

A Synchronous Coordinates Approach in Position and Speed Estimation for Permanent Magnet Synchronous Machines

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Abstract—A novel and simple observer scheme is proposed for rotor speed and position reconstruction for Permanent Magnet Synchronous Machines. The reference frame adopted in the observer is pushed toward the synchronous one by forcing it to be intrinsically aligned with the estimated back-emf vector and by designing suitable adaptations law for its speed and angle along with the back-emf amplitude. Stator flux dynamics are not used in this approach, leading to an improved robustness with respect to voltage and current measurement uncertainties. Stability analysis is carried out by using singular perturbation approach. Effective tuning rules are drawn exploiting insightful linearization of the proposed nonlinear adaptive observer. Simulations show the properties of the presented method.

I. INTRODUCTION

Permanent Magnet Synchronous Machines (PMSM) are a class of electrical machines commonly used for a wide range of applications. Vector control methods are usually adopted to ensure an efficient regulation; they require the knowledge of the rotor position, and, when a speed loop is implemented, the rotor speed feedback is also needed.

The desire of reducing the cost and the number of components, improving, at the same time, the system reliability, has stimulated the research towards *sensorless* control algorithms. A natural choice is to augment the system with an observer and feed the controller with the estimated variables. An intense research activity has been carried out to cope with this problem, it has also been discussed in some monographs ([1], [2], [3]) concerning nonlinear and adaptive control solutions applied to electrical drives regulation. Despite the topic is mature, and many practical applications have been successfully implemented, there is still room for improving the estimation scheme performance, particularly under some well known critical conditions.

Literature on estimation of PMSM parameters is divided into two main categories. The first, usually referred as *signal-based*, includes all the approaches based on high frequency voltage signals injection, used to get complete position information exploiting the *magnetic saliency* (see [4], for instance). Differently, the second category, usually referred as *model-based*, covers methods where nominal models of PMSM are exploited in different ways to reconstruct the rotor magnet position and speed through the back-emf induced on stator windings.

The solution presented in this paper belongs to the last

family, which is commonly adopted, especially in medium or high speed range of operation. In fact, it's well known that at low speed values, the performance of model-based methods abruptly decreases due to a lack of observability of the system (see [5], [6] for more details on observability analysis). Another common problem of these approaches is the sensitivity to parameters uncertainty, again particularly relevant at low speed. These two drawbacks are even more significant when a linear approximation of the machine model is taken to design the estimation system, hence the research effort has been devoted to develop nonlinear observers for this kind of application. Beside the classical solutions based on open-loop integrations of some system dynamics (typically the stator flux dynamics) or on extended Kalman filters (see [7] among the others), some other interesting solutions, aimed to cope with the above stated issues, have been proposed. In [8], [9], [10] significant approaches, concerning analysis and improvement of robustness with respect to parameters uncertainties, are presented along with some discussions on the stability properties of the adopted nonlinear schemes. Among the others, the two solutions recently presented in [5] and [11] are particularly attractive since, exploiting modern nonlinear observer design techniques ([12], [13]), rigorous stability analysis has been carried out.

In this paper, a novel and simple position and speed observer for PMSM is formulated. Taking the cue from the approach proposed in [14], the idea is to build an observer in a generic reference frame imposing a representation for the back-emf vector equivalent to the one it would have in the so-called synchronous coordinates [1]. In this way, an implicit alignment is forced between the adopted reference frame and the estimated back-emf vector, while its amplitude along with the rotor speed and position can be suitably adapted to obtain asymptotic reconstruction of the stator currents. No pure integration of the stator dynamics is adopted, since the stator current dynamics are exploited as indirect measurement of the back-emf vector. This leads to an intrinsic robustness to many kinds of voltage and current measurement uncertainties. Time scale separation between the stator current dynamics and the remainder of the observer dynamics is exploited to provide practical semiglobal asymptotic stability.

The paper is organized as follows. In Section II the standard model of PMSM is recalled to highlight its representation in the so-called synchronous reference frame. Then, the general objectives for position and speed sensorless reconstruction are reported. Particular attention is paid to the bandwidth requirements, necessary to compensate the lack of a detailed mechanical model, and the robustness issues

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with respect to uncertainties. In Section III the proposed observer is presented and a stability analysis is proposed using singular perturbation approach ([15], [16]). Tuning rules are derived in Section IV by using linearization of the resulting estimation error dynamics. In Section V significative simulation tests are reported to testify the properties of the proposed method in different scenarios. Finally, some conclusions and future work directions are depicted in the last section.

II. PMSM MODEL AND OBSERVER PROBLEM STATEMENT

According to standard planar representation of three-phase electric motors [1], in the following the PMSM electromagnetic model is reported in a generic 2-phase u-v reference frame rotated by an angle ϵ_0 with respect to a static reference frame aligned to the stator windings

$$\begin{aligned} \dot{i}_u &= -\frac{R}{L}i_u + \omega_0 i_v + \frac{\omega\phi_v}{L} + \frac{u_u + d_u}{L} \\ \dot{i}_v &= -\frac{R}{L}i_v - \omega_0 i_u - \frac{\omega\phi_u}{L} + \frac{u_v + d_v}{L} \\ \dot{\phi}_u &= -(p\omega - \omega_0)\phi_v \\ \dot{\phi}_v &= (p\omega - \omega_0)\phi_u \end{aligned} \quad (1)$$

