

IDENTIFICATION AND CONTROL OF PERIODIC DISTURBANCES

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Abstract— A new Adaptive Feed-Forward strategy is presented, that identifies the periodic disturbances of known frequencies. These disturbance sources can be external or internal. Internal disturbances are also called self-excited vibrations that come from the load or its parameter variations. These variations depend periodically on the angle of rotation particularly in the case of rotary machines. The identified parameters of the disturbance model are used to design feed forward controller to counteract the disturbance effect on the system output, this is to be with a minimum interaction with an already existing set point tracking feedback controller.

I. INTRODUCTION

SET-POINT tracking and disturbance rejection are the main goals to achieve in control system theory and its applications. Periodic signal is a special disturbance case that appears frequently in rotary machines when some of the load parameters are angle dependent. This can be because of an eccentricity, non-uniform machine parts or the internal design of the load machine. This type of effect can be modeled as internal disturbance (self-excited vibration). On the other hand, when the disturbance parameters, the amplitude and the frequency, are not dependent on the angular position or on any other variable/parameter of the machine, then this disturbance type is modeled as an external disturbance.

Although the problem here is the vibration (periodic disturbance), that comes from an external source or an internal source for self-excited vibration system, the environment fits the general control problem format with set point tracking as a primary goal to achieve, and disturbance rejection as a secondary goal also needed to be achieved but with most minimum interaction with the primary goal. This separation could be done by preserving a feedback controller mainly to achieve the set point tracking demands, and a feed forward controller as a vibration compensator.

Most of the *Adaptive Feed-Forward Control* (AFFC) strategies, e.g. *Least Mean Squares* (LMS) family methods, developed for pure *Active Vibration Control* (AVC) applications do not suit in particular this control problem because of stability issue, parameter convergence and the online adaptation (tuning) problems [1, 2]. This means that as long as the algorithm does not converge, the system

operates with very poor performance and that will be until and only if the parameters converge.

Paper [3] has developed an *Adaptive Eccentricity Compensation* algorithm, which is an AFFC, [4] has also developed an AFFC to cancel the ripples of a PMSM, but both of them are with a direct controller parameter (tuning) optimization algorithm. While in this work the problem of Feed-Forward Control (FFC) adaptation will be transferred into identification (parameter optimization) problem, where the identification model is constructed by two parts, dynamic part in terms of linear or nonlinear differential or difference equation and a disturbance part in terms of sine cosine sum [2], this will allow the identification to gather information online (offline as priori) about the process and to pass the converged parameters to use in the FFC law, even when the FFC is deactivated, particularly at the starting phase as the dynamic and disturbance parameters are far from the optimal ones.

In the following, section II discusses briefly the conditions for good set point tracking and disturbance rejection using only feedback control methods, while section III introduces the general strategy of set point tracking feedback control and periodic disturbance compensation by feed forward control, section IV describes a periodically disturbed process, V introduces how to model the externally and internally disturbed system for discrete, continuous, nonlinear and linear in parameter models, section VI and VII give simulation and experimental examples respectively to the introduced strategy, a conclusion is presented in section VIII.

II. PERIODIC DISTURBANCE REJECTION USING ONLY FEEDBACK CONTROL

The typical feedback control block diagram structure is shown in Fig. (1). The system has the set point tracking as a main goal here, besides the process is periodically disturbed and needs to be rejected also by the use of feedback control.

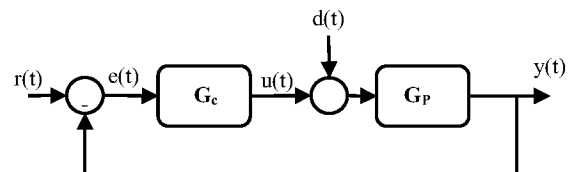


Fig. 1. Feedback control of disturbed process

Now, for a good set point tracking behavior the condition

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($G_p G_c \gg 1$) should be valid over the interested bandwidth, this is for the primary objective. Moreover for a good disturbance rejection also the second condition ($G_c \cong \infty$) should be valid at the disturbance frequency, which can be done according to the *Internal Model Principle* [2, 5] by *Notch Filter* or *Repetitive Control* for single or infinite multi-harmonic disturbance [6] respectively. The design interference between the set point tracking and disturbance rejection demands will be obvious if the disturbance frequencies are outside of the set point bandwidth, since adding an internal model in the controller to reject these frequencies will actually add extra lag to the loop, this will of course deteriorate the set point tracking (relative stability) demands.

