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Multi-Decision Prognosis: Decentralized Architectures Cooperating for Predicting Failures in Discrete Event Systems

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Abstract— **A new framework, called multi-decision prognosis, has been recently developed, which consists in using several decentralized architectures working in parallel and whose prognoses are combined to obtain an effective prognosis. In this paper, we analyze and improve multi-decision prognosis in several ways. Firstly, we present a generic form of multi-decision prognosis which is independent on the architectures in parallel and on how their decisions a combined. Secondly, we solve in a rigorous way a problem of decomposing infinite languages which arises with multi-decision prognosis. Thirdly, we identify and solve a so-called hesitation problem that arises in multi-decision prognosis.**

I. INTRODUCTION

Prognosis of a plant modeled as a discrete event system (DES) consists in predicting failure events of the plant. We approach failure prognosis as it has been studied by the supervisory control community, where the objective is to predict a failure whose occurrence in a bounded future is certain. [1] presents a prognosis framework, where a prognoser issues a verdict on whether a failure will occur, based on the partial observation it has of the plant. The notion of predictability (or *prognosability*) is defined formally to characterize the class of languages for which: 1) every failure is prognosed before its occurrence, and 2) every failure will certainly occur after it has been prognosed. In [2], an off-line polynomial-time and an on-line algorithms are proposed to check prognosability.

In [3], a decentralized prognosis architecture is proposed, where several local prognosers cooperate to predict failures. Each local prognoser has a partial observation of the plant and predicts a failure when it is *certain* that it will occur. Otherwise, it issues a "don't know" decision. The local prognosers transmit their prognoses to a fusion system that synthesizes an *effective* prognosis. [4], [5] study two complementary decentralized architectures qualified as disjunctive and conjunctive. The disjunctive one is also studied in [3].

[3] is generalized in [6] by developing an inference-based architecture, where each local prognoser associates to its prognosis a level of ambiguity. When the fusion system receives several concurrent prognoses from the local prognosers, it will select the prognosis with the lowest ambiguity level.

[4] presents a new framework called multi-decision prognosis. Its principle is to use several decentralized architectures working in parallel and whose global prognoses are combined in a certain way to obtain an effective prognosis. Two cases are considered: the architectures in parallel are disjunctive (resp. conjunctive) and their prognoses are combined conjunctively (resp. disjunctively). These two cases generalize the disjunctive and conjunctive architectures of [4], [5], respectively.

In this paper, we analyze and improve multi-decision prognosis of [4]. Compared to [4], our main contributions are:

- To help for a more fundamental understanding of multi-decision prognosis, we present it in a generic form which is independent on the architectures in parallel and on how their decisions a combined.
- With multi-decision prognosis, we are confronted to a problem of decomposing infinite languages. We solve the problem more rigorously than in [4], by an automaton-based approach.
- We identify and solve a hesitation problem that arises in multi-decision prognosis. More precisely, we prove that in each of the two multi-decision architectures of [4], the decision taken by the prognoser may oscillate between 1 (failure predicted) and 0 (no failure predicted) before stabilizing. Then, we propose a solution to this problem.

The remainder of the paper is organized as follows. In Section II, we present preliminaries and notations and define formally the prognosis objectives. Section III introduces a generic form of multi-decision prognosis. In Section IV (resp. V), we study multi-decision prognosis in the specific case where several disjunctive (resp. conjunctive) architectures have their decisions combined conjunctively (resp. disjunctively). Section VI concludes.

II. PRELIMINARIES AND NOTATIONS

We denote by Σ the finite set of events (*alphabet*) that can be executed by the DES to be prognosed. Let the term *trace* mean a finite sequence of events. Σ^* is the set of all traces of events of Σ, including the empty trace ε. For any trace $\lambda \in \Sigma^*$, $|\lambda|$ denotes the length of λ. $\Sigma^{\geq k}$ is the subset of Σ^* containing the traces whose length is $\geq k$. For any traces $\lambda, \mu \in \Sigma^*$, μ is said a prefix of λ, if $\exists \nu \in \Sigma^*$ s.t. $\lambda = \mu\nu$. The set of all prefixes of a language K is denoted $\overline{K}$, and K is said prefix-closed if $K = \overline{K}$. Let $\mathbf{N}^+ = \{1, 2, 3, \cdots\}$ denote the set of strictly positive integers.

The DES to be prognosed, which is called *plant*, is assumed modeled by a prefix-closed language $\mathcal{L}$. The latter consists of a *failure* part and a non-failure (or *healthy*) part, modeled by $\mathcal{F}$ and $\mathcal{H}$, respectively. We have $\mathcal{L} = \mathcal{H} \cup \mathcal{F}$ and $\mathcal{H} \cap \mathcal{F} = \emptyset$. Typically, $\mathcal{F}$ and $\mathcal{H}$ are related to a set of failure

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events $\Sigma_f \subset \Sigma$ as follows: $\mathcal{F}$ is the part of $\mathcal{L}$ which contains the traces with at least one failure event, and $\mathcal{H}$ contains the traces without failure event, hence $\mathcal{H}$ is prefix-closed. We will use the following notions defined in [3], [6]:

Definition 2.1: Consider a prefix-closed language $\mathcal{L}$ and its faulty and healthy parts, $\mathcal{H}$ and $\mathcal{F}$.

- A *boundary* trace of $\mathcal{H}$ w.r.t $\mathcal{L}$ is a trace of $\mathcal{H}$ for which a failure in a next step is possible. The set of boundary traces of $\mathcal{H}$ w.r.t $\mathcal{L}$ is formally defined by: $\partial = \{\lambda \in \mathcal{H} \mid \{\lambda\}\Sigma \cap \mathcal{F} \neq \emptyset\}$.
- An *indicator* trace of $\mathcal{H}$ w.r.t $\mathcal{L}$ is a trace of $\mathcal{H}$ for which a failure in future is guaranteed. The set of indicator traces of $\mathcal{H}$ w.r.t $\mathcal{L}$ is formally defined by: $\Im = \{\lambda \in \mathcal{H} \mid \exists k \in \mathbf{N}^+ : \{\lambda\}\Sigma^{\geq k} \cap \mathcal{H} = \emptyset\}$.
- A *non-indicator* trace of $\mathcal{H}$ w.r.t $\mathcal{L}$ is a trace of $\mathcal{H}$ for which a failure in future is not guaranteed. The set of non-indicator traces of $\mathcal{H}$ w.r.t $\mathcal{L}$ is formally defined by: $\Upsilon = \mathcal{H} \setminus \Im = \{\lambda \in \mathcal{H} \mid \forall k \in \mathbf{N}^+ : \{\lambda\}\Sigma^{\geq k} \cap \mathcal{H} \neq \emptyset\}$.

