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A Comprehensive Evaluation of Three Robust Adaptive Control Methodologies

Vahid Hassani and António M. Pascoal

Abstract— **Using Monte-Carlo simulations, the performance of three recently developed multiple-controller robust adaptive control methods is compared for different time-varying uncertain parameter waveforms, using a mass-spring-dashpot benchmark example. We further examine the performance of these different adaptive methods with that of the best possible robust nonadaptive design using the same physical example. Whenever possible, we also compare the adaptive methods' performance with that of the (unrealizable) "Perfect Model Identification (PM.ID)". In order to have a "fair" comparison of the performance obtained with the different control laws we used the same bank of local controllers and tuned each approach for its best performance. The Monte-Carlo simulations highlight the strength and aptitude of each method as well as its shortcomings and drawbacks.**

I. INTRODUCTION

In many practical applications of control theory, it is virtually impossible to obtain a highly accurate mathematical model of the physical process of interest. Moreover, all practical systems are subjected to uncertainty caused by a number of reasons that may include unmodeled dynamics, uncertain and unknown system parameters, plant disturbances, measurement noise, changes in operating conditions, failure or degradation of components, and unexpected changes in the system dynamics. Consequently, a practical controller should be able to provide stability and performance in the presence of the above types of uncertainty. Modern control theories, e.g. LQR, LQG, H_∞ and μ-synthesis [1], [2], provide powerful tools to control such systems when the model uncertainties are small enough; however, in the case of "large" uncertainty, a single fixed LTI controller that robustly stabilizes the system and achieves satisfactory closed-loop performance may not exist. Adaptive control is often required to control plants with large parameter uncertainty. Adaptive control has the potential to deal with large parametric uncertainty by adapting controller gains thanks to real-time parameter identification, but the mapping from estimated plant parameters to controller gains in the case of H_∞ and μ designs is not clear; this often curtails the use of well developed theories of H_∞ and μ-synthesis in conventional adaptive control. To overcome this hurdle, a multi-controller approach is often adopted. By using off the shelf candidate controllers, designed off-line, a multi-controller architecture avoids real-time controller synthesis and provides an attractive framework for combining adaptive and modern robust tools. Different versions of adaptive control using multi-controllers have been reported in the last decade. Numerous results based on these methods can be found for example in [3]–[7]. The adaptive architectures proposed consist of a set of candidate controllers and an identification module. At least one of the candidate controllers is assumed to meet desired closed-loop stability and performance requirements. The identification module tries to select a controller whose performance is "better" than the other controllers in the candidate set of controllers.

One of the promising multi-controller approaches is based on the so-called robust multiple model adaptive control (RMMAC) methodology that provides guidelines for designing both the bank of candidate controllers (using mixed-μ synthesis tools) and the identification module which consists of a bank of Kalman filters [5]. At some point, inspired by the RMMAC approach, Kuipers and Ioannou developed the so called multiple model adaptive control with mixing (MMAC-WM) [6], [8]. While the MMAC-WM exploits the same bank of controllers as the one in the RMMAC, the identification module in MMAC-WM consists of a parameter estimator (or a bank of parameter estimators) to estimate an unknown parameter and select the corresponding controller. More recently, a similar approach called multiple controller robust adaptive control using extended Kalman filter (MCRAC-EKF) was developed [7]. In MCRAC-EKF, an extended Kalman filter is designed to estimate uncertain parameters and the estimates are then used to compute the dynamic weights which blend adaptively the candidate control laws.

We feel that a fair comparison of the above mentioned approaches can shed light into the strengths and limitations of each method and call attention for future research. In this article, the benchmark example of Mass-Spring-Dashpot (MSD) [5], [8] is used to examine the performance comparison of the abovementioned adaptive control methods with the help of Monte-Carlo simulations. Different scenarios with different time-varying uncertain parameter waveforms and constant disturbance intensities are examined. Furthermore, for the sake of a better comparison the performance of two more controllers is also discussed: the (unrealizable) "Perfect Model Identification (PM.ID)", that selects the local controller according the real value of the uncertain parameter, and the best possible robust nonadaptive controller.

The structure of the paper is as follows. The benchmark example of a mass spring dashpot is introduced II. A brief description of each robust adaptive control methodology is given in section III. In Section IV we present the results of simulations with stochastic signals that illustrate the performance of adaptive controllers and the best non adaptive one. Conclusions and suggestions for future research are summarized in Section V.

