Trajectory optimization and Adaptive Fuzzy based Launch Vehicle Attitude Control

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Abstract— **This paper presents a methodology based on direct adaptive fuzzy controller to control the pitch attitude of Satellite Launch Vehicle (SLV). To generate reference trajectories of a four-stage Launch Vehicle, Trajectory Optimization has been performed offline using Genetic Algorithms. The optimization routine takes into account the maximum angle of attack along with normal and lateral overload constraints. To efficiently follow the reference pitch profile, a fuzzy based controller is proposed in which an adaptation law is devised which automatically adjusts the tunable parameter. This tunable parameter is the consequent part of fuzzy IF-THEN rules. Lyapunov principle is incorporated in the adaptation law for ensuring system's stability. It is observed that although the Lyapunov approach make sure that the system is asymptotically stable, it is very critical to select appropriately the value of semi positive definite matrix. To analyze the performance of proposed control scheme, 6DOF simulation model is developed in Simulink.**

I. INTRODUCTION

ADAPTIVE control has been attaining significant attention and advancement for the class of non linear uncertain systems, as the main aspiration of this technique is to achieve optimal system performance even when non linearities, uncertainties and variation in plant parameters are present. A large number of adaptive control laws [1][2][3] have been proposed to deal with uncertainties of non linear systems. The benefits that we could attain over non adaptive control techniques are automatic adjustment of control parameters in accordance with changing surroundings and its ability to find out the dynamics of the plant in real time [3] without having detailed information of the mathematical model.

Current research shows that the domain of fuzzy logic made a significant progress for opening new approaches of its implementation for practical systems. For various control applications many adaptive Neural Network and Fuzzy Logic based controllers with closed- loop stability have been presented [4][5]. Direct integration of linguistic information in the controller without knowing the detailed mathematical of the system are the main advantages of Fuzzy systems. By employing linguistic information, fuzzy controllers are highly robust against uncertainties and variations in plant parameters and established themselves to be universal approximators [6][7].

In fuzzy Logic, various adaptive laws have been developed recently [8][9]. Generally there are two types of approaches for designing adaptive fuzzy controller i.e. Indirect Adaptive Fuzzy control and Direct Adaptive fuzzy control [10][11]. In the former case, the adaptive controller is used to model the certain parameters of the plant however in the latter case, an adaptive law is derived which ensures the convergence of the control output. To guarantee system's stability, Lyapunov Stability principle is utilized in adaptive fuzzy control, which affirms the asymptotical stability of the system when the Lyapunov function is converging. By utilizing complex approximation function, the adaptive fuzzy system provides stabilized controller based on Lyapunov function to cater all the non linearities and uncertainties present in the system.

In this work, direct adaptive fuzzy controller is proposed to control the pitching profile of SLV. Initially point mass model has been developed for trajectory optimization which is carried out offline by using Genetic Algorithm (GA) for acquiring reference trajectories. In order to precisely follow the reference pitch attitude profile, direct adaptive fuzzy controller is designed which possess advantages such as a) no need of the knowledge of the mathematical model of the plant, b) linguistic knowledge directly incorporated in the controller and c) Lyapunov stability criteria is employed to ensure system stability. To analyze the performance and efficiency of proposed scheme, a 6 degree of freedom (6DOF) simulation model for four-stage Satellite Launch Vehicle has been developed in Simulink MATLAB. The paper is organized as follows:

Manuscript received May 14, 2012.

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978-1-4673-2529-5/12/$31.00

In section 2, Flight program is discussed in which optimization is carried out using Genetic Algorithm. Section 3 describes the detailed description of design process of Adaptive Fuzzy controller aided with Lyapunov principle to guarantee system's stability. Simulation results and conclusion are presented in section 4 and 5 respectively.

II. TRAJECTORY OPTIMIZATION

The prime objective of trajectory optimization is to restrict aerodynamic heating and loading due to structural constraints of the system. To accomplish this task, flight program has been designed in such a way that in high dynamic pressure region, the Angle of Attack (AOA) remains minimal to meet all the constraints. The constraints are applied on axial and lateral overloads to stay under 12g and 1.5g respectively. Maximum AOA should not exceed above 6 Deg and should be zero during transonic region. The flight program contains three different phases as listed below [12][13]:

1. Vertical Flight Phase ($0 \sim t_1$)
2. Turning Flight Phase ($t_1 \sim t_2$)
3. Turning Phase with Zero AOA ($t_2 \sim t_{end}$)