In (1), $\omega_0 = \dot{\epsilon}_0$ is the angular speed of the arbitrary selected reference frame u-v; p are the pole pairs; R, L are stator winding resistance and inductance; ω is the actual rotor mechanical speed; i_u, i_v are the stator currents; ϕ_u, ϕ_v are the components given by the projection in the considered reference frame of the rotor magnet flux vector, whose amplitude will be indicated as Φ . In this framework, θ , such that $\dot{\theta} = \omega$ and θ_e , such that $\dot{\theta}_e = p\omega$, can be used respectively to represent the mechanical and the so-called electrical angle of the rotor magnet flux vector w.r.t the static stator-aligned reference frame. Finally, $u_u + d_u$ and $u_v + d_v$ give the voltages applied to the stator windings. It is worth noting that these voltages have been represented by the sum of the expected voltages u_u, u_v and unknown signals d_u, d_v . Usually, stator voltages are actuated by means of switching power converters (commonly referred as *inverters*) and direct measurements are not available or not highly accurate. Therefore, d_u, d_v account for measurement errors and/or inverter non-idealities (such as Dead-Time effect, IGBT/MOS voltage drop).

From now on, and with no loss of generality, $p = 1$ will be taken, therefore ω will be the so-called electrical rotor speed and the mechanical angle, θ , and the electric one, θ_e , will be the same.

As it is well known, [1], defining a reference frame d-q with the d-axis aligned with the rotor flux vector, the model (1) reads as follows.

$$\begin{aligned} \dot{i}_d &= -\frac{R}{L}i_d + \omega i_q + \frac{u_d + d_d}{L} \\ \dot{i}_q &= -\frac{R}{L}i_q - \omega i_d - \frac{\omega\Phi}{L} + \frac{u_q + d_q}{L} \\ \dot{\phi}_d &= 0 \\ \dot{\phi}_q &= 0 \end{aligned} \quad (2)$$

For this kind of coordinate frame, the speed ω_0 and angle ϵ_0 become exactly the electrical rotor speed ω and the rotor flux vector angle θ . Therefore, this reference frame is usually referred as *synchronous* reference frame, and d and q stand for *direct* and *quadrature* axis.

In sensorless control of PMSM a fundamental issue is to achieve an estimation of the rotor flux vector angle θ and speed ω , since no direct measurements are available. This is crucial to build standard and also some kind of advanced speed-torque controllers based on field-orientation concepts, [1]. Usually, in model-based approaches, speed and position estimation task is performed by defining a suitable observer exploiting the electromagnetic model of the PMSM, while no relevant information is assumed available on the mechanical model of what connected to the machine rotor. On the other hand, the speed dynamics is assumed much slower than the electromagnetic one, therefore the speed is assumed constant (or slowly varying) in stating the above-mentioned estimation problem. Beside the basic problem, also the estimation of the amplitude of the rotor magnetic flux vector is often considered to enable very accurate torque control (the flux amplitude can vary along time and depending on the working temperature [1]).

Bearing in mind the previous considerations, the following general objectives can be defined for an observer based on the electromagnetic model of PMSM.

- 1) Guaranteeing estimation of rotor magnet vector position, θ , and speed ω along with its amplitude Φ , under constant rotation conditions, assuming stator currents and expected stator voltages available from measures and actuations, respectively, and considering null voltage uncertainties (these conditions will be referred as *nominal conditions*).
- 2) Achieving as large as possible bandwidth in the estimation of the speed ω in order to compensate for the lack of knowledge of the mechanical model and cope with variable speed conditions.
- 3) Obtaining large voltage disturbances rejection, i.e. attenuation of the d_d, d_q disturbances.

III. NONLINEAR OBSERVER BASED ON SYNCHRONOUS REFERENCE FRAME

The basic idea of the proposed approach is to build the observer starting by imposing in a generic reference frame the model (2) which is valid only in the synchronous one. Then, feedback estimation laws are designed in order to push the angle and the speed of the adopted reference toward θ and ω of the synchronous frame. Therefore, the proposed observer reference frame can be seen as an estimation of the synchronous one and, from now on, it will be referred as $\hat{d} - \hat{q}$ with angle $\hat{\theta}$ and speed $\hat{\omega}$ w.r.t static stator reference frame.

An additional important step in the line defined above is a coordinate changing highlighting the back-emf as state variables. Therefore, defining $\chi_{\hat{d}} = \omega\phi_{\hat{d}}$ and $\chi_{\hat{q}} = \omega\phi_{\hat{q}}$ the synchronous model (2) can be revised leading to the

following observer model in the $\hat{d} - \hat{q}$ reference frame

$$\begin{aligned}\dot{i}_{\hat{d}} &= -\frac{R}{L}i_{\hat{d}} + \hat{\omega}i_{\hat{q}} + \frac{u_{\hat{d}}}{L} + \eta_d \\ \dot{i}_{\hat{q}} &= -\frac{R}{L}i_{\hat{q}} - \hat{\omega}i_{\hat{d}} - \frac{\hat{A}}{L} + \frac{u_{\hat{q}}}{L} + \eta_q \\ \dot{\hat{A}} &= \nu_a \\ \dot{\hat{\omega}} &= \eta_\omega \\ \dot{\hat{\omega}} &= \hat{\omega} + \nu_\omega \\ \dot{\hat{\theta}} &= \hat{\omega}\end{aligned}\quad (3)$$

where $i_{\hat{d}}$, $i_{\hat{q}}$ and $u_{\hat{d}}$, $u_{\hat{q}}$ are the stator currents and expected voltages, available from measurements and actuator commands and reported in \hat{d} - \hat{q} frame; \hat{A} is the estimation of the back-emf $\omega\Phi$ in (2); while the meaning of $\hat{i}_{\hat{d}}$, $\hat{i}_{\hat{q}}$, $\hat{\theta}$ and $\hat{\omega}$ is straightforward. Differently, η_d , η_q , ν_a , η_ω and ν_ω are feedback terms defined as follows, for observer convergence