In general, the solution of this contradictory problem is always by compromising between the demands of good set point tracking and perfect disturbance rejection, this is a major *Robust Control Theory* applications. Therefore, in the next section, a strategy is introduced that decouple the conflicts between the demands and ensure them.

III. SET-POINT TRACKING FEEDBACK CONTROL AND DISTURBANCE COMPENSATION FEED FORWARD CONTROL

The strategy utilizes an identification algorithm to identify the *identification model* that represents (approximates) the *disturbed process* by dynamic model plus input from internal and external disturbance model. By this type of modeling and identification, the adaptation can practically be done for both controllers, where the identified dynamic parameters can be used to compute the feedback control law, while the identified disturbance parameters are used to construct the feed-forward controller, this strategy, AFBC for set point tracking and AFFC for disturbance compensation, is schematically presented in Fig. (2). Nevertheless, the rest of the paper is only concerned with AFFC. The identification model structure can be parameterized and identified by any parametric identification technique [7, 8 ... 13], for example *Output Error*, *Prediction Error* or even with the *Simple Recursive Least Squares* method, provided that the model structure has linear in parameter form. The choice of the identification procedure depends on many conditions, some of them are model structure, measurement noise, type of model based or state based control strategy, conditions for identification in closed loop and identification for control, etc. If the parameters converge, this means the parameters of the system dynamics and the disturbance model, then these parameters of disturbance model can be used to generate an anti-disturbance signal that can be fed forward to the system input to compensate the disturbance action on the system.

In general, the stability of the procedure is directly dependent on the identification algorithm, so if the identification algorithms converge to parameters, and these parameters represent the real dynamics and disturbance of

the real system at the interested frequencies (bandwidth), then these parameters can be used to compensate the disturbance. Otherwise, the parameters are useless, since they do not represent the real process.

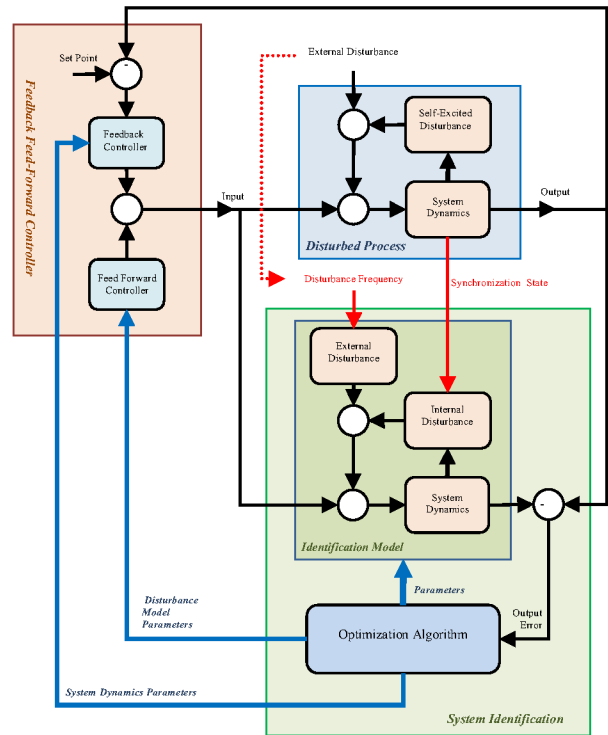


Fig. 2. The general strategy of adaptive set-point tracking feedback and disturbance feed-forward controller

In the next section IV, a disturbed process is introduced, while in section V, its corresponding identification model is constructed in state space format (subsection A) and in discrete linear in parameter ARX format (subsection B).

