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Output Harmonic Disturbance Compensation for Nonlinear Plant

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Abstract – **The paper deals with the problem of compensation of unmeasured output biased harmonic disturbance for nonlinear plant. We propose a frequency identifier for unmeasured disturbance $\bar{y}(t)=\bar{\sigma}_0+\bar{\sigma}\sin(\bar{\omega}t+\bar{\varphi})$. Using this identifier the disturbance compensation algorithm was designed. This algorithm is based on observing the extended plant model and designing a control signal for the disturbance compensation by measured states of the designed observer. It is demonstrated that the proposed algorithm is applicable under variable disturbance frequency.**

I. Introduction

THE problem of adaptive compensation of sinusoidal disturbances is a point of interest discussed in number of papers. If the frequencies are known it can be solved according to the internal model principle [1]. If a frequency is (or frequencies are) unknown, one may try different approaches proposed during last decades. Most of them concern the case of linear plant (stable or minimum phase) [2–5]. As far as nonlinear systems are concerned, several results are available under the minimum phase assumption (MP), namely: semiglobal output regulation problem is studied in [6] for the systems with unknown parameters in the exosystem under known restrictions on the frequencies and the disturbances; global robust state feedback control scheme is presented in [7] for the systems affected by both unknown structured disturbances and noise. Global regulation algorithm for a non-MP nonlinear systems is presented in [8] assuming that the number of frequencies disturbing the system is known. [9] presents a solution to the problem of designing an output feedback compensator for the class of non-MP nonlinear systems assuming that only an upper bound of the number of sinusoidal disturbances is known. As [9] relates to this paper let us consider its results.

The following class of nonlinear systems is considered:

$$\begin{cases}\dot{x}=\Phi(y)+A_n x+bu+D\omega\\ y=C_n x\end{cases},\qquad \dot{\omega}=R\omega$$

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where $x\in R^n$ is a state, $u\in R$ is a control input, $\omega\in R^{2m+1}$ is an exosystem state, $y\in R$ is a measurable output, $b=\left[-b_{n-1}\ \ldots\ \text{-b}_0\right]^T$ is known constant vector, $\Phi(\cdot)$ is known smooth vector function of its argument, $A_n\in R^n\times R^n$, $A_n=\begin{bmatrix}0 & I_{n-1}\\ 0 & 0\end{bmatrix}$ and $C_n\in R^n$, $C_n=\begin{bmatrix}1 & 0 & \cdots & 0\end{bmatrix}_{1\times n}$. The disturbance $D\omega$ to be compensated is generated by the unknown exosystem $\dot{\omega}=R\omega$ that has $2m+1$ different eigenvalues on the imaginary axis (i.e. $\omega(t)$ is a biased sum of m sinusoidal parts with different frequencies). Also for undisturbed system ($\omega=0$) there exists an output feedback controller, such that the closed loop system is exponentially stable. Purpose of the paper is to design the disturbance compensation algorithm. This algorithm is based on the frequency identification. Only the upper bound M of parameter m is known, but not the parameter itself, so an algorithm of detecting the number of frequencies is proposed before. When this number is known authors design the compensation algorithm of dynamic order $3n+6m+2$. Main disadvantage of this algorithm is its complexity caused by using complex frequency identification algorithm. One can find many different papers dedicated to the problem of frequency identification. In [10–16] the minimal dimension of dynamic order of the algorithm is 4, and in [4] dimension of the algorithm is 9. In [17] new algorithm of frequency identification of the measured biased harmonic signal with dynamic order of 3 was proposed.

This paper expands the result of [18] to unmeasured biased harmonic output disturbance in nonlinear system. This paper considers the case if disturbance has only one frequency (i.e. $m=1$), but it can be easily extended to the case of multi-harmonic disturbance [19]. The algorithm proposed in this paper has dynamic order $n+5m$.

This paper consists of two parts. At first one we identify frequency of unmeasured disturbance using input and output signals. An example of frequency identification for the disturbance affecting a magnetic spring based drive illustrates transients in the proposed algorithm. Also it is demonstrated that proposed method is applicable to the case of variable frequency with known limits. In the second part we introduce an extended plant model that includes the internal disturbance model. Using the result of identification we can design observer for this extended model and then design a control signal for compensation. Second part of the example for the same plant illustrates the disturbance compensation.

