Preprints of the 2012 20th Mediterranean Conference on
Control & Automation (MED), Barcelona, Spain, July 3-6,
2012

Experimental study of an active control for a faulty perturbed flexible structure

Leonardo Acho, Gisela Pujol and Josep M. Rossell

Abstract— **In this paper, a linear dynamic control is developed for a faulty flexible structure subject to external ground perturbation. This active controller is based on H_∞ theory and is designed using linear matrix inequality (LMI) theory. Lyapunov theory is invoked to validate the control design. According to experiments, where a two levels flexible building with active mass damper and external perturbation is employed, show that this strategy improves controller performance when it is compared with a given controller. This design faces the nonlinear structural system too.** *20th Mediterranean Conference on Control and Automation (MED12)*.

I. INTRODUCTION

The purpose of this paper is to design a linear robust controller able to attenuate vibrations (produced by the external ground perturbation) on a flexible structure using the active mass damper controller. This is conceptually similar to active mass damper being studied in earthquake mitigation research facilities to reduce damage from earthquakes on high rise buildings ([3], [9]). For the purpose of maintaining the seismic response of structures within safety levels, service and comfort limits, the combination of passive base isolators and feedback controllers has been proposed in recent years ([3], [2], [11], [14]).

For linear systems, H_∞ control theory offers the possibility of including robustness considerations explicitly in the design and the opportunity to formulate physically meaningful performance objectives that can be expressed as H_∞ design specifications. In the state-space formulation, the problem of reducing the H_∞ norm or, equivalently, its induced L_2-norm, of the closed-loop system is viewed as a solution of the algebraic Riccati inequalities arising in linear quadratic differential theory (see [8] for references on the H_∞ design). In order to solve these Riccati inequalities, they are expressed in terms of linear matrix inequalities (LMI) [10].

In this paper, the design of a local controller that stabilizes the two levels flexible structure, in the presence of extern disturbances, is studied (see Figure 1). The main objective is to demonstrate how a solution to such a complex control problem can be obtained by applying the efficient tools of LMI theory. To show the robustness of the proposed controller, laboratory experiments are carried on. The solution is presented using the robust H_∞ control theory [8] by the method of linear matrix inequality ([1], [5], [10]), that can be applied to structural control ([18], [4]). Our objective is to present an easy control algorithm that solves a complex nonlinear problem. Moreover, an advantage of the active control based on LMI techniques is the robustness to external disturbance, as it is well known, but also its simple design ([12], [16]). In this sense, the law is defined a priori, before doing the experiments, and it only depends on the linearized structural system. It is relatively easy to design the controller because it does not rely on online controller parameters adjustment.

Fig. 1. Quanser Shake Table II without structure fault.

An experimental shake table is used to simulate earthquakes exciting the flexible modes of a tall structure [17]. It is composed by a flexible two levels building with an active mass damper at the top of the building. Only acceleration measurements at level one and two are available, and the position measurement of the cart (the active mass damper) situated at the building top (see Figure 1). In [13], the authors present a solution where chattering control theory is used. Now, a simpler controller is designed and a modification of the experiment is introduced to study the performance in front of additional flexible devices and model uncertainties. Our faulty perturbed flexible structure experiment is as follows (see Figures 2-4). Floor one is disjointed from floor two, where a cylindric roller is placed between them. Then, elastic links are used to fix the floors one and two. Figure 2 pictures

---

This work was supported by grant DPI2011-27567-C02-02 and DPI2011-25822 of the Spanish Ministry of Science and Innovation.

L. Acho and G. Pujol are with the Department of Applied Mathematic III, Escola Universitària d'Enginyeria Tècnica Industrial de Barcelona, Universitat Politecnica de Catalunya - BarcelonaTech, Urgell 187, 08036 Barcelona, Spain.