Let $\mathrm{X}(\lambda)$ denote the prognosis issued after the execution of the trace λ. We consider the case where the objective of a prognoser is to respect the following three properties:

No missed prediction: Predict a failure if its occurrence is possible in the next step (which does not forbid to predict it also earlier). Formally:

$$\forall\lambda \in \partial : \mathrm{X}(\lambda) = 1 \qquad (1)$$

No false prediction: Predict a failure only if it will certainly occur, that is, predict no failure if no failure is certain.

$$\forall\lambda \in \Upsilon : \mathrm{X}(\lambda) = 0 \qquad (2)$$

No changed positive prediction: After a failure is predicted, the prognoser continues to predict it until its occurrence. Formally:

$$\forall\lambda, \mu \in \Sigma^* s.t. \lambda\mu \in \mathcal{H} : \mathrm{X}(\lambda) = 1 \Rightarrow \mathrm{X}(\lambda\mu) = 1 \qquad (3)$$

Eqs. (1) and (2) are defined w.r.t ∂ and Υ, respectively. We will say that Eqs. (1,2) are defined w.r.t (∂, Υ).

Eq. (3) is not used as objective in [4], [3], [6]. However, it is shown in [5] that the architecture of [3] respects by construction our Eq. (3).

In decentralized prognosis of [3], [6], [5], n local prognosers $(\mathrm{Prog}_i)_{1\leq i\leq n}$ observe the plant and generate local prognoses that are combined (or fused) by an operator F to generate an effective prognosis. Each Prog_i has a partial view of the plant, its set of observable events is $\Sigma_{o,i} \subseteq \Sigma$. Let $\Sigma_o = \bigcup_{1\leq i\leq n} \Sigma_{o,i}$ and $\Sigma_{uo} = \Sigma \setminus \Sigma_o$. Therefore, an event of Σ_o is observable by at least one Prog_i, and no Prog_i can observe an event of Σ_{uo}. We denote by P_i the natural projection that hides the events of $\Sigma \setminus \Sigma_{o,i}$ from any trace $\lambda \in \Sigma^*$.

III. INTRODUCTION TO MULTI-DECISION PROGNOSIS

The authors of [3], [4], [5] develop a decentralized architecture called disjunctive, where the local prognoses of the local prognosers are fused disjunctively to synthesize an effective prognosis. The authors of [4], [5] propose also a conjunctive architecture where the local prognoses are fused conjunctively. Assume now that we are in a situation where none of these architectures provides a prognoser that satisfies Eqs. (1,2,3). A solution would be to try the inference-based architecture proposed in [6]. Assume also that we are in a situation where the inference-based architecture also provides no solution. In order to tackle this problem, we have developed a solution called *multi-decision* prognosis.

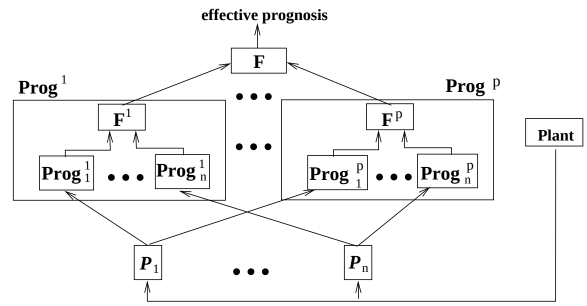

Fig. 1. Multi-decision prognosis architecture

The principle of multi-decision is to use several (say p) decentralized prognosers $(\mathrm{Prog}^j)_{j=1\cdots p}$ running in parallel and whose global prognoses $(\mathrm{X}^j)_{j=1\cdots p}$ are combined by a *global* module F to obtain an effective prognosis, as illustrated in Figure 1. Note that each prognoser Prog^j is itself a decentralized architecture as developed in [3], [4], [5], i.e. it consists of several local prognosers $(\mathrm{Prog}_i^j)_{i=1\cdots n}$ whose local prognoses $(\mathrm{X}_i^j)_{i=1\cdots n}$ are combined by a *local* fusion module F^j. More precisely, after the execution of a trace $\lambda \in \mathcal{H}$, each global prognosis $\mathrm{X}^j(\lambda)$ is computed from the local prognoses $(\mathrm{X}_i^j(P_i(\lambda)))_{i=1\cdots n}$ by Eq. (4), and the effective prognosis $\mathrm{X}(\lambda)$ is computed from the global prognoses $(\mathrm{X}^j(\lambda))_{j=1\cdots p}$ by Eq. (5).

$$\forall j \in \{1, \cdots, p\} : \mathrm{X}^j(\lambda) = \mathrm{F}^j(\mathrm{X}_1^j(P_1(\lambda)), \cdots, \mathrm{X}_n^j(P_n(\lambda))) \qquad (4)$$

$$\mathrm{X}(\lambda) = \mathrm{F}(\mathrm{X}^1(\lambda), \cdots, \mathrm{X}^p(\lambda)) \qquad (5)$$

Definition 3.1: A prognoser/architecture obtained by using local fusions $(\mathrm{F}^j)_{j=1\cdots p}$ and a global fusion F is called F-$(\mathrm{F}^1, \cdots, \mathrm{F}^p)$-prognoser/architecture. When all the $(\mathrm{F}^j)_{j=1\cdots p}$ are equal to the same module Ψ, we obtain a so-called F-Ψ^p-prognoser/architecture.