IV. THE DISTURBED PROCESS

A rotary Drive-Load system is considered to be as the disturbed process in Fig. (2), which is simply a motor that drives a load through a mechanical link with a gear. The mechanical load machine has a self-excited vibration that depends on the load angular position and independent vibration as shown in the following Fig. (3):

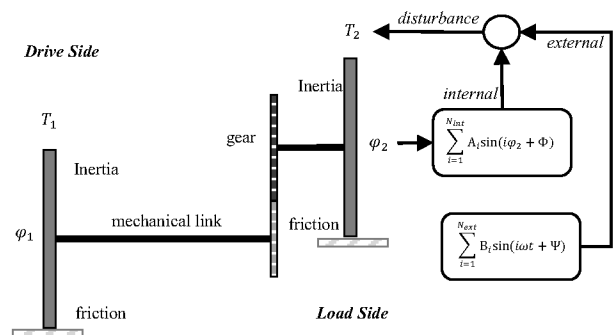


Fig. 3. Drive-Load system with periodic disturbances

The disturbance torque acting on the load side is given by:

$$T_2 = \sum_{i=1}^{N_{int}} A_i \sin(i\varphi_2 + \Phi) + \sum_{i=1}^{N_{ext}} B_i \sin(i\omega t + \Psi) \quad (1)$$

This can be extendable to cases where the amplitude of the self-excited vibration depends (non-) linearly on some of the process states (e.g. angular velocity). The process is to be constructed as velocity control servo system to follow the desired velocity set point and to reject these periodic disturbances.

V. THE IDENTIFICATION MODEL OF AN EXTERNALLY AND INTERNALLY DISTURBED PROCESS

The structure of the identification model and its parameterization should generally take into consideration all of the available priori knowledge about the real (physical) plant as much as possible, for example, the model of internal disturbances can be made (non)linearly dependent on some of the system state(s). Moreover, the disturbance model is to be constructed so that all disturbances are added to the input, i.e. input disturbance form see Fig. (4), this makes the use of identified disturbance model and its application very direct and without extra computations, even if the real system has another form, e.g. output disturbance form, as long as these disturbances are periodic.

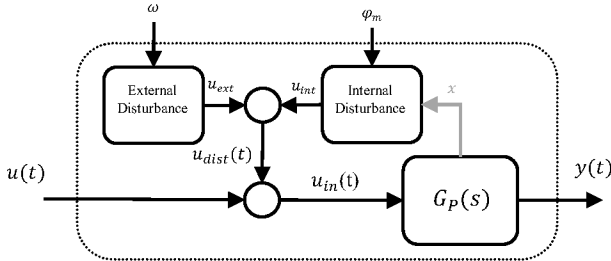


Fig. 4. Identification model of an internally and externally disturbed process

A. Construction of an Identification model for the Rotary Drive-Load Process

The input of the *identification model* is defined by the control variable plus the disturbance, while the disturbance input is defined as internal and external part, as following:

$$\begin{aligned} u_{in}(t) &= u(t) + u_{dist}(t) \\ u_{dist}(t) &= u_{ext}(t) + u_{int}(t) \end{aligned} \quad (2)$$

Since the disturbances are periodic, they can be represented (approximated) by their Fourier Expansion as:

$$\begin{aligned} u_{dist}(t) &= \sum_{i=1}^{N_{ext}} \alpha_i \sin(i\omega t) + \beta_i \cos(i\omega t) \\ &+ \sum_{i=1}^{N_{int}} [\gamma_i \sin(i\varphi_m) + \rho_i \cos(i\varphi_m)] \end{aligned} \quad (3)$$

Note: Since the output variable of the process is the measured angular velocity of the load side ($y(t) = \dot{\varphi}_m = \dot{\varphi}_2$), the angular position of the load side ($\varphi_m = \varphi_2$) will be unobservable and has to be measured.

Equation (3) can be written in compact vector format as:

$$u_{dist}(t) = \begin{bmatrix} \underline{\alpha}^T & \underline{\beta}^T & \underline{\gamma}^T & \underline{\rho}^T \end{bmatrix} \begin{bmatrix} \sin(\omega t) \\ \cos(\omega t) \\ \sin(\varphi_m) \\ \cos(\varphi_m) \end{bmatrix} \quad (4)$$

