Preprints of the 2012 20th Mediterranean Conference on Control & Automation (MED), Barcelona, Spain, July 3-6, 2012

On Lyapunov stability of nonlinear adaptive control based on Neural Networks Emulator and Controller

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Abstract— **This paper adresses a Lyapunov stability analysis of nonlinear systems control. We consider an adaptive control scheme based on recurrent neural networks emulator and controller with decoupled adaptive rates. Lyapunov sufficient stability conditions for decoupled adaptive rates of the emulator and controller are proposed. In order to guarantee the fast convergence, the optimal adaptive rate of controller is derived in the sense of Lyapunov exponential stability taking into account the stability of the emulator. The good performances of the proposed stable control design are shown with numerical simulations results.**

I. INTRODUCTION

Adaptive control scheme based on recurrent neural networks seems a promising solution for uncertain or unknown nonlinear and multi-variable dynamical systems [1], [2], [3]. Thanks to their approximation capabilities, neural networks have been successfully applied to nonlinear plants control [4], [6], [5]. Indeed, the development of neural controllers requires efficient adaptation algorithms. The most popular algorithms used for parameters adaptation are the gradient back propagation learning algorithm and numerous variants [7], [8]. However, the gradient descent method is known for its slowness and its frequent confinement to local minima. So, it is very difficult to achieve stable and online controls in presence of unknown external disturbances and parameters perturbations.
Stability of adaptive neural controllers served authors interests [9], [10]. The adaptive control strategies consider the Lyapunov approach for stability analysis. The main drawback of the proposed methods is the necessity to fulfill some conditions so that the methods can be applied with numerous processes. In this paper, we propose a stable adaptive control design using Lyapunov stability analysis. We provide Lyapunov sufficient stability conditions for decoupled adaptive rates of the emulator and controller. An adaptation strategy based on exponential stability approach is developed to improve closed-loop performances.

The paper is planned as follows. In section 2, the adaptive control scheme is described. Stability of neural controller is studied in the sense of Lyapunov asymptotic strategy, in section 3. Section 4 presents an adaptation strategy based on Lyapunov exponential stability. Simulations results prove the effectiveness of the proposed approach. Section 5 concludes the paper.

II. ADAPTIVE NEURAL CONTROLLER

An indirect adaptive control scheme based on recurrent neural networks has been recently proposed for the control design of unknown or uncertain nonlinear systems [11]. This scheme is composed of a neural emulator (NE) and a neural controller (NC). Networks parameters adapt themselves thanks to an adaptation algorithm based on Real Time Recurrent Learning (RTRL) [7]. The proposed scheme NE-NC neither include any learning phase, nor particular initialization and nor explicit knowledge about the dynamical system. Moreover, NE and NC sizes are small and depend only on the number of inputs and outputs. The adaptation algorithm of networks parameters is based on the gradient of the emulation error. This algorithm also includes the updating of the adaptive rate and time parameter that are used to change the adaptation dynamics [11]. The good performances of the proposed adaptive scheme have been illustrated for the control of SISO and MIMO square systems [12], [13].
In this paper, we continue the investigation of controller based on recurrent neural networks (Fig. 1) with decoupled adaptive rates [14]. NE is used to emulate the instantaneous outputs of the process (i.e. to map the inputs-outputs relationships over a small size time window). NC is used to drive the plant outputs according to regulation or tracking applications.

The parameters of NE and NC are updated independently. The subscripts e and c are used to distinguish the NE and NC parameters respectively. In the following, we consider square MIMO systems (ie. $N = N_{IN} = N_{OUT}$, and $IN = OUT = 1,\dots,N$). The total number of neurons N_e of NE is chosen equal to $2N$ so that any node is either an input node or an output node but not both at the same time in order to avoid a direct link from any input signal to an output one [11]. For NC, N_c is also equal to 2N. The NC inputs are the N output error functions and the N desired outputs. Inputs and outputs signals for NE, NC and plant are normalized in the range [-1, 1]. NE and NC are developed with fully connected recurrent neural networks. Concerning NE (resp. NC), the dynamics