Note that the global prognoses $(\mathrm{X}^j(\lambda))_{j=1\cdots p}$ (computed by Eq. (4)) are just intermediate results used to compute the prognosis $\mathrm{X}(\lambda)$ (using Eq. (5)), the latter being the effective prognosis that is actually made.

For example, consider the $\vee$-$(\vee, \wedge)$-architecture, which means that $p = 2$, $\mathrm{F}^1 = \vee$, $\mathrm{F}^2 = \wedge$, and $\mathrm{F} = \vee$. Concretely, we have a disjunctive (F^1) and a conjunctive (F^2) architectures running in parallel and whose global prognoses are combined disjunctively (F). to obtain an effective prognosis.

Note that the disjunctive (resp. conjunctive) architecture studied in [5] is a special case of F-$(\mathrm{F}^1, \cdots, \mathrm{F}^p)$-architecture, obtained by taking $p = 1$, $\mathrm{F}^1 = \vee$ (resp. $\mathrm{F}^1 = \wedge$) and F =identity operator. For $p > 1$, two cases of global combination are considered: F $= \wedge$ (conjunctive) and F $= \vee$ (disjunctive). We will see that the two cases are based on decomposing the sets Υ and ∂, respectively. They are studied in Sections IV and V, respectively.

IV. $\wedge$-$\vee^p$-ARCHITECTURE: GENERALIZATION OF THE DISJUNCTIVE ARCHITECTURE

A. $\wedge$-$\vee^p$-*prognoser*

We study the case where $\mathrm{F} = \wedge$ and $\mathrm{F}^j = \vee, \forall j \in \{1, \cdots, p\}$, i.e we have a $\wedge$-$(\vee, \cdots, \vee)$-prognoser, or more concisely a $\wedge$-$\vee^p$-prognoser. In other words, p decentralized disjunctive prognosers are running in parallel, and their global prognoses are fused conjunctively to synthesize the effective prognosis. The notation $\wedge$-$\vee^{\geq 1}$-prognoser means any $\wedge$-$\vee^p$-prognoser, for some unspecified $p \in \mathbf{N}^+$. The obtained $\wedge$-$\vee^p$-architecture generalizes the disjunctive architecture of [4], [3], [5], because the latter is obtained by taking $p=1$.

The global and effective prognoses are computed by the following Eqs. (6,7) that specialize Eqs. (4,5).

$$\forall j \in \{1, \cdots, p\} : \mathrm{X}^j(\lambda) = \bigvee_{i=1 \cdots n} \mathrm{X}_i^j(P_i(\lambda)) \tag{6}$$

$$\mathrm{X}(\lambda) = \bigwedge_{j=1 \cdots p} \mathrm{X}^j(\lambda) \tag{7}$$

B. *Decomposing* Υ *to compute the local prognoses*

We have explained that, given $p \in \mathbf{N}^+$, a $\wedge$-$\vee^p$-prognoser X is obtained by using p disjunctive prognosers Prog^j running in parallel, whose global prognoses are combined conjunctively to synthesize the effective prognosis. Eqs. (6,7) are used to compute the effective prognosis $\mathrm{X}(\lambda)$ from the local prognoses $(\mathrm{X}_i^j(P_i(\lambda)))_{i=1 \cdots n}$. Let us propose a rule to compute each local prognosis $\mathrm{X}_i^j(P_i(\lambda))$. We assume that we are given a decomposition $(\Upsilon^1, \cdots \Upsilon^p)$ of Υ, i.e. $\Upsilon = \Upsilon^1 \cup \cdots \cup \Upsilon^p$. Since each Prog^j is a disjunctive prognoser, its local prognoses $(\mathrm{X}_i^j(P_i(\lambda)))_{i=1 \cdots n}$ are computed as in [4], [5] for the disjunctive architectures, but by using Υ^j instead of Υ. Formally:

$$\forall j \in \{1, \cdots, p\}, \forall i \in \{1, \cdots, n\} : \\ \mathrm{X}_i^j(P_i(\lambda)) = \begin{cases} 0, & \text{if } P_i(\lambda) \in P_i(\Upsilon^j) \\ 1, & \text{otherwise} \end{cases} \tag{8}$$

The idea behind this computation is that Eqs. (6,8) imply that the global prognosis X^j of each disjunctive prognoser Prog^j satisfies Eqs. (1,2), but w.r.t (∂, Υ^j) instead of w.r.t (∂, Υ). And by fusing conjunctively all the global prognoses $(\mathrm{X}^j)_{j=1 \cdots p}$ (using Eq. (7)), we obtain an effective prognosis that satisfies Eqs. (1,2) w.r.t (∂, Υ). The question that arises is: how is Eq. (3) guaranteed ? We solve this question by requiring that once a prognosis $\mathrm{X}(\lambda) = 1$ is issued, the prognoser is blocked on this positive prognosis. This can be easily implemented by using a variable X_c which is initialized to 0 and switches to 1 the first time $\mathrm{X}(\lambda) = 1$ is issued. When $\mathrm{X}_c = 0$, the prognoser computes $\mathrm{X}(\lambda)$ by Eq. (7). When $\mathrm{X}_c = 1$, the prognoser issues $\mathrm{X}(\lambda) = 1$ without any computation. Formally, we must replace Eq. (7) by the following equation:

$$\mathrm{X}(\lambda) = \mathrm{X}_c \vee (\bigwedge_{j=1 \cdots p} \mathrm{X}^j(\lambda)) \tag{9}$$