With:

$$\begin{aligned} \underline{\alpha} &= \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{N_{ext}} \end{bmatrix}; \underline{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{N_{ext}} \end{bmatrix} \\ \underline{\gamma} &= \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_{N_{int}} \end{bmatrix}; \underline{\rho} = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_{N_{int}} \end{bmatrix} \end{aligned} \quad (5)$$

$$\begin{aligned} \underline{\sin(\omega t)} &= \begin{bmatrix} \sin(\omega t) \\ \sin(2\omega t) \\ \vdots \\ \sin(N_{ext}\omega t) \end{bmatrix}; \underline{\cos(\omega t)} = \begin{bmatrix} \cos(\omega t) \\ \cos(2\omega t) \\ \vdots \\ \cos(N_{ext}\omega t) \end{bmatrix} \\ \underline{\sin(\varphi_m)} &= \begin{bmatrix} \sin(\varphi_m) \\ \sin(2\varphi_m) \\ \vdots \\ \sin(N_{int}\varphi_m) \end{bmatrix}; \underline{\cos(\varphi_m)} = \begin{bmatrix} \cos(\varphi_m) \\ \cos(2\varphi_m) \\ \vdots \\ \cos(N_{int}\varphi_m) \end{bmatrix} \end{aligned} \quad (6)$$

So the dynamics, external and internal disturbance models of the *identification model* can be written in the following continuous (or discrete) state space format as:

$$\begin{aligned} \dot{\underline{x}}(t) &= \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \\ -a_0 & -a_1 & \cdots & -a_{n-1} \end{bmatrix} \underline{x}(t) + \end{aligned} \quad (7)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \underline{\alpha}^T & \underline{\beta}^T & \underline{\gamma}^T & \underline{\rho}^T \end{bmatrix} \begin{bmatrix} u(t) \\ \sin(\omega t) \\ \cos(\omega t) \\ \sin(\varphi_m) \\ \cos(\varphi_m) \end{bmatrix}$$

$$y(t) = [b_0 \quad b_1 \quad \cdots \quad b_{n-1}] \underline{x}(t) \quad (8)$$

Where $\underline{x}(t)$, $y(t)$, $u(t)$ are the state vector, the output and the input. n, N_{ext}, N_{int} are the system order, the external and internal number of harmonics. The parameter vector:

$$\theta^T = \begin{bmatrix} -a_1 & -a_2 & \cdots & -a_{n-1} & \vdots & b_1 & b_2 & \cdots & b_{n-1} \\ \alpha_1 & \alpha_2 & \cdots & \alpha_{N_{ext}} & \vdots & \beta_1 & \beta_2 & \cdots & \beta_{N_{ext}} \\ \gamma_1 & \gamma_2 & \cdots & \gamma_{N_{int}} & \vdots & \rho_1 & \rho_2 & \cdots & \rho_{N_{int}} \end{bmatrix} \quad (9)$$

These parameters can be identified by any nonlinear in parameter identification algorithm, for example *Output Error*, *Prediction Error* or even with *Dual (Extended)*

Kalman Filter to predict the system states and its parameters in a stochastic environment. Actually this formulation fits the general format of (adaptive) model based control design for example state feedback control strategies.

B. Linear in Parameter Discrete Identification Model Structure to use with RLS-Method

The disturbed process has external and internal state dependent disturbance (vibration), as shown in Fig. (3). This process is going to be modeled as *Auto Regressive with eXogenous* (ARX) input discrete model, so that a *Recursive Least Squares* (RLS) identification algorithm can be used to identify the discrete dynamics and the disturbance model in the form of sine-cosine sum function parameters, and to use the identified parameters to generate the anti-disturbance signal in order to compensate the system vibrations. The discrete linear dynamics are described by the following difference equation:

$$y(t) = a_1y(t-1) + a_2y(t-2) + \dots + a_{No}y(t-No) + b_1u_{in}(t-1) + b_2u_{in}(t-2) + \dots + b_{No}u_{in}(t-No) \quad (10)$$

Where No is the order of the system. The input signal is defined previously as in equations (2 & 3). For example, if ($No = 1$) with single harmonic external disturbance, the *identification model* will be as:

$$y(t) = a_1y(t-1) + b_1[u(t-1) + \alpha \sin(\omega(t-1)) + \beta \cos(\omega(t-1))] \quad (11)$$

In vector format:

$$y(t) = \begin{bmatrix} y(t-1) \\ u(t-1) \\ \sin(\omega(t-1)) \\ \cos(\omega(t-1)) \end{bmatrix}^T \begin{bmatrix} a_1 \\ b_1 \\ b_1\alpha \\ b_1\beta \end{bmatrix} \quad (12)$$

