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Modeling and Twin Nonlinear Controller Design for ac/dc Voltage Source Converters Driven dc Series Motors

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Abstract— **Modeling, control design and stability of series-connected dc motors fed by three-phase ac/dc voltage source converters are investigated. The design developed in this paper results in a twin nonlinear controller structure each acting as an oscillator with damped frequency and complementary tasks, i.e. to achieve precise motor speed regulation and operation with unity power factor. An extended passivity-based analysis shows that the proposed approach guarantees the system damping which is essential for stability analysis. To this end, by using the sequence of linear time-varying approximations method, it is proven that the system with the external unknown input is attracted to the desired equilibrium. Furthermore, it becomes clear that the proposed controller does not need any measurement or knowledge of the system conditions and parameters. Stability analysis and these properties constitute the main advantages of this control design approach with respect to other existing schemes. Simulation results verify the controller performance under speed reference and load torque changes.**

I. Introduction

In industry drive applications, direct current (dc) motors are still in use because of their high performance characteristics, such as accurate and fast response. In many cases, such as rail train or load levitating systems, where high starting torque is required, the series-connected dc motor is preferred [1], [2], [3]. However, since the model of a series-connected dc motor is nonlinear, an effective controller design for this case is not an easy task.

Nowadays, the most common power device for driving a dc motor is the three-phase pulse width modulated (PWM)-regulated ac/dc voltage source converter (VSC). Various control strategies have been proposed in the last decades for controlling a three-phase ac/dc VSC power device. For the analysis, it has been shown in [4], [5] that the average modeling provides an adequate description of an ac/dc converter, resulting in a nonlinear model which however increases the control design difficulties. Traditional proportional-integral (PI) controllers [6] have been proposed which in the most common scheme are in cascaded form with main tasks to achieve unity power factor, while a cascaded PI controller is used to regulate the dc voltage output. In this case, only the inner current loop PI controller schemes can be taken into account for the stability analysis [7]. Intelligent control methods such as fuzzy control methods [8] are used to improve the converter performance, which however lack in theoretical analysis and cannot guarantee a stable closed-loop operation. Sliding mode [9] and feedback linearization techniques [10] show an enhanced performance in simulating environments but are sensitive to model uncertainties. Since the average VSC model has been derived through Hamiltonian modeling, several researchers have proposed passivity-based control (PBC) methods [11], [12] that include Lyapunov functions used in system energy-shaping, which however are directly dependent on the system parameters.

Most of the above controllers can be effectively extended to include a separately-excited dc motor due to its simple linear model which can be easily handled and implemented. Limited research work has been addressed for the control of series-connected dc motors [1]. Especially, in the case where the motor is driven by a three-phase VSC power converter, the difficulty of the overall system increases since the rank of the system with nonlinearities increases. Only when VSC devices are not taken into account, the Hamiltonian modeling of the series-connected dc motor makes nonlinear controllers suitable for motor speed regulation [3], [13].

In this paper, we consider the overall system of a three-phase ac/dc VSC with a series-connected dc motor. The complete model has been derived using Hamiltonian and passivity analysis. Our main task is to achieve precise motor speed regulation and power factor correction. Therefore, we propose a new twin nonlinear control scheme suitable to guarantee closed-loop passivity and system stability. First, we prove that the closed-loop system preserves the damping characteristics of the initial open-loop model, and based on this analysis we prove stability of the closed-loop solution. Using a new approach based on linear time-varying approximations [14] we show that the solution remains bounded and according to [15] we conclude that the system converges to the desired equilibrium. Simulation results verify the proposed approach under several changes of the reference speed and sudden changes of the load torque.

The paper is organized as follows. In Section II, the nonlinear dynamic model of the converter-motor system is presented and the desired equilibrium is derived. In Section III, the proposed twin nonlinear controller is presented and the closed-loop system is analyzed. In Section IV, extended simulation results are presented and finally in Section V some conclusions are drawn.