Consider the example of Fig. 2, where $\Sigma_{uo} = \{f\}$, $\Sigma_{o,1} = \{a_1, \sigma\}$, $\Sigma_{o,2} = \{a_2, \sigma\}$. $\mathcal{F} = \sigma^*(a_1 + a_2)\sigma\sigma\sigma f f^*$ contains the traces with f, and $\mathcal{H} = \sigma^*(a_1 + a_2)\sigma\sigma\sigma + \sigma^*(a_1 a_2 + a_2 a_1)\sigma^*$ contains the traces without f. $\Upsilon = \sigma^* + \sigma^* a_1 + \sigma^* a_1 a_2 \sigma^* + \sigma^* a_2 + \sigma^* a_2 a_1 \sigma^*$ is accepted by the FSA $\mathcal{A}_\Upsilon$ obtained from Fig. 2 by removing states 4-7 and marking all the other states; it contains the traces of $\mathcal{H}$ for which f is not guaranteed in future. $\partial = \sigma^*(a_1 + a_2)\sigma\sigma\sigma$ is accepted by the FSA $\mathcal{A}_\partial$ obtained from Fig. 2 by removing states 7-11 and marking state 6; it contains the traces of $\mathcal{H}$ for which f is possible in the next step. Consider first the case where Υ is not decomposed, i.e. $p = 1$ and $\Upsilon^1 = \Upsilon$. The $\wedge$-$\vee^1$-prognoser defined by Eqs. (6,8,9) corresponds to the disjunctive architecture of [4], [3], [5]. We have checked that it does not satisfy Eqs. (1,2).

Let us consider the following decomposition of Υ: $\Upsilon^1 = \sigma^*(a_1 a_2 + a_2 a_1)(\sigma + \sigma\sigma\sigma\sigma^*)$ corresponding to states 9 and 11, and $\Upsilon^2 = \sigma^*(a_1 a_2 + a_2 a_1) + \sigma^*(a_1 a_2 + a_2 a_1)\sigma\sigma$ corresponding to states 1, 2, 3, 8, 10. Table I outlines $\mathrm{X}_i^j(P_i(\lambda))$, $\mathrm{X}^j(\lambda)$ and $\mathrm{X}(\lambda)$ of the $\wedge$-$\vee^2$-prognoser computed by Eqs. (6,8,9) for traces $\lambda \in \mathcal{H}$. In the sequel, a trace λ is said symmetrical to a trace μ, if each of them is obtained from the other by switching a_1 and a_2. Since the example is symmetrical, we do not list all the traces of $\mathcal{H}$. The missing traces are symmetrical to the traces listed in lines 2-9. For traces $\sigma^* a_1 \sigma\sigma$ (line 4), in the last column we indicate the two prognoses obtained with Eqs. (7) and (9) respectively. For the other traces, the two equations generate the same prognosis.

We see that "$\mathrm{X}(\lambda) = 1$" for $\sigma^* a_1 \sigma\sigma\sigma$ (line 5), and by symmetry we have the same prognosis for the symmetrical traces. Hence, "$\mathrm{X}(\lambda) = 1$" for all traces of ∂, i.e. Eq. (1) is satisfied. We see that "$\mathrm{X}(\lambda) = 0$" is issued in lines 1, 2, 6-9, and by symmetry we have the same prognosis for the symmetrical traces. Hence, "$\mathrm{X}(\lambda) = 0$" for all traces of Υ, i.e. Eq. (2) is satisfied. The other traces (lines 3 and 4 and their symmetrical) belong to $\mathcal{H} \setminus (\Upsilon \cup \partial)$. In lines 3-4, we see that with Eq. (7), the prognoser changes its mind since its decision passes from 1 to 0. With Eq. (9), the decision remains 1, hence Eq. (3) is satisfied. By this example, we have shown the necessity of using Eq. (9) instead of Eq. (7) to respect Eq. (3)

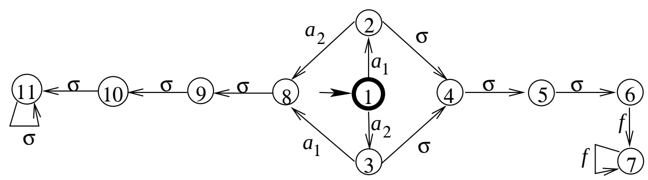

Fig. 2. Example for illustrating the $\wedge$-$\vee^2$ prognosis

C. *Existence Results*

We are using $\wedge$-$\vee^p$-architectures defined by Eqs. (6,9) and the objective is to satisfy Eqs. (1,2,3). As explained in

λ	$P_1(\lambda)$	$P_2(\lambda)$	$\mathrm{X}_1^1(P_1(\lambda))$	$\mathrm{X}_2^1(P_2(\lambda))$	$\mathrm{X}^1(\lambda)$	$\mathrm{X}_1^2(P_1(\lambda))$	$\mathrm{X}_2^2(P_2(\lambda))$	$\mathrm{X}^2(\lambda)$	$\mathrm{X}(\lambda)$
σ^*	σ^*	σ^*	1	1	1	0	0	0	0
$\sigma^* a_1$	$\sigma^* a_1$	σ^*	1	1	1	0	0	0	0
$\sigma^* a_1 \sigma$	$\sigma^* a_1 \sigma$	$\sigma\sigma^*$	0	1	1	1	0	1	1
$\sigma^* a_1 \sigma\sigma$	$\sigma^* a_1 \sigma\sigma$	$\sigma\sigma\sigma^*$	1	1	1	0	0	0	0, 1
$\sigma^* a_1 \sigma\sigma\sigma$	$\sigma^* a_1 \sigma\sigma\sigma$	$\sigma\sigma\sigma\sigma^*$	0	1	1	1	0	1	1
$\sigma^* a_1 a_2$	$\sigma^* a_1$	$\sigma^* a_2$	1	1	1	0	0	0	0
$\sigma^* a_1 a_2 \sigma$	$\sigma^* a_1 \sigma$	$\sigma^* a_2 \sigma$	0	0	0	1	1	1	0
$\sigma^* a_1 a_2 \sigma\sigma$	$\sigma^* a_1 \sigma\sigma$	$\sigma^* a_2 \sigma\sigma$	1	1	1	0	0	0	0
$\sigma^* a_1 a_2 \sigma\sigma\sigma\sigma^*$	$\sigma^* a_1 \sigma\sigma\sigma\sigma^*$	$\sigma^* a_2 \sigma\sigma\sigma\sigma^*$	0	0	0	1	1	1	0

TABLE I

$\wedge$-$\vee^2$ PROGNOSIS RESULTS FOR THE EXAMPLE OF FIG. 2

Section IV-B, Eq. (3) is guaranteed by the use of X_c of Eq. (9). Therefore, the question that remains to be answered is:

Question 4.1: Do there exist $p \in \mathbf{N}^+$ and a $\wedge$-$\vee^p$-architecture defined by Eqs. (6,9) and satisfying Eqs. (1,2)?