So the disturbance parameters are calculated by:

$$\alpha = \frac{b_1\alpha}{b_1}; \beta = \frac{b_1\beta}{b_1} \quad (13)$$

Generally for ($No > 1$), the disturbance model parameters can be calculated by the average of all the available redundant parameters as following:

$$\alpha_i = \frac{1}{No} \sum_{j=1}^{No} \frac{b_j\alpha_i}{b_j}; \beta_i = \frac{1}{No} \sum_{j=1}^{No} \frac{b_j\beta_i}{b_j} \quad (14)$$

$$\gamma_i = \frac{1}{No} \sum_{j=1}^{No} \frac{b_j\gamma_i}{b_j}; \rho_i = \frac{1}{No} \sum_{j=1}^{No} \frac{b_j\rho_i}{b_j}$$

VI. SIMULATION EXAMPLES

A. Simulation Example 1: Single-Harmonic External Periodic Disturbance

The disturbed process is assumed to be the rotary machine with a stiff mechanical link and an external periodic disturbance that modeled by the following linear differential equation:

$$\dot{\vartheta}(t) + \vartheta(t) = T_{in}(t) + T_{dis}(t) \quad (15)$$

And the disturbance source is defined by:

$$T_{dis}(t) = \sin(t) \quad (16)$$

Both equations can be combined to form the following system as:

$$\dot{\vartheta}(t) + \vartheta(t) = T_{in}(t) + \sin(t) \quad (17)$$

Where $\vartheta(t)$ is the angular velocity “rad/sec”, $T_{in}(t)$ is the input (motor) torque “N.m”. Now, if it is required that the system to be more slowly than it is, but keeping at the same time the steady state error to step input equal to zero, this can be achieved by changing the dynamics of the system by using a dynamic compensator or simply a PI feedback controller. By using the following PI parameter set A (P = 0.10, I = 0.10), these parameters will yield set point and disturbance responses shown in Fig. (5), where the formally stated demands are met by this parameter set A.

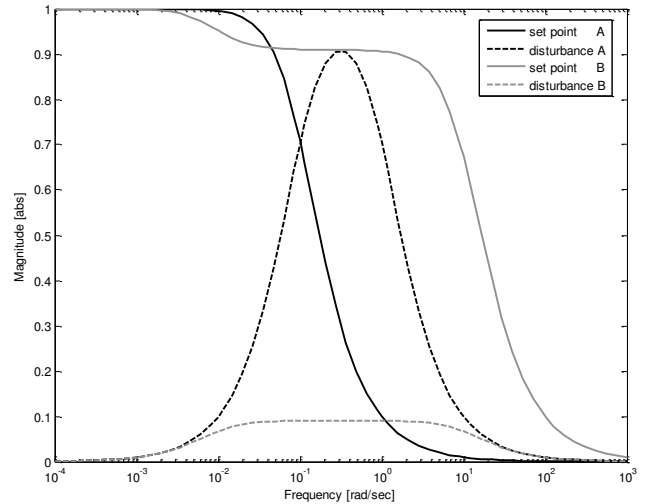


Fig. 5. Closed loop frequency response to the set point and the disturbance, case A and B

On the other hand, the resulted disturbance rejection characteristics become very poor around the region between 0.1 to 1 “rad/sec”. So this case has contradictory demands between slow response and good disturbance rejection which demands the opposite that the system should have faster response or wider bandwidth. If the main objective is just only interested in good disturbance rejection and fast

response then the PI controller parameter set B ($P = 10.0$, $I = 0.10$) is suggested. Fig. (5) shows the frequency response of the closed loop system under PI controller for parameter set B. So in case A, the method of feed-forward compensation can be used to complement the job of feedback controller to cancel any undesirable disturbances especially outside the working bandwidth of the feedback controller. Fig. (6) shows the system step response with PI-Feedback controller with parameter set A, where it is clear that the disturbance is out of its working bandwidth, the same figure shows the Feed-Forward Disturbance Compensation (FFDC) with the identified disturbance model parameters.