$$\begin{aligned}\eta_d &= k_p \tilde{i}_{\hat{d}}, \quad \eta_q = k_p \tilde{i}_{\hat{q}}, \quad \nu_a = -Lk_1 k_p \tilde{i}_{\hat{q}}, \\ \eta_\omega &= \gamma \frac{\hat{A}}{Lk_p} \tilde{i}_{\hat{d}}, \quad \nu_\omega = k_2 \frac{\hat{A}}{Lk_p} \tilde{i}_{\hat{d}}\end{aligned}\quad (4)$$

where $\tilde{i}_{\hat{d}} = i_{\hat{d}} - \hat{i}_{\hat{d}}$, $\tilde{i}_{\hat{q}} = i_{\hat{q}} - \hat{i}_{\hat{q}}$. It is worth noting that a sort of PI structure has been adopted for the $\hat{\theta}$ estimation, but just $\hat{\omega}$ will be considered as output speed estimation of the proposed observer.

Finally, the PMSM model can be expressed in the observer reference frame as follows

$$\begin{aligned}\dot{i}_{\hat{d}} &= -\frac{R}{L}i_{\hat{d}} + \hat{\omega}i_{\hat{q}} + \frac{\chi_{\hat{q}}}{L} + \frac{u_{\hat{d}} + d_{\hat{d}}}{L} \\ \dot{i}_{\hat{q}} &= -\frac{R}{L}i_{\hat{q}} - \hat{\omega}i_{\hat{d}} - \frac{\chi_{\hat{d}}}{L} + \frac{u_{\hat{q}} + d_{\hat{q}}}{L} \\ \dot{\chi}_{\hat{d}} &= -(\omega - \hat{\omega})\chi_{\hat{q}} + \frac{\dot{\omega}}{\omega}\chi_{\hat{d}} \\ \dot{\chi}_{\hat{q}} &= (\omega - \hat{\omega})\chi_{\hat{d}} + \frac{\dot{\omega}}{\omega}\chi_{\hat{q}}\end{aligned}\quad (5)$$

where $\chi_{\hat{d}}$, $\chi_{\hat{q}}$ enlighten the back-emf projections in the considered frame, according to previous definitions.

In model (5) also $d_{\hat{d}}$, $d_{\hat{q}}$ and $\dot{\omega}$ are reported to highlight the effect of voltage uncertainties and non-constant speed conditions.

A. STABILITY ANALYSIS BASED ON SINGULAR PERTURBATION

In this Subsection the stability analysis of the proposed solution is sketched. This analysis is carried out assuming nominal conditions defined in Objective 1 at the end of Section II, hence the disturbances on the actuated voltages and the perturbation introduced by a non constant rotor speed, appearing in (5), have been neglected. These additional input signals will be considered in next Section for observer gains tuning, according to Objectives 2-3 at the end of Section II. A model to suitably represent the behavior of the observation error can be defined by considering the current errors $\tilde{i}_{\hat{d}}$ and $\tilde{i}_{\hat{q}}$, previously introduced, and adding the following errors variables related to the estimation of the back-emf components and speed

$$\tilde{\chi}_{\hat{d}} = \chi_{\hat{d}} - \hat{A}, \quad \tilde{\chi}_{\hat{q}} = \chi_{\hat{q}}, \quad \tilde{\omega} = \omega - \hat{\omega}\quad (6)$$

By subtracting (3) from (5), neglecting $d_{\hat{d}}$, $d_{\hat{q}}$, $\dot{\omega}$, the dynamics of the above defined estimation errors is the following

$$\begin{aligned}\dot{\tilde{i}}_{\hat{d}} &= -\eta_d + \frac{\tilde{\chi}_{\hat{q}}}{L} \\ \dot{\tilde{i}}_{\hat{q}} &= -\eta_q - \frac{\tilde{\chi}_{\hat{d}}}{L} \\ \dot{\tilde{\chi}}_{\hat{d}} &= -(\tilde{\omega} - \nu_\omega)\tilde{\chi}_{\hat{q}} - \nu_a \\ \dot{\tilde{\chi}}_{\hat{q}} &= (\tilde{\omega} - \nu_\omega)(\tilde{\chi}_{\hat{d}} + \hat{A}) \\ \dot{\tilde{\omega}} &= -\eta_\omega\end{aligned}\quad (7)$$

Exploiting the adaptation laws defined in (4) and defining the following change of coordinates $\tilde{\chi}_{\hat{d}_1} = \tilde{\chi}_{\hat{d}}/Lk_p$, $\tilde{\chi}_{\hat{q}_1} = \tilde{\chi}_{\hat{q}}/Lk_p$, $\nu_{a_1} = \nu_a/Lk_p$, $\hat{A}_1 = \hat{A}/Lk_p$ and $\epsilon = \frac{1}{k_p}$, the system (7) can be expressed as

$$\begin{aligned}\dot{\epsilon\tilde{i}}_{\hat{d}} &= -\tilde{i}_{\hat{d}} + \tilde{\chi}_{\hat{q}_1} \\ \dot{\epsilon\tilde{i}}_{\hat{q}} &= -\tilde{i}_{\hat{q}} - \tilde{\chi}_{\hat{d}_1} \\ \dot{\tilde{\chi}}_{\hat{d}_1} &= -(\tilde{\omega} - k_2 \hat{A}_1 \tilde{i}_{\hat{d}})\tilde{\chi}_{\hat{q}_1} + k_1 \tilde{i}_{\hat{q}} \\ \dot{\tilde{\chi}}_{\hat{q}_1} &= (\tilde{\omega} - k_2 \hat{A}_1 \tilde{i}_{\hat{d}})(\tilde{\chi}_{\hat{d}_1} + \hat{A}_1) \\ \dot{\tilde{\omega}} &= -\gamma \hat{A}_1 \tilde{i}_{\hat{d}}\end{aligned}\quad (8)$$