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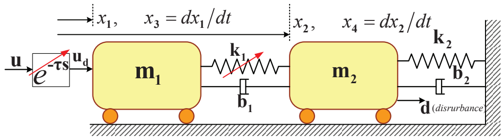

Fig. 1. The two-cart MSD system. The spring-constant k_1 is uncertain.

II. BENCHMARK TEST-BED: THE MASS SPRING DASHPOT

The mass-spring-dashpot (MSD) system will be used as a test-bed for performance comparison of different adaptive control design. In the early 1990's Dennis Bernstein used this testbed example as a control challenge problem and for years the MSD has been a well known benchmark for robust control design [9]. A slightly different version of the MSD was introduced in [5] as benchmark test-bed of robust adaptive control methodologies. In the MSD testbed example explored here, the control signal is noncollocated with the performance variable; furthermore, it is also coupled by an uncertain (time-varying) spring.

Many practical applications are similar to the MSD testbed sharing the noncollocated control property such as: i) active automotive suspension systems, ii) flexible robot arms, where the control is away from the robot tip, and 3) flexible space structures where the controls (say, control-moment gyros) are away from the radar/telescope that must be pointed accurately. All of these physical applications involve noncollocated controls and flexible structures between the control and the desired rectilinear or angular position. Different adaptive control methodologies have been applied to the MSD benchmark example [6], [8], [10].

The two-cart mass-spring-damper (MSD) system is shown in Fig. 1. The plant disturbance $d(t)$ is a low-frequency, stationary stochastic process acting on m_2 that is generated by shaping a continuous-time white gaussian process as follows

$$d(s) = \frac{a}{s+a}\xi(s),$$

where $\xi(t)$ has zero mean and unit intensity ($\Xi = 1$), and $a > 0$ is the disturbance bandwidth (in this work taken $a = 0.1$ rad/sec). The only measurement available $y(t)$ is the position of cart m_2 corrupted by measurement noise $\theta(t)$, i.e. $y(t) = x_2(t) + \theta(t)$, where the statistics of the sensor noise $\theta(t)$ are known to be zero mean with intensity $E\{\theta(t)\theta(\tau)\} = 10^{-6}\delta(t-\tau)$. The control $u(t)$ is applied to m_1 through a control channel with an uncertain delay τ. The maximum delay through this channel is known to be 0.05 sec.

The performance variable is given by

$$z(s) = \frac{A_p}{s+a}x_2(s)$$

and the control objective is to keep it small for the largest value of the performance index A_p. A state-space representation of the plant, including the disturbances and noise inputs, is given by

$$\dot{x}(t) = Ax(t) + Bu_d(t) + L\xi(t), \quad (1)$$
$$y(t) = Cx(t) + \theta(t), \quad (2)$$

with state vector

$$x(t) = [x_1(t)\ \dot{x}_2(t)\ x_1(t)\ \dot{x}_2(t)\ d(t)]^T$$

and

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \frac{-k_1}{m_1} & \frac{k_1}{m_1} & \frac{-b_1}{m_1} & \frac{b_1}{m_1} & 0 \\ \frac{k_1}{m_2} & \frac{-(k_1+k_2)}{m_2} & \frac{b_1}{m_2} & \frac{-(b_1+b_2)}{m_2} & \frac{1}{m_2} \\ 0 & 0 & 0 & 0 & -0.1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix},$$

$$L^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0.1 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix},$$

where $m_1 = m_2 = 1\,$Kg; $k_2 = 0.15\,$N/m; $b_1 = b_2 = 0.1\,$Ns/m; and k_1 is an unknown parameter assumed to have a value in the interval $[0.25,\ 1.75]$.

III. BRIEF REVIEW OF RMMAC, MMAC-WM, & MCRAC-EKF

In what follows we summarize the results of local controller design for the RMMAC, MMAC-WM, and MCRAC-EKF methodologies. For a detailed description of each methodology see [5]–[7]. The uncertainty set is broken down into four smaller intervals and, associated with each subinterval, a robust controller is designed using the mixed-μ synthesis [2], [5]. These controllers are called local non-adaptive robust controllers (LNARC). Moreover, the global non-adaptive robust controllers (GNARC) which is the best non-adaptive robust controllers that can provide robust stability and performance is designed using the mixed-μ synthesis. The design summary can be found in Table I. Here, we should emphasize that the RMMAC, MMAC-WM, and MCRAC-EKF use the same bank of controllers.[1] Common to all these methods is the use of information obtained online to decide on appropriate control actions. The adaptive architectures proposed consist of a set of candidate controllers and an identification module. The identification module tries to blend the control laws of local controllers in the bank. The rationale is to associate a higher gain to the controller whose performance is "better" than the other controllers in the candidate set of controllers. All the above mentioned techniques use a model-based approach. The overall structure of the above mentioned controllers is depicted in Fig. 2.