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II. Model Of A Series-Connected DC motor Driven By A Three-Phase AC/DC Converter

Figure 1 shows the complete dc drive system which includes a symmetrical three-phase ac power supply, a boosting inductor circuit, a dc filter capacitor, a three-phase ac/dc voltage source converter and a series-connected dc motor. Parameters R and L are the resistance and inductance of the boosting inductor, while C is the dc-side filter capacitor. R_a and L_a are the total armature resistance and inductance of the dc motor, K_e is the motor constant, J is the total machine and load inertia and b is the friction coefficient. The converter input voltages and currents after the boosting inductor are expressed by the notations V_i and I_i, $i = a,b,c$ respectively.

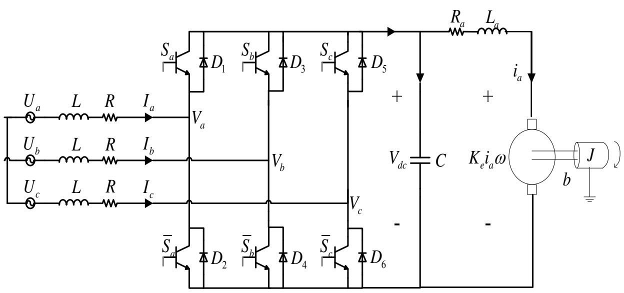

Fig. 1 Power circuit of a three-phase ac/dc converter and dc series motor

Since the original three-phase model of the system contains discontinuous input terms due to the switching functions caused by the PWM operation, we have to use the averaging analysis [4] in order to overcome this problem. Indeed, for control purposes, both the average models of PWM regulated converters and the ac circuit representation on the $d-q$ reference frame are used [5].

Particularly, using the synchronous rotating, voltage oriented, $d-q$ reference frame, well-known as Park's Transformation [16], all the sinusoidal quantities are transformed into dc quantities in steady-state [11], [12]. Adding to this model the dynamic equations of the series-connected dc motor [1], the complete converter-motor system becomes:

$$\begin{aligned} L\dot{i}_d &= -Ri_d + \omega_s L i_q - m_d V_{dc} + U_d \\ L\dot{i}_q &= -Ri_q - \omega_s L i_d - m_q V_{dc} + U_q \\ C\dot{V}_{dc} &= \frac{3}{2}\left(m_d i_d + m_q i_q\right) - i_a \\ L_a \dot{i}_a &= -R_a i_a - K_e i_a \omega + V_{dc} \\ J\dot{\omega} &= K_e i_a^2 - b\omega - T \end{aligned} \tag{1}$$

where the state vector is $x = \begin{bmatrix} i_d & i_q & V_{dc} & i_a & \omega \end{bmatrix}^T$, ω_s is the angular frequency of the power source, i_d, i_q are the $d-$ and $q-$axis components of the line current, U_d and U_q are the $d-$ and $q-$ axis components of the supply voltage (which are constant uncontrolled inputs, see [16]), T is the load torque and m_d and m_q are the $d-$ and $q-$axis controlled input components, namely the switching duty ratio $d-q$ components for which it holds true that [16]:

$$m_a = 2\sqrt{m_d^2 + m_q^2} \tag{2}$$

with m_a being the switching duty ratio of phase $-a$ in a period under PWM regulation and $\Delta\varphi$ is the phase shifting between the voltages U_a and V_a in phase $-a$ (Fig. 1):

$$\Delta\varphi = \arctan\left(\frac{m_d}{m_q}\right) \tag{3}$$

However, since U_d, U_q are constant and T is the disturbance input, the only controlled inputs are the duty ratios m_d, m_q which appear in nonlinear terms.

Also, in the $d-q$ reference frame, the active and reactive power at the supply input correspondingly are:

$$P = \frac{3}{2}\left(U_d I_d + U_q I_q\right) \tag{4}$$

and

$$Q = \frac{3}{2}\left(U_q I_d - U_d I_q\right) \tag{5}$$

Considering, now, the $q-$axis to be aligned with the phase $-a$ of the supply voltage, we have

$$U_d = 0 \text{ and } U_q = U_m \tag{6}$$

where U_m is the maximum voltage of the supply phase voltage.