We will show further that answering this question amounts to answering the following question:

Question 4.2: Do there exist an integer $p \in \mathbf{N}^+$ and a decomposition $(\Upsilon^j)_{j=1\cdots p}$ of Υ such that:
$\forall j = 1, \cdots, p : \bigcap_{i=1,\cdots n}[P_i^{-1} P_i(\Upsilon^j)] \cap \partial = \emptyset$?

With Question 4.2, we are confronted to the difficult problem of decomposing an infinite set, namely Υ. To solve this problem, we use the following requirement which transforms the problem of decomposing the infinite regular language Υ into the problem of decomposing the *finite* set of marked states of an FSA $\mathcal{A}_\Upsilon$ accepting Υ.

Requirement R1: Given an FSA $\mathcal{A}_\Upsilon$ accepting Υ, the only eligible decompositions $(\Upsilon^j)_{j=1\cdots}$ of Υ are such that every Υ^j consists of the traces leading to one or several marked states of $\mathcal{A}_\Upsilon$. In other words, every Υ^j corresponds to a subset of the set of marked states of $\mathcal{A}_\Upsilon$.

Let us return to the example of Fig 2, we have obtained an FSA $\mathcal{A}_\Upsilon$ accepting Υ from the FSA of Fig 2 by removing states 4-7 and marking the other states. Decomposing Υ amounts therefore to decomposing the finite set of marked states $\{1, 2, 3, 8, 9, 10, 11\}$. The number of decompositions is therefore finite. In Section IV-B, we have used the following decomposition: $\Upsilon^1 = \sigma^*(a_1 a_2 + a_2 a_1)(\sigma + \sigma\sigma\sigma\sigma^*$ corresponding to states 9 and 11, and $\Upsilon^2 = \sigma^*\overline{(a_1 a_2 + a_2 a_1)} + \sigma^*(a_1 a_2 + a_2 a_1)\sigma\sigma$ corresponding to states 1, 2, 3, 8, 10.

The previous Question 4.2 is thus strengthened into checking the following notion of $\wedge$-$\vee^p$-COPROG w.r.t an FSA $\mathcal{A}_\Upsilon$:

Definition 4.1: Consider a pair $(\mathcal{L}, \mathcal{H})$ of prefix-closed languages with $\mathcal{H} \subseteq \mathcal{L}$ and their corresponding (Υ, ∂), $p \in \mathbf{N}^+$, and an FSA $\mathcal{A}_\Upsilon$ accepting Υ. (Υ, ∂) is said $\wedge$-$\vee^p$-COPROG w.r.t $\mathcal{A}_\Upsilon$ if there exists a decomposition $(\Upsilon^j)_{j=1\cdots p}$ of Υ satisfying **R1** w.r.t $\mathcal{A}_\Upsilon$ and such that:

$$\forall j = 1, \cdots, p : \bigcap_{i=1,\cdots n} [P_i^{-1} P_i(\Upsilon^j)] \cap \partial = \emptyset \qquad (10)$$

And (Υ, ∂) is said $\wedge$-$\vee^{\geq 1}$-COPROG if there exists $p \in \mathbf{N}^+$ for which (Υ, ∂) is $\wedge$-$\vee^p$-COPROG.

The link between Questions 4.1 and 4.2 is stated by the following proposition:

Proposition 4.1: Consider a pair $(\mathcal{L}, \mathcal{H})$ of prefix-closed languages with $\mathcal{H} \subseteq \mathcal{L}$ and their corresponding (Υ, ∂), $p \in \mathbf{N}^+$ and an FSA $\mathcal{A}_\Upsilon$ accepting Υ. There exists a $\wedge$-$\vee^p$-prognoser defined by Eqs. (6,7) and satisfying Eqs. (1,2), if (Υ, ∂) is $\wedge$-$\vee^p$-COPROG w.r.t $\mathcal{A}_\Upsilon$.

If in Prop. 4.1, we add Eq. (3) and replace Eq. (7) by Eq. (9), we obtain the following existence theorem:

Theorem 4.1: Consider a pair $(\mathcal{L}, \mathcal{H})$ of prefix-closed languages with $\mathcal{H} \subseteq \mathcal{L}$ and their corresponding (Υ, ∂), $p \in \mathbf{N}^+$ and an FSA $\mathcal{A}_\Upsilon$ accepting Υ. There exists a $\wedge$-$\vee^p$-prognoser defined by Eqs. (6,9) and satisfying Eqs. (1,2,3), if (Υ, ∂) is $\wedge$-$\vee^p$-COPROG w.r.t $\mathcal{A}_\Upsilon$.

The following theorem implies that it is not restrictive to consider as solution uniquely the $\wedge$-$\vee^p$-prognoser defined by Eqs. (6,8,9), instead of any $\wedge$-$\vee^p$-prognoser defined by Eqs. (6,9).

Theorem 4.2: Consider a pair $(\mathcal{L}, \mathcal{H})$ of prefix-closed languages with $\mathcal{H} \subseteq \mathcal{L}$ and their corresponding (Υ, ∂), $p \in \mathbf{N}^+$, and an FSA $\mathcal{A}_\Upsilon$ accepting Υ. Consider also any decomposition $(\Upsilon^j)_{1 \leq j \leq p}$ of Υ satisfying Eq. (10) and Requirement **R1** w.r.t $\mathcal{A}_\Upsilon$.[1] The corresponding $\wedge$-$\vee^p$-prognoser defined by Eqs. (6,8,9) satisfies Eqs. (1,2,3), if (Υ, ∂) is $\wedge$-$\vee^p$-COPROG w.r.t $\mathcal{A}_\Upsilon$.