This can be easily seen as a standard singular perturbation model [15], where the time scale separation between the current error dynamics and the back-emf and speed error dynamics is parametrized by the gain k_p . Therefore, assuming a sufficiently high value of k_p has been chosen (more details will be given in the next Section), the problem of the estimates convergence can be approached by considering the overall system as the interconnection of a *fast subsystem*, represented by the current error variables ($\tilde{i}_{\hat{d}}$, $\tilde{i}_{\hat{q}}$), and a *slow subsystem* given by the other dynamics ($\tilde{\chi}_{\hat{d}_1}$, $\tilde{\chi}_{\hat{q}_1}$, $\tilde{\omega}$).

According to [15] and [16], we start by studying the so-called *boundary layer* system related to the fast dynamics. First, define the *quasi steady-state* value for the current errors, obtained as the solution of the fast subsystem when $\epsilon = 0$, it results $\tilde{i}_{\hat{d}} = \tilde{\chi}_{\hat{q}_1}(t)$, $\tilde{i}_{\hat{q}} = -\tilde{\chi}_{\hat{d}_1}(t)$. Then, defining $y_d = \tilde{i}_{\hat{d}} - \tilde{\chi}_{\hat{q}_1}$, $y_q = \tilde{i}_{\hat{q}} + \tilde{\chi}_{\hat{d}_1}$, $t = \epsilon\tau$, after some computation, consisting in *freezing* the slow varying variables by setting $\epsilon = 0$, the following boundary layer system is obtained

$$\frac{dy_d}{d\tau} = -y_d, \quad \frac{dy_q}{d\tau} = -y_q\quad (9)$$

It's trivial to verify that the origin of (9) is globally exponentially stable, uniformly in both the slow variables and the time, since it's a LTI system with Hurwitz state matrix.

Again according to [15] and [16], we focus now on the reduced dynamics obtained by substituting the fast variables $\tilde{i}_{\hat{d}}$, $\tilde{i}_{\hat{q}}$ with their quasi steady-state, $\tilde{i}_{\hat{d}} = \tilde{\chi}_{\hat{q}_1}(t)$, $\tilde{i}_{\hat{q}} = -\tilde{\chi}_{\hat{d}_1}(t)$, in the slow dynamics given by the last three equations in (8). Note that the quasi steady-state definition enlightens how the current errors can be used as indirect measure of the back-emf estimation errors, thanks to time scale separation imposed by k_p . After some computation the following reduced system results

$$\begin{aligned}\dot{\tilde{\chi}}_{\hat{d}_1} &= -(\tilde{\omega} - k_2 \hat{A}_1 \tilde{\chi}_{\hat{q}_1})\tilde{\chi}_{\hat{d}_1} - k_1 \tilde{\chi}_{\hat{d}_1} \\ \dot{\tilde{\chi}}_{\hat{q}_1} &= (\tilde{\omega} - k_2 \hat{A}_1 \tilde{\chi}_{\hat{q}_1})(\tilde{\chi}_{\hat{d}_1} + \hat{A}_1) \\ \dot{\tilde{\omega}} &= -\gamma \hat{A}_1 \tilde{\chi}_{\hat{q}_1}\end{aligned}\quad (10)$$

To investigate the stability of (10) consider the Lyapunov candidate function $V = \frac{1}{2}(\tilde{\chi}_{\hat{d}_1}^2 + \tilde{\chi}_{\hat{q}_1}^2 + \frac{\tilde{\omega}^2}{\gamma})$, taking its derivative along the system trajectories we obtain

$$\dot{V} = -k_1\tilde{\chi}_{\hat{d}_1}^2 - k_2\hat{A}_1^2\tilde{\chi}_{\hat{q}_1}^2 \quad (11)$$

This function is negative semi-definite for any positive values of k_1, k_2 . In addition, from direct application of Barbalat's lemma ([15]), it can be stated that $\lim_{t \rightarrow \infty} \dot{V} = 0$, $\lim_{t \rightarrow \infty} \tilde{\chi}_{\hat{d}_1} = 0$ and $\lim_{t \rightarrow \infty} \tilde{\chi}_{\hat{q}_1} = 0$. Therefore, the origin of the reduced dynamics is globally asymptotically stable.

From the previous considerations and using the singular perturbation results as formulated in [16] (which actually covers also averaging and ISS analysis), the following properties for the overall error dynamics (8) can be drawn.