Under this assumption and in order to achieve unity power factor, i.e. operation with $Q = 0$, equation (5) constrains i_d to be zero [11], [12]. Then, for motor speed regulating at ω_{ref} the desired steady-state equilibrium point of (1) is given by:

$$\begin{aligned} x^* &= \begin{bmatrix} 0 & i_q^* & V_{dc}^* & i_a^* & \omega_{ref} \end{bmatrix}^T \\ &= \begin{bmatrix} 0 & i_q^* & R_a i_a^* + K_e i_a^* \omega_{ref} & \sqrt{\dfrac{b\omega_{ref} + T}{K_e}} & \omega_{ref} \end{bmatrix}^T \end{aligned} \tag{7}$$

for some steady-state input values m_d^* and m_q^* with i_q^* given by [9], [12]:

$$i_q^* = \frac{1}{2}\left(\frac{U_m}{R} - \frac{\sqrt{U_m^2 - \frac{8}{3} R i_a^* V_{dc}^*}}{R}\right) \tag{8}$$

From (8), it is obvious that a limit on the dc armature current i_a exists as follows:

$$i_a^* < \frac{3U_m^2}{8RV_{dc}^*}.$$

III. THE PROPOSED TWIN CONTROLLER DESIGN

A. Nonlinear dynamic control scheme

The main tasks of the control of a three-phase ac/dc converter driving a dc motor are to achieve precise motor speed regulation and operation with unity power factor. To this end, we propose the following twin nonlinear control scheme as described in *(i)* and *(ii)*, each taking the following general form

$$\begin{bmatrix} \dot{z}_j \\ \dot{z}_{j+1} \end{bmatrix} = \begin{bmatrix} 0 & k_j\left(r - r_{ref}\right) \\ -k_j\left(r - r_{ref}\right) & 0 \end{bmatrix} \begin{bmatrix} z_j \\ z_{j+1} \end{bmatrix}$$

where z_j, z_{j+1} are the control states, k_j is a constant gain and r, r_{ref} represent the controlled state variable and its desired reference value respectively.

i) for the motor speed regulation

The controlled input m_q is:

$$m_q = z_j \text{ with } j = 1 \tag{9}$$

where $r = \omega$ represents the measured motor speed and $r_{ref} = \omega_{ref}$ is the desired motor speed of the dc motor.

One can see that the proposed nonlinear controller (9) is independent from all system parameters, the supply voltage and the load torque. This fact increases the robustness of the proposed approach under usual circumstances.

ii) for the power factor regulation

As mentioned in Section II, to achieve power factor equal to 1, the control task is to maintain zero steady-state value of the $d-$axis current $i_d = 0$. Therefore, we propose the following nonlinear control scheme for the input m_d:

$$m_d = z_j \text{ with } j = 3 \tag{10}$$

where $r = i_d$ is the measured $d-$axis current and $r_{ref} = 0$ is its desired value. Once again, nonlinear controller (10) is independent from the system parameters.

After applying both controllers (9) and (10), the closed-loop system takes the form:

$$\begin{aligned}
L\dot{i}_d &= -Ri_d + \omega_s L i_q - z_3 V_{dc} \\
L\dot{i}_q &= -Ri_q - \omega_s L i_d - z_1 V_{dc} + U_m \\
C\dot{V}_{dc} &= \frac{3}{2}\left(z_3 i_d + z_1 i_q\right) - i_a \\
L_a \dot{i}_a &= -R_a i_a - K_e i_a \omega + V_{dc} \\
J\dot{\omega} &= K_e i_a^2 - b\omega - T \\
\dot{z}_1 &= k_1\left(\omega - \omega_{ref}\right) z_2 \\
\dot{z}_2 &= -k_1\left(\omega - \omega_{ref}\right) z_1 \\
\dot{z}_3 &= k_3 i_d z_4 \\
\dot{z}_4 &= -k_3 i_d z_3
\end{aligned} \tag{11}$$

where the new state vector of the closed-loop system is $\tilde{x} = \begin{bmatrix} x^T & \vdots & z^T \end{bmatrix}^T = \begin{bmatrix} i_d & i_q & V_{dc} & i_a & \omega & \vdots & z_1 & z_2 & z_3 & z_4 \end{bmatrix}^T$.