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I. PROBLEM STATEMENT AND DISTURBANCE FREQUENCY IDENTIFICATION

Consider nonlinear system of the form (see Figure 1)

$$\dot{x}(t) = Ax(t) + Bu(t) + Df(g(t)), \quad g(t) = Cx(t) + \bar{y}(t) \tag{1}$$

or

$$g(t) = \frac{b(p)}{a(p)}u(t) + \frac{d(p)}{a(p)}f(g(t)) + \bar{y}(t) \tag{1b}$$

where vector $x \in R^n$ is unmeasured; A, B, C and D are known matrices; p is differentiation operator; $g(t)$ is the output; $u(t)$ is control; $f(g(t))$ is nonlinearity, $f(g)$ is a smooth function and there exists a Laplace transformation $F(s) = L\{f(g(t))\}$, $a(p)$ is polynomial (possibly unstable), $\deg a(p) = n$; degrees of polynomials $b(p)$ and $d(p)$ are less than n; coefficients of polynomials $a(p)$, $b(p)$ and $d(p)$ are known; $g(t)$, $u(t)$ and $f(g)$ are known; $\bar{y}(t) = \bar{\sigma}_0 + \bar{\sigma}\sin(\bar{\omega}t + \bar{\varphi})$ is the output disturbance with unknown bias $\bar{\sigma}_0$ amplitude $\bar{\sigma}$, frequency $\bar{\omega}$ and phase $\bar{\varphi}$. Also polynomial $a(p)$ does not have pure imaginary roots $\pm j\bar{\omega}$.

Let us state the purpose of this part as design of identification algorithm satisfying the condition

$$\lim_{t\to\infty}\left|\bar{\omega} - \hat{\bar{\omega}}(t)\right| = 0 \tag{2}$$

where $\hat{\bar{\omega}}(t)$ is a current estimation of parameter $\bar{\omega}$ for any $\bar{\sigma}_0$, $\bar{\sigma}$, $\bar{\varphi}$ and $\bar{\omega} > 0$.

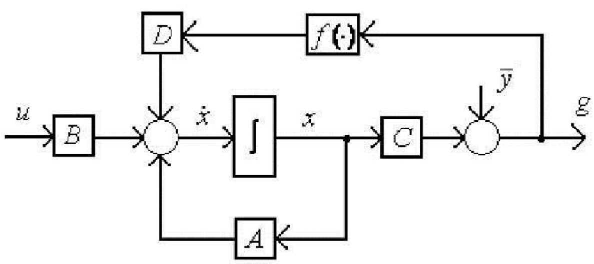

Fig. 1. Structural scheme of the plant

Passing to Laplace images in (1) we obtain

$$G(s) = \frac{b(s)}{a(s)}U(s) + \frac{d(s)}{a(s)}F(s) + Y(s) + \frac{H(s)}{a(s)} \tag{3}$$

where s is a complex variable, $G(s) = L\{g(t)\}$, $U(s) = L\{u(t)\}$, and $Y(s) = L\{\bar{y}(t)\}$ are Laplace images of functions $g(t)$, $u(t)$, $f(g(t))$, $\bar{y}(t)$ respectively, polynomial $H(s)$ stands for the sum of all terms containing nonzero initial conditions. Let us transform model (3) in the following way

$$(s+\lambda)^n G(s) = a_1(s)G(s) + b(s)U(s) + d(s)F(s) + a(s)Y(s) + H(s) \tag{4}$$

whence

$$G(s) = \frac{a_1(s)}{(s+\lambda)^n}G(s) + \frac{b(s)}{(s+\lambda)^n}U(s) + \frac{d(s)}{(s+\lambda)^n}F(s) + \frac{a(s)}{(s+\lambda)^n}Y(s) + \frac{H(s)}{(s+\lambda)^n} \tag{5}$$

where parameter $\lambda > 0$ and $a(s) = (s+\lambda)^n - a_1(s)$. After inverting Laplace transform (5) takes the form

$$g(t) = \frac{a_1(p)}{(p+\lambda)^n}g(t) + \frac{b(p)}{(p+\lambda)^n}u(t) + \frac{d(p)}{(p+\lambda)^n}f(g(t)) + \frac{a(p)}{(p+\lambda)^n}\bar{y}(t) + \varepsilon_y(t) \tag{6}$$

where $\varepsilon_y(t) = L^{-1}\left\{\frac{H(s)}{(s+\lambda)^n}\right\}$ is exponentially decaying function of time caused by nonzero initial conditions, and it is possible to accelerate its convergence to zero by increasing parameter λ.