J.M. Rossell is with Department of Applied Mathematic III, Escola Politecnica Superior d'Enginyeria de Manresa, Universitat Politecnica de Catalunya - BarcelonaTech, Av. Bases de Manresa 61-73, 08240 Manresa, Spain. {`leonardo.acho, gisela.pujol, josep.maria.rossell`}`@upc.edu`

978-1-4673-2529-5/12/$31.00 ©2012 IEEE

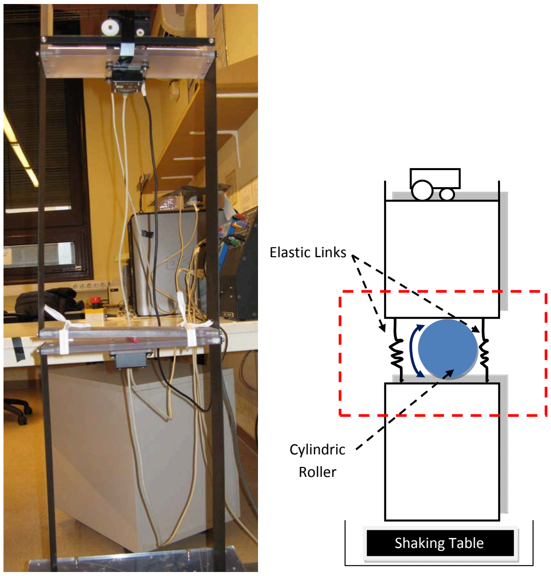

Fig. 2. Modification of Quanser Shake Table II (left: real experiment; right: scheme).

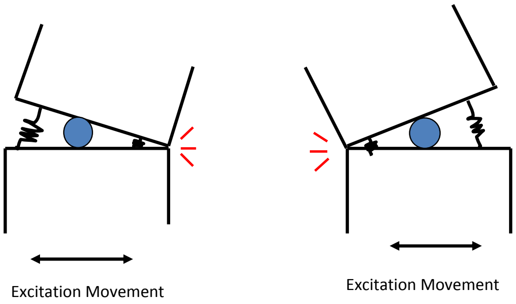

Fig. 3. Movement of shake table under ground excitation.

this faulty experimental modification. Experiments are done to study the effectiveness of the linear H_∞ dynamic controller (based on LMI techniques), when sinusoidal chirp external perturbation is present. This scenario can appear in structural control in transport industry (containerization). For instance, suppose that we have two staked containers fixed together by elastic links. During the transportation, the top container can slide similarly to the present experiment. Figures 3 and 4 show the possible movements of the containers or levels under transportation excitation. Moreover, this fault is too similar to the dead-zone appearing in mechanical systems with impacts [6].

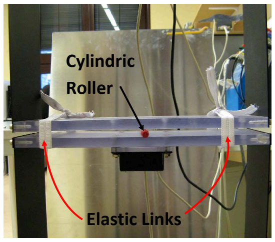

Fig. 4. Zoom of modified experiment presented in Figure 2.

The paper is structured as follows. Section II presents the linear description of the two-floors structure with a single active mass damper located at the top of the structure. The control objective is stated, displaying the dynamic H_∞ control solution in terms of LMI. In Section III experimental results are shown, proven the effectiveness of the methodology. Finally, conclusions and future work are stated in Section IV.

II. CONTROL DESIGN

A. Dynamic unfaulty structure model

This section provides the statement of the differential equation of motion for the two levels building. For small floor deflection angles, both floors are modeled as standard linear spring-mass systems [4]. In our approach, the structure viscous damping coefficients are neglected. The model is derived using Lagrangian formulation, where the dynamic equations are obtained and then a linear model is derived by linearizing about the quiescent (latent) point [17]. The states are defined as:

$$x^T(t) = [x_c(t)\ x_{f_1}(t)\ x_{f_2}(t)\ \dot{x}_c(t)\ \dot{x_{f_1}}(t)\ \dot{x_{f_2}}(t)] \ , \qquad (1)$$

where $x_{f_i}(t)$ is the position of the floor $i = 1,2$ (floor deflection), and $x_c(t)$ is the cart position (see Figure 1). The available measurements are $x_c(t)$, $\ddot{x}_{f_1}(t)$ and $\ddot{x}_{f_2}(t)$. That means $y(t) = [x_c(t)\ ,\ \ddot{x}_{f_1}(t)\ ,\ \ddot{x}_{f_2}(t)]^T$ must be introduced as the measured output. The linear model about the quiescent point is defined by:

$$\begin{aligned}
& m_c m_{f_2} \ddot{x}_c - m_c k_{f_2} x_{f_2} = (m_c + m_{f_2}) u(t) \\
& m_{f_1} \ddot{x}_{f_1} + k_{f_1} x_{f_1} - k_{f_2} x_{f_2} = 0 \\
& m_{f_1} m_c m_{f_2} r_{mp}^2 \ddot{x}_{f_2} - m_{f_2} m_c m_{f_2} r_{mp}^2 k_{f_1} x_{f_1} + \\
& \qquad + \left(m_{f_1} m_c + m_{f_2} m_c\right) r_{mp}^2 k_{f_2} x_{f_2} = -m_{f_1} m_c r_{mp}^2 u(t)
\end{aligned} \qquad (2)$$

where m_c, m_{f_1} and m_{f_2} are the cart, first and second floor mass; k_{f_1} and k_{f_2} are the linear stiffness constants of both floors; r_{mp} is the cart motor pinion radius; and the driving force of the motor cart is $u(t)$ (control input).Taking into account (2), the state space representation results to be:

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t) + Du(t) \ .
\end{aligned} \qquad (3)$$

The matrices are defined as [17]:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 278.43 & -18.69 & 0 & 0 \\ 0 & -431.03 & 431.03 & 0 & 0 & 0 \\ 0 & 431.03 & -766.49 & 5.98 & 0 & 0 \end{bmatrix} ,$$

$$B = [0\ \ 0\ \ 0\ \ 3.01\ \ 0\ \ -0.96]^T ,$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -431.03 & 431.03 & 0 & 0 & 0 \\ 0 & 431.03 & -766.49 & 5.98 & 0 & 0 \end{bmatrix} ,$$

$$D = \begin{bmatrix} 0 \\ 0 \\ -0.96 \end{bmatrix} . \tag{4}$$

B. Linear dynamic control

In the structure, the active damper is located at the top of the building. For this reason, we want to study the effect of external disturbance $w(t) \in \mathscr{L}_2$ on the cart position $x_c(t)$ and floor two acceleration $\ddot{x}_{f_2}(t)$, where the cart is located. Therefore, a performance variable (virtual output) $z(t)$ is defined as $z^T(t) = \left[x_c(t)\ \ddot{x}_{f_2}(t)\ \ u(t)\right]$. The state-space representation of system (3)-(4) yields

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_1 w(t) , \\ z(t) = C_1 x(t) + D_{12} u(t) , \\ y(t) = Cx(t) + Du(t) + D_{21} w(t) , \end{cases} \tag{5}$$

where

$$B_1 = [0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0] ,$$

$$C_1 = \begin{bmatrix} 100 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} ,$$

$$D_{12} = [0\ \ 0\ \ 0.1]^T \ \text{ and } \ D_{21} = [0\ \ 0\ \ 1]^T .$$

Matrix B_1 is defined to take into account that the external perturbation $w(t)$ is produced on the ground. Matrices C_1 and D_{12} are defined to increase the weight of the cart position (where the controller is located) in front of the external perturbation. The above dynamic model satisfies the following standard H_∞ assumptions[1] [8]:

1) (A, B, C) is stabilizable and detectable;
2) (A, B_1, C_1) is stabilizable and detectable;
3) $D_{12}^T(C_1\ D_{12}) = (0\ \alpha)$ with $\alpha > 0$;
4) $\begin{bmatrix} B_1 \\ D_{21} \end{bmatrix} D_{21}^T = \begin{bmatrix} 0 \\ I \end{bmatrix}$.

The problem of robust controller with guaranteed H_∞ performance is addressed to answer this question: Does there exist a feedback control such that the H_∞ norm of the closed-loop system from input disturbance $w(t) \in \mathscr{L}_2$ to output $z(t)$ is less than some prescribed value $\gamma \in \mathbf{R}^+$? In order to solve this problem, the LMI techniques are used.