Proposition 1: For the system (8), replacing for simplicity current coordinates, $\tilde{i}_{\hat{d}}, \tilde{i}_{\hat{q}}$, with the above defined $y_d = \tilde{i}_{\hat{d}} - \tilde{\chi}_{\hat{q}_1}$, $y_q = \tilde{i}_{\hat{q}} + \tilde{\chi}_{\hat{d}_1}$, there exist two class \mathcal{KL} functions β_f and β_s such that, for each $\delta > 0$ and for every compact sets $\Omega_f \subset \mathbb{R}^2$ and $\Omega_s \subset \mathbb{R}^3$, there exists ϵ^* such that $\forall \epsilon = k_p^{-1} \in (0, \epsilon^*]$, the following inequalities hold

$$\| [y_d(t), y_q(t)]^T \| \leq \beta_f \left(\| [y_d(0), y_q(0)]^T \|, t/\epsilon \right) + \delta \quad (12)$$

$$\forall [y_d(0), y_q(0)]^T \in \Omega_f$$

$$\| [\tilde{\chi}_{\hat{d}_1}(t), \tilde{\chi}_{\hat{q}_1}(t), \tilde{\omega}(t)]^T \| \leq \beta_s \left(\| [\tilde{\chi}_{\hat{d}_1}(0), \tilde{\chi}_{\hat{q}_1}(0), \tilde{\omega}(0)]^T \|, t \right) + \delta \quad (13)$$

$$\forall [\tilde{\chi}_{\hat{d}_1}(0), \tilde{\chi}_{\hat{q}_1}(0), \tilde{\omega}(0)]^T \in \Omega_s$$

Hence, semiglobal practical stability can be stated for the overall error dynamics (8), provided that a sufficiently large k_p has been selected.

IV. TUNING RULES FOR THE PROPOSED SOLUTION

In this Section the tuning rules are defined according to time scale separation requirements derived in Subsection III-A and the general objectives defined in Section II. A preliminary step toward this goal is to rewrite the error dynamics (7) taking into account the voltage disturbances and the perturbation given by non-constant speed as follows

$$\begin{aligned} \dot{\tilde{i}}_{\hat{d}} &= k_p(-\tilde{i}_{\hat{d}} + \tilde{\chi}_{\hat{q}_1}) + \frac{d_{\hat{d}}}{L} \\ \dot{\tilde{i}}_{\hat{q}} &= -k_p(\tilde{i}_{\hat{q}} + \tilde{\chi}_{\hat{d}_1}) + \frac{d_{\hat{q}}}{L} \\ \dot{\tilde{\chi}}_{\hat{d}_1} &= -(\tilde{\omega} - \nu_{\omega})\tilde{\chi}_{\hat{q}_1} - \nu_{\omega_1} + \frac{\dot{\omega}}{\omega}(\tilde{\chi}_{\hat{d}_1} + \hat{A}_1) \\ \dot{\tilde{\chi}}_{\hat{q}_1} &= (\tilde{\omega} - \nu_{\omega})(\tilde{\chi}_{\hat{d}_1} + \hat{A}_1) + \frac{\dot{\omega}}{\omega}\tilde{\chi}_{\hat{q}_1} \\ \dot{\tilde{\omega}} &= -\eta_{\omega} + \dot{\omega} \end{aligned} \quad (14)$$

The origin of the system, $[\tilde{\chi}_{\hat{d}_1} \tilde{\chi}_{\hat{q}_1} \tilde{\omega}] = 0$, is an equilibrium point, and linearizing the system near the origin we obtain the following LTI system

$$\begin{aligned} \dot{\tilde{i}}_{\hat{d}} &= k_p(-\tilde{i}_{\hat{d}} + \tilde{\chi}_{\hat{q}_1}) + \frac{d_{\hat{d}}}{L}, \quad \dot{\tilde{i}}_{\hat{q}} = k_p(-\tilde{i}_{\hat{q}} - \tilde{\chi}_{\hat{d}_1}) + \frac{d_{\hat{q}}}{L} \\ \dot{\tilde{\chi}}_{\hat{d}_1} &= -\nu_{\omega_1} + \dot{\omega}\Phi_1, \quad \dot{\tilde{\chi}}_{\hat{q}_1} = (\tilde{\omega} - \nu_{\omega})\Phi_1\omega \\ \dot{\tilde{\omega}} &= -\eta_{\omega} + \dot{\omega} \end{aligned} \quad (15)$$

where $\Phi_1 = \Phi/Lk_p$. The variable $\dot{\omega}$ can be seen as an input acting on $\tilde{\omega}$ and $\tilde{\chi}_{\hat{d}_1}$ and it is useful to evaluate the sensitivity

of the error variables to the variable speed, i.e. the observer bandwidth. Other inputs in the dynamics (15) are the voltage disturbances $d_{\hat{d}}$ and $d_{\hat{q}}$, also the sensitivity to such variables will be considered.

Applying to the error system (15) the results deriving from singular perturbation properties enlightened in Subsection III-A, the following quasi-steady state equations can be considered (with some abuse of notation)

$$-\tilde{i}_{\hat{d}} + \tilde{\chi}_{\hat{q}_1} + \frac{d_{\hat{d}}}{Lk_p} \approx 0, \quad -\tilde{i}_{\hat{q}} - \tilde{\chi}_{\hat{d}_1} + \frac{d_{\hat{q}}}{Lk_p} \approx 0 \quad (16)$$