Let us neglect the exponentially decaying term $\varepsilon_y(t)$ and parameterize model (6). Consider auxiliary filters of the following form

$$\upsilon_1(t) = \frac{1}{(p+\lambda)^n}g(t), \tag{7}$$

$$\upsilon_2(t) = \frac{1}{(p+\lambda)^n}u(t), \tag{8}$$

$$\upsilon_3(t) = \frac{1}{(p+\lambda)^n}f(g(t)). \tag{9}$$

Substituting (7)-(9) into (6), obtain

$$g(t) = a_1(p)\upsilon_1(t) + b(p)\upsilon_2(t) + d(p)\upsilon_3(t) + y(t) \tag{10}$$

where $y(t) = \frac{a(p)}{(p+\lambda)^n}\bar{y}(t)$. For (10) we have

$$y(t) = g(t) - \mu(t) \tag{11}$$

where $\mu(t) = a_1(p)\upsilon_1(t) + b(p)\upsilon_2(t) + d(p)\upsilon_3(t)$.

Consider signal $y(t) = a(p)(p+\lambda)^{-n}\bar{y}(t)$. As polynomial $(p+\lambda)^n$ is Hurwitz and $\bar{y}(t) = \bar{\sigma}_0 + \bar{\sigma}\sin(\bar{\omega}t + \bar{\varphi})$, the signal $y(t)$ can be rewritten in the following form: $y(t) = \sigma_0 + \sigma\sin(\omega t + \varphi)$ and $\omega = \bar{\omega}$. So, the problem of frequency identification of the unmeasured signal $\bar{y}(t)$ can be reduced to the problem of frequency identification of measured biased harmonic signal $y(t)$ and new term to attain is

$$\lim_{t\to\infty}\left|\omega - \hat{\omega}(t)\right| = 0 \tag{12}$$

where $\hat{\omega}(t)$ is a current estimate of parameter ω for any $\bar{\sigma}_0$, σ, φ and $\omega > 0$.

It is known that for generating the signal (11) it is possible to use differential equation of form (13) (see [12], [14])

$$\dddot{y}(t) = -\omega^2\dot{y}(t) = \theta\dot{y}(t) \tag{13}$$

where $\theta = -\omega^2$ is a constant parameter.

Lemma. Consider second-order filter

$$\begin{cases}\dot{\varsigma}_1(t) = \varsigma_2(t),\\ \dot{\varsigma}_2(t) = -2\alpha\varsigma_2(t) - \alpha^2\varsigma_1(t) + y(t),\\ \varsigma(t) = \varsigma_1(t)\end{cases} \tag{14}$$

or

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$$\varsigma(t)=\frac{1}{(p+\alpha)^2}y(t) \tag{14b}$$

where p is the differentiation operator and number $\alpha>0$. Then differential equation (13) can be rewritten in the form

$$\dot{y}(t)=2\alpha\ddot{\varsigma}(t)+\alpha^2\dot{\varsigma}(t)+\theta\dot{\varsigma}(t)+\bar{\varepsilon}_y(t) \tag{15}$$

where $\bar{\varepsilon}_y(t)$ is an exponentially decaying function of time caused by nonzero initial conditions. Proof of this lemma can be found in appendix A.

Remark 1. Exponentially decaying function $\bar{\varepsilon}_y(t)$ depends on the parameter α, so it is possible to accelerate convergence of $\bar{\varepsilon}_y(t)$ to zero by increasing α.