It is remarkable that the desired closed-loop equilibrium of (11) for $\omega = \omega_{ref}$ and unity power factor ($i_d = 0$) is still given by (7) for x while for z it holds true that since $\omega \to \omega_{ref}$ and $i_d \to 0$ as $t \to \infty$, then z tends to some value z^* that correspond to $m_q^* = z_1^*$ and $m_d^* = z_3^*$ respectively with $z_1^*, z_3^* \neq 0$.

As it will be shown in the stability analysis which follows, a suitable choice of the initial conditions for the control state variables always forces the closed-loop system to obtain the desired equilibrium avoiding convergence to zero for the controller dynamics z.

B. Closed-loop system passivity analysis

For the passivity analysis of the closed-loop system, consider the storage function:

$$\begin{aligned} V = {} & \frac{1}{2}Li_d^2 + \frac{1}{2}Li_q^2 + \frac{1}{3}CV_{dc}^2 + \frac{1}{3}L_a i_a^2 + \frac{1}{3}J\omega^2 + \\ & + \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2}z_3^2 + \frac{1}{2}z_4^2 \end{aligned} \tag{12}$$

Then the derivative of the storage function V is calculated as:

$$\begin{aligned} \dot{V} = {} & Li_d\dot{i}_d + Li_q\dot{i}_q + \frac{2}{3}CV_{dc}\dot{V}_{dc} + \frac{2}{3}L_a i_a \dot{i}_a + \frac{2}{3}J\omega\dot{\omega} + \\ & + z_1\dot{z}_1 + z_2\dot{z}_2 + z_3\dot{z}_3 + z_4\dot{z}_4 \end{aligned}$$

Substituting, now, $\dot{\tilde{x}}$ from the dynamic model (11) we arrive at:

$$\begin{aligned} \dot{V} &= -Ri_d^2 - Ri_q^2 - \frac{2}{3}R_a i_a^2 - \frac{2}{3}b\omega^2 + i_q U_m - \frac{2}{3}\omega T \\ &\leq i_q U_m - \frac{2}{3}\omega T \end{aligned} \tag{13}$$

Assuming, as output the vector $y = \begin{bmatrix} i_q & \omega \end{bmatrix}^T$ and as input the vector $u = \begin{bmatrix} U_m & -\frac{2}{3}T \end{bmatrix}^T$, inequality (13) becomes:

$$\dot{V} \leq y^T u$$

which according to the passivity theorem [17], proves that the closed-loop system remains passive under the proposed control scheme. This is a fundamental property since it proves that the proposed control scheme does not change the damping characteristics of the complete ac/dc converter-motor system.

C. Closed-loop system stability analysis

Although the passivity property of the closed-loop system ensures damping, stability is not yet justified. To this end, we provide the following theorem which proves boundedness of the closed-loop system solution and convergence to the desired equilibrium.

Theorem 1. For the control scheme (9) and (10) with initial conditions $z_1(0)=0$, $z_2(0)=0.5$ and $z_3(0)=0$, $z_4(0)=0.5$ respectively, the solution $\tilde{x}(t)$ of the closed-loop system (11) remains bounded and converges to the desired equilibrium $\tilde{x}^*$.

Proof. Closed-loop system (11) can be written into the following nonlinear matrix form:

$$\dot{\tilde{x}} = A\left(\tilde{x}\right)\tilde{x} + B\left(\tilde{x}\right)u \tag{14}$$

where $A\left(\tilde{x}\right)=$

$$\left[\begin{array}{ccccc:cc:cc}
-\frac{R}{L} & \omega_s & -\frac{z_3}{L} & 0 & 0 & & & & \\
-\omega_s & -\frac{R}{L} & -\frac{z_1}{L} & 0 & 0 & & & & \\
\frac{3z_3}{2C} & \frac{3z_1}{2C} & 0 & -\frac{1}{C} & 0 & \multicolumn{2}{c:}{0_{5\times 2}} & \multicolumn{2}{c}{0_{5\times 2}} \\
0 & 0 & \frac{1}{L_a} & -\frac{R_a}{L_a} & -\frac{K_e i_a}{L_a} & & & & \\
0 & 0 & 0 & \frac{K_e i_a}{J} & -\frac{b}{J} & & & & \\
\hdashline
\multicolumn{5}{c:}{\multirow{2}{*}{$0_{2\times 5}$}} & 0 & -k_1(\omega-\omega_{ref}) & \multicolumn{2}{c}{\multirow{2}{*}{$0_{2\times 2}$}} \\
 & & & & & k_1(\omega-\omega_{ref}) & 0 & & \\
\hdashline
\multicolumn{5}{c:}{\multirow{2}{*}{$0_{2\times 5}$}} & \multicolumn{2}{c:}{\multirow{2}{*}{$0_{2\times 2}$}} & 0 & -k_3 i_d \\
 & & & & & & & k_3 i_d & 0
\end{array}\right]$$