Hence, the following linearized reduced error dynamics can be derived

$$\begin{aligned} \dot{\tilde{\chi}}_{\hat{d}_1} &= k_1(-\tilde{\chi}_{\hat{d}_1} + d_{\hat{q}}/Lk_p) + \dot{\omega}\Phi_1 \\ \dot{\tilde{\chi}}_{\hat{q}_1} &= \omega\Phi_1\tilde{\omega} - k_2(\omega\Phi_1)^2(\tilde{\chi}_{\hat{q}_1} + d_{\hat{q}}/Lk_p) \\ \dot{\tilde{\omega}} &= -\gamma\omega\Phi_1(\tilde{\chi}_{\hat{q}_1} + d_{\hat{q}}/Lk_p) + \dot{\omega} \end{aligned} \quad (17)$$

with the following state matrix A_R and input matrix B_R w.r.t. the input vector $[d_{\hat{d}} \ d_{\hat{q}} \ \dot{\omega}]^T$

$$A_R = \begin{bmatrix} -k_1 & 0 & 0 \\ 0 & -k_2(\omega\Phi_1)^2 & \Phi_1\omega \\ 0 & -\gamma\omega\Phi_1 & 0 \end{bmatrix}, \quad B_R = \frac{1}{Lk_p} \begin{bmatrix} 0 & k_1 & \Phi \\ -k_2(\omega\Phi_1)^2 & 0 & 0 \\ -\gamma\omega\Phi_1 & 0 & Lk_p \end{bmatrix} \quad (18)$$

State matrix A_R in (18) has the following eigenvalues

$$\lambda_1 = -k_1, \quad \lambda_{2,3} = \frac{k_2(\omega\Phi_1)^2}{2} \left[-1 \pm \sqrt{1 - \frac{4\gamma}{k_2^2(\omega\Phi_1)^2}} \right] \quad (19)$$

It is possible to find the value of k_2, γ to impose damping (δ) and angular natural frequency (ω_n) for the eigenvalues $\lambda_{2,3}$ using the following equations

$$k_2 = \frac{2\omega_n\delta}{(\omega\Phi_1)^2}, \quad \gamma = \frac{1}{(\omega\Phi_1)^2} \left[\omega_n^2(1 - \delta^2) + \frac{k_2^2(\omega\Phi_1)^4}{4} \right] \quad (20)$$

With these formulas at hand, and bearing in mind the introduction of this section the tuning parameter must be chosen to cope with:

- 1) **Frequency separation**, between fast dynamics of $\tilde{i}_{\hat{d}_1}, \tilde{i}_{\hat{q}_1}$ and slow dynamics of $\tilde{\chi}_{\hat{d}_1}, \tilde{\chi}_{\hat{q}_1}, \tilde{\omega}$.
- 2) **High Bandwidth Observer**, for good estimation during speed variations, i.e. low sensitivity to $\dot{\omega}$.
- 3) **Disturbance rejection** for high robustness to common disturbances due to Inverter non-idealities, i.e. low sensitivity to $d_{\hat{d}}, d_{\hat{q}}$.

Obviously, frequency separation can be obtained choosing large k_p . The upper bound for this parameter is usually related to the common discrete time realization of the observer. In fact k_p represents the bandwidth for the current $\hat{i}_{\hat{d}}, \hat{i}_{\hat{q}}$ reconstruction.

High observer bandwidth can be obtained acting on k_1, k_2 and γ but, actually, this is in contrast with disturbance rejection requirement.

First of all, a good practice is to chose k_1, k_2 and γ such that they identify three distinct eigenvalues for the matrix A_R , to avoid ill conditioned problem. k_1 is related only on the bandwidth of the $\tilde{\chi}_{\hat{d}_1}$ dynamic. Its value must be chosen to be lower than the faster dynamic imposed by k_p (e.g. $k_1 = k_p/50$), recalling that a low value for this parameter produces a low sensitivity to $\dot{\omega}$.

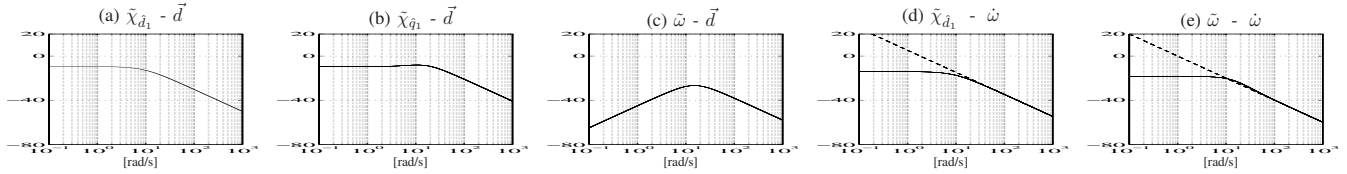


Fig. 1. \vec{d} -Sensitivity Bode diagrams (a,b,c) ($\vec{d} = [d_{\hat{d}} d_{\hat{q}}]^T$) and $\dot{\omega}$ -Sensitivity Bode diagrams (d,e).

k_2 and γ can be chosen to impose damping (δ) and angular natural frequency (ω_n) of $\lambda_{2,3}$ eigenvalues of the reduced order system, as reported in (20). For the value of w_n , the same considerations as for k_1 hold, i.e. w_n must be lower than the fast dynamic imposed by k_p (e.g. $w_n = k_p/80$), but not too low to not compromise the observer bandwidth. The damping of the eigenvalues $\lambda_{2,3}$ ($\delta < 1$) can be chosen to lightly augment the frequency of the eigenvalue related to it, but its major effect is to introduce a resonant frequency behavior, giving low disturbance rejection for a particular disturbance band frequency.

For what concerns the disturbance rejection, a preliminary task is the identification of the disturbance band frequency. Inverter non-idealities introduce voltage disturbances with frequencies n -times the actual electrical frequency, and from practical experiments for the main disturbance component $n = 6$. The worst case is at low electrical frequency, for which also disturbances are at low frequency.