Now, on the base of the lemma results one can state a scheme of unknown parameter θ identification. At first, let us assume that function $\dot{y}(t)$ is measured. Then neglect the exponentially decaying item $\bar{\varepsilon}_y(t)$, and write an ideal identification algorithm in the following form

$$\begin{aligned}&\dot{\hat{\theta}}(t)=k\dot{\varsigma}^2(t)(\theta-\hat{\theta}(t))=k\dot{\varsigma}(t)z(t)-k\dot{\varsigma}^2(t)\hat{\theta}(t)\\&\hat{\omega}(t)=\sqrt{\left|\hat{\theta}(t)\right|}\end{aligned} \tag{16}$$

where function $z(t)=\dot{y}(t)-2\alpha\ddot{\varsigma}(t)-\alpha^2\dot{\varsigma}(t)$ and number $k>0$. The following statement proves efficiency of ideal identification algorithm for satisfying (2).

Proposition. Let the algorithm of identification of unknown parameter θ have the form

$$\dot{\hat{\theta}}(t)=k\dot{\varsigma}^2(t)(\theta-\hat{\theta}(t)),$$

where number $k>0$, and function $\varsigma(t)$ is a solution of differential equation (14). Then condition (12) is satisfied. Proof of the proposition can be found in appendix B.

Remark 2. Function $\hat{\theta}(t)$ converges faster to parameter θ if increase coefficient *k*. Respectively, it is possible to adjust the convergence rate of the tuned parameter to its real value in identification algorithm (16) with changing coefficient *k*.

However, here only the signal *y*(*t*) is measured but not its derivatives. To derive applicable scheme of identification algorithm let us consider the following variable:

$$\chi(t)=\hat{\theta}(t)-k\dot{\varsigma}(t)y(t) \tag{17}$$

Differentiating (17), obtain

$$\begin{aligned}\dot{\chi}(t)&=\dot{\hat{\theta}}(t)-k\ddot{\varsigma}(t)y(t)-k\dot{\varsigma}(t)\dot{y}(t)=\\&=k\dot{\varsigma}(t)(\dot{y}(t)-2\alpha\ddot{\varsigma}(t)-\alpha^2\dot{\varsigma}(t))-\\&-k\dot{\varsigma}^2(t)\hat{\theta}(t)-k\ddot{\varsigma}(t)y(t)-k\dot{\varsigma}(t)\dot{y}(t)=\\&=k\dot{\varsigma}(t)(-2\alpha\ddot{\varsigma}(t)-\alpha^2\dot{\varsigma}(t))-k\dot{\varsigma}^2(t)\hat{\theta}(t)-\\&-k\ddot{\varsigma}(t)y(t)\end{aligned} \tag{18}$$

From equations (17), (18) derive applicable identification algorithm of the following form

$$\begin{aligned}&\dot{\chi}(t)=k\dot{\varsigma}(t)(-2\alpha\ddot{\varsigma}(t)-\alpha^2\dot{\varsigma}(t))-\\&-k\dot{\varsigma}^2(t)\hat{\theta}(t)-k\ddot{\varsigma}(t)y(t)\\&\hat{\theta}(t)=\chi(t)+k\dot{\varsigma}(t)y(t)\\&\hat{\omega}(t)=\sqrt{\left|\hat{\theta}(t)\right|}\end{aligned} \tag{19}$$

$$\begin{cases}\dot{\varsigma}_1(t)=\varsigma_2(t),\\ \dot{\varsigma}_2(t)=-2\alpha\varsigma_2(t)-\alpha^2\varsigma_1(t)+y(t),\\ \varsigma(t)=\varsigma_1(t).\end{cases} \tag{20}$$

II. EXAMPLE PART 1

Let us consider the plant (see Figure 2)

$$g(t)=\frac{10}{p^2+p}u(t)+\frac{-2.5}{p^2+p}500\sin(g(t))+\bar{y}(t), \tag{21}$$

where $\bar{y}(t)=1+3\sin(5\cdot 2\pi t)$. This structure is typical for magnetic spring based drives or if the gravitation force exerts robotic arm. If there is no disturbance ($\bar{y}(t)=0$) one can control this plant using simple PID controller

$$u_1(t)=\frac{2.9p^2+30p+100}{p(0.001p+1)}. \tag{22}$$

But if disturbance is present, it may cause oscillations in the output value, and to compensate it one has to identify frequency of this disturbance. Figure 3 shows step response for the plant (21) with control (22) with $\bar{y}(t)=0$ (dashed line) and $\bar{y}(t)=1+3\sin(5\cdot 2\pi t)$ (solid line). Step size is 45.

