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Observer design for Fault Diagnosis for the Takagi-Sugeno model with unmeasurable premise variables

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Abstract— **This paper deals with the problem of the state estimation and the sensor faults detection for discrete time nonlinear systems described by Takagi-Sugeno (TS) fuzzy models with unmeasurable premise variables. Indeed, a TS observer is synthesized, in descriptor form, to estimate both the system states and the sensor faults simultaneously. The idea of the proposed approach is to introduce the sensor fault as an auxiliary variable in the state vector. Besides, the multiple model with unmeasurable premise variables is reduced to a perturbed model with measurable variables.**

Convergence conditions are established with Lyapunov theory and the $\mathcal{L}_2$ optimization in order to guarantee the convergence of the state estimation error. These conditions are expressed in terms of Linear Matrix Inequalities (LMIs). The gains matrices of the multi-observers are characterized using the solution existence of the LMI conditions. Finally, the model of an hydraulic system with three tanks is used to validate the proposed approach.

Index Terms— **Nonlinear systems, multiple model approach, descriptor observer, state estimation, fault detection, unmeasurable premise variables, $\mathcal{L}_2$ optimization, LMIs.**

I. Introduction

Nonlinear models are widely used in many engineering and science processes to describe the dynamic behavior of real world processes. Therefore, algorithms for diagnosis of nonlinear systems took an important consideration. Indeed, the fault diagnosis problem has attracted much attention recently in several researchers ([8], [9], [15], [18], [20], [26]). The model based fault diagnosis becomes more difficult to achieve. As it is delicate to synthesize an observer for a nonlinear system, we propose to represent these systems into several linear fuzzy models based on the Takagi-Sugeno (TS) model structure ([27], [28]). This TS approach has been extensively used to modelize nonlinear systems. The basic idea for the TS model structure, sometimes known as multiple model structure, is the decomposition of the operating space of a nonlinear system into a finite number of operating zones. The behavior of the system in each zone is represented by a local linear model. Each local model is then quantified by means of weighting function. In fact, the multiple model structure presented in ([16], [22], [24], [29]) show that as an elegant and a powerful tool with several uses in the fields of identification, control and diagnosis of a large class of complex systems. The interests to use this structure is already well known, most of the analysis tools available for linear systems can be extended to the analysis of nonlinear systems, such as the observers synthesis.

The fault detection can be realized using observer based on this method. Indeed, it exists many works on observer design for multiple model systems. The work of Thau [30] was proposed an extension of the Luenberger observer [19] to nonlinear systems. In [1], [2] and [7], sliding mode observers for the linear systems, were transposed to the multiple model. Moreover, the unknown input observers designed for linear systems were implemented in [3] and [17], which are transposed, in the same way, into the case of nonlinear systems described by the multiple model structure. In [9] and [13], a descriptor observer is developed.

However, the works cited above, the authors suppose that the weighting functions depend on measurable premise variables (like the input or the output of the system). In the literature, only a few works are devoted to the case of unmeasurable premise variables, however, we can cite [5], [6], [7] where the authors proposed an observer which is an extension of the Thau-Luenberger observer. In this context, we can cite some works based on this approach as [5], [17], [23] and [25]. In this paper, a new method is proposed for state and sensor fault estimation of discrete time nonlinear systems. It is based on the use of the TS model approach to design descriptor observers for nonlinear systems which the weighting functions depending on unmeasurable premise variables (like the system state). Using the Lyapunov method and the $\mathcal{L}_2$ optimization, the stabilization of the multi-observer is performed. The convergence conditions may be expressed in Linear Matrix Inequalities (LMIs).

The outline of this paper is as follows. After the introduction, we give the multiple model structure in section II. In section III, we propose the multiple descriptor observer and the mains results are given under LMI formulation. To assure the convergence conditions, we can use the Lyapunov theory and the $\mathcal{L}_2$ techniques. In section IV, the proposal method is applied to the three tank system for state estimation and sensor fault detection. Finally, section V gives some conclusions.