To perform trajectory optimization, a 3DOF simulation model is developed in Simulink MATLAB, whose equations are listed below [12][13]:

$$\dot{V} = \frac{4P\cos\alpha - D}{m} - g\sin\vartheta \tag{1}$$

$$\dot{\vartheta} = \frac{4P\sin\alpha + L}{mV} - \frac{g\cos\vartheta}{V} + \frac{V}{R_e + h}\cos\vartheta \tag{2}$$

$$\dot{h} = V\sin\vartheta \tag{3}$$

$$\dot{l} = \left(\frac{R_e}{R_e + h}\right)V\cos\vartheta \tag{4}$$

$$\varphi = \alpha - \eta + \vartheta \tag{5}$$

$$\eta = \frac{l}{R_e} \tag{6}$$

$$\alpha = \alpha_{pro} \quad \text{(Flight Program)} \tag{7}$$

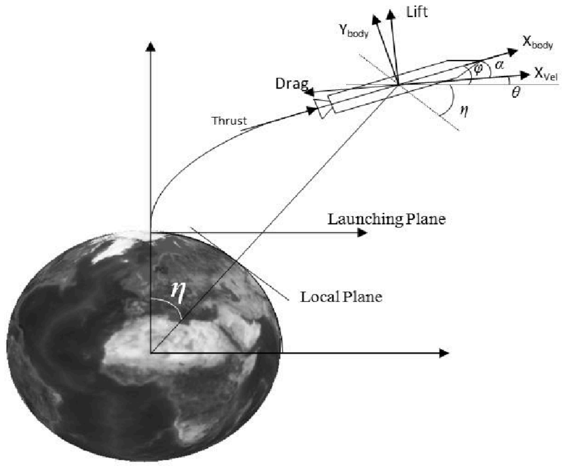

Fig. 1 Symbol's definitions for 3 DOF

In order to perform the trajectory optimization, a data of four-stage launch Vehicle is used [14]. In the optimization model several assumptions have been made such as constant thrust and constant mass flow rate whereas all the aerodynamic parameters are calculated by using DATCOM. Table 1 indicates all the major parameters of four-stage Launch Vehicle data used to develop Trajectory simulation model.

Table 1
Data of Satellite Launch Vehicle

Parameters	Stage-1	Stage-2	Stage-2	Stage-4
Liftoff mass (*ton*)	31.34	9.31	2.87	0.91
Propellant mass (*ton*)	18.76	5.55	1.72	0.54
Thrust force (*kN*)	593	262	94	30
Mass flow rate (*kg*/*s*)	265	103	37	13
Burning time (*s*)	70.7	53.8	46.9	46.9
Stage diameter (*m*)	1.3	1.0	0.7	0.5

A. Genetic Algorithms

In the past decade, Genetic algorithms gained significant importance for finding global optimal solution. They are based on the procedure of natural selection and fittest survival. They are used for solving both constrained and unconstrained optimization problems. Each individual of population is characterized by a Fitness function. Based on their fitness, parents are selected to reproduce offspring for a new generation. This process continues until the population will converge to optimal solution [15]. The objective of GA is to optimize the parameters used for trajectory optimization such as maximum AOA i.e. $\alpha_{\max}$, time to start pitch over i.e. t_1, time at which AOA becomes zero i.e. t_2 and the time at which max AOA occurs i.e. t_3 by minimizing the following objective function [13]:

$$Obj = G_1\left|H_{req} - H_{act}\right| + G_2\left|V_{req} - V_{act}\right| + G_3\left|\vartheta_{req} - \vartheta_{act}\right| \tag{8}$$

The objective function is based on the summation of absolute error of orbital height, velocity and the injection angle. After optimization, the values of these parameters along with AOA profile is shown in fig.2

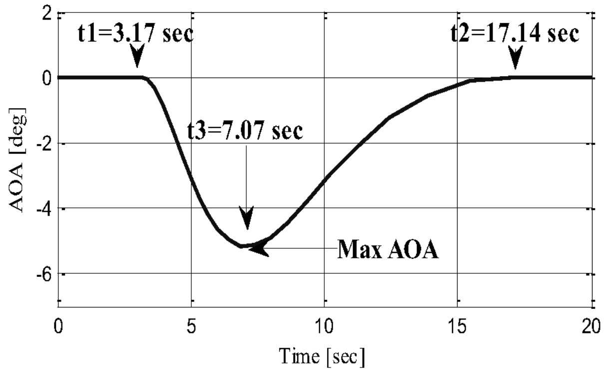

Fig. 2 Flight program vs. time

The optimized orbital parameters after trajectory optimization are depicted in table 2.

