = \begin{cases} 0.5 & \text{for } 0 \leq t < T_{obs}/4, \\ 0 & \text{for } T_{obs}/4 \leq t < T_{obs}/2, \\ -0.5 & \text{for } T_{obs}/2 \leq t < 3T_{obs}/4, \\ 1 & \text{for } 3T_{obs}/4 \leq t < T_{obs}. \end{cases} \tag{36}$$

As in the previous cases, the promptness of the system is put in evidence and the rejection action is shown (see Figs 10-11)

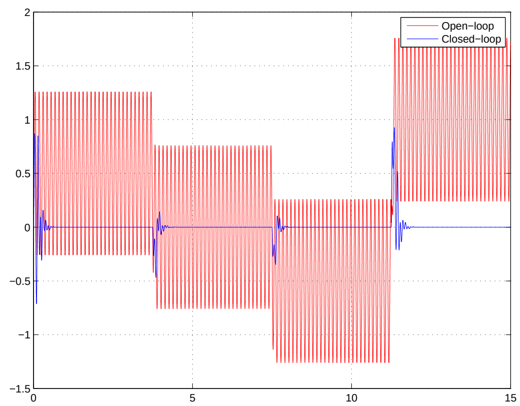

Fig. 10. **Example** 3**.** Plant outputs.

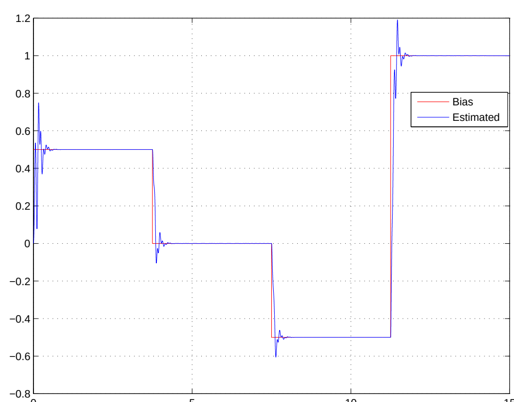

Fig. 11. **Example** 3. Bias term estimation.

V. Conclusions

In this paper, a novel approach to the biased sinusoidal rejection problem has been presented. The discussed method makes use of an orthogonal signals generator based on a third-order generalized integrator to estimate the frequency of the unknown disturbance. The estimation is performed via an adaptation of the TOGI resonant frequency. An averaging analysis has been conducted to prove the convergence of the proposed adaptive law. The control input is generated according to the outputs of the TOGI system and the plant estimate. Once the resonant frequency converges to the unknown one, it has been proved that the system completely rejects the input disturbance even in presence of rough plant estimates under the assumption that the estimated plant guarantees the stability of the overall system. Numerical experiments of the ANC-TOGI method have been proposed to demonstrate the strength of the method. Frequency steps and frequency sweep cases have been simulated. The case of offset steps has been also covered.

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