Notation. X^T and X^{-1} are the transpose and the inverse of matrix, respectively. I is the identity matrix with appropriate dimension. The symbol $*$ in a symmetric matrix denotes the transposed block in the symmetric position.

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II. Multiple Model Approach

The multiple model approach allows to represent the behavior of a discrete time nonlinear system as multiple local linear models. Each sub-model contributes to the global presentation through a weighting function $\mu_i(\xi(k))$. The multiple model structure is given by :

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{r} \mu_i(\xi(k)) A_i x(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \tag{1}$$

where

$$\mu_i(\xi(k)) = \frac{\lambda_i(\xi(k))}{\sum_{i=1}^{r} \lambda_i(\xi(k))}, \quad \lambda_i(\xi(k)) = \prod_{j=1}^{g} M_{ji}(\xi(k)) \tag{2}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^m$ is the input vector and $y(k) \in \mathbb{R}^p$ is the output vector. A_i is the state matrix, B is the input matrix and C represent the output matrix.
In this paper, we consider TS models in discrete time affected by sensor faults. Then, the system (1) becomes :

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{r} \mu_i(\xi(k)) A_i x(k) + Bu(k) \\ y(k) &= Cx(k) + D_s f_s(k) \end{aligned} \tag{3}$$

where $f_s(k) \in \mathbb{R}^s$ is the fault vector and D_s is a matrix with appropriate dimension and it's assumed full column rank.

The representation of TS model is very interesting which allows to simplify the stability studies of nonlinear systems. Indeed, we find for example in [11] and [13] the tools inspired directly from the study of linear systems. In [4], [21], the authors worked on the problem of state estimation and application for diagnosis of TS fuzzy systems. However, in the works, the authors suppose that the premise variable $\xi(t)$ is measurable i.e. $\xi(t) = u(t)$ or $\xi(t) = y(t)$. In the diagnosis problem, this hypothesis oblige to conceive a bank of observers based on multiple models which the weighting functions depending on the input $u(t)$, for the detection and isolation of the sensor faults, or the output $y(t)$ for the detection and isolation of the actuator fault. This necessitates the elaboration of two different TS models for the same nonlinear system. To eliminate this problem, it's interesting to develop only one TS model which considers the system state as premise variables for the nonlinear systems.
Thus, consider the system (3) with weighting functions depending on the system state :

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{r} \mu_i(x(k)) A_i x(k) + Bu(k) \\ y(k) &= Cx(k) + D_s f_s(k) \end{aligned} \tag{4}$$

where $f_s(k)$ represents the additive sensor fault and D_s is a matrix of appropriate dimensions.
The multiple model with unmeasurable variables (4) can be reduced to a perturbed multiple model with measurable variables as follows :

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{r} \mu_i(\hat{x}(k)) A_i x(k) + Bu(k) + w(k) \\ y(k) &= Cx(k) + D_s f_s(k) \end{aligned} \tag{5}$$

where

$$w(k) = \sum_{i=1}^{r} \big(\mu_i(x(k)) - \mu_i(\hat{x}(k))\big)\big(A_i x(k)\big) \tag{6}$$

In this paper, an augmented system is constructed using the descriptor technique. Then, the system (5) with measurable premise variable and sensor faults can be written as follows :

$$\begin{aligned} \bar{E}\bar{x}(k+1) = & \sum_{i=1}^{r} \mu_i(\hat{x}(k)) \bar{A}_i \bar{x}(k) + \bar{B}u(k) + \bar{w}(k) \\ & + \bar{D}x_s(k) \\ y(k) = & \bar{C}\bar{x}(k) = C_0\bar{x}(k) + x_s(k) \end{aligned} \tag{7}$$