Remark 4.1: In Prop. 4.1 and Theorems 4.1 and 4.2, the fact that (Υ, ∂) is $\wedge$-$\vee^p$-COPROG w.r.t $\mathcal{A}_\Upsilon$ is a sufficient condition which may be unnecessary. We obtain a necessary and sufficient condition if: 1) we do not consider Eq. (3) and replace Eq. (9) by (7); and 2) we remove Requirement **R1**, i.e. all decompositions of Υ are eligible. But recall that **R1** is useful because it permits to transform the difficult problem of decomposing the infinite language Υ into the easy problem of decomposing a finite state set.

Let us return to the example of Fig. 2, where $\partial = \sigma^*(a_1 + a_2)\sigma\sigma\sigma$ and $\Upsilon = \sigma^* + \sigma^* a_1 + \sigma^* a_1 a_2 \sigma^* + \sigma^* a_2 + \sigma^* a_2 a_1 \sigma^*$. In Section IV-B, we have noted that without decomposition of Υ, the $\wedge$-$\vee^1$-prognoser defined by Eqs. (6,8,9) does not satisfy Eqs. (1,2). From Theorem 4.2, we should have that (Υ, ∂) is not $\wedge$-$\vee^1$-COPROG, i.e. $\bigcap_{i=1,2}[P_i^{-1} P_i(\Upsilon)] \cap \partial \neq \emptyset$. We have effectively computed $\bigcap_{i=1,2}[P_i^{-1} P_i(\Upsilon)] \cap \partial = \partial \neq \emptyset$.

Consider now the decomposition $\Upsilon^1 = \sigma^*(a_1 a_2 + a_2 a_1)(\sigma + \sigma\sigma\sigma\sigma^*$ and $\Upsilon^2 = \sigma^*\overline{(a_1 a_2 + a_2 a_1)} + \sigma^*(a_1 a_2 + a_2 a_1)\sigma\sigma$. We compute: $\bigcap_{i=1,2}[P_i^{-1} P_i(\Upsilon^1)] \cap$

[1]Note that there exists at least one such decomposition, by definition of $\wedge$-$\vee^p$-COPROG w.r.t $\mathcal{A}_\Upsilon$.

$\partial = \bigcap_{i=1,2}[P_i^{-1}P_i(\Upsilon^2)] \cap \partial = \emptyset$. That is, (Υ, ∂) is $\wedge$-$\vee^2$-COPROG w.r.t $\mathcal{A}_\Upsilon$ (Def. 4.1). Therefore, from Theor. 4.2, the $\wedge$-$\vee^2$-prognoser defined by Eqs. (6,8,9) satisfies Eqs. (1,2,3). This is confirmed in Table I which outlines the prognoses of such a prognoser.

V. $\vee$-$\wedge^p$-ARCHITECTURE: GENERALIZATION OF THE CONJUNCTIVE ARCHITECTURE

In Sect. IV, we proposed a $\wedge$-$\vee^p$-architecture that generalizes the disjunctive architecture of [4], [3], [5]. Let us now propose a $\vee$-$\wedge^p$-architecture that generalizes the conjunctive architecture of [4], [5]. The two architectures of Sects. IV-V are dual, in the sense that one is obtained from the other essentially by switching: 1) between $\vee$ and $\wedge$ operators, and 2) between decomposing ∂ and decomposing Υ.

A. $\vee$-$\wedge^p$-prognoser

We take $\mathrm{F} = \vee$ and $\mathrm{F}^j = \wedge, \forall j \in \{1, \cdots, p\}$, i.e. we have a $\vee$-$\wedge^p$-prognoser. In other words, p decentralized conjunctives prognosers are running in parallel, and their global prognoses are fused disjunctively to synthesize the effective prognosis. The notation $\vee$-$\wedge^{\geq 1}$-prognoser denotes any $\vee$-$\wedge^p$-prognoser, for some unspecified $p \in \mathbf{N}^+$.

The global and effective prognoses are computed by the following Eqs. (11,12) that specialize Eqs. (4,5) for $\mathrm{F} = \vee$ and $\mathrm{F}^j = \wedge, \forall j \in \{1, \cdots, p\}$.

$$\forall j \in \{1, \cdots, p\} : \mathrm{X}^j(\lambda) = \bigwedge_{i=1\cdots n} \mathrm{X}_i^j(P_i(\lambda)) \tag{11}$$

$$\mathrm{X}(\lambda) = \bigvee_{j=1\cdots p} \mathrm{X}^j(\lambda) \tag{12}$$

B. Decomposing ∂ to compute the local prognoses

Using the same approach as in Section IV-B, we use a decomposition $(\partial^j)_{j=1\cdots p}$ of ∂, i.e. $\partial = \partial^1 \cup \cdots \cup \partial^p$. The local prognoses $\mathrm{X}_i^j(P_i(\lambda))$ are computed from this decomposition as follows:

$$\forall j \in \{1, \cdots, p\}, \forall i \in \{1, \cdots, n\} : \quad \mathrm{X}_i^j(P_i(\lambda)) = \begin{cases} 1, & \text{if } P_i(\lambda) \in P_i(\partial^j) \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

The idea behind this computation is that Eqs. (11,13) guarantee that the global prognosis X^j of each conjunctive prognoser Prog^j satisfies Eqs. (1,2), but w.r.t (∂^j, Υ) instead of w.r.t (∂, Υ). And by fusing disjunctively the global prognoses $(\mathrm{X}^j)_{j=1\cdots p}$ (using Eq. (12)), we obtain an effective prognosis that satisfies Eqs. (1,2) w.r.t (∂, Υ).