Let's consider an H_∞ controller $u(t)$ designed as a dynamic control strictly proper [1]:

$$K : \begin{cases} \dot{\varphi}(t) = A_k \varphi(t) + B_k y(t) \\ u(t) = C_k \varphi(t) . \end{cases} \tag{6}$$

Considering the variable $\eta(t)^T = (x(t)^T, \varphi(t)^T)$, the closed loop system (5)-(6) is defined by:

$$\begin{aligned} \dot{\eta}(t) &= \underbrace{\begin{bmatrix} A & BC_k \\ B_k C & A_k + B_k D C_k \end{bmatrix}}_{\mathscr{A}} \eta(t) + \underbrace{\begin{bmatrix} B_1 \\ B_k D_{21} \end{bmatrix}}_{\mathscr{B}} w(t), \\ z(t) &= \underbrace{(C_1\ \ D_{12}C_k)}_{\mathscr{C}_\infty} \eta(t) , \\ y(t) &= \underbrace{(C\ \ DC_k)}_{\mathscr{C}_{2\infty}} \eta(t) + D_{12} w(t) . \end{aligned} \tag{7}$$

Our first control objective is to design a dynamic control (6) such that it stabilizes the closed loop system (7) under $\mathscr{L}_2$ disturbances, employing H_∞-LMI theory. A practical way to solve this problem is to consider a Lyapunov function $V_1(t)$ such that for any nonzero $\eta(t)$ and input $w(t) \in \mathscr{L}_2$, the following condition holds (see [1] for details):

$$\frac{d}{dt} V_1(t) + \gamma^{-1}\, z^T(t) z(t) - \gamma\, w^T(t) w(t) < 0 . \tag{8}$$

Then, an H_∞ performance bound for the closed-loop system (7) is ensured.

Definition 1 (H_∞ Controller [1]): If there exists a matrices A_k, B_k and C_k such that (8) holds, the control law $u(t)$ (6) is said to be an H_∞ *controller* for the system (5), that is, the system is internally stable[2] with H_∞ norm less than γ, i.e., $\|z\|_\infty \leq \gamma^2 \|w\|_\infty$ for $w \in \mathscr{L}_2$.

The H_∞ characterization of the robust control begins by considering the next Lyapunov function:

$$V_1(t) = \eta(t)^T P\, \eta(t) , \tag{9}$$

with $P > 0$. Imposing the H_∞ condition (8), we obtain:

$$\begin{aligned} &(\mathscr{A}\eta + \mathscr{B}w) P \eta + \eta^T P^T (\mathscr{A}\eta + \mathscr{B}w)^T + \\ &\quad + \gamma^{-1} \eta^T \mathscr{C}_\infty^T \mathscr{C}_\infty \eta - \gamma w^T w < 0 . \end{aligned} \tag{10}$$

We can rewrite inequality (10) as a matrix inequality by applying the Schur complement [1]:

$$\begin{bmatrix} \mathscr{A}^T P + P\mathscr{A} & P\mathscr{B} & \mathscr{C}_\infty^T \\ \mathscr{B}^T P & -\gamma & 0 \\ \mathscr{C}_\infty & 0 & -\gamma \end{bmatrix} < 0 . \tag{11}$$

This matrix inequality is feasible and solvable numerically using LMI-techniques included in the robust control toolbox

---

[1] In [8], it is assumed that $D = 0$. A transformation from D nonzero to $D = 0$ is explained in [15].

[2] Internally stable means that the closed-loop system is asymptotically stable when $w = 0$.

of Matlab [7]. Given the continuous-time plant (5), the sub-optimal H_∞ performance γ is computed, as well as an H_∞ controller K (6) that internally stabilizes the plant, yielding a closed-loop gain no larger than γ.

III. EXPERIMENTS: TESTING ROBUSTNESS

A. Experimental setup

Shake Table II is an instrumental shake table developed by Quanser Inc. [17]. The system is comprised by a shake table, a universal power module, a data acquisition card (DAC) along with its external terminal board, and a PC running control software. The PC sends and receives signals through the DAC using the WinCon software. Designed to simulate earthquakes and evaluate the performance of active mass damper, the shake table consists of an 1 Hp brushless servo motor driving a lead screw ([17], [4]). The lead screw drives a circulating ball nut which is coupled to the 18” × 18” table (see Figure 5). The table itself slides on a low friction linear ball bearings on 2 ground-hardened shafts. It can drive a 15 Kg. mass at 2.5 g. Maximum travel is ±7cm. In this paper, the external disturbance considered is a sine chirp wave (Quanser Chirp input $w(t)$ in Figure 6) with increasing frequency from 0.1Hz to 0.7Hz and target time 40s, the total time of the experiments.