To give an example of a realistic parameter tuning process following the rules mentioned above, the data of a realistic set-up reported in the second column of Tab.I has been taken (these data will be exploited later for simulations). Supposing for the motor a speed range of $[33 - 70]$ [rad/s], the main disturbances frequency is about $[200 - 420]$ [rad/s]. Recalling [14] for the analysis of sensitivity to disturbances for the linearized system (18), it is possible to verify that the sinusoidal voltage disturbances can be taken counter-rotating or rotating equivalently, thanks to the structure of the couple A_R, B_R . Three transfer functions must be drawn, each for one of the error variables $\tilde{\chi}_{\hat{d}_1}, \tilde{\chi}_{\hat{q}_1}, \tilde{\omega}$ of the reduced system (18). To have a good static disturbance rejection k_p can be set as 900, higher values for this parameter are not allowed due to its discrete time realization ($f_s = 2.5$ [kHz]). The parameter k_1 can be chosen as 10, i.e. $k_1 \approx k_p/100$, and with $k_2 = 0.0075$ and $\gamma = 0.0627$ we obtain $\delta = 0.9$ and $\omega_n = 15$ supposing a rotor speed of 33[rad/s]. With these parameters, the three Bode diagrams of the sensitivity transfer functions related to voltage disturbances and the two diagrams for evaluating the observer bandwidth are reported in in Fig.1. From plots (a), (b) and (c), it can be noted that, in the disturbances frequency band, we have a minimum attenuation of -36 dB for $\tilde{\chi}_{\hat{d}_1}$, -27 dB for $\tilde{\chi}_{\hat{q}_1}$ and -44 dB for $\tilde{\omega}$.

To better evaluate the observer bandwidth, in plots (d) and (e) of Fig.1 the transfer function obtained with no feedback actions in the observer has been added. That is Φ_1/s for the Bode diagram of $\tilde{\chi}_{\hat{d}_1} - \dot{\omega}$, and $1/s$ for the $\tilde{\omega} - \dot{\omega}$. From these plots a bandwidth of 10[rad/s] can be inferred.

V. SIMULATION RESULTS

In this section the performance of the proposed observer are simulated in two different scenarios. First, according to standard field-oriented control schemes the estimated speed ($\hat{\omega}$) and angular position ($\hat{\theta}$) are used to feed a standard speed controller designed in the $\hat{d} - \hat{q}$ reference frame, the simulations are carried out in a time-continuous domain assuming an ideal scenario. Then, a second and more realistic set-up is considered by adopting a discrete time version of the observer ($f_s = 2.5$ [kHz]) used along with a torque controller. For this set-up a more accurate model of the electrical converter has been implemented, introducing PWM technique ($f_{PWM} = 2.5$ [kHz]) and switching devices non-idealities (dead-times, solid state devices voltage drop and turn ON/OFF time delays), in order to validate the robustness of the estimator with respect to the most common disturbances in practical applications.

Simulations for the first and second scenarios are performed using machine parameters reported respectively in the first and second column of Tab.I. The observer parameters, tuned according to linear analysis discussed in IV, are: $k_p = 3030$, $k_1 = 60.6$, $k_2 = 4503$, $\gamma = 927050$ for the first scenario, and $k_p = 928.8$, $k_1 = 9.29$, $k_2 = 0.0058$, $\gamma = 0.0375$ for the second one. In the first scenario the initial estimate value is set to zero for all the estimated variables. For the second scenario an estimate for the initial rotor speed is known, hence a good estimate for the initial value of the rotor speed itself ($\hat{\omega}$) and *bemf* (\hat{A}) are known.

Fig.2 shows the obtained results, a variable speed reference trajectory ω^* and load torque (T_L) steps have been reproduced in order to test the observer dynamic behavior. The angular speed and position estimates are quickly recovered during reference and load torque changes, even when slow speed region is crossed, consequently also the speed controller is able to ensure a good tracking response.

Motor inertia J [Kgm^2]	0.04	<i>n.a.</i>
Nominal angular speed ω_{nom} [rad/s]	300	1.60
Rotor flux Φ [Wb]	1	5.50
Nominal torque T_{nom} [Nm]	5	680×10^3
Stator resistance R [Ω]	2.5	9.0×10^{-3}
Stator inductance L [H]	0.1	3.0×10^{-3}
Number of pole pairs p	1	50

TABLE I
MOTOR PARAMETERS, IDEAL (FIRST COLUMN) AND REALISTIC (SECOND COLUMN) SCENARIOS.

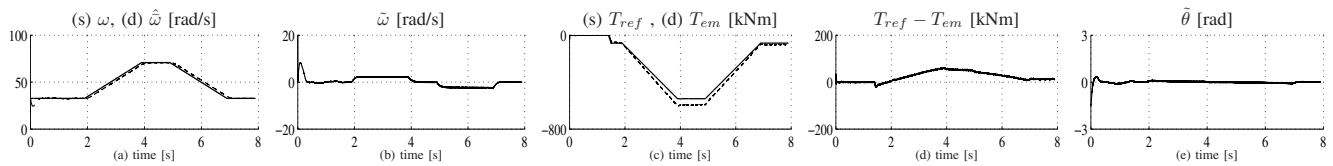


Fig. 3. (s) solid, (d) dashed. Torque tracking performance in a carachteristic profile for the second scenario.

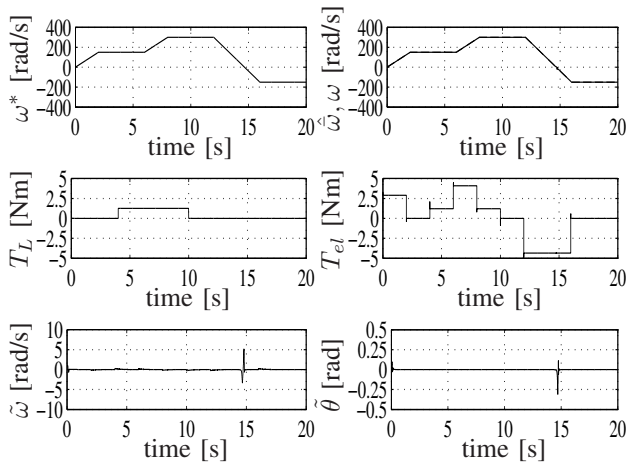


Fig. 2. Simulation results of sensorless speed control using the proposed observer.