$$B\left(\tilde{x}\right)=\left[\begin{array}{ccccc:cc:cc}
0 & \frac{1}{L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{3}{2J} & 0 & 0 & 0 & 0
\end{array}\right]$$

and the input vector is given as in the passivity analysis $u=\begin{bmatrix} U_m & -\frac{2}{3}T \end{bmatrix}^T$.

It can be easily proven that matrices $A\left(\tilde{x}\right)$ and $B\left(\tilde{x}\right)$ satisfy the local Lipschitz condition and according to [14] system (14) can be approximated by a sequence of linear time-varying systems:

$$\dot{\tilde{x}}^{[0]}(t) = A(\tilde{x}_0)\tilde{x}^{[0]}(t) + B(\tilde{x}_0)u^{[0]}(t)\ , \quad \tilde{x}^{[0]}(0)=\tilde{x}_0 \tag{15a}$$

and for $i \geq 1$,

$$\dot{\tilde{x}}^{[i]}(t) = A(\tilde{x}^{[i-1]}(t))\tilde{x}^{[i]}(t) + B(\tilde{x}^{[i-1]}(t))u^{[i]}(t)\ , \ \tilde{x}^{[i]}(0)=\tilde{x}_0 \tag{15b}$$

Since the nonlinear system can be expressed through a sequence of linear time-varying systems, then the dynamics of the two control schemes can be investigated separately due to the "decoupling" structure of matrices $A\left(\tilde{x}\right)$ and $B\left(\tilde{x}\right)$, i.e. for the motor speed regulation:

$$\begin{bmatrix} \dot{z}_1^{[i]}(t) \\ \dot{z}_2^{[i]}(t) \end{bmatrix} = \begin{bmatrix} 0 & k_1\left(\omega^{[i-1]}(t)-\omega_{ref}\right) \\ -k_1\left(\omega^{[i-1]}(t)-\omega_{ref}\right) & 0 \end{bmatrix} \begin{bmatrix} z_1^{[i]}(t) \\ z_2^{[i]}(t) \end{bmatrix} \tag{16}$$

Thus, handling (16) as an independent subsystem, one could consider the Lyapunov function:

$$W_1^{[i]}(t) = \left(z_1^{[i]}(t)\right)^2 + \left(z_2^{[i]}(t)\right)^2 \geq 0 \tag{17}$$

Then, taking the time derivative of $W_1^{[i]}(t)$ we arrive at:

$$\begin{aligned} \dot{W}_1^{[i]}(t) &= 2k_1\left(\omega^{[i-1]}(t)-\omega_{ref}\right) z_1^{[i]}(t) z_2^{[i]}(t) + \\ &\quad -2k_1\left(\omega^{[i-1]}(t)-\omega_{ref}\right) z_1^{[i]}(t) z_2^{[i]}(t) = 0\ , \quad \forall i \geq 1 \end{aligned} \tag{18}$$

which results that system (16) is Lyapunov stable. From the above equation one can result that:

$$W_1^{[i]}(t) = W_1^{[i]}(0)\ , \quad \forall t \geq 0 \tag{19}$$

According to (17), $W_1^{[i]}(t)$ represents a circle in $z_1^{[i]} - z_2^{[i]}$ plane, while equation (19) implies that $z_1^{[i]}(t)$ and $z_2^{[i]}(t)$ will stay for all future time on circle $W_1^{[i]}(0)$ with radius dependent on the initial values $z_1^{[0]}(0)=z_1^{[1]}(0)=...=z_1(0)$ and $z_2^{[0]}(0)=z_2^{[1]}(0)=...=z_2(0)$. Thus, it has been proven that $z_1^{[i]}(t)$ and $z_2^{[i]}(t)$ are all bounded $\forall i \geq 1$.