At first apply auxiliary filters (7)-(9) and measured harmonic signal of form (11). For auxiliary filters we choose $\lambda=30$. Then use (11) and (19), (20) for frequency identification. Figure 4 shows the result of the identification $\hat{\omega}(t)$. Oscillation at first seconds can be eliminated if identification starts after the start of auxiliary filters (when transient processes in auxiliary filters are complete).

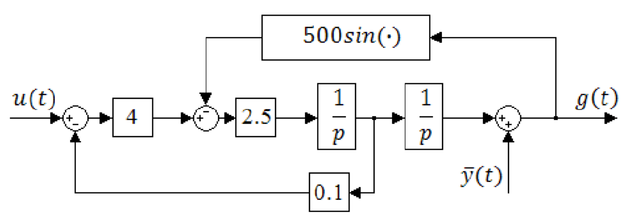

Fig. 2. Structural scheme of plant (21).

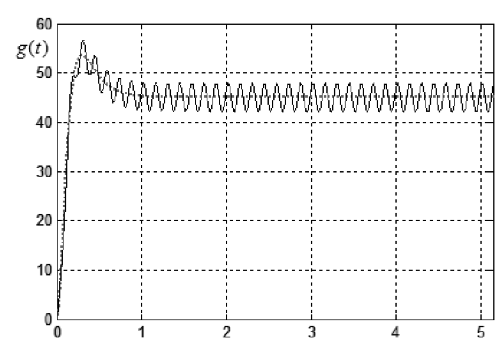

Fig. 3. Step response for plant (21) with control (22).

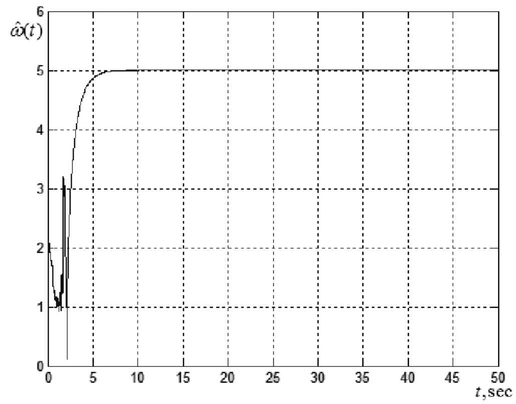

Fig. 4. Plot of $\hat{\omega}(t)$ for $\overline{y}(t)=1+3\sin(5\cdot 2\pi t)$.

In some systems the disturbance frequency may vary during system operation. In this case let us demonstrate that the frequency identifier proposed in this work also identifies frequency if it varies between two constant values. Figure 5 shows frequency $\omega(t)$ that changes from one value to another (dashed line) and $\hat{\omega}(t)$ (solid line).

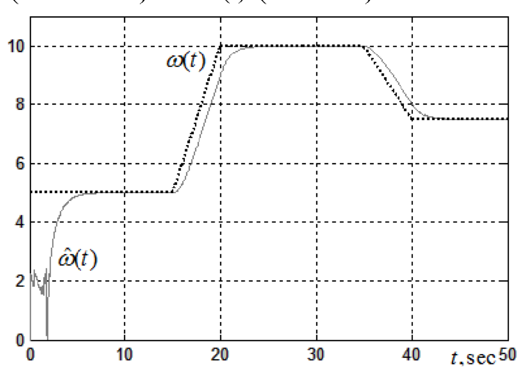

Fig. 5. Plot of $\hat{\omega}(t)$ for inconstant frequency.

III. COMPENSATION OF THE DISTURBANCE

Typically the constant component of disturbance can be easily compensated by classical regulators, and this problem is not of interest. In this section we consider the problem of compensation of the harmonic disturbance component. Let us rewrite signal $\overline{y}(t)$ (1) in the form:

$$\overline{y}(t)=\overline{\sigma}_0+y_2(t), \tag{23}$$

$$y_2(t)=\overline{\sigma}\sin(\overline{\omega}t+\overline{\varphi}). \tag{24}$$

Let us solve the problem for compensation signal $y_2(t)$.