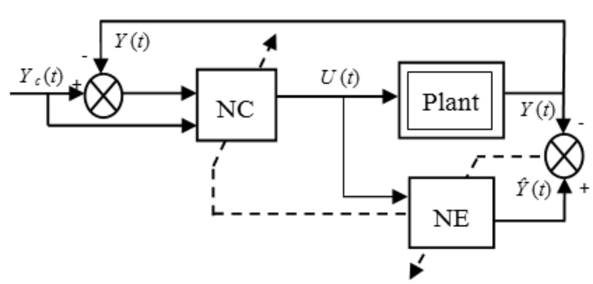

Fig. 1. Adaptive control scheme NE - NC

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of the N_e (resp. N_c) neurons are defined by (1) (resp. (2)), for $i=1,\dots,N_e$ (resp. $i=1,\dots,N_c$), in continuous time:

$$\frac{1}{|\tau_e(t)|}\frac{d(s_i(t))}{dt} = -s_i(t)+\tanh\left(\sum_{j=1}^{N_e} w_{ij}(t)s_j(t)+x_i(t)\right) \quad (1)$$

$$\frac{1}{|\tau_c(t)|}\frac{d(o_i(t))}{dt} = -o_i(t)+\tanh\left(\sum_{j=1}^{N_c} \phi_{ij}(t)o_j(t)+z_i(t)\right) \quad (2)$$

$s_i(t)$ is the i^{th} neuron state of emulator and $x_i(t)$ its input, $x_i(t)=u_i(t)$ if $i\in\{1,\dots,N\}$; $x_i(t)=0$ if $i\in\{N+1,\dots,2N\}$; $\hat{y}_{i-N}(t)=s_i(t)$ if $i\in\{N+1,\dots,2N\}$ is the i^{th} NE output, $w_{ij}(t)$ are the emulator weights from neuron j to neuron i and $1/|\tau_e(t)|$ is the emulator time parameter. $o_i(t)$ is the i^{th} neuron state of controller, $u_i(t)=o_i(t)$, if $i\in\{1,\dots,N\}$, is the i^{th} controller output, $\phi_{ij}(t)$ is the controller weight from neuron j to neuron i and $1/|\tau_c(t)|$ is the controller time parameter. $z_i(t)=y_{c_i}(t)-y_i(t)$ if $i\in\{1,\dots,N\}$; $z_i(t)=y_{c_{i-N}}(t)$ if $i\in\{N+1,\dots,2N\}$ where, $y_{c_i}(t)$ is the i^{th} desired output and $y_i(t)$ is the i^{th} process output.

NE and NC weights are updated according to an autonomous algorithm inspired from the RTRL. The major advantage of such method is that it doesn't depend on any preliminary knowledge about dynamics. A similar algorithm is used to update the rates and the time parameters of both networks. NE parameters $\tau_e(t)$ and $\eta_e(t)$ and NC ones $\tau_c(t)$ and $\eta_c(t)$ are updated independently. $\eta_c(t)$ increases more quickly than $\eta_e(t)$ during the starting stage of the control algorithm. It results a quicker evolution of controller parameters that acts as an accelerator for the control process [14]. According to NC adaptive rate $\eta_c(t)$, sufficient stability conditions will be established in the following section.

III. Lyapunov asymptotic stability

In this section, sufficient conditions are proposed in order to ensure the asymptotic stability of the controlled system. The actual system is incorporated in the stability analysis by means of the control and emulation errors that depend on the system output.

A. Stability sufficient conditions on η_c

We consider the tracking error $e_c(t)$, as Lyapunov function:

$$e_c(t)=\frac{1}{2}\|E_c(t)\|^2 \quad (3)$$

where $E_c(t)=(e_{c_l}(t))\in\Re^{N\times 1}$ is a column vector: $e_{c_l}(t)=y_{c_l}(t)-y_l(t)$, $y_l(t)$ is the plant output and $y_{c_l}(t)$ is the desired one. By comparison with our previous work [12], this one considers decoupled adaptive rates for NE and NC. According to the Lyapunov stability theorem, the asymptotic stability is ensured if $e_c(t)$ is positive definite and $\dot{e}_c(t)<0$,