where

$\bar{x}(k) = \begin{pmatrix} x(k) \\ x_s(k) \end{pmatrix}$, $\bar{w}(k) = \begin{pmatrix} w(k) \\ 0 \end{pmatrix}$, $\bar{E} = \begin{pmatrix} I_n & 0 \\ 0 & 0 \end{pmatrix}$,
$\bar{A}_i = \begin{pmatrix} A_i & 0 \\ 0 & -I_n \end{pmatrix}$, $x_s(k) = D_s f_s(k)$, $\bar{B} = \begin{pmatrix} B^T & 0 \end{pmatrix}^T$,
$\bar{D} = \begin{pmatrix} 0 & I_p \end{pmatrix}^T$, $C_0 = \begin{pmatrix} C & 0 \end{pmatrix}$, and $\bar{C} = \begin{pmatrix} C & I_p \end{pmatrix}$

III. Multiple Observer Structure

In this section, we propose a TS observer to simultaneously estimate the inaccessible states and the sensor faults which the weighting functions depending on the estimated state :

$$\begin{aligned} Ez(k+1) &= \sum_{i=1}^{r} \mu_i(\hat{x}(k)) N_i z(k) + \bar{B}u(k) \\ \hat{\bar{x}}(k) &= z(k) + Ly(k) \\ \hat{y}(k) &= C_0\hat{\bar{x}}(k) = C\hat{x}(k) \end{aligned} \tag{8}$$

where $z(k) \in \mathbb{R}^{n+p}$ is an auxiliary state vector of the observer, $\hat{\bar{x}}(k) \in \mathbb{R}^{n+p}$ is the state estimation vector of the augmented system (7). E , $N_i \in \mathbb{R}^{(n+p)\times(n+p)}$ and $L \in \mathbb{R}^{(n+p)\times p}$ are the design parameters of the observer.
In order to establish the convergence conditions of the observer (8), we define the state estimation error :

$$\bar{e}(k) = \bar{x}(k) - \hat{\bar{x}}(k) \tag{9}$$

For the stabilization problem, combining the equations (7) and (8) gives the following expression :

$$\begin{aligned} (\bar{E} + EL\bar{C})\bar{x}(k+1) - E\hat{\bar{x}}(k+1) = \sum_{i=1}^{r} \mu_i(\hat{x}(k)) \\ \Big(\big(\bar{A}_i + N_i L C_0\big)\bar{x}(k) - N_i\hat{\bar{x}}(k) + \bar{D}x_s(k) + F_i L x_s(k) + \bar{w}(k) \Big) \end{aligned} \tag{10}$$

If the following conditions are respected :

$$\begin{aligned} N_i &= \bar{A}_i + N_i L C_0 \\ \bar{D} &= -N_i L \\ E &= \bar{E} + EL\bar{C} \end{aligned} \tag{11}$$

Then, the error dynamics (10) can be written as follows :

$$E\bar{e}(k+1) = \sum_{i=1}^{r} \mu_i(\hat{x}(k))\Big(N_i\bar{e}(k) + \bar{w}(k)\Big) \tag{12}$$

In order to satisfy the conditions (11), a solution of the observer parameters is given as follows :

$$N_i = \begin{bmatrix} A_i & 0 \\ -C & -I_p \end{bmatrix},\ L = \begin{bmatrix} 0 \\ I_p \end{bmatrix},\ E = \begin{bmatrix} I_n & 0 \\ RC & R \end{bmatrix} \tag{13}$$

where $R \in \mathbb{R}^{p\times p}$ is a full-rank matrix, that is a parameter to determine. By choosing the matrix R non singular, the dynamic error can be rewritten as follows :

$$\bar{e}(k+1) = \sum_{i=1}^{r} \mu_i(\hat{x}(k))\Big(S_i\bar{e}(k) + G\bar{w}(k)\Big) \tag{14}$$

where $S_i = E^{-1}N_i$ for $i = 1,...,r$ and $G = E^{-1}$.
One can calculate :

$$E^{-1} = \begin{bmatrix} I_n & 0 \\ -C & R^{-1} \end{bmatrix} \tag{15}$$

It can be shown that :

$$S_i = \begin{bmatrix} A_i & 0 \\ -CA_i - R^{-1}C & -R^{-1} \end{bmatrix},\ G = \begin{bmatrix} I_n & 0 \\ -C & R^{-1} \end{bmatrix} \tag{16}$$

The asymptotic convergence of the error dynamics (14) can be formulated by the following theorem.