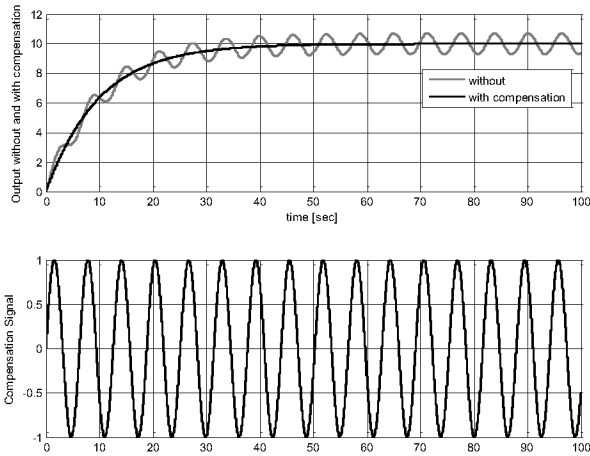


Fig. 6. PI parameter set A, with and without FFDC

B. Simulation Example 2: Internal Disturbance (Self-Excited Vibration)

The system is the rotary Drive-Load process but with a flexible mechanical link, and the load has an angle dependent parameter that produces the self-excited vibration, see the following Fig. (7).

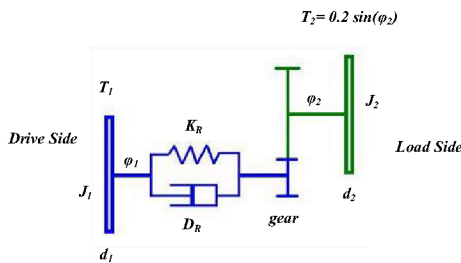


Fig. 7. Flexibly linked drive-load system

The system parameters are given as following: motor side parameters ($J_1=0.1$ "kgm²", $d_1=0.1$ "Nmsec/rad"), the link parameters ($K_R=1000.0$ "Nm/rad", $D_R=0.01$ "Nmsec/rad"), the gear ration ($\varphi_1/\varphi_2 = 1.5$), the load side parameters ($J_2=1.0$ "kgm²", $d_2=0.1$ "Nmsec/rad"), the self-exciting disturbance (α & $\beta = [0.2 \ 0]$). The system dynamic and disturbance model are described by the following equations:

$$\dot{\varphi}_2(s) =$$

$$\frac{(0.15s + 15000) T_1(s) + (s^2 + 1.1s + 10000) T_2(s)}{s^3 + 1.223s^2 + 12250s + 3250} \quad (18)$$

$$T_2(t) = 0.2 \sin(\varphi_2)$$

With poles: $(-0.48 + 110.68i, -0.48 - 110.68i, -0.2653)$

Although the system has a third order dynamics, the identification model is simply set to a first order. Nevertheless, the real system frequency response also shows that the dynamics before the resonance frequency are merely a first order one, see Fig. (8), this is why the identification and compensation algorithm has worked and gave useful parameters, but if the machine is driven to the resonance region then the identification model should have the ability to approximate this region, which can be only by third order model or higher, otherwise the compensation will not succeed because of the mismatched parameters.

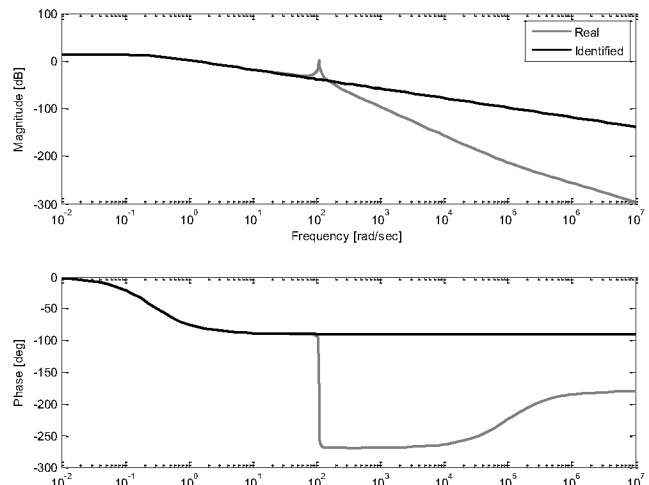


Fig. 8. Real and identified dynamics

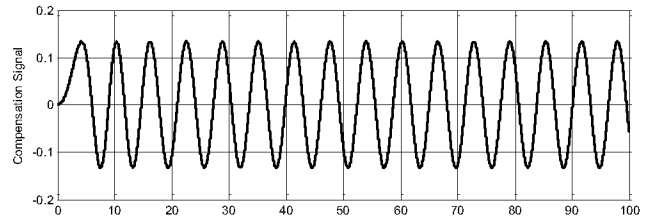
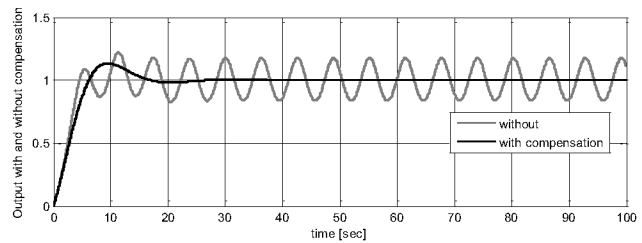