From (13) derive the disturbance model in state-space form with nonzero initial values:

$$\begin{aligned}&\dot{\psi}=A_\psi\psi(t), y_2(t)=C_\psi\psi, A_\psi=\begin{bmatrix}0 & 1\\ \theta_{fix} & 0\end{bmatrix},\\ &C_\psi=\begin{bmatrix}1 & 0\end{bmatrix}, \psi(0)=\begin{bmatrix}\overline{\sigma}\sin(\overline{\varphi})\\ \overline{\omega}\overline{\sigma}\cos(\overline{\varphi})\end{bmatrix}.\end{aligned} \tag{25}$$

Remark 3. In (19) $\hat{\theta}(t)$ is variable and for obtaining time invariant model (25) we have to fix signal $\hat{\theta}(t)$ as constant value $\theta_{fix}=\hat{\theta}(t_{fix})$. The value t_{fix} can be defined as time when the signal $\hat{\theta}(t)$ is in interval $\left[\theta_{fix}-\Delta\theta;\theta_{fix}+\Delta\theta\right]$ over time Δt, where $\Delta\theta>0, \Delta t>0$ are arbitrary values. If the disturbance frequency varies between two constant values, changing the signal $\hat{\theta}(t)$ indicates change of the disturbance frequency. In this case compensation stops until the transients in the identifier are complete. Then a new value θ_{fix} should be set to resume the disturbance compensation. The value θ_{fix} can be obtained in some other ways.

Using disturbance model we can write the extended form of (1):

$$\begin{aligned}&\dot{x}_{ex}(t)=A_{ex}x_{ex}(t)+B_{ex}u(t)+D_{ex}f(g(t)),\\ &g(t)=C_{ex}x_{ex},\end{aligned} \tag{26}$$

where $x_{ex}(t)=\begin{bmatrix}x(t)\\ \psi(t)\end{bmatrix}$, $A_{ex}=\begin{bmatrix}A & 0\\ 0 & A_\psi\end{bmatrix}$, $B_{ex}=\begin{bmatrix}B\\ 0\end{bmatrix}$, $C_{ex}=\begin{bmatrix}C & C_\psi\end{bmatrix}$, $D_{ex}=\begin{bmatrix}D\\ 0\end{bmatrix}$. As all the parameters are known we can design observer for (26) in the form

$$\begin{aligned}&\dot{\hat{x}}_{ex}(t)=A_{ex}\hat{x}_{ex}(t)+B_{ex}u(t)+D_{ex}f(g(t))+\\ &+L(g(t)-\hat{g}(t)), \hat{g}(t)=C_{ex}\hat{x}_{ex}(t)\end{aligned}, \tag{27}$$

where $\hat{x}_{ex}(t)$ and $\hat{g}(t)$ are observation of signals $x_{ex}(t)$ and $g(t)$, respectively. Differentiating observation error $\tilde{x}_{ex}(t)=x_{ex}(t)-\hat{x}_{ex}(t)$, we obtain

$$\begin{aligned}&\dot{\tilde{x}}_{ex}(t)=A_{ex}\tilde{x}_{ex}(t)-L(g(t)-g(t))=\\ &=(A_{ex}-LC_{ex})\tilde{x}_{ex}(t)\end{aligned}. \tag{28}$$

One can choose matrix L that provides eigenvalues of the matrix $(A_{ex}-LC_{ex})$ below zero, so (28) is exponentially stable. As (28) is exponentially stable we can write for $t\to\infty$

$$\tilde{x}_{ex}(t)\to 0, \hat{x}_{ex}(t)\to x_{ex}(t), \hat{\psi}(t)\to\psi(t). \tag{29}$$

We will divide control signal $u(t)$ in (1) into two parts $u(t)=u_1(t)+u_2(t)$, where $u_2(t)$ provides compensation of the harmonic disturbance (24) and $u_1(t)$ provides control over undisturbed plant (e.g. stabilization). Designing of $u_1(t)$ strongly depends from current plant; it is nonlinearity and is out of scope of this paper. We assume that signal $u_1(t)$ for undisturbed plant is known and are to design only $u_2(t)$. Let us rewrite (1b)

$$\begin{aligned}&g(t)=\frac{b(p)}{a(p)}(u_1(t)+u_2(t))+\\ &+\frac{d(p)}{a(p)}f(g(t))+\sigma_0+y_2(t)\end{aligned}. \tag{30}$$