TABLE I
SUMMARY OF DESIGNING THE CONTROLLERS

Controller	Uncertainty Interval	A_p
GNARC	[0.25 1.75]	50.75
LNARC # 1	[0.25 0.40]	694.5
LNARC # 2	[0.40 0.64]	694.5
LNARC # 3	[0.64 1.02]	694.5
LNARC # 4	[1.02 1.75]	694.5

In RMMAC, the identification module consists of a bank of Kalman filters running in parallel and a posterior probability evaluator. At each sample time, the output estimation error of each Kalman filter is analyzed. By running a dynamic

[1] Since the RMMAC, the MMAC-WM, and the MCRAC-EKF share the same bank of controllers, throughout this paper we do not present any comparison on the control actions among the above mentioned adaptive controllers.

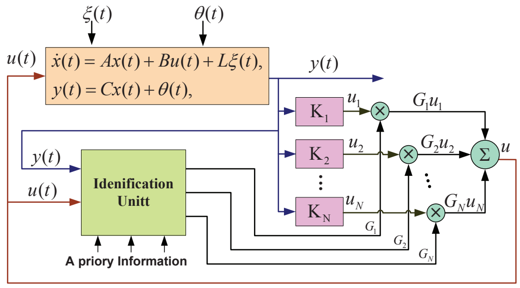

Fig. 2. Multi-Controller Robust Adaptive Control: the basic architecture.

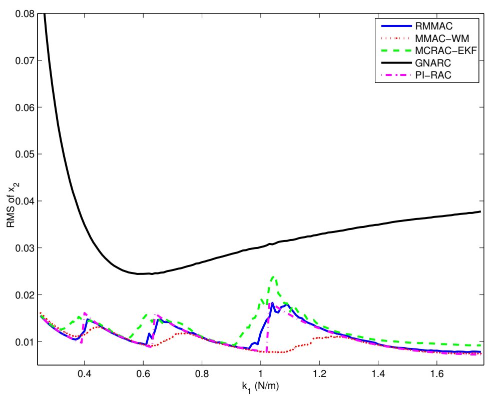

Fig. 3. Comparison of the RMS of the output signal x_2 for different values of k_1 and different controllers.

hypothesis testing, the filter that "best" approximates the uncertain plant is assigned a higher weight and, consequently, the associated local controller is given a higher weight among other local controllers in the bank of local controllers.

In MMAC-WM the identification module consists of a parameter estimator to estimate an unknown parameter and select the corresponding controller. This approach is limited to linear parametric model (LPM). One of the potential drawbacks of this methodology is the presence of a possibly large number of tuning parameters. Moreover, the approach is limited to single parametric uncertainty. A comprehensive description of the method can be found in [6], [8].

In MCRAC-EKF, the identification unit is an extended Kalman filter (EKF). In this approach, an EKF is used to estimate the uncertain parameters. Consequently the control actions are blended based on the estimated parameters. No stability proof is available for this methodology, yet. However, the method can in principle deal with two or even more uncertain parameters. Furthermore, there are no tuning parameters and the design of the identification unit is straightforward.

IV. SIMULATION RESULTS

In this section we apply the RMMAC, MMAC-WM, and MCRAC-EKF methodologies to stabilize and control the MSD plant. All the results in this paper are result of averaging 10 Monte Carlo simulations, and the sample time of 10 ms is used. Also one sample time delay in the control channel is used in all the simulations.

Fig. 3 illustrates the result of a Monte-Carlo simulation where, for a hundred different values of k_1 (uniformly selected from the uncertainty interval), the average of 10 Monte-Carlo simulations is shown. The RMS value of the output x_2 is compared under three different controllers: RMMAC, MMAC-WM, and MCRAC-EKF. In order to have a better understanding of the performance of the mentioned adaptive controllers, we define the "Perfect Identification Robust Adaptive Control" (PI-RAC) which simply generates the plant control $u(t)$ in Fig. 1 by switching in the correct local controller at the instant that the parameter k_1 crosses the model boundaries; clearly this controller is infeasible in practice because it needs access to the real value of uncertain parameter, but it helps to better evaluate the performance of the different adaptive approach. Moreover, to highlight the performance improvement of the adaptive controllers, the results of another controller is included: the global non-adaptive robust control (GNARC) which is the best nonadaptive robust controller that can provide robust stability and robust performance, designed with mixed-μ synthesis [2], [5].