Table 2
Optimization Results

Injection orbit	Desired values	Optimal results
Height (km)	500.0	500.406
Velocity (m/s)	7612.60	7612.585
Angle of Injection (deg)	0	0.0000055

The reference trajectories of pitch angle, height, velocity and injection angle generated offline by using GA as global optimizer are depicted in fig.3.

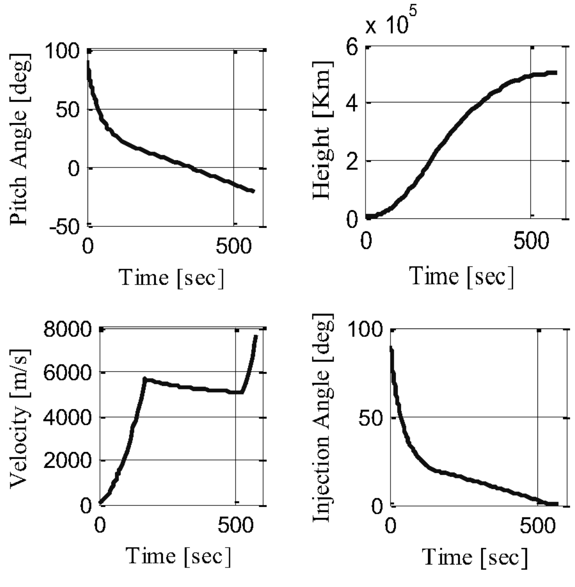

Fig. 3 Reference trajectories vs. time

In the subsequent sections, detailed 6DOF modeling and the design of fuzzy based adaptive controller for pitch channel of Satellite Launch Vehicle are discussed along with it performance evaluation.

III. DESIGNING OF DIRECT ADAPTIVE FUZZY BASED PITCH AUTOPILOT

In order to follow the reference pitch profile generated offline, 6DOF simulation model of Launch Vehicle is developed in Simulink, in which direct Adaptive Fuzzy control is employed in autopilot loop in order to analyze its performance.

A. 6DOF Trajectory Model

In 6DOF simulation model, six differential equations are modeled by assuming rigid body dynamics and 1976 standard atmosphere is taken [16].

$$m V \dot{\theta} = 4P \sin\alpha \cos\beta - mg \cos\vartheta + 2P\delta_{\varphi} \cos\alpha + L - ma_{cy} - ma_{ey} \tag{9}$$

$$m\dot{V} = 4P\cos\alpha\cos\beta - mg\sin\vartheta\cos\sigma - D - ma_{cx} - ma_{ex} \tag{10}$$

$$-mV\dot{\sigma} = -4P\sin\beta - mg\sin\vartheta\sin\sigma - 2P\delta_{\psi}\cos\beta + C - ma_{cz} - ma_{ez} \tag{11}$$

$$J_x\dot{\omega}_x - (J_y - J_z)\omega_y\omega_z = -P\delta_{\gamma} z_r \tag{12}$$

$$J_y\dot{\omega}_y - (J_z - J_x)\omega_x\omega_z = -C(x_d - x_z) - 2P(x_R - x_z)\delta_{\psi} \tag{13}$$

$$J_z\dot{\omega}_z - (J_x - J_y)\omega_y\omega_x = -L(x_d - x_z) - 2P(x_R - x_z)\delta_{\varphi} \tag{14}$$

$$\varphi = \theta + \alpha \tag{15}$$

$$\vartheta = \theta + \eta \tag{16}$$

$$\sigma = \beta + \psi \tag{17}$$

B. Designing of Direct Adaptive Fuzzy Autopilot

In fuzzy control, linguistic information is incorporated which is effectively applied to many practical systems with the intention of acquiring improved control performance [17]. Due to the presence of uncertainties and non linearities in the system, some adaptive mechanism is required to adjust the control parameters itself in accordance with varying environment. Therefore it is recommended to have adaptive fuzzy system through which better performance can be achieved. This paper demonstrates the design process of direct adaptive fuzzy technique to control the attitude of launch vehicle. The detailed block diagram of Adaptive law based on fuzzy logic to control pitch attitude of SLV is shown in figure.4

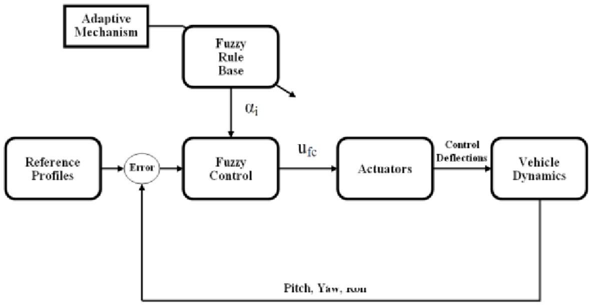

Fig. 4 Adaptive Fuzzy based Pitch autopilot.