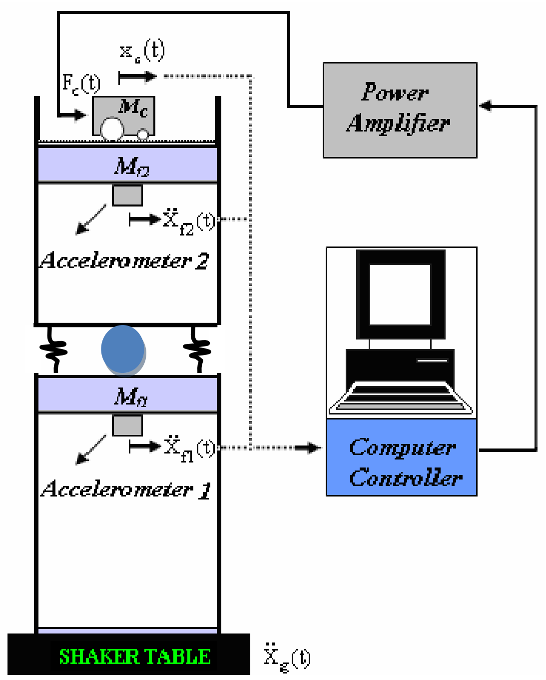

Fig. 5. Experimental set-up.

The shake table operating simulink program is presented in Figure 6, where the proposed dynamic controller (6) is implemented. Using Matlab’s Robust Control Toolbox [7] to compute (6), the performance index $\gamma = 6.03$ is obtained and the control matrices are:

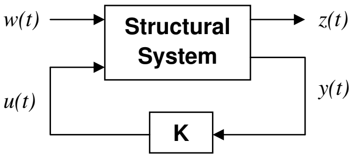

Fig. 6. Block diagram of simulink program for Quanser Shake Table II experiments.

$$A_k =$$

$$\begin{bmatrix} -57.99 & -4.89 & -27.66 & 303.85 & -212.17 & 2.12 \\ 0.18 & -3.24 & -1.32 & 52.52 & 4.54 & -0.29 \\ -41.11 & -16.74 & -28.22 & 392.42 & -162.13 & -8.47 \\ 247.39 & 26.78 & 137.73 & -1655.66 & 967.55 & 4.66 \\ 245.41 & 49.72 & 129.82 & -1935.56 & 786.63 & 4.01 \\ 800.71 & 463.07 & 537.60 & -11120.36 & 2231.03 & -99.36 \end{bmatrix}$$

$$B_k = \begin{bmatrix} -0.09 & 1.90 & -0.28 \\ 0.06 & -0.29 & -0.19 \\ -2.87 & 2.50 & -3.33 \\ 14.54 & 1.55 & 16.70 \\ -32.72 & -9.90 & 17.81 \\ 24.16 & -160.28 & 76.91 \end{bmatrix}, \tag{12}$$

$$C_k = [13.13 \;\; 1.06 \;\; 8.41 \;\; -68.68 \;\; 72.10 \;\; -0.84] \,.$$

B. Linear dynamic versus PD sample controls

To compare the performance of linear dynamic control (6) against the faulty structure case, we consider the sample controller provided in [17], a PD Position Control that implements the proportional-derivative feedback loop and calculates the motor input current needed to move the stage to the desired position. To simplify the presentation of figures, we will call it as Sample control, and the controller (6) with matrices (12) will be named after as Dynamic control. The two control strategies are compared in order to study the performance. In this paper, the external disturbance considered is a sine chirp wave (Quanser Chirp).

Figures 7 to 9 picture the time history plots under chirp input, showing open loop (uncontrolled case), dynamic and sample controls behavior. The accelerations of floor 1 and 2 are similar under both controllers (Figures 7 and 8) but the control performance of dynamic controller (6) is better than the sample controller. In Figure 9, where control performance is shown, we note that the control performance of dynamic control (6) improves the sample control [17], almost halving the control effort.