Fig. 9. Disturbance Compensation

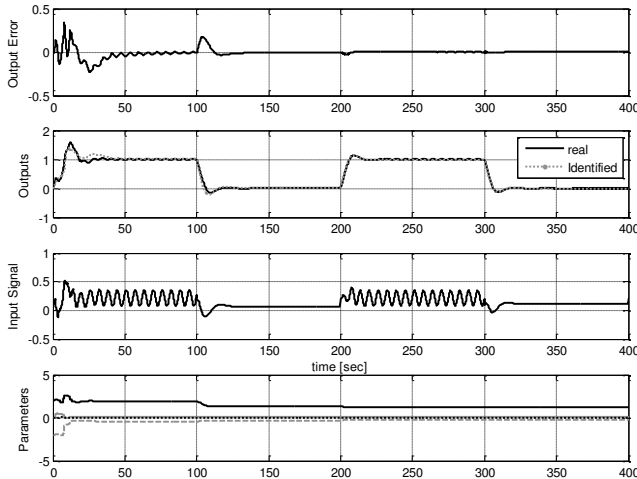


Fig. 10. The Adaptive Run

The final parameter vector of the identification run is given by $[-0.2653 \quad 1.2245 \quad 0.1334 \quad -0.0000]^T$. The system is identified first then the converged disturbance parameters are used in the feed-forward controller to generate the anti-vibration signal. Fig. (8) shows the frequency response of the real system and the converged identified parameters. Fig. (9) shows the (1 “rad/sec”) step response of the system with and without internal periodic disturbance compensation. With the same parameters and conditions instead of first identifying the parameters and then using them latter for compensation, the adaptive strategy of *certainty equivalence principle* [14] is applied, see Fig. (10), this shows that, the algorithm has the ability to cope with real application cases.

VII. EXPERIMENTAL EXAMPLES

A. Test Bench Description

It is a rotary Drive-Load system, see Fig. (3), with a main motor as drive motor flexibly linked to an inertial load through a gear plus a small motor works as disturbance motor. The system is constructed as velocity control servo-system with an internal disturbance that depends on the load angular position, which actually emulates the behavior of self-excited system as in the simulation example (2).

B. Experimental Example 1

The experiment is done by applying the presented adaptive Feed-Forward algorithm to cancel the self-excited vibrations of the system with a set point PI FB controller. Although, there are no enough priori about the system, nevertheless, the identification model is chosen to be as first order model, and the initial parameters of the system dynamics are arbitrary chosen to represent stable dynamics. The initial disturbance parameters are set to zeros. Fig. (11) shows the experiment run graphs, where before the time equal to 40 sec only the identification is working, after that the FFC is activated to compensate the self-excited vibration.