By using the same reasoning as in Section IV-B, we guarantee Eq. (3) by replacing Eq. (12) by the following equation:

$$\mathrm{X}(\lambda) = \mathrm{X}_c \vee (\bigvee_{j=1\cdots p} \mathrm{X}^j(\lambda)) \tag{14}$$

Consider the example of Fig. 3, where $\Sigma_{uo} = \{f\}$, $\Sigma_{o,1} = \{a_1, \sigma\}$, $\Sigma_{o,2} = \{a_2, \sigma\}$. $\mathcal{F} = a_1(\sigma + \sigma\sigma\sigma)fa_1^* + a_2(\sigma + \sigma\sigma\sigma)fa_2^*$, and $\mathcal{H} = (a_1 + a_2)\sigma\overline{\sigma\sigma} + \sigma^*\overline{(a_1a_2 + a_2a_1)\sigma^*}$. $\Upsilon = \sigma^* + \overline{(a_1a_2 + a_2a_1)\sigma^*}$ is accepted by the FSA $\mathcal{B}_\Upsilon$ obtained from Fig. 3 by marking the states 1, 2, 3 and 4, and removing the other states. $\partial = (a_1+a_2)(\sigma+\sigma\sigma\sigma)$ is accepted by the FSA $\mathcal{B}_\partial$ obtained from Fig. 3 by removing states 4, 8, 9, 13, 14, and marking states 5, 7, 10, 12. Consider first the case where ∂ is not decomposed, i.e. $p = 1$ and $\partial^1 = \partial$. The $\vee$-$\wedge^1$-prognoser defined by Eqs. (11,13,14) corresponds to the conjunctive architecture of [4], [5]. We have checked that it does not satisfy Eqs. (1,2).

Let us consider the following decomposition of ∂: $\partial^1 = a_1(\sigma + \sigma\sigma\sigma)$ corresponding to states 5 and 7, and $\partial^2 = a_2(\sigma + \sigma\sigma\sigma)$ corresponding to states 10 and 12. Table II outlines $\mathrm{X}_i^j(P_i(\lambda))$, $\mathrm{X}^j(\lambda)$ and $\mathrm{X}(\lambda)$ of the $\vee$-$\wedge^2$-prognoser computed by Eqs. (11,13,14) for traces $\lambda \in \mathcal{H}$. Recall that a trace λ is said symmetrical to a trace μ, if each of them is obtained from the other by switching a_1 and a_2. Since the example is symmetrical, we do not list all the traces of $\mathcal{H}$. The missing traces are symmetrical to the traces listed in lines 6-14. For the trace $a_1\sigma\sigma$ (line 8), in the last column we indicate the two prognoses obtained with Eqs. (12) and (14) respectively. For the other traces, the two equations generate the same prognosis.

We see that "$\mathrm{X}(\lambda) = 1$" in lines 7 and 9, and by symmetry we have the same prognosis for the symmetrical traces. Hence, "$\mathrm{X}(\lambda) = 1$" for all traces of ∂, i.e. Eq. (1) is satisfied. We see that "$\mathrm{X}(\lambda) = 0$" is issued in lines 1-6 and 10-14, and by symmetry we have the same prognosis for the symmetrical traces. Hence, "$\mathrm{X}(\lambda) = 0$" for all traces of Υ, i.e. Eq. (2) is satisfied. The other traces (line 8 and its symmetrical) belong to $\mathcal{H} \setminus (\Upsilon \cup \partial)$. In lines 7-8, we see that with Eq. (12), the prognoser changes its mind since its decision passes from 1 to 0. With Eq. (14), the decision remains 1, hence Eq. (3) is satisfied. By this example, we have shown the necessity of using Eq. (14) in place of Eq. (12).

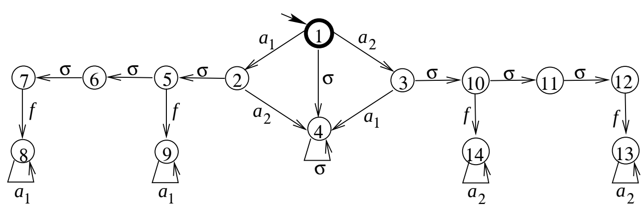

Fig. 3. Example for illustrating the $\vee - \wedge^2$ prognosis

C. Existence Results

Using the same approach as for the $\wedge$-$\vee^p$-architecture in Sect. IV, we consider the following requirement **R2**:

Requirement R2: Given an FSA $\mathcal{A}_\partial$ accepting ∂, the only eligible decompositions $(\partial^j)_{j=1\cdots}$ of ∂ are such that every ∂^j consists of the traces leading to one or several marked states of $\mathcal{A}_\partial$. In other words, every ∂^j corresponds to a subset of the set of marked states of $\mathcal{A}_\partial$.

Let us return to the example of Fig 3, we have seen that we obtain an FSA $\mathcal{B}_\partial$ accepting ∂ from the FSA of Fig 3 by removing states 4, 8, 9, 13, 14 and marking states 5, 7, 10, 12. Decomposing ∂ amounts therefore to decomposing

λ	$P_1(\lambda)$	$P_2(\lambda)$	$\mathrm{X}_1^1(P_1(\lambda))$	$\mathrm{X}_2^1(P_2(\lambda))$	$\mathrm{X}^1(\lambda)$	$\mathrm{X}_1^2(P_1(\lambda))$	$\mathrm{X}_2^2(P_2(\lambda))$	$\mathrm{X}^2(\lambda)$	$\mathrm{X}(\lambda)$
ε	ε	ε	0	0	0	0	0	0	0
σ	σ	σ	0	1	0	1	0	0	0
$\sigma\sigma$	$\sigma\sigma$	$\sigma\sigma$	0	0	0	0	0	0	0
$\sigma\sigma\sigma$	$\sigma\sigma\sigma$	$\sigma\sigma\sigma$	0	1	0	1	0	0	0
$\sigma\sigma\sigma\sigma\sigma^*$	$\sigma\sigma\sigma\sigma\sigma^*$	$\sigma\sigma\sigma\sigma\sigma^*$	0	0	0	0	0	0	0
a_1	a_1	ε	0	0	0	0	0	0	0
$a_1\sigma$	$a_1\sigma$	σ	1	1	1	0	0	0	1
$a_1\sigma\sigma$	$a_1\sigma\sigma$	$\sigma\sigma$	0	0	0	0	0	0	0, 1
$a_1\sigma\sigma\sigma$	$a_1\sigma\sigma\sigma$	$\sigma\sigma\sigma$	1	1	1	0	0	0	1
a_1a_2	a_1	a_2	0	0	0	0	0	0	0
$a_1a_2\sigma$	$a_1\sigma$	$a_2\sigma$	1	0	0	0	1	0	0
$a_1a_2\sigma\sigma$	$a_1\sigma\sigma$	$a_2\sigma\sigma$	0	0	0	0	0	0	0
$a_1a_2\sigma\sigma\sigma$	$a_1\sigma\sigma\sigma$	$a_2\sigma\sigma\sigma$	1	0	0	0	1	0	0
$a_1a_2\sigma\sigma\sigma\sigma\sigma^*$	$a_1\sigma\sigma\sigma\sigma\sigma^*$	$a_2\sigma\sigma\sigma\sigma\sigma^*$	0	0	0	0	0	0	0