$$\dot{e}_c(t)=E_c^T(t)\cdot\dot{E}_c(t) \quad (4)$$

where, $(.)^T$ represents the transpose. At each time, $\dot{e}_c(t)\approx\hat{\dot{e}}_c(t)=E_c^T(t)\cdot\hat{\dot{E}}_c(t)$ and $\hat{\dot{e}}_{c_l}(t)=\dot{y}_{c_l}(t)-\dot{\hat{y}}_l(t)$. The estimated tracking errors' variations with respect to the set of parameters of NE and NC are given by:

$$\begin{aligned}\hat{\dot{e}}_{c_l}(t)=\frac{dy_{c_l}(t)}{dt}-\sum_{i=1}^{N_c}\sum_{j=1}^{N_c}\left(\frac{\partial\hat{y}_l}{\partial\phi_{ij}}\frac{d\phi_{ij}(t)}{dt}\right)\\ -\frac{\partial\hat{y}_l}{\partial\eta_c}\frac{d\eta_c(t)}{dt}-\frac{\partial\hat{y}_l}{\partial\tau_c}\frac{d\tau_c(t)}{dt}-\frac{\partial\hat{y}_l}{\partial\eta_e}\frac{d\eta_e(t)}{dt}\\ -\frac{\partial\hat{y}_l}{\partial\tau_e}\frac{d\tau_e(t)}{dt}-\sum_{i=1}^{N_e}\sum_{j=1}^{N_e}\left(\frac{\partial\hat{y}_l}{\partial w_{ij}}\frac{dw_{ij}(t)}{dt}\right)\end{aligned} \quad (5)$$

Let us define the following sensitivity functions:
$v_l=\partial\hat{y}_l/\partial\eta_e=\partial\hat{y}_l/\partial\tau_e$, $V_u=(v_1,\dots,v_N)^T$,

$r_l=\partial\hat{y}_l/\partial\eta_c=\partial\hat{y}_l/\partial\tau_c$, $R_u=(r_1,\dots,r_N)^T$,

$P_{lij}(t)=\partial\hat{y}_l(t)/\partial w_{ij}(t)$, $P_{ij}=\left(P_{lij}\right)\in\Re^{N\times 1}$,

$\theta_{lij}=\partial\hat{y}_l/\partial\phi_{ij}$, $G_{ij}=\left(\theta_{lij}\right)\in\Re^{N\times 1}$ and the following

matrices $\boldsymbol{M}_1=(F_{1l})\in\Re^{N\times N}$ and $\boldsymbol{M}_2=(F_{2l})\in\Re^{N\times N}$

with $F_{1l}=(\psi_{1l\nu})\in\Re^{N\times 1}$, $F_{2l}=(\psi_{2l\nu})\in\Re^{N\times 1}$ and:

$$\psi_{1l\nu}=\sum_{i=1}^{N_c}\sum_{j=1}^{N_c}\left(\theta_{lij}\theta_{\nu ij}\right)\in\Re$$

$$\psi_{2l\nu}=\sum_{i=1}^{N_e}\sum_{j=1}^{N_e}\left(P_{lij}P_{\nu ij}\right)\in\Re$$

We note that, $\boldsymbol{M}_1=\boldsymbol{M}_1^T$ and $\boldsymbol{M}_2=\boldsymbol{M}_2^T$.
Networks parameters updating is given by:

$$\frac{dw_{ij}(t)}{dt}=-|\eta_e(t)|E_e^T(t)P_{ij}(t) \quad (6)$$

$$\frac{d\phi_{ij}(t)}{dt}=|\eta_c(t)|E_c^T(t)G_{ij}(t) \quad (7)$$

$$\frac{d\eta_e(t)}{dt}=-E_e^T(t)V_u(t) \quad (8)$$

$$\frac{d\tau_e(t)}{dt}=-|\eta_e(t)|E_e^T(t)V_u(t) \quad (9)$$

$$\frac{d\eta_c(t)}{dt}=E_c^T(t)R_u(t) \quad (10)$$

$$\frac{d\tau_c(t)}{dt} = |\eta_c(t)|\, E_c^T(t) R_u(t) \tag{11}$$

Let us introduce, $V_e(t) = V_u(t)E_e^T(t)V_u(t)$ and $R_c(t) = R_u(t)E_c^T(t)R_u(t)$, then, the derivative of the Lyapunov function defined by (4) can be written as:

$$\begin{aligned} \dot{e}_c(t) &= |\eta_e(t)|\, E_c^T(t)\left(\left(E_e^T(t)\boldsymbol{M}_2(t)\right)^T + V_e(t)\right) \\ &+ |\eta_c(t)|\, E_c^T(t)\left(-\left(E_c^T(t)\boldsymbol{M}_1(t)\right)^T - R_c(t)\right) \\ &+ E_c^T(t)\left(\dot{Y}_c(t) - R_c(t) + V_e(t)\right) \end{aligned} \tag{12}$$

According to the asymptotic stability theory, the adaptive control scheme is Lyapunov asymptotically stable if $\eta_c(t)$ satisfy this condition:

$$a_c(t) + b_c(t)\,|\eta_c(t)| < 0 \tag{13}$$

where $a_c(t)$ and $b_c(t)$ are two functions defined by (14) and (15) depending only on the networks parameters and the output errors.

$$\begin{aligned} a_c(t) &= |\eta_e(t)|\, E_c^T(t)\left(\left(E_e^T(t)\boldsymbol{M}_2(t)\right)^T + V_e(t)\right) \\ &+ E_c^T(t)\left(\dot{Y}_c(t) - R_c(t) + V_e(t)\right) \end{aligned} \tag{14}$$

$$b_c(t) = E_c^T(t)\left(-\left(E_c^T(t)\boldsymbol{M}_1(t)\right)^T - R_c(t)\right) \tag{15}$$

Let us introduce a test function $f_c(t)$ that check if $\eta_c(t)$ satisfies the sufficient stability conditions when the tracking error is used as a Lyapunov function. It is reasonable to state that the stability of the control error dynamics implies stability of the controlled system. However, in the calculation, it is assumed that $\dot{e}_c(t)$ is approximately equal to $\dot{\hat{e}}_c(t)$, where $\hat{e}_c = y_c - \hat{y}$ i.e. the deviation between reference input and emulator output. In the sequel, we prove stability for $\hat{e}_c$ instead of e_c. This is not sufficient to prove the stability of the controlled system. It only shows, that the controlled model is stable. So, to conclude that the controlled plant is stable, in this work, to show that (e_e) i.e. the deviation between plant output and emulator output is stable too.

B. Stability sufficient conditions on η_e

The asymptotic stability of e_e is ensured if $e_e(t)$ is positive definite and $\dot{e}_e(t) < 0$. The emulation error $e_e(t)$, used as Lyapunov function is given by:

$$e_e(t) = \frac{1}{2}\,\|E_e(t)\|^2 \tag{16}$$

where $E_e(t) = (e_{e_l}(t)) \in \Re^{N\times 1}$ is a column vector: $e_{e_l}(t) = \hat{y}_l(t) - y_l(t)$, $\hat{y}_l(t)$ is the estimated output. The derivative of the Lyapunov function is:

$$\dot{e}_e(t) = E_e^T(t)\cdot \dot{E}_e(t) \tag{17}$$

The estimated emulation errors' variations with respect to the set of parameters of NE are given by:

$$\begin{aligned} \dot{e}_{e_l}(t) &= \sum_{i=1}^{N_e}\sum_{j=1}^{N_e}\left(\frac{\partial \hat{y}_l}{\partial w_{ij}}\frac{dw_{ij}(t)}{dt}\right) \\ &+ \frac{\partial \hat{y}_l}{\partial \eta_e}\frac{d\eta_e(t)}{dt} + \frac{\partial \hat{y}_l}{\partial \tau_e}\frac{d\tau_e(t)}{dt} - \frac{dy_l(t)}{dt} \end{aligned} \tag{18}$$

Then, the equation defined by (17) can be written as:

$$\begin{aligned} \dot{e}_e(t) &= -|\eta_e(t)|\, E_e^T(t)\left(\left(E_e^T(t)\boldsymbol{M}_2(t)\right)^T + V_e(t)\right) \\ &- E_e^T(t)\left(V_e(t) + \dot{Y}(t)\right) \end{aligned} \tag{19}$$