The detailed derivation of Adaptive Fuzzy controller is established here. In the design procedure, the resulting component of the fuzzy rule is considered to be adjustable parameter by assuming that the Membership functions remain fixed [10]. Now the main goal of adaptive law is to recognize the adjustable parameter by maintaining the stability using Lyapunov principle. The nth-order representation of nonlinear system is written as:

$$x^n = f\left(x, \dot{x}, \ldots, x^{(n-1)}\right) + bu \qquad (18)$$
$$y = x$$

In the above equation, f is a continuous function, u is input of the system, y is the system's output and b is a positive constant. The prime objective of the controller is to follow the reference command signal y_m in order to meet the desired performance. Now considering that the system parameters are known i.e. f and b, at that time it would be feasible to derive the perfect feedback linearization control law as defined below:

$$u^* = \frac{1}{g(x)}\left[-f(x) + y_m^{(n)} + K^T e\right] \qquad (19)$$

However in reality it would not be possible to precisely develop the mathematical model including all the non linearities and uncertainties. That is why in order to design a controller for such scenario, an adaptive mechanism is required that contains some tunable parameters to make the designed controller approaching the optimal feedback control law u^*.

The prime objective of the control law is to develop the adaptive fuzzy control system which contains an adjustable parameter, such that the designed controller u_{fc}^* approaches the optimal feedback control u^*. In our system the input to the fuzzy system is pitch angle error and its rate, so objective of fuzzy is to execute mapping between input states and the controller output u_{fc}. Since the system has two inputs, each input containing five membership functions, so the fuzzy rule base contains 25 rules in total. Five fuzzy sets are defined for each input i.e. Pitch angle error and its rate over the interval [-5 5] and [-1.5 1.5] respectively.

The i_{th} rule in the design process is written as:

$$Rule\, i: IF\ e\ is\ F_e^i\ and\ \dot{e}\ is\ F_{\dot{e}}^i, THEN\ u_{fc}\ is\ \alpha_i$$

For each input variable, IF part is constructed by using triangular shaped membership functions. The subsequent element of individual fuzzy rule is the singleton control action denoted by $\alpha_i\ i = 1, 2, \ldots, m$ where m represents the number of fuzzy rules. Center of Gravity method is used for defuzzification of the control output as described below.

$$u_{fc}(e,\alpha) = \frac{\sum_{i=1}^{m} w_i \times \alpha_i}{\sum_{i=1}^{m} w_i} \qquad (20)$$

Where firing weight is denoted by w_i for the i_{th} rule. In the proposed method the singleton control action α_i is selected to be a tunable parameter. The above equation of control output u_{fc} which is a function of error and adjustable parameter can be written as:

$$u_{fc}(e,\alpha) = \alpha^T \xi \qquad (21)$$

In the above equation, $\alpha = \left[\alpha_1, \alpha_2, \ldots, \alpha_m\right]^T$ and $\xi = \left[\xi_1, \xi_2, \ldots, \xi_m\right]^T$ representing the parameter vector and regressive vector respectively with ξ_i defined as

$$\xi_i = \frac{w_i}{\sum_{i=1}^{m} w_i} \qquad (22)$$

The optimal fuzzy control system $u_{fc}^*\left(e, \alpha^*\right)$ in accordance with the universal approximation theorem [8] is written in the following form:

$$u^*(t) = u^*_{fc}\left(e, \alpha^*\right) + \varepsilon = \alpha^{*T}\xi + \varepsilon \qquad (23)$$

Where ε is the approximation error defined as:

$$\varepsilon = u^* - u_{fc}^*\left(e|\alpha^*\right) \qquad (24)$$

In order to approximate the optimal feedback control law u^*, the estimated fuzzy control law $\hat{u}_{fc}\left(e|\hat{\alpha}\right)$ is written as

$$\hat{u}_{fc}\left(e|\hat{\alpha}\right) = \hat{\alpha}^T \xi \qquad (25)$$

Where $\hat{\alpha}$ is the estimation of α^*. After incorporating this, the direct adaptive fuzzy control law is represented as,

$$u(t) = \hat{u}_{fc}\left(e|\hat{\alpha}\right) = \hat{\alpha}^T \xi \qquad (26)$$