To compare more accurately the performance of the different control strategies, the H_∞ index is estimated as follows:

$$J_2 = \int_0^T \|u(t)\|_2^2 \, dt \,. \tag{13}$$

978-1-4673-2529-5/12/$31.00 ©2012 IEEE

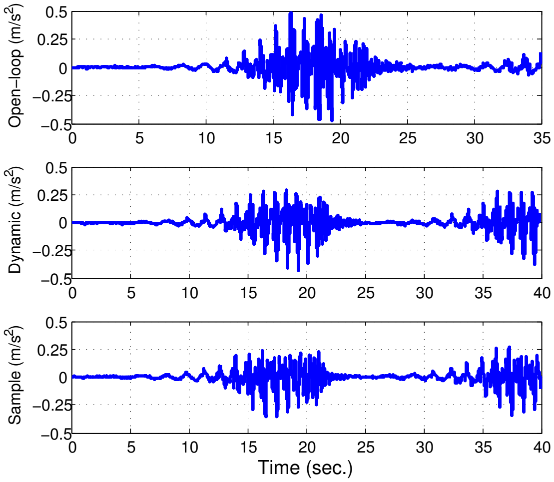

Fig. 7. Acceleration on Floor 1 under Chirp input.

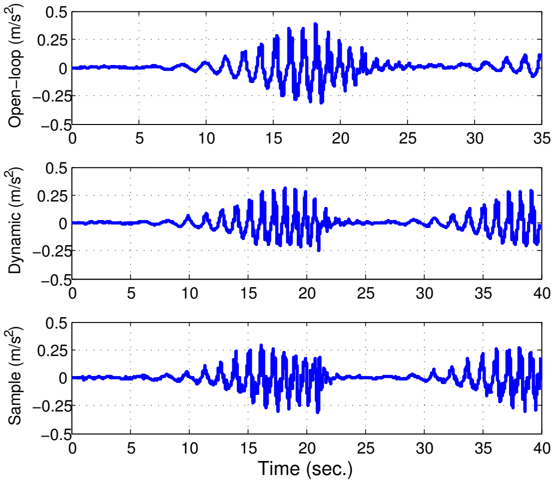

Fig. 8. Acceleration on Floor 2 under Chirp input.

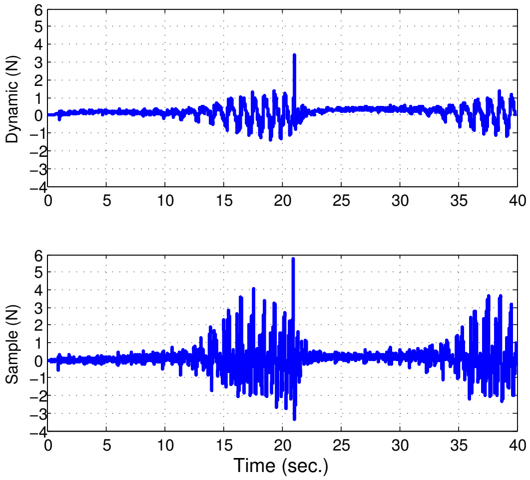

Fig. 9. Controls under Chirp input.

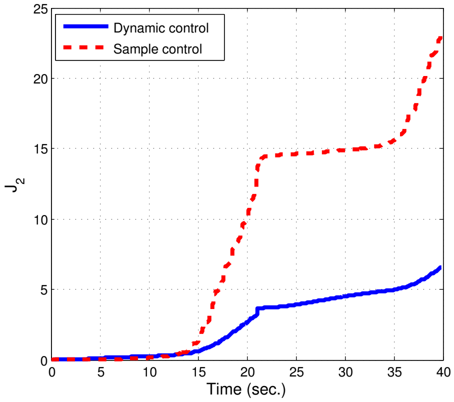

Fig. 10. H_∞ index J_2 under Chirp input.

This index J_2 is evaluated for all the time interval. Due to the numerical evaluation, the design has to be careful implemented on initial conditions.

Figure 10 pictures the H_∞ index J_2 when a Quanser Chirp wave is introduced as perturbation. As Figure 10 pictures, the energy index J_2 is worst in the sample case. In [4], an H_∞ is also designed for the same experiment. Note that the H_∞ weighting matrices induces a greater power consumption. In this sense, our control design is cheeper than [4].