According to the asymptotic stability approach, the adaptive control scheme is Lyapunov asymptotically stable if $\eta_e(t)$ satisfy (20):

$$a_e(t) + b_e(t)\,|\eta_e(t)| < 0 \tag{20}$$

where $a_e(t)$ and $b_e(t)$ are two functions defined by (21) and (22) depending only on the networks parameters and the output errors.

$$a_e(t) = -E_e^T(t)\left(V_e(t) + \dot{Y}(t)\right) \tag{21}$$

$$b_e(t) = -E_e^T(t)\left(\left(E_e^T(t)\boldsymbol{M}_2(t)\right)^T + V_e(t)\right) \tag{22}$$

Let us define the test function $f_e(t)$ that check if $\eta_e(t)$ satisfies the sufficient stability conditions when the emulation error is used as a Lyapunov function. Then, we can conclude the stability of the controlled plant according to the intersection of the sufficient conditions on η_e (20) and η_c (13). So, $f_{c-e}(t)$ is a test function obtained from the intersection between the test functions $f_e(t)$ and $f_c(t)$.

Remark

Let us mention that the system global error $e_t(t)$ defined by (23) can also be considered as a Lyapunov function.

$$e_t(t) = e_e(t) + e_c(t) \tag{23}$$

In that case, equation (13) still hold with a new definition of the function $a_c(t)$:

$$\begin{aligned} a_c(t) &= |\eta_e(t)|\, E_c^T(t)\left(\left(E_e^T(t)\boldsymbol{M}_2(t)\right)^T + V_e(t)\right) \\ &- |\eta_e(t)|\, E_e^T(t)\left(\left(E_e^T(t)\boldsymbol{M}_2(t)\right)^T + V_e(t)\right) \\ &+ E_c^T(t)\left(\dot{Y}_c(t) - R_c(t) + V_e(t)\right) \\ &- E_e^T(t)\left(\dot{Y}(t) - V_e(t)\right) \end{aligned} \tag{24}$$

We state that $f_t(t)$ is a test function that check if $\eta_c(t)$ satisfies the sufficient stability conditions when the global error (the sum of emulation and tracking errors) is used as a Lyapunov function.

C. Simulations

In this section, a nonlinear system, described by Fig. 2, is considered [12]. The parameters η_e and η_c are adapted respectively by (8) and (10). The terms $\varepsilon_e = (dv_l/dt)|_{t=0} > 0$ and $\varepsilon_c = (dr_l/dt)|_{t=0} > 0$ are chosen respectively equal to 0.1 and 0.2. These terms are used to ensure the process starting with a minimum normalized mean square error (NMSE). Simulations are made using Matlab (version7.4, R2007a) with sampling period $\Delta T = 0.1s$.

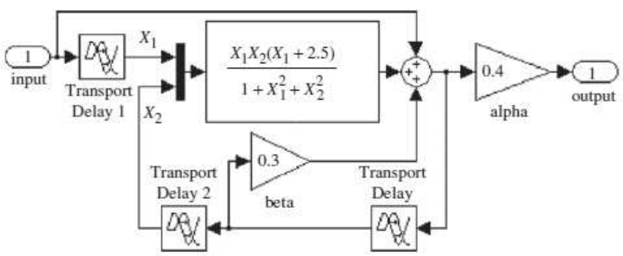

Fig. 2. Non-linear system.

Parametric perturbations are applied during the tracking (100-150s) and regulation (300-350s) phases (Fig. 3). We observe that parametric perturbations affect the control system performances and lead to instabilities (Fig. 3(a)) in the variations of the system output. Unexpected oscillations appear and persist even when perturbations disappear. Fig. 3(b), (c), (d) and (e) plot respectively the test functions $f_e(t)$, $f_c(t)$, $f_{c-e}(t)$ and $f_t(t)$. These test functions may take 3 values: the value 0 means that no solution exists for inequality (13) or (20), the value 1 denotes that $\eta_c(t)$ (resp. $\eta_e(t)$) does not satisfy inequality (13) (resp. (20)) and the value 2 indicates that $\eta_c(t)$ (resp. $\eta_e(t)$) satisfies inequality (13) (resp. (20)). The values of $f_{c-e}(t)$ are obtained from $f_e(t) \times f_c(t)$ (Table I). We note that stability conditions are generally satisfied by the parameter $\eta_c(t)$ (when the global error (Fig. 3(e)) is used as a Lyapunov function) despite the apparent system instability (Fig. 3(a)). But, the test function obtained for the tracking error oscillates between the values 1 and 2 (Fig. 3(d)).