A similar analysis can show that for the power factor controller the same properties hold true and $z_3^{[i]}(t)$ and $z_4^{[i]}(t)$ will stay for all future time on another circle $W_2^{[i]}(0)$ with radius dependent on the initial values $z_3^{[0]}(0)=z_3^{[1]}(0)=...=z_3(0)$ and $z_4^{[0]}(0)=z_4^{[1]}(0)=...=z_4(0)$, i.e. they also remain bounded $\forall i \geq 1$.

Now, in order to prove stability, i.e. state boundedness of the whole closed-loop system, the rest of the nonlinear system that interacts with the controllers, i.e. the converter-motor system, should also be analyzed. Obviously, the converter-motor system is:

$$\begin{bmatrix} \dot{i}_d^{[i]}(t) \\ \dot{i}_q^{[i]}(t) \\ \dot{V}_{dc}^{[i]}(t) \\ \dot{i}_a^{[i]}(t) \\ \dot{\omega}^{[i]}(t) \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \omega_s & -\frac{z_3^{[i-1]}(t)}{L} & 0 & 0 \\ -\omega_s & -\frac{R}{L} & -\frac{z_1^{[i-1]}(t)}{L} & 0 & 0 \\ \frac{3z_3^{[i-1]}(t)}{2C} & \frac{3z_1^{[i-1]}(t)}{2C} & 0 & -\frac{1}{C} & 0 \\ 0 & 0 & \frac{1}{L_a} & -\frac{R_a}{L_a} & -\frac{K_e i_a^{[i-1]}(t)}{L_a} \\ 0 & 0 & 0 & \frac{K_e i_a^{[i-1]}(t)}{J} & -\frac{b}{J} \end{bmatrix} \begin{bmatrix} i_d^{[i]}(t) \\ i_q^{[i]}(t) \\ V_{dc}^{[i]}(t) \\ i_a^{[i]}(t) \\ \omega^{[i]}(t) \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{L} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{3}{2J} \end{bmatrix}^T \begin{bmatrix} U_m \\ -\frac{2}{3}T \end{bmatrix}$$

which is in the form:

$$\dot{x}^{[i]}(t) = A_s\left(\tilde{x}^{[i-1]}(t)\right)x^{[i]}(t) + B_s u\ , \ \tilde{x}^{[i]}(0)=\tilde{x}_0 \tag{20}$$

where A_s is the 5×5 submatrix of $A\left(\tilde{x}(t)\right)$, matrix B_s is the 5×2 submatrix of B which is constant. Furthermore, input u is usually constant or piecewise constant in industrial applications assuming symmetrical three-phase supply and constant load.

Assuming $i=0$, we first consider the linear time-invariant system:

$$\dot{x}^{[0]}(t) = A_s\left(\tilde{x}_0\right)x^{[0]}(t) + B_s u\ , \ \tilde{x}^{[0]}(0)=\tilde{x}_0 \tag{21}$$

Due to the initial conditions of $z_1(0)$ and $z_3(0)$, one can easily prove that the eigenvalues of $A_s\left(\tilde{x}_0\right)$ have negative real parts independently from the system parameters and the initial condition $i_a(0)$. As a result the state $x^{[0]}(t)$ of system (21) is bounded.

Consider, now, the first system ($i=1$) of the sequence:

$$\dot{x}^{[1]}(t)=A_s\left(\tilde{x}^{[0]}(t)\right)x^{[0]}(t)+B_s u,\ \tilde{x}^{[1]}(0)=\tilde{x}_0 \tag{22}$$

For system (22) we first investigate the following system without the external input u:

$$\dot{x}^{[1]}(t)=A_s\left(\tilde{x}^{[0]}(t)\right)x^{[0]}(t),\ \tilde{x}^{[1]}(0)=\tilde{x}_0 \tag{23}$$