The estimated error between the adaptive fuzzy law and the optimal control containing the following form:

$$\tilde{u}_{fc} = \hat{u}_{fc} - u^{*} = \hat{u}_{fc} - u_{fc}^{*} - \varepsilon \qquad (27)$$

Where estimation of tunable parameter is defined as:

$$\tilde{\alpha} = \hat{\alpha} - \alpha^{*} \qquad (28)$$

To guarantee system's stability, Lyapunov function is used in order to force the variables $e(t)$ and $\tilde{\alpha}$ tend to zero in the following form:

$$V = \frac{1}{2} e^{T} P e + \frac{b}{2\gamma} \tilde{\alpha}^{T} \tilde{\alpha} \qquad (29)$$

Where γ is a positive constant. The time derivative of above equation is written as:

$$\dot{V} = -\frac{1}{2} e^{T} Q e + e^{T} P b \left[\left(\alpha^{*} - \alpha \right)^{T} \xi - \varepsilon \right] - \frac{b}{\gamma} \left(\alpha^{*} - \alpha \right)^{T} \dot{\alpha} \qquad (30)$$

Or

$$\dot{V} = -\frac{1}{2} e^{T} Q e + \frac{b}{\gamma} \left(\alpha^{*} - \alpha \right)^{T} \left[\gamma e^{T} P_{n} \xi - \dot{\alpha} \right] - e^{T} P_{n} b \varepsilon \qquad (31)$$

Where $b = [0, 0, \ldots, b]^{T}$, P is a positive semi definite matrix whose last column is denoted by P_{n}.

For achieving $\dot{V} < 0$ the adaptation law is taking the following form:

$$\dot{\alpha} = \gamma e^{T} P_{n} \xi \qquad (32)$$

Therefore,

$$\dot{V} = -\frac{1}{2} e^{T} Q e - e^{T} P_{n} b \varepsilon \qquad (33)$$

Since sufficient fuzzy rules are designed, therefore we can say that $\boldsymbol{\varepsilon}$ can be very small, thus

$$\left| e^{T} P_{n} b \varepsilon \right| < \frac{1}{2} e^{T} Q, \ therefore\ \dot{V} < 0 \qquad (34)$$

Hence it is sufficient condition to declare that the system is stable.

IV. SIMULATION RESULTS

In 6DOF simulation model, various simulations have been carried out to measure the performance of Fuzzy based adaptive control law in the presence of the following disturbances listed below in table 3, which consequently affect the pitching profile of the Launch Vehicle.

Table 3
Trajectory Dispersion Sources

Error sources	Deviation
Thrust misalignment	0.25 deg
Wind	60.96 m/s max.
Drag coefficient	6.7%
Thrust variation	2%

In adaptive law, the positive definite matrix used in the Lyapunov Function whose last column is denoted by P_{n} should be selected appropriately to guarantee system stability. Several simulations are carried out by considering various values of P_{n} to see its effect as depicted in fig 5.

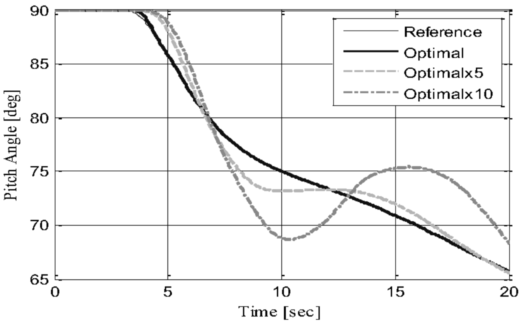

Fig. 5 Variation of P_{n} on controller performance.

The above figure depicts the performance comparison of using different values of P_{n} in initial pitch over. It is evident that the performance has been detoriated as values of column vector P_{n} is changed from the optimal values. Hence it can be concluded that during design process, column vector P_{n} must be selected properly, otherwise instability will be observed if selected inappropriately.

Figures 6 and 7 depict the controlled pitch profile and the control deflection respectively in order to show the efficacy of proposed control algorithm in initial pitch over phase by accurately following the reference pitch angle. The complete pitching profile and the control deflection from launch till injection are shown in fig. 8 and 9.