For control signal perturbations, the conclusions are similar. The system output (Fig. 4(a)) presents oscillations even after disappearance of the perturbations and the stability conditions based on global error are less restrictive. The use of the criterion based on tracking error is more efficient to solve instability problems than the one based on global error.

In the next section, we shall consider the tracking error as Lyapunov function and it will be compared to the results obtained from the intersection between stability of e_c and e_e.

TABLE I
VALUES OF $f_{c-e}(t)$

$f_e(t) \times f_c(t)$	0	1	2
0	0	0	0
1	0	1	1
2	0	1	2

IV. LYAPUNOV EXPONENTIAL STABILITY

A. Principle

In this section, we propose a new method to update the controller adaptive rate $\eta_c(t)$ in order to improve the closed-loop performances in terms of stability. Lyapunov function (3) based on the tracking error is considered. Condition on $\eta_c(t)$ will be established so that the Lyapunov function satisfies the relation (25):

$$\dot{e}_c(t) = -\rho e_c(t) < 0 \qquad (25)$$

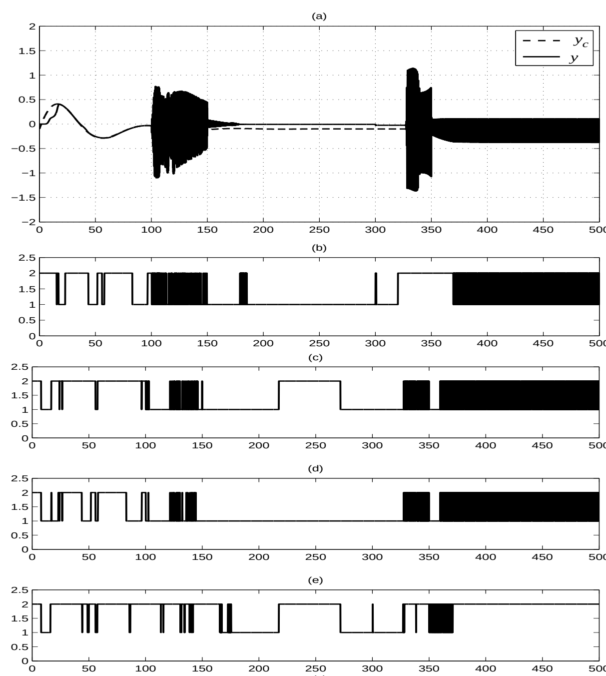

Fig. 3. Simulation in presence of parametric perturbations: (a) desired and system outputs, (b) test in case of emulation error Lyapunov function $f_e(t)$, (c) test in case of tracking error Lyapunov function $f_c(t)$, (d) test in case of intersection between emulation and tracking errors Lyapunov functions $f_{e-c}(t)$, (e) test in case of global error Lyapunov function $f_t(t)$.