Theorem1 The system (16) is stable and the $\pounds_2$ gain of the transfer from $\bar{w}(t)$ to the state estimation error $\bar{e}(t)$ is bounded by γ, if there exist two positive definite symmetric matrices $\bar{P}_1$, $\bar{P}_2$, a matrix Z_2 and a positif scalar γ such that the following LMI :

$$\begin{pmatrix} -\bar{P}_1 & 0 & \bar{P}_1A_i & 0 & \bar{P}_1 & 0 & 0 & 0 \\ * & -\bar{P}_2 & -\bar{P}_2CA_i - Z_2C & -Z_2 & -\bar{P}_2C & Z_2 & 0 & 0 \\ * & * & -\bar{P}_1 & 0 & 0 & 0 & I_n & 0 \\ * & * & * & -\bar{P}_2 & 0 & 0 & 0 & I_p \\ * & * & * & * & -\gamma I_n & 0 & 0 & 0 \\ * & * & * & * & * & -\gamma I_p & 0 & 0 \\ * & * & * & * & * & * & -\gamma I_n & 0 \\ * & * & * & * & * & * & 0 & -\gamma I_p \end{pmatrix} < 0 \tag{17}$$

The observer parameters are defined by (13) where R is defined by :

$$R = Z_2^{-1}P_2 \tag{18}$$

Proof : To prove the convergence of the estimation error toward zero, let as consider the quadratic function of Lyapunov :

$$V(\bar{e}(k)) = \bar{e}^T(k)P\bar{e}(k) \tag{19}$$

where P is a symmetric positive definite matrix.
The variation of V along the trajectory of (14) is given by :

$$\Delta V = \bar{e}^T(k+1)P\bar{e}(k+1) - \bar{e}^T(k)P\bar{e}(k) \tag{20}$$

and by using (14) :

$$\Delta V = \sum_{i=1}^{r} \mu_i(\hat{x}(k))\Big(\bar{e}^T(k)S_i^TPS_i\bar{e}(k) + \bar{e}^T(k)S_i^TPG\bar{w}(k) + \bar{w}^T(k)G^TPS_i\bar{e}(k) + \bar{w}^T(k)G^TPG\bar{w}(k)\Big) - \bar{e}^T(k)P\bar{e}(k) \tag{21}$$

The $\pounds_2$ gain of the transfer from $\bar{w}(k)$ to the state estimation error $\bar{e}(k)$ is bounded by γ such that :

$$\Delta V + \bar{e}^T(k)\bar{e}(k) - \gamma^2\bar{w}^T(k)\bar{w}(k) < 0 \tag{22}$$

By subsisting the equation (21) into the equation (22), we obtain :

$$\sum_{i=1}^{r} \mu_i(\hat{x}(k))\Big(\bar{e}^T(k)S_i^TPS_i\bar{e}(k) + \bar{e}^T(k)S_i^TPG\bar{w}(k) + \bar{w}^T(k)G^TPS_i\bar{e}(k) + \bar{w}^T(k)G^TPG\bar{w}(k)\Big) - \bar{e}^T(k)\bar{e}(k) + \bar{e}^T(k)\bar{e}(k) - \gamma^2\bar{w}^T(k)\bar{w}(k) < 0$$

which implies :