In the RMMAC methodology there is no need for tuning parameters thanks to systematic and performance based design of the controller. The MMAC-WM has few parameters to tune, and in this experiment we have tuned these parameters to have the highest performance when the uncertain parameter is constant [11]. The only tuning parameter in the MCRAC-EKF is the intensity of the fake white noise.

In Fig. 3 it can be easily seen that RMMAC, MMAC-WM, and MCRAC-EKF have almost the same performance level as that of PI-RAC. The performance improvements of adaptive controllers over GNARC is evident in Fig. 3.

We stress that the performance of any adaptive system must be evaluated not only for constant unknown parameters, but also for time-varying parameters that undergo slow or rapid time-variations. In what follows we examine different scenarios where the unknown parameter, k_1, changes fast or slowly.

Case I: k_1 changing slowly

We start by analyzing the behavior of the RMMAC, the MMAC-WM, and the MCRAC-EKF controller under the spring coefficient time-variation shown in the Fig. 4. In this simulation the spring coefficient is steady for 75 s. Then, it changes linearly until it is within the next model, keeping steady for another 75 s, and repeating the procedure. These changes take 25 s each, so the slope is, in general, not always the same. This is shown in the Fig. 4, which also shows (using the dashed lines) the model boundaries as defined by Table I. Furthermore, the estimated parameter $\hat{k}_1$ by MCRAC-EKF and MMAC-WM are also depicted for the sake of completeness. The RMS of the regulated state (x_2) under different controllers is computed in Table II. We have also computed the performance improvement of the adaptive approaches over GNARC and the performance degradation (if any) of adaptive controllers compared with PI-RAC.

To quantify the performance improvement of RMMAC, MMAC-WM, and MCRAC-EKF over GNARC and also to evaluate performance degradation of using them rather than

PI-RAC, we introduce the following percentage comparisons

$$\%E = \frac{|\text{adaptive}_{RMS(x_2)}| - |\text{PI-RAC}_{RMS(x_2)}|}{|\text{PI-RAC}_{RMS(x_2)}|} \times 100 \quad (3)$$

$$\%F = \frac{|\text{GNARC}_{RMS(x_2)}| - |\text{adaptive}_{RMS(x_2)}|}{|\text{adaptive}_{RMS(x_2)}|} \times 100 \quad (4)$$

Thus, %E measures the performance degradation of the RMMAC compared to that of the (unrealizable) instantaneous correct identification of the PM.ID. Similarly, %F measures the performance degradation of the non-adaptive GNARC design as compared to that of the RMMAC.

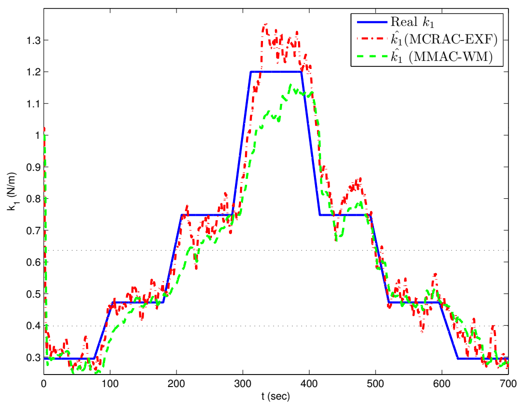

Fig. 4. Time variation of k_1 and its estimated values (case I).

TABLE II
THE RMS VALUE OF THE OUTPUTS (CASE I)

	x_2 (RMS)	%E	%F
GNARC	0.044	249 %	0 %
PI-RAC	0.012	0 %	249 %
RMMAC	0.013	3.76 %	236 %
MMAC-WM	0.013	3.76 %	236 %
MCRAC-EKF	0.013	4.34 %	235 %

The RMS of the output ($x_2(t)$) experience 249 % performance improvement by using (infeasible) PI-RAC over GNARC. The interesting observation is that all three adaptive controllers have similar performance improvement of (almost) 236 % over nonadaptive controller. Similarly the performance degradation of all adaptive controllers comparing with PI-RAC is negligible, ((almost) 3 % in this case).