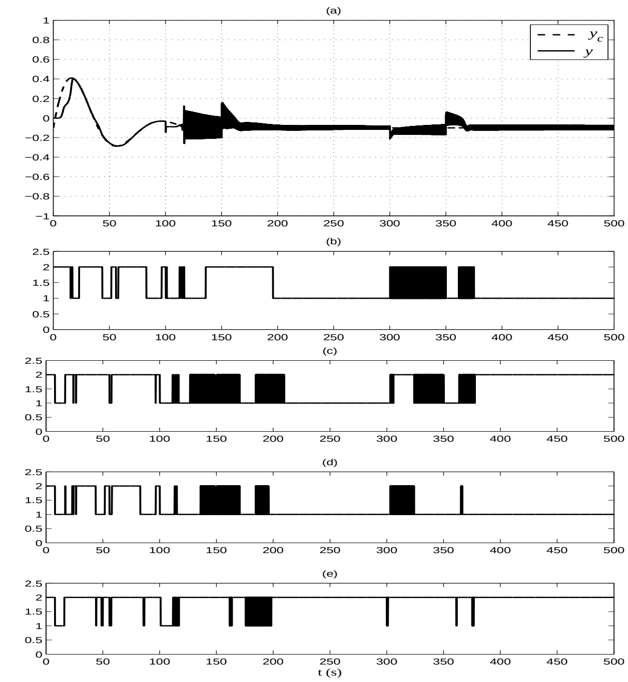

Fig. 4. Simulation in presence of control signal perturbations: (a) desired and system outputs, (b) test in case of emulation error Lyapunov function $f_e(t)$, (c) test in case of tracking error Lyapunov function $f_c(t)$, (d) test in case of intersection between emulation and tracking errors Lyapunov functions $f_{e-c}(t)$, (e) test in case of global error Lyapunov function $f_t(t)$.

with, $\rho > 0$. As a consequence, exponential stability results by imposing the tracking error dynamic.

Equation (25) guaranties a minimum dissipation rate proportional to energy. Using (25), an adaptation method is proposed to update $\eta_c(t)$ with the gradient algorithm and stability analysis. To improve control design, the dynamic of tracking error is imposed according to relations (3-12) and (25). The derivative of Lyapunov function can be written as:

$$\begin{aligned} &|\eta_e(t)|E_c^T(t)\left(\left(E_e^T(t)\boldsymbol{M}_2(t)\right)^T + V_e(t)\right) \\ &+|\eta_c(t)|E_c^T(t)\left(-\left(E_c^T(t)\boldsymbol{M}_1(t)\right)^T - R_c(t)\right) \\ &+E_c^T(t)\left(\dot{Y}_c(t) - R_c(t) + V_e(t)\right) = -\rho e_c(t) \end{aligned} \tag{26}$$

Consequently, Lyapunov exponential stability is guaranteed if $\eta_c(t)$ satisfies (27):

$$\begin{aligned} |\eta_c(t)| = &\frac{E_c^T(t)\left(-\dot{Y}_c(t) - V_e(t) + R_c(t)\right) - \rho e_c(t)}{E_c^T(t)\left(-\left(E_c^T(t)\boldsymbol{M}_1(t)\right)^T - R_{\mathbf{c}}(t)\right)} \\ &\frac{-|\eta_e(t)|\left(E_c^T(t)\left(\left(E_e^T(t)\boldsymbol{M}_2(t)\right)^T + V_e(t)\right)\right)}{E_c^T(t)\left(-\left(E_c^T(t)\boldsymbol{M}_1(t)\right)^T - R_c(t)\right)} \end{aligned} \tag{27}$$

Equation (27) is then preferred to the relation (10) to update $\eta_c(t)$ when the condition (13) is not satisfied. To ensure the stability of the controlled plant, stability condition on $\eta_e(t)$ (20) is also considered.

B. Simulations

To avoid instabilities, Lyapunov exponential stability is used for the controller updating. The nonlinear system (Fig. 2) and the same simulation conditions are considered again. The constant ρ, chosen equal to 5, is used to accelerate controller convergence. The performances of the proposed strategy based on the stability of the controlled plant are evaluated and compared to the results obtained for controller updating using only the stability of the controlled model and doesn't take into account the stability of the emulator.

Fig. 5(a) (relative to the controlled plant stability) shows a quicker convergence to the desired output and a good rejection of parametric perturbations. The proposed updating strategy prevent system instabilities (Fig.3). Corresponding tracking error, plotted in logarithmic scale, confirms this comment in Fig. 5(b). In Fig. 5(c), emulation error is plotted in logarithmic scale. The proposed strategy based on conditions on $\eta_e(t)$ and $\eta_c(t)$ improves the closed-loop system performances.

Control signal perturbations are also considered during intervals [100-150s] and [300-350s]. The effectiveness of the controller with the proposed method is illustrated in Fig.6. The controller ensure a better perturbations rejection by comparison with Fig.4.