Fig.3 allows performance evaluation of the proposed observer in a torque tracking control scheme during a typical torque-speed profile for wind-turbines applications. The observer is turn on with a non null motor speed, after a brief transient time the speed and the torque request augment reaching their respective nominal values. To better evaluate the observer performance some error variables are reported, speed error (plot b), torque error (plot d) and angle error (plot e).

VI. CONCLUSIONS AND FUTURE WORKS

A novel nonlinear observer devoted to the rotor speed and angular position reconstruction for PMSM has been presented. A nominal synchronous model of the back-emf vector is imposed in the observer reference frame, then suitable adaptation laws for the speed and position of such frame, along with back-emf amplitude estimation, are designed to achieve the effective alignment with the actual synchronous frame.

This solution does not require a perfect knowledge of the flux rotor amplitude, and direct integration of the stator dynamics is avoided leading to some intrinsic robustness. A suitable time scale separation between the observer dynamics is imposed to guarantee stability properties. From a practical viewpoint, it allows to consider current estimation errors as indirect measure of the flux error variables.

Linearization of the observation error model has been exploited to achieve accurate tuning of the observer gains. The objectives considered in this task are the rejection of disturbances produced by the power converter non-idealities, and the tracking of variable speed trajectories.

The proposed approach can be extended in different directions; simulation results show the efficiency of the solution when the observer is used alongside with both a speed and a torque feedback controller. However, a rigorous formal discussion on the stability and robustness properties of the overall closed loop system, obtained replacing the actual variables with their estimates, should be carried out, in order to deeply validate the considered solution. Furthermore the main idea, reported here for the PMSM, can be exploited and suitably adapted to design observers for other kind of electrical machines, such as the induction machines.

REFERENCES

- [1] P. Vas, *Sensorless Vector and Direct Torque Control*, Oxford, New York (USA), 1998.
- [2] F. Khorrami, P. Krishnamurthy, and H. Melkote, *Modeling and Adaptive Nonlinear Control of Electric Motors*, Springer, Heidelberg (Germany), 2003.
- [3] R. Marino, P. Tomei, C. M. Verrelli, *Induction Motor Control Design*, Springer-Verlag, London (UK), 2010.
- [4] S. Ichikawa, M. Tomita, S. Doki, and S. Okuma, "Sensorless control of PMSM using on-line parameter identification based on systems identification theory", *IEEE Transactions on Industrial Electronics*, vol. 53, no. 2, pp. 363-373, 2006.
- [5] F. Poulain, L. Praly, and R. Ortega, "An observer for PMSM with application to sensorless control", *IEEE Conference on Decision and Control*, pp. 5390-5395, Cancun, Mexico, 2008.
- [6] S. Ibarra, J. Moreno, and G. Espinosa-Perez, "Global observability analysis of sensorless induction motor", *Automatica*, vol. 40, pp. 1079-1085, 2004.
- [7] A. Bado, S. Bolognani and M. Zigliotto, "Effective estimation of speed and rotor position of a PM synchronous motor drive by a Kalman filtering technique", *IEEE Power Electronics Specialists Conference*, pp. 951-957, Toledo, Spain, 1992.
- [8] B. Nahid Mobarakeh, F. Meibody-Tabar, and F. Sargos, "Robustness study of a model-based technique for mechanical sensorless control of PMSM", *IEEE Power Electronics Specialists Conference*, pp. 811-816, Vancouver, Canada, 2001.
- [9] N. Matsui, "Sensorless PM brushless DC motor drives", *IEEE Transactions on Industrial Electronics*, vol. 43, no. 2, pp. 300-308, 1996.
- [10] J. Solsona, M. I. Valla, and C. Muravchik, "A nonlinear reduced order observer for PMSM", *IEEE Transactions on Industrial Electronics*, vol. 43, no. 4, pp. 492-497, 1996.
- [11] R. Ortega, L. Praly, A. Astolfi, J. Lee, and K. Nam, "Estimation of rotor position and speed of permanent magnet synchronous Motors With Guaranteed Stability", *IEEE Transactions on Control Systems Technology*, vol. 19, pp. 601-614, 2010.
- [12] N. Kazantzis, C. Kravaris, "Nonlinear observer design using Lyapunov auxiliary theorem", *Systems & Control Letters*, vol. 34, pp. 241-247, 1998.
- [13] A. Astolfi, D. Karagiannis, and R. Ortega, *Nonlinear and Adaptive Control with Applications*, Springer-Verlag, Berlin (Germany), 2007.
- [14] A. Tili, C. Conficoni, "Adaptive observer for three phase line voltage under unbalanced conditions", *Nonlinear Control Systems Symposium*, pp. 1981-1986, Bologna, Italy, 2010.
- [15] H. K. Khalil, *Nonlinear Systems 3rd Ed.*, Prentice-Hall Inc., Upper Saddle River (USA), 2002.
- [16] A. R. Teel, L. Moreau and D. Nesić, "A unified framework for input-to-state stability in systems with two time scales", *IEEE Transactions on Automatic Control*, vol. 48, pp. 1526-1544, 2003.