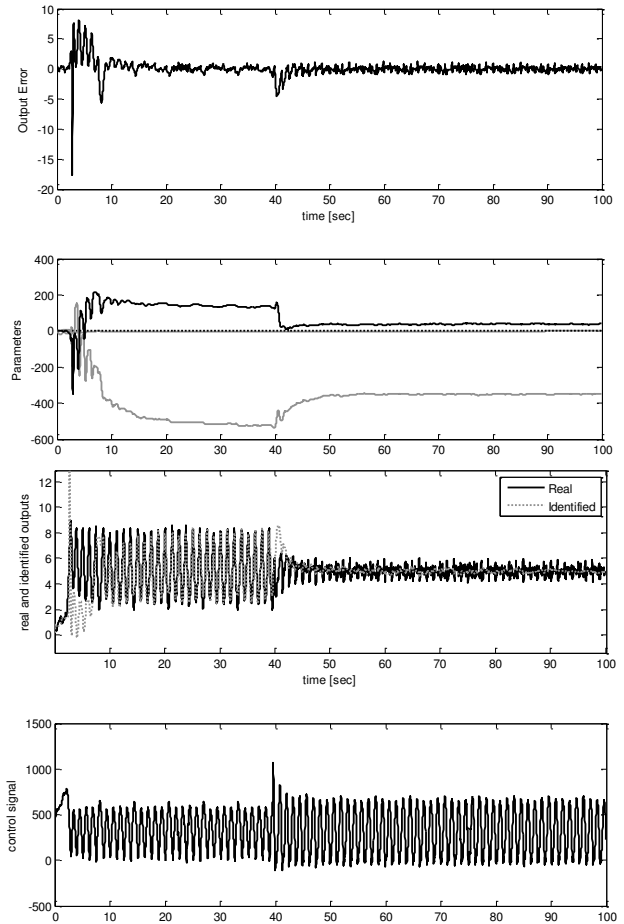


Fig. 11. Experiment 1 run

C. Experimental Example 2

The test bed structure is the same as in experimental example (1). But in this example first order discrete model is used as an identification model, equations (10-13). Fig (12) shows the run of the experiment, where until the time about (40 sec) the system is only under the PI set point tracking feedback controller, plus the identification algorithm. Only after that (40 sec) the feed forward controller is activated for compensation and that by using the online identified disturbance model parameters to generate the anti-disturbance (vibration) signal.

VIII. CONCLUSION

Since the usual periodic vibration adaptive feed-forward Controller, which is only directly online tuned for example by one of LMS-Methods, whom already developed by signal processing society for Active Noise and Vibration Control applications, does not suit control application demands especially if there is a set point tracking problem by feedback in terms of stability and performance. A new strategy is developed, that uses system identification algorithm as a tuning (optimization) method to tune the

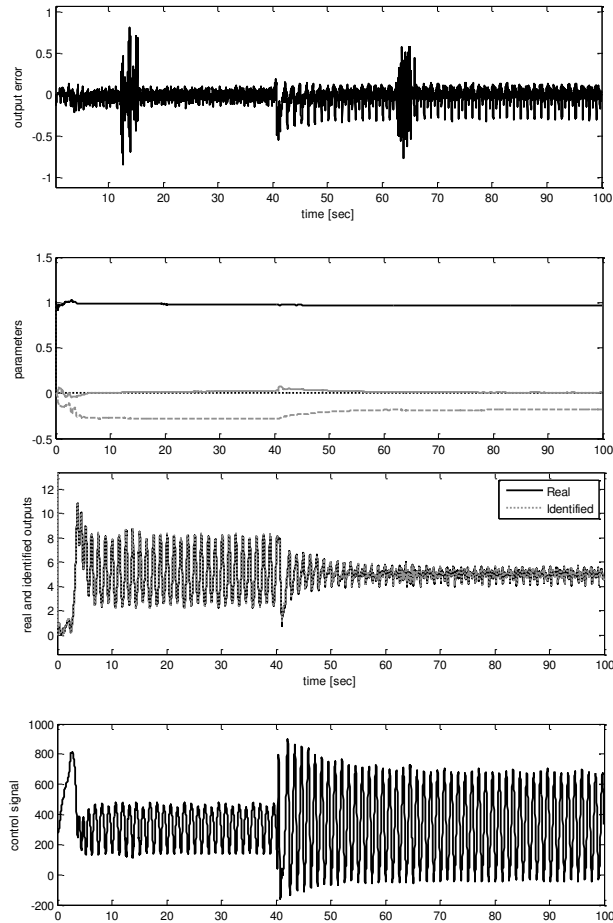


Fig. 12. Experiment 2 run

parameters of the feed-forward controller either directly (online) or virtually indirectly (offline) to cancel the external and/or the internal (self-excited vibrations) disturbances.

This is done by modeling the identification model as separate dynamic and disturbance model parts. An advantage of the algorithm is that its formulation fits the general frame work of Modern Control Theory by means of developing the states and the parameter estimates for model dependent control design applications, for example state feedback control strategy (e.g. pole-placement), as well as the feed forward controller for the periodic disturbance compensation objective that does not interfere with the design of the already existing feedback controller. The potential of the algorithm is demonstrated as an offline or online tuning method by both the simulation and the experimental examples.

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