Case II: k_1 with step changes (stable region)
Next, a slightly different scenario is analyzed. The spring coefficient time-variation is the staircase function shown in the Fig. 5, which means that it stays constant for 100 s and then jumps to another constant value. According to [5] the local controller is still able to stabilize the new value of k_1. Furthermore, the estimated parameter $\hat{k}_1$ by MCRAC-EKF and MMAC-WM are also depicted in Fig. 5. It can be seen that both approaches can follow the changes of the uncertain parameter. Table III shows again the superior performance of all three adaptive controllers (RMMAC, MMAC-WM, and MCRAC-EKF) over the GNARC is obvious.

Again the RMS of the output ($x_2(t)$) experience 253 % performance improvement by using (infeasible) PI-RAC

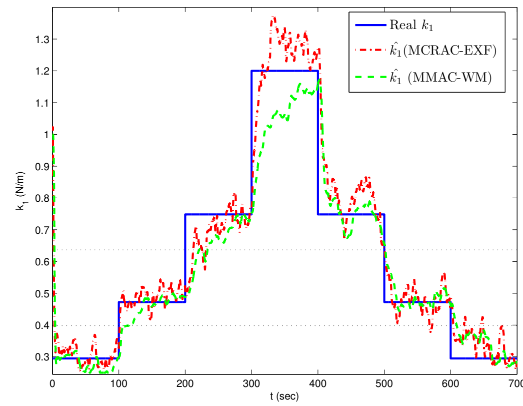

Fig. 5. Time variation of k_1 and its estimated values (case II).

TABLE III
THE RMS VALUE OF THE OUTPUTS (CASE II)

	x_2 (RMS)	%E	%F
GNARC	0.046	253 %	0 %
PI-RAC	0.013	0 %	253 %
RMMAC	0.012	-0.75 %	255 %
MMAC-WM	0.013	3.30 %	242 %
MCRAC-EKF	0.013	4.47 %	238 %

over the GNARC. The interesting observation is that the RMMAC yields better performance even compared with that obtained with PI-RAC. The reason is soft switching nature of the RMMAC, thanks to blending local control actions with posterior probabilities.[2] The MCRAC-EKF and the MMAC-WM show comparable performance.

Case III: k_1 with step changes (unstable region)
Next we consider a case where the parameter jumps result in temporary instability. The spring coefficient time-variation is the staircase function shown in the Fig. 6, which means that it stays constant for 100 s and then jumps to another constant value. In this experiment the new value of k_1 after each jump is not in the stability interval of the local controller which was in the loop before the jump. *In fact, this experiment illustrates a forced instability situation.* Table IV summarizes the results.

TABLE IV
THE RMS VALUE OF THE OUTPUTS (CASE III)

	x_2 (RMS)	%E	%F
GNARC	0.074	357 %	0 %
PI-RAC	0.016	0 %	357 %
RMMAC	0.040	150 %	82 %
MMAC-WM	6.686	41205 %	-99 %
MCRAC-EKF	0.033	104 %	124 %

In this simulation, the MMAC-WM yields very poor performance, even worse than that obtained with the non adaptive controller. Also, Fig. 6 shows that MMAC-WM cannot estimate the unknown parameter well. To avoid this,

[2]This is due to the fact that in the RMMAC (and also in MCRAC-EKF and MMAC-WM) the control action is a weighted sum of the local control actions (the LNARCs) and the switching between the local controllers is carried out smoothly. However, in PI-RAC the switching between two local controllers is done instantly.

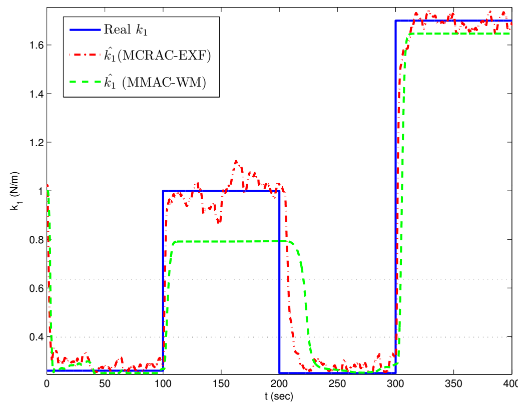

Fig. 6. Time variation of k_1 and its estimated values (case III).

we retuned the parameters in the MMAC-WM to make it capable of following faster parameter changes (see [11] for how to tune the parameters in the MMAC-WM). We repeated the simulation with new parameters in the MMAC-WM.