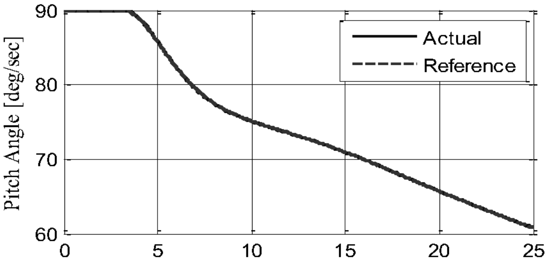

Fig. 6 Pitch control in initial pitch over

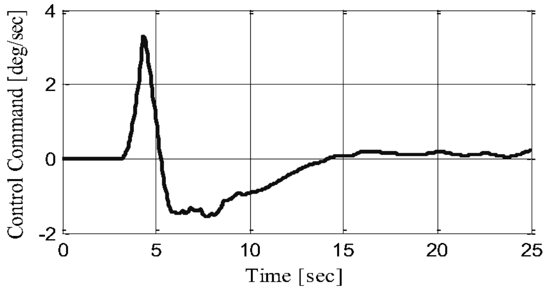

Fig. 7 Control deflection in initial pitch over

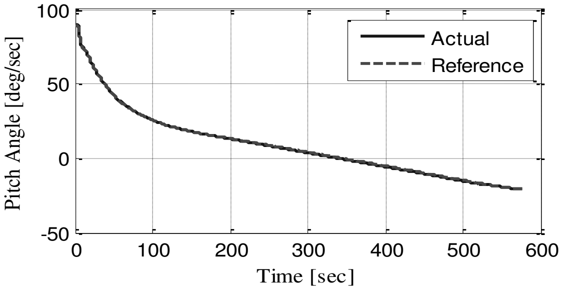

Fig. 8 Pitch angle vs. time

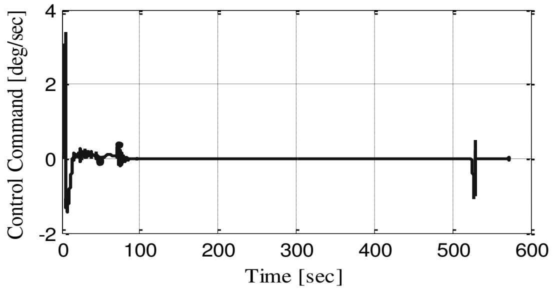

Fig. 9 Control deflection vs. time

To evaluate the robustness of the designed controller, the aerodynamic coefficients such as drag and lift coefficients, center of gravity and center of pressure has been varied $\pm 20\%$ from their nominal values. The simulation result of these variations in initial pitch over phase is shown in fig. 10.

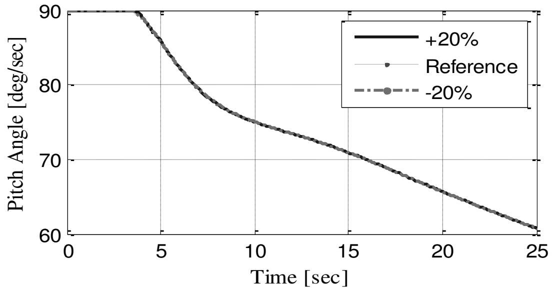

Fig. 10 Robustness of pitch autopilot

The controller effectively follows the reference pitch profile catering all the variations due to parametric uncertainties. Hence it is deduced that by using adaptive fuzzy control aided with Lyapunov principle, better results can be achieved with in tolerable limits.

V. CONCLUSION

This paper presents the design process of direct adaptive fuzzy controller to efficiently control the pitch angle of SLV. To generate the reference trajectories, point mass model is developed; in which trajectory optimization has been done successfully using Genetic Algorithms by taking account all the constraints. To analyze the performance of adaptive fuzzy controller, 6DOF simulation model is developed in Simulink. The proposed methodology has effectively verified the design process of the fuzzy controller using adaptive law for attitude control of Launch Vehicle. In the design process, Lyapunov stability principle is utilized to maintain system stability. The variation of P, which is a positive semi definite matrix, should be selected properly required in the Lyapunov method. The adaptive law adjusts the tunable parameter accordingly to achieve desired controller performance. The simulation result shows the effectiveness of Adaptive fuzzy control law which efficiently tracks the desired pitch profile. The presented approach based on adaptive law is efficient, effective and has a considerable potential for a variety of control problems.

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