Since $\tilde{x}^{[0]}(t)$ is already proven to be bounded, then $A^{[0]}(\infty)=\lim_{t\to\infty}A\left(\tilde{x}^{[0]}(t)\right)$ exist. Furthermore, one can once again see that the eigevalues of $A\left(\tilde{x}^{[0]}(t)\right)$ have negative real parts independently from the values of $z_1^{[0]}(t)$, $z_3^{[0]}(t)$ and $i_a^{[0]}(t)$. As a result, according to [14], system (23) is asymptotically stable. It is obvious that any initial condition $\tilde{x}_0$ will result in an asymptotically stable system at the origin independently from t_0. Thus, the origin is uniformly asymptotically stable and therefore for the transition matrix $\Phi^{[1]}(t,\tau)$ of (22) or (23) it holds true that:

$$\begin{gathered}\exists\ c_1^{[1]},c_2^{[1]},m^{[1]}\geq 0:\ \left\|\Phi^{[1]}(t,t_0)\right\|\leq c_1^{[1]}e^{-c_2^{[1]}(t-t_0)}\\ \int_{t_0}^{t}\left\|\Phi^{[1]}(t,\tau)\right\|d\tau\leq m^{[1]},\ \forall t\geq t_0\geq 0\end{gathered} \tag{24}$$

Since input u is assumed to be constant (or piecewise constant) and matrix B_s is constant, then according to [18] system (22) is totally stable, i.e. solution $x^{[1]}(t)$ is bounded yielding that $\tilde{x}^{[1]}(t)$ is bounded.

Proceeding with $i>1$, a similar analysis can be sequentially applied and result that $z_1^{[i]}(t)$, $z_2^{[i]}(t)$ and $z_3^{[i]}(t)$, $z_4^{[i]}(t)$ will move on circles $W_1^{[i]}(t)$ and $W_2^{[i]}(t)$ respectively, while the remaining states $x^{[i]}(t)$ will be bounded. As a result the solution $\tilde{x}^{[i]}(t)$ of every system in the sequence is proven to be bounded. Eventually, as $i\to\infty$, the solution $\tilde{x}(t)$ of the closed-loop system (14) is bounded since $\tilde{x}(t)=\lim_{i\to\infty}\tilde{x}^{[i]}(t)$.

Now let $\tilde{x}^*$ be the desired equilibrium corresponding to the duty ratio values $m_q^*=z_1^*$ and $m_d^*=z_3^*$. These desired values for the control inputs are represented by two lines l_1 and l_2 on z_1-z_2 and on z_3-z_4 planes respectively as shown in Figure 2. Due to the initial conditions of z_1 and z_2, they are initially defined on a point $A\left(0,0.5\right)$ on z_1-z_2 plane and as imposed by the analysis, they move on circle $W_1(0)$ with center the origin and radius equal to 0.5 (Fig. 2). Similarly, z_3 and z_4 are also initially defined on a point $C\left(0,0.5\right)$ on z_3-z_4 plane and move on circle $W_2(0)$ with center the origin and radius equal to 0.5 (Fig. 2).

Since the converter is preferred to work in linear mode [16] ($m_a\leq 1$), the two circles $W_1(0)$ and $W_2(0)$ have definitely intersection points with lines l_1 and l_2 respectively, represented by points B and D in Figure 2. As a result, the state vector $\tilde{x}(t)$ of the closed-loop system is bounded in a region where the desired equilibrium exists. Furthermore, the system is passive and since the external input vector u is assumed to be constant (or piecewise constant) in a common application, therefore all requirements imposed in [15] are satisfied and as a result the closed-loop system converges to the desired equilibrium $\tilde{x}^*$ as it is proven in [15]. □