IV. CONCLUSIONS AND FUTURE WORKS

A. *Conclusions*

In this paper, the design of an active controller that stabilizes a structural system was studied, in the presence of a faulty study case and external disturbances. A linear dynamic H_∞ control was developed for active mass damper systems subject to sinusoidal perturbation. This robust control was designed using linear matrix inequality (LMI) theory. According to experiments, where a flexible two levels building with active mass damper seismically excited was employed, it was appreciated a good performance of our linear dynamic design, when it is compared to a given PD controller, evidencing our control robustness. The experimental results, with external disturbance, demonstrated that the performance was satisfactory.

B. *Future Works*

To obtain a cheapest LMI control, a local controller will be designed, where only cart position can be used.

REFERENCES

[1] P. Apkarian, H.D. Tuan, J. Bernussou. Continuos-Time analysis, eigenstructure assignment and H_2 synthesis with enhanced LMI characterizations, *IEEE Transactions on Automatic Control*, 46(12),1941–1946, 2001.

[2] A. Bahar, F. Pozo, L. Acho, J. Rodellar, A. Barbat. Hierarchical semi-active control of base isolated structures using a new inverse model of magnetorheological dampers. *Computers and Structures*, 88(78), 483–496, 2010.

[3] A. Barbat, J. Rodellar, E. Ryan, N. Molinares. Active control of nonlinear base-isolated buildings. *ASCE Journal of Engineering Mechanics*, 121(6), 676–684, 1995.

[4] Borges R., D. Domingues, C.R. Marqui and V. Lopes. H_∞ controller with output feedback using linear matrix inequalities for application in building model, *ABCM Symposium Serires in Mechatronics*, 3, 56–65, 2008.

[5] S. Boyd, L. El Ghaoui, E. Feron, V. Balakrishnan. *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, USA; 1994.

[6] B. Brogliato, *Nonsmooth Impact Mechanics. Models, Dynamics and Control*, Lecture Notes in Control and Information Sciences 220, Springer-Verlag, 1996.

[7] R. Y. Chiang, M.G. Safonov. *Matlab Robust Control Toolbox User's Guide version 2*, The MathWorks Inc, MA, USA; 1998.

[8] J. C. Doyle, K. Glover, P. Khargonekar, B. A. Francis. State-Space solutions to standard H_2 and H_∞ control problems, *IEEE Transactions on Automatic Control*, 34(8), 831–847, 1989.

[9] W. Lan, D. Wang. Structural design of composite nonlinear feedback control for linear systems with actuator constraint. *Asian Journal of Control*, 12(5), 616–625, 2010.

[10] R.C.L.F. Oliveira, P.L.D. Peres. LMI conditions for robust stability analysis based on polynomial parameter-dependent Lyapunov functions, *Systems and Control Letters*, 55(1), 52–61, 2006.

[11] F. Pozo, L. Acho, J. Rodellar, J.M. Rossell, A velocity-based seismic control for base-isolated building structures, *American Control Conference*, St. Louis (Missouri), United States, 2009.

[12] G. Pujol, L. Acho. Stabilization of the Furuta pendulum with backlash using H1-LMI technique: experimental validation. *Asian Journal of Control*, 12(4), 460-467, 2010.

[13] G. Pujol, L. Acho. Robust control design of structural systems with experimental study under faults. *IASTED International Conference Intelligent Systems and Control*, DOI: 10.2316/P.2011.744-041, 162–168, Cambridge, United Kingdom, 2011.

[14] A. Rodriguez, F. Pozo, A. Bahar, L. Acho, Y. Vidal, J. Rodellar. Force-derivative feedback semi-active control of base-isolated buildings using large-scale MR fluid dampers. *Structural Control and Health Monitoring*, DOI:10.1002/stc.430, 2011.

[15] Safonov M.G., Limeber D.J.N., Simplifying the H_∞ theory via loop shifting,*in 27th IEEE Conference on Decision and Control*, Austin, TX, USA, 1988.

[16] M. Sun, Y. Jia. Delay-Dependent Robust H_2/H_∞ control for time-delay systems with polytopic uncertainty. *Asian Journal of Control*, 12(5), 661–666, 2010.

[17] User Manual: Shake Table II. *Quanser specialty experiment series*, Quanser Inc., Canada; 2007.

[18] J.-C. Wu, H.-H. Chih, C.-H. Chen. A robust control method for seismic protection of civil frame building, *Journal of Sound and Vibration*, 294, 314–328, 2006.