One can see that for compensation of harmonic disturbance $y_2(t)$ we can write $u_2(t)$ as

$$u_2(t)=-\frac{a(p)}{b(p)}y_2(t). \tag{31}$$

This equation cannot be solved directly as $y_2(t)$ is unmeasured and degree of polynomial $b(p)$ is less then degree of polynomial $a(p)$, but we can represent it as

$$u_2(t) = \frac{a(p)}{b(p)} \hat{y}_2(t), \qquad (32)$$

where $\hat{y}_2(t) = C_\psi \hat{\psi}(t)$. As $\hat{\psi}(t) \to \psi(t)$ with $t \to \infty$, so $\hat{y}_2(t) \to y_2(t)$. As signal $\hat{y}_2(t)$ is provided by model (25), its derivatives are measured. And as it has a harmonic form it can be used for obtaining compensation signal (32) for any degrees $a(p)$ and $b(p)$.

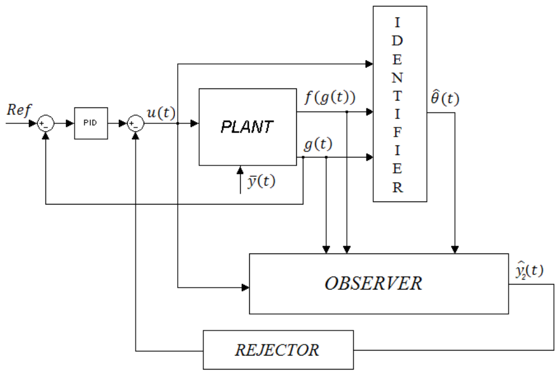

Fig. 6. Structural scheme of the closed loop system.

IV. EXAMPLE PART 2

For plant (21) and disturbance frequency estimation $\theta = -\omega^2 = -(5 \cdot 2\pi)^2$ we can write extended model (26) and observer (27). Matrix $L = [1.8 \quad 3.4 \quad 37.2 \quad -1376.8]^T$ provides closed loop eigenvalues $\lambda_i = -10, i = \overline{1,4}$. At figure 6 one can see structural scheme of the closed loop system with frequency identifier, observer and compensator. At figure 7 one can see the system output $g(t)$. Here at the time $t = 0$ transactions in auxiliary filters start. At the time $t = 1$ identification stars and at the time $t = 7$ (see figure 4) we can fix value θ_{fix}. At this moment matrix L can be calculated and observation starts. Transient processes in observer take less than 2 seconds and at the time $t = 9$ disturbance compensation starts.

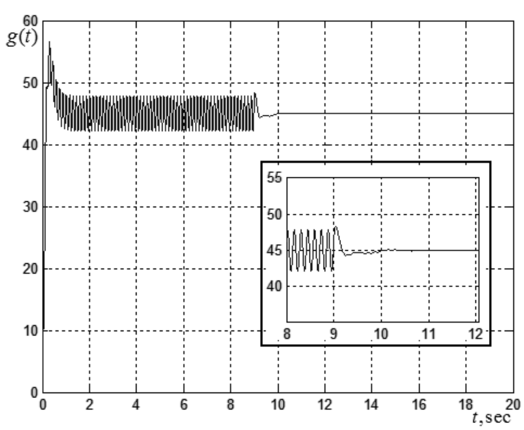

Fig. 7. Output signal of the closed loop system.

V. CONCLUSION

The problem of compensation of unmeasured output biased harmonic disturbance for nonlinear plant was considered. Compensation is separated into two steps: frequency identification (19)–(20) and disturbance compensation with extended state observer (27), (32). Proposed method has dynamic order less than most of known results, it can be used for nonlinear plant with known nonlinearity, it doesn't require measuring internal states of the plant, it allows tuning rate of convergence of frequency estimation. Identifying frequency when it varies between two constants is also possible.