TABLE II

$\vee$-$\wedge^2$ PROGNOSIS RESULTS FOR THE EXAMPLE OF FIG. 3

the finite set of marked states $\{5, 7, 10, 12\}$. The number of decompositions is therefore finite. In Section V-B, we have used the following decomposition: $\partial^1 = a_1(\sigma + \sigma\sigma\sigma)$ corresponding to states 5 and 7, and $\partial^2 = a_2(\sigma + \sigma\sigma\sigma)$ corresponding to states 10 and 12.

We use the following definition:

Definition 5.1: Consider a pair $(\mathcal{L}, \mathcal{H})$ of prefix-closed languages with $\mathcal{H} \subseteq \mathcal{L}$, their corresponding (Υ, ∂), $p \in \mathbf{N}^+$, and an FSA $\mathcal{A}_\partial$ accepting ∂. (Υ, ∂) is said $\vee$-$\wedge^p$-COPROG w.r.t $\mathcal{A}_\partial$ if there exists a decomposition $(\partial^j)_{j=1\cdots p}$ of ∂ satisfying **R2** w.r.t $\mathcal{A}_\partial$ and such that:

$$\forall j = 1, \cdots, p : \bigcap_{i=1,\cdots n} [P_i^{-1} P_i(\partial^j)] \cap \Upsilon = \emptyset \qquad (15)$$

And (Υ, ∂) is said $\vee$-$\wedge^{\geq 1}$-COPROG if there exists $p \in \mathbf{N}^+$ for which (Υ, ∂) is $\vee$-$\wedge^p$-COPROG.

By analogy with the $\wedge$-$\vee^p$-architecture of Sect. IV, we obtain results similar to Prop. 4.1, Theorems 4.1-4.2 and Remark 4.1. For space limit, we present uniquely the result similar to Theorem 4.2, which is the most useful in practice:

Theorem 5.1: Consider a pair (Υ, ∂) of prefix-closed languages with $\mathcal{H} \subseteq \mathcal{L}$ and their corresponding (Υ, ∂), $p \in \mathbf{N}^+$, and an FSA $\mathcal{A}_\partial$ accepting ∂. Consider also any decomposition $(\partial^j)_{1\leq j\leq p}$ of ∂ satisfying Eq. (15) and requirement **R2** w.r.t $\mathcal{A}_\partial$.[2] The corresponding $\vee$-$\wedge^p$-prognoser defined by Eqs. (11,13,14) satisfies Eqs. (1,2,3), if (Υ, ∂) is $\vee$-$\wedge^p$-COPROG w.r.t $\mathcal{A}_\partial$.

Let us return to the example of Fig. 3, where $\Upsilon = \sigma^* + (a_1a_2 + a_2a_1)\sigma^*$, and $\partial = (a_1 + a_2)(\sigma + \sigma\sigma\sigma)$. In Section V-B, we have noted that without decomposition of ∂, the $\vee$-$\wedge^1$-prognoser defined by Eqs. (11,13,14) does not satisfy Eqs. (1,2). From Theorem 5.1, we should have that (Υ, ∂) is not $\vee$-$\wedge^1$-COPROG, i.e. $\bigcap_{i=1,2}[P_i^{-1}P_i(\partial)] \cap \Upsilon \neq \emptyset$. We have effectively computed $\bigcap_{i=1,2}[P_i^{-1}P_i(\partial)] \cap \Upsilon \neq \emptyset$.

Consider now the decomposition $\partial^1 = a_1(\sigma + \sigma\sigma\sigma)$ and $\partial^2 = a_2(\sigma + \sigma\sigma\sigma)$. We compute: $\bigcap_{i=1,2}[P_i^{-1}P_i(\partial^1)] \cap \Upsilon = \bigcap_{i=1,2}[P_i^{-1}P_i(\partial^2)] \cap \Upsilon = \emptyset$. That is, (Υ, ∂) is $\vee$-$\wedge^2$-COPROG w.r.t $\mathcal{B}_\partial$ (Def. 5.1). Therefore, from Theor. 5.1, the $\wedge$-$\vee^2$-prognoser defined by Eqs. (11,13,14) satisfies Eqs. (1,2,3). This is confirmed in Table II which outlines the prognoses of such a prognoser.

[2]Note that there exists at least one such decomposition, by definition of $\vee$-$\wedge^p$-COPROG w.r.t $\mathcal{A}_\partial$.

VI. CONCLUSION

We have analyzed and improved a framework called multi-decision prognosis previously developed in [4]. Compared to [4], our main contributions are:

- To help understanding of multi-decision prognosis, we have first presented it in a generic form which is independent on the architectures in parallel and on how their decisions a combined.
- We have solved more rigorously than in [4], a problem of decomposing infinite languages which arises in multi-decision prognosis.
- We have identified and solved a "hesitation" problem, where the prognosis may oscillate between 1 and 0.

For future work, we plan to develop an efficient method to verify coprognosability associated to the two multi-decision architectures studied in this work. We also plan to apply the multi-decision framework to the inference-based architectures of [6].

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