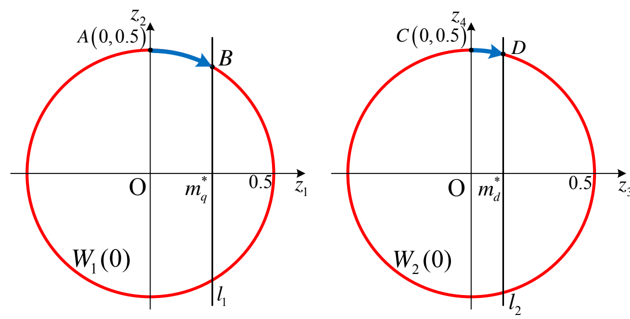

Fig. 2 Convergence to the desired equilibrium on z_1-z_2 and on z_3-z_4 planes

Remark 1. Since the control state variables z_1, z_2 and z_3, z_4 can move only on circles $W_1(0)$ and $W_2(0)$ respectively with radius 0.5, then obviously any initial conditions satisfying $\left(z_1(0)\right)^2+\left(z_2(0)\right)^2=0.25$ and $\left(z_3(0)\right)^2+\left(z_4(0)\right)^2=0.25$ will result in a stable operation that leads the closed-loop system (11) to converge to the desired equilibrium.

IV. Simulation Results

To verify the effectiveness of the proposed control scheme, the response of the three-phase ac/dc converter-motor system is simulated. The parameters of the ac/dc converter are $U_m=80V$, $R=0.1Ohm$, $L=3.3mH$, $C=100\mu F$, $\omega_s=2\pi 60 rad/\sec$ while the series-connected dc motor parameters are $P_n=1.5kW$, $R_a=2.5Ohm$, $L_a=0.3H$, $J=0.08kg\,m^2$, $K_e=0.183Vs/rad/A$, $b=0.001Nm/rad/s$. The controller gains are selected $k_1=0.0075\left(rad/\sec\right)^{-1}$, $k_3=10A^{-1}$ while the initial conditions of the control states are $z_1(0)=0.47$, $z_2(0)=0.1706$, $z_3(0)=0.07$, $z_4(0)=0.4951$ satisfying Remark 1.

The system is assumed to be at any initial state with power factor different from 1. At time-instant $t = 0\sec$ the reference speed is set to $\omega_{ref} = 160 rad/\sec$ while at $t = 15\sec$ it rises to $\omega_{ref} = 190 rad/\sec$. Finally, at $t = 30\sec$, a 15% step increase is assumed for the load torque T (from $5Nm$ to $5.75Nm$).

Figure 3 shows the time response of an ac/dc converter-motor system. Starting from an arbitrary initial point, one can observe that the proposed nonlinear controller effectively regulates the $d-$axis current to zero achieving unity power factor while simultaneously drives the motor speed to the reference value. Figure 4 shows the time response of control inputs m_q and m_d which are regulated at the desired values for any change of the reference speed or the load torque verifying the mathematical analysis.

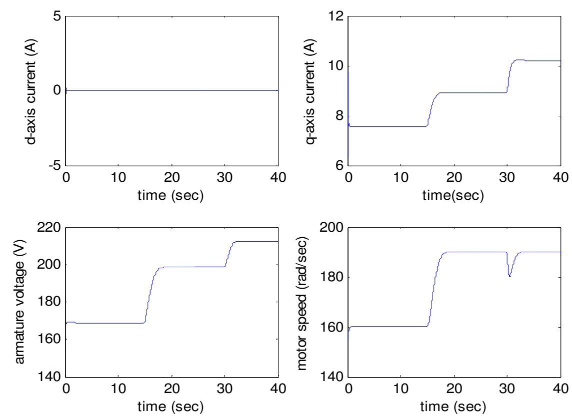

Fig. 3 Response of the three phase ac/dc converter with series-connected dc motor drive system

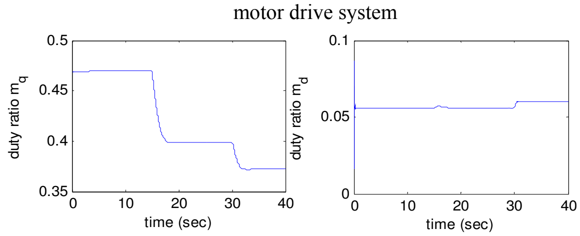

Fig. 4 Response of the control inputs

V. Conclusion

A new simple twin dynamic nonlinear control scheme for a series-connected dc motor drive fed by a three-phase ac/dc VSC has been developed and analyzed. Extended mathematical analysis has proven that the proposed parameter-free controller regulates the system at the desired equilibrium. Simulation results verify its excellent performance and efficiency, under speed reference signal changes and sudden load disturbances.

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