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Appendix A. PROOF OF THE LEMMA

Applying the Laplace transform to (13) we obtain

$$sY(s) = \frac{s}{(s+\alpha)^2}\theta Y(s) + \frac{2\alpha s^2 + \alpha^2 s}{(s+\alpha)^2}Y(s) + \frac{Q(s)}{(s+\alpha)^2}$$

where s is a complex variable, $Y(s) = L\{y(t)\}$ is Laplace image of signal $y(t)$, and polynomial $Q(s)$ denotes sum of all terms, containing nonzero initial conditions. Thus we get

$$\dot y(t) = \frac{p}{(p+\alpha)^2}\theta\, y(t) + \frac{2\alpha p^2 + \alpha^2 p}{(p+\alpha)^2}y(t) + \bar\varepsilon_y(t) \qquad (33)$$

where exponentially decaying function of time $\bar\varepsilon_y(t) = L^{-1}\{Q(s)/(s+\alpha)^2\}$ is determined by nonzero initial conditions. Substituting (14b) into (33) we obtain

$$\dot y(t) = 2\alpha\ddot\varsigma(t) + \alpha^2\dot\varsigma(t) + \theta\dot\varsigma(t) + \bar\varepsilon_y(t).$$

Q.E.D.

Appendix B. PROOF OF THE PROPOSITION

Consider estimation error of parameter θ of the following form: $\tilde\theta(t) = \theta - \hat\theta(t)$. After differentiation we have

$$\dot{\tilde\theta} = \dot\theta - \dot{\hat\theta}(t) = 0 - k\dot\varsigma^2(t)\tilde\theta(t) = -k\dot\varsigma^2(t)\tilde\theta(t) \qquad (34)$$

Solving differential equation (34) we obtain

$$\tilde\theta(t) = \tilde\theta(t_0)e^{-k\gamma(t,t_0)} \qquad (35)$$

where function

$$\gamma(t,t_0) = \int_{t_0}^{t}\dot\varsigma^2(\tau)d\tau \qquad (36)$$

It is obvious that as polynomial $(p+\alpha)^2$ is Hurwitz, function $\varsigma(t)$ takes the form

$$\varsigma(t) = \tilde\sigma_0 + \tilde\sigma\sin(\omega t + \tilde\varphi) + \Delta \qquad (37)$$

where $\tilde\sigma_0$, $\tilde\sigma$ and $\tilde\varphi$ are constant coefficients depending on parameters of signal $y(t) = \sigma_0 + \sigma\sin(\omega t + \varphi)$ and number α, and Δ is an exponentially decaying item, caused by transients.

Neglecting Δ and differentiating (37) we obtain $\dot\varsigma(t) = \tilde\sigma\omega\cos(\omega t + \tilde\varphi)$. Substituting $\dot\varsigma(t) = \tilde\sigma\omega\cos(\omega t + \tilde\varphi)$ into (36) we have

$$\gamma(t,t_0) = \int_{t_0}^{t}\dot\varsigma^2(\tau)d\tau = \tilde\sigma^2\omega^2\int_{t_0}^{t}(\cos(\omega\tau + \tilde\varphi))^2 d\tau =$$
$$= \frac{\tilde\sigma^2\omega^2 t}{2} - \frac{\tilde\sigma^2\omega^2 t_0}{2} + \frac{\tilde\sigma^2\omega^2\sin(2\omega t + 2\tilde\varphi)}{4\omega} -$$
$$- \frac{\tilde\sigma^2\omega^2\sin(2\omega t_0 + 2\tilde\varphi)}{4\omega} = \gamma_0 t + \gamma_1(t,t_0) \qquad (38)$$

where function

$$\gamma_1(t,t_0) = -\frac{\tilde\sigma^2\omega^2 t_0}{2} + \frac{\tilde\sigma^2\omega^2\sin(2\omega t + 2\tilde\varphi)}{4\omega} -$$
$$- \frac{\tilde\sigma^2\omega^2\sin(2\omega t_0 + 2\tilde\varphi)}{4\omega}$$

is bounded for any *t*, and number $\gamma_0 = \frac{\tilde\sigma^2\omega^2}{2} > 0$.

Let us substitute (38) into (35):

$$\tilde\theta(t) = \tilde\theta(t_0)e^{-k\gamma_0 t}e^{-k\gamma_1(t,t_0)}. \qquad (39)$$

It follows from (39) that $\lim_{t\to\infty}\tilde\theta = 0$, and hence $\hat\omega(t) = \sqrt{\left|\hat\theta(t)\right|} \to \omega(t)$ for $t \to \infty$. Q.E.D.