$$\sum_{i=1}^{r} \mu_i(\hat{x}(k)) \begin{pmatrix} \bar{e} \\ \bar{w} \end{pmatrix}^T \Omega \begin{pmatrix} \bar{e} \\ \bar{w} \end{pmatrix} < 0 \tag{23}$$

where

$$\Omega = \begin{pmatrix} S_i^TPS_i - P + I & S_i^TPG \\ * & G^TPG - \gamma^2 I \end{pmatrix} \tag{24}$$

which implies the following conditions :

$$\Omega < 0 \tag{25}$$

Applying the Schur complement again to the inequality (25), we obtain :

$$\begin{pmatrix} S_i^T\bar{P}S_i - \bar{P} & S_i^T\bar{P}G & I \\ * & G^T\bar{P}G - \gamma I & 0 \\ * & * & -\gamma I \end{pmatrix} < 0 \tag{26}$$

where : $\bar{P} = \gamma^{-1}P$
Based on the work of Gahinet [12], the condition (26) is equivalent to :

$$\begin{pmatrix} -\bar{P}^{-1} & S_i & G & 0 \\ * & -\bar{P} & 0 & I \\ * & * & -\gamma I & 0 \\ * & * & * & -\gamma I \end{pmatrix} < 0 \tag{27}$$

This inequality is nonlinear, so we multiply the both sides by $diag(\bar{P}, I, I, I)$, we obtain :

$$\begin{pmatrix} -\bar{P} & \bar{P}S_i & \bar{P}G & 0 \\ * & -\bar{P} & 0 & I \\ * & * & -\gamma I & 0 \\ * & * & * & -\gamma I \end{pmatrix} < 0 \tag{28}$$

Replacing S_i, supposing that $\bar{P} = diag(\bar{P}_1, \bar{P}_2)$ in the condition (28) and considering the change of variable defined by :

$$Z_2 = P_2R^{-1} \tag{29}$$

we obtain the LMI (17).

According to Theorem 1, the fuzzy observer can be obtained by the following algorithm :

Algorithm :

1) Solve the LMI (17) to give P_1, P_2 and Z_2.
2) Compute the matrix R using (18).
3) Compute N_i, L and E by (13).
4) By using the observer, we can obtain the estimation vector $\hat{\bar{x}}(k)$ of the augmented state $\bar{x}(k)$. Furthermore, the following estimated state is equal to :

$$\hat{x}(k) = \begin{bmatrix} I_n & 0 \end{bmatrix} \hat{\bar{x}}(k) \quad (30)$$

If we assume that matrix D_s is full column rank , we can calculate the estimate of $f_s(k)$ as follows

$$\hat{f}_s(k) = (D_s^T D_s)^{-1} D_s^T \begin{bmatrix} 0 & I_n \end{bmatrix} \hat{\bar{x}}(k) \quad (31)$$

IV. Application for the three tank system

In this section, we present the laboratory system that will be used to test the proposed methodology. Its nonlinear model is also derived.

A. Process description

The three tank system is presented in Fig. 1 [14]. The section of the three tanks R1, R2 and R3, with identically height equal to 80 cm, are respectively equal to 19x19, 21x21 and 26x26 (cm). Two pumps allow the water supply of tanks R1 and R2 by adjustment of the control inputs u1 and u2 respectively. There have an outflow including approximately 30 and 100 liters per minute. These tanks R1 and R2 are also connected through a manual valve to perturbed the system. Two electronic valves EV1 and EV2 are used to fill tank R3, and an electronic valve EV3 is used to evacuate the water. Three ultrasonic sensors find in each tank. It delivers an analogical output voltage varying from 0 to 10V. The measurement bracket lies between 10cm and 60cm. It is fed by a tension equal to 24VDC.