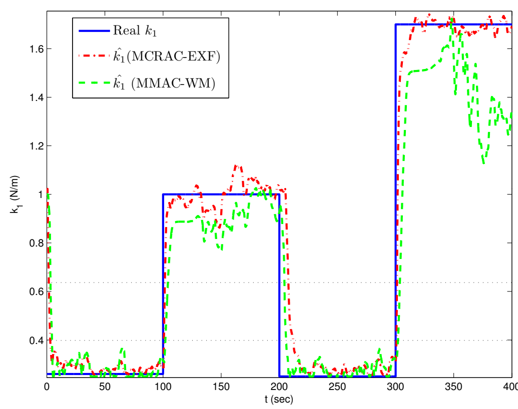

Fig. 7. Time variation of k_1 and its estimated values (case III-After tuning MMAC-WM).

TABLE V

THE RMS VALUE OF THE OUTPUTS (CASE III-AFTER TUNING MMAC-WM)

	x_2 (RMS)	%E	%F
GNARC	0.074	356 %	0 %
PI-RAC	0.016	0 %	356 %
RMMAC	0.043	164 %	73 %
MMAC-WM	1.480	9067 %	-95 %
MCRAC-EKF	0.033	105 %	123 %

The MMAC-WM problem is alleviated slightly, see Fig. 7 and Table V. In this experiment we observe that the RMMAC and MCRAC-EKF recover rapidly from the forced instability while the MMAC-WM needs more time to adapt, which lead to performance degradation. Moreover, MCRAC-EKF shows the best performance among the adaptive controllers while the MMAC-WM has the poorest performance.

Case IV: Sinusoidal time-varying spring coefficient
This experiment shows another simulation in which k_1 varies according to a sinusoidal waveform.

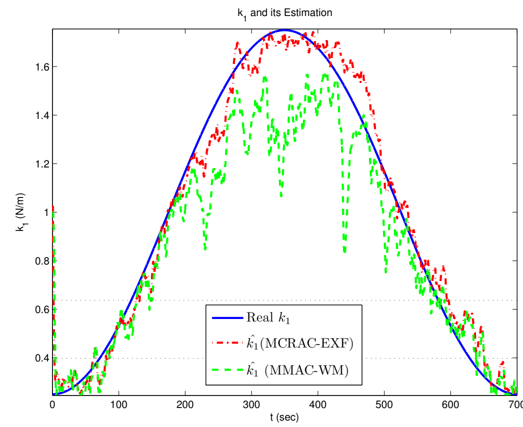

Fig. 8. Time variation of k_1 and its estimated values (case IV-Slow Sinusoid).

TABLE VI

THE RMS VALUE OF THE OUTPUTS (CASE IV-SLOW SINUSOID)

	x_2 (RMS)	%E	%F
GNARC	0.045	273 %	0 %
PI-RAC	0.012	0 %	273 %
RMMAC	0.011	-10 %	316 %
MMAC-WM	0.013	8 %	246 %
MCRAC-EKF	0.013	13 %	231 %

Fig. 8 shows the time-variation of k_1 and its estimated value by MMAC-WM and MCRAC-EKF. The simulation result, summarized in Table VI, shows that all adaptive controllers have comparable performance. Fig. 9 shows the results of a simulation with a faster sinusoidal waveform. See also Table VII. The results of this experiment are smmerized in Table VII. No performance improvement is experienced by using MMAC-WM over non adaptive controller, while the RMMAC and the MCRAC-EKF have about 50 % performance improvement. Furthermore, our simulation shows that when the unknown parameter changes very fast, the adaptive controllers cannot follow the varying parameter.

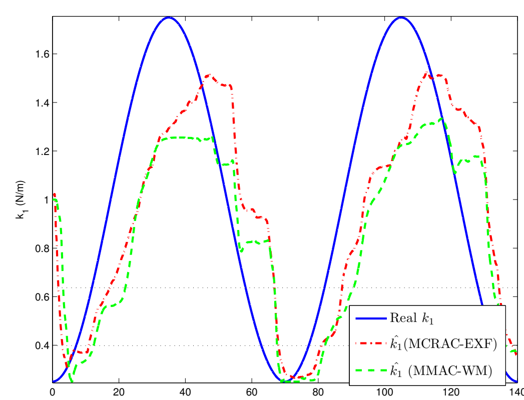

Fig. 9. Time variation of k_1 and its estimated values (case IV-Fast Sinusoid).