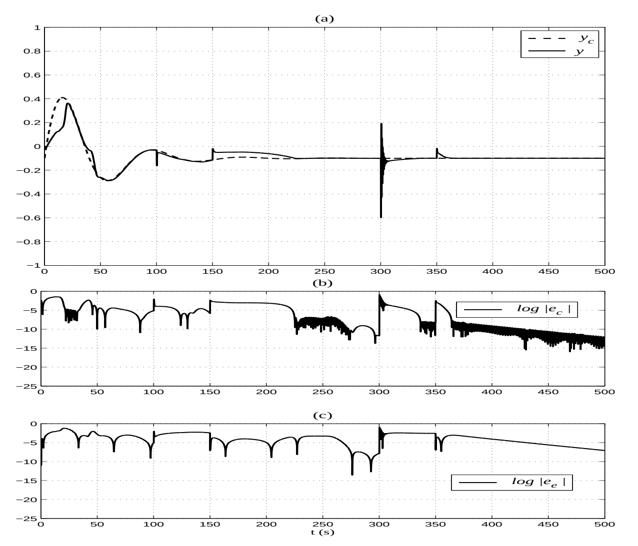

Fig. 5. Simulation in presence of parametric perturbation: (a) desired and system outputs, (b) tracking error, (b) emulation error.

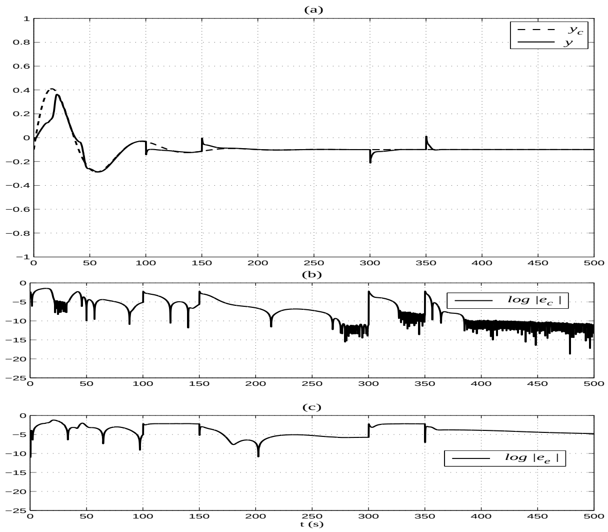

Fig. 6. Simulation in presence of control signal perturbation: (a) desired and system outputs, (b) tracking error, (b) emulation error.

Tracking error (Fig. 6(b)) and emulation error (Fig. 6(c)) confirm these conclusions. Obtained results show that stability analysis of the controlled system is performed and efficient for nonlinear system control with disturbances.
To establish the potential of the developed strategy, Normalized Mean Square Error (*NMSE*) is computed with (28):

$$NMSE = \frac{\sum_l \left(y_{c_l} - y_l\right)^2}{\sum_l \left(y_{c_l}\right)^2} \tag{28}$$

Results are summed up in Table II. Adaptation method based on exponential stability lead to good performances by comparison with the basic gradient updating that doesn't consider the stability criteria. The proposed strategy that ensure the stability of the controlled system improves the system performances in presence of perturbations. Then, we note that stability of the controlled model is not sufficient to prove the stability of the controlled system.

TABLE II
NMSE COMPUTED FOR THE PROPOSED ADAPTATION STRATEGY

NMSE	Parametric perturbations	Control signal perturbations
η_c updated with (10) and η_e with (8)	7.65	0.30
η_c updated with (27) and η_e with (8)	0.073	0.032
η_c updated with (27) and η_e with (20)	0.061	0.023

V. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

Stability of the controlled plant is proved according to the obtained stability conditions on emulator and controller rates. The effectiveness of the new updating of controller rate based on Lyapunov exponential stability approach and taking into account the stability of emulator is shown with numerical simulations results. Stability of the controlled plant ensures better performances in tracking and regulation problems.

B. Future Works

An objective criteria based on the minimization of the system energy will be proposed and applied to real MIMO processes.

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