The nonlinear model of the three tank system can be written in the following form :

$$\begin{aligned} x(k+1) &= A(x(k))x(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \quad (32)$$

where

$$x(k) = \begin{pmatrix} n_1(k) & n_2(k) & n_3(k) \end{pmatrix}^T$$

$$A(x(k)) = \begin{pmatrix} -\frac{p_1}{\sqrt{n_1(k)}} & 0 & 0 \\ 0 & -\frac{p_2}{\sqrt{n_2(k)}} & 0 \\ \frac{p_1}{\sqrt{n_1(k)}} & \frac{p_2}{\sqrt{n_2(k)}} & -\frac{p_3}{\sqrt{n_3(k)}} \end{pmatrix}$$

Fig. 1. Three tank system

$$B = \begin{pmatrix} p_{11} & 0 \\ 0 & p_{22} \\ 0 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where p_1, p_2, p_3, p_{11} and p_{22} are constants.
To consider the nonlinearity terms $\xi_j(k) = \frac{1}{\sqrt{n_i(k)}} \in [\alpha_i, \beta_i] (i = 1, 2, 3)$ of the matrix $A(x(k))$, the nonlinear model (32) can be represented by $(2^3 = 8)$ rules as :

$$A_1 = \begin{pmatrix} -p_1\beta_1 & 0 & 0 \\ 0 & -p_2\beta_2 & 0 \\ p_1\beta_1 & p_2\beta_2 & -p_3\beta_3 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} -p_1\beta_1 & 0 & 0 \\ 0 & -p_1\beta_2 & 0 \\ p_1\beta_1 & p_2\beta_2 & -p_3\alpha_3 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} -p_1\beta_1 & 0 & 0 \\ 0 & -p_2\alpha_2 & 0 \\ p_1\beta_1 & p_2\alpha_2 & -p_3\beta_3 \end{pmatrix}$$

$$A_4 = \begin{pmatrix} -p_1\beta_1 & 0 & 0 \\ 0 & -p_2\alpha_2 & 0 \\ p_1\beta_1 & p_2\alpha_2 & -p_3\alpha_3 \end{pmatrix}$$

$$A_5 = \begin{pmatrix} -p_1\alpha_1 & 0 & 0 \\ 0 & -p_2\beta_2 & 0 \\ p_1\alpha_1 & p_2\beta_2 & -p_3\beta_3 \end{pmatrix}$$

$$A_6 = \begin{pmatrix} -p_1\alpha_1 & 0 & 0 \\ 0 & -p_2\beta_2 & 0 \\ p_1\alpha_1 & p_2\beta_2 & -p_3\alpha_3 \end{pmatrix}$$

$$A_7 = \begin{pmatrix} -p_1\alpha_1 & 0 & 0 \\ 0 & -p_2\alpha_2 & 0 \\ p_1\alpha_1 & p_2\alpha_2 & -p_3\beta_3 \end{pmatrix}$$

$$A_8 = \begin{pmatrix} -p_1\alpha_1 & 0 & 0 \\ 0 & -p_2\alpha_2 & 0 \\ p_1\alpha_1 & p_2\alpha_2 & -p_3\alpha_3 \end{pmatrix}$$

and $B_1 = B_2 = B_3 = B_4 = B_5 = B_6 = B_7 = B_8 = B$
The considered faults sensors are :

$$f_s = \begin{pmatrix} f_{s1} \\ f_{s2} \\ f_{s3} \end{pmatrix},$$

with

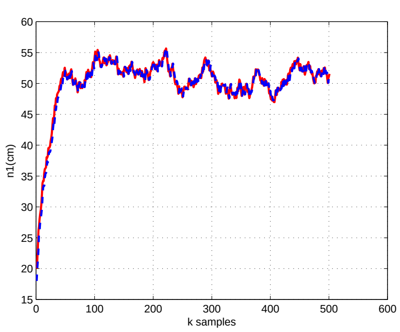
$$f_{s1} = \begin{cases} 0 & k < 200 \\ 0.08 * k & 200 \leq k < 250 \\ 0 & k \geq 250 \end{cases}$$

$$f_{s2} = \begin{cases} 0 & k < 230 \\ 3sin(0.5(k-1)) & 230 \leq k < 300 \\ 0 & k \geq 300 \end{cases}$$

$$f_{s3} = \begin{cases} 0 & k < 250 \\ 0.06(k-2) & 250 \leq k < 300 \\ 0 & k \geq 300 \end{cases}$$