We repeated this simulation with an even faster sinusoid; see Fig 10 and Table VIII where all the adaptive controllers

TABLE VII
THE RMS VALUE OF THE OUTPUTS (CASE IV-FAST SINUSOID)

	x_2 (RMS)	%E	%F
GNARC	0.029	140 %	0 %
PI-RAC	0.012	0 %	140 %
RMMAC	0.017	42 %	69 %
MMAC-WM	0.029	143 %	-1 %
MCRAC-EKF	0.017	48 %	62 %

have worse performance than non-adaptive one; specially MMAC-WM exhibits very poor performance.

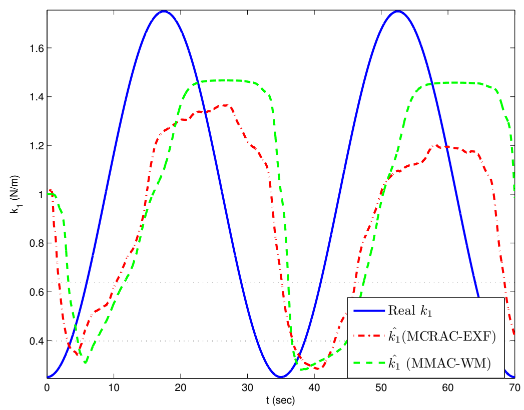

Fig. 10. Time variation of k_1 and its estimated values (case IV-Very Fast Sinusoid).

TABLE VIII
THE RMS VALUE OF THE OUTPUTS (CASE IV-VERY FAST SINUSOID)

	x_2 (RMS)	%E	%F
GNARC	0.025	79 %	0 %
PI-RAC	0.012	0 %	79 %
RMMAC	0.031	164 %	-32 %
MMAC-WM	1.26	10430 %	-98 %
MCRAC-EKF	0.026	113 %	-16 %

To conclude, we repeated the first simulation with a constant uncertain parameter, for the case where the MMAC-WM was tuned for a fast changing parameter. Fig. 11 illustrates the result of a Monte-Carlo simulation where, for a hundred different values of k_1 (uniformly selected from the uncertainty interval), the average of 10 Monte-Carlo simulations is shown and the RMS value of the output x_2 is compared under three different controllers: RMMAC, MMAC-WM, and MCRAC-EKF. It is clear that when the MMAC-WM is tuned for fast parameter changes, then it cannot identify the correct value of spring coefficient for soft springs. The second sub-figure in Fig. 11 is a zoom-in view of the first sub-figure.

V. CONCLUSIONS

Using Monte-Carlo simulations, the performance of three recently developed multiple-controller robust adaptive control methods was compared for different time-varying uncertain parameter waveforms, using the test example of a mass-spring-dashpot system. The Monte-Carlo simulations highlighted the strength and aptitude of each method as well

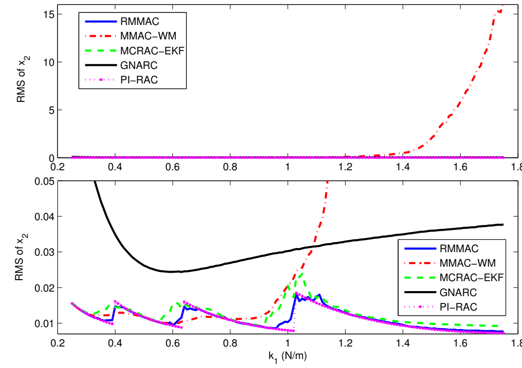

Fig. 11. Comparison of the RMS of the output signal x_2 for different values of k_1 and different controllers (MMAC-WM is tuned to fast parameter changes).

as its shortcomings and drawbacks. We showed, with the help of simulations, that when the uncertain parameter is constant all three adaptive controllers have similar performance. In the case of time varying uncertain parameters, when the uncertain parameter changed slowly all controllers could stabilize the plant but some parameters in the MMAC-WM needed to be tuned while the MCRAC-EKF and RMMAC needed no modifications. Moreover, our simulations showed that when MMAC-WM was tuned for fast parameter changes, then it could not cope with constant uncertain parameters (soft spring).

VI. ACKNOWLEDGMENTS

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