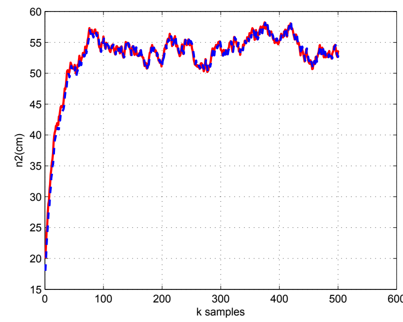
B. Simulation results

We propose to apply the descriptor observer, which the weighting functions depending on the estimated state, to estimate both the system states and the sensors faults. In fact, some simulation results are shown in Figs. 2-3 to exhibit the states and the sensor faults and their estimated, respectively. It can be seen that the estimation of the states is good. In the same way, the sensor fault estimator gives a satisfying result. Consequently, we can confirm the convergence to zero of the estimation errors. Then, these results show the effectiveness for the proposed method.



V. Conclusion

This paper addresses a method to design an observer for fault diagnosis in discrete time Takagi-Sugeno systems with unmeasurable premise variables. In order to estimate both the state and the sensor faults, we have designed a descriptor observer where the sensor faults are considered as an auxiliary variable in the state vector. For the convergence



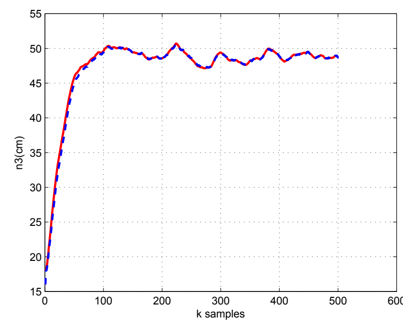
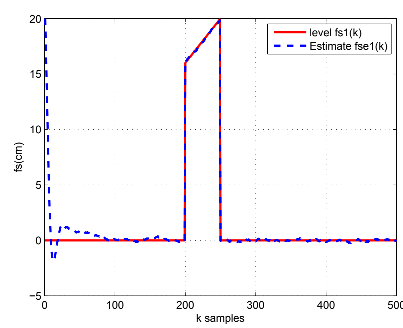
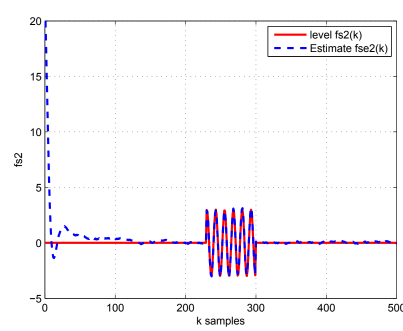


Fig. 2. State responses and their estimates n_1, n_2 and n_3





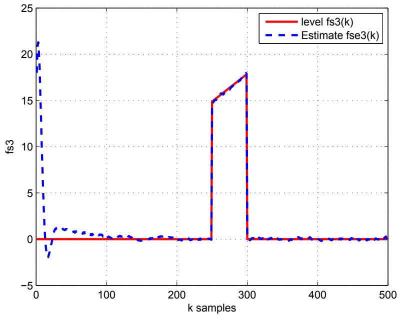


Fig. 3. Sensor faults and their estimates f_{s1}, f_{s2} and f_{s3}

of the estimation errors, we considered a quadratic function of Lyapunov and the £2 approach. The sufficient conditions for existence of the robust estimator is provided in terms of Linear Matrix Inequalities(LMI). The proposed method is applied successfully to three tank system. The method is based on a representation of the nonlinear system by a multiple model based on the convex property of the sum of the weighting functions.

VI. Acknowledgments

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