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# A Multiple-Model Approach to Fault Tolerant Tracking Control for Non-linear Systems

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***Abstract*—This paper presents a new approach to active fault-tolerant control for non-linear systems, based on fault estimation and compensation for time-varying faults. The work is a motivation for utilising the extension of the well known so-called fast *adaptive fault estimation strategy* to non-linear systems via a multiple-model strategy involving Takagi-Sugeno (T-S) fuzzy observer-based estimation and linear model-reference state tracking control. Moreover, the proposed strategy handles the case of unmatched uncertainty in the tracking error dynamics. The stability proof of the appropriate non-linear augmented system is derived via a Lyapunov condition and formulated as an efficient one step Linear Matrix Inequality (LMI) problem constraining the time-varying system eigenvalues to lie within specified regions in the complex plane. The fault compensation strategy is illustrated using a non-linear example of an inverted pendulum which includes Stribeck friction and a loss-of effectiveness actuator fault.**

## I. INTRODUCTION

THE increase in demand for maintaining controlled system performances and stability under different operation conditions and in the presence of malfunctions or faults has led to the development of the subject of Fault-Tolerant Control (FTC). The increasing rate of recent publications on FTC shows strong evidence of popularity of this topic with some literature also considering strategies for application to real engineering problems often driven by the safety-critical requirements in the aerospace industry [1-5].

FTC loops can be classified as either Passive Fault Tolerant Control (PFTC) or Active Fault Tolerant Control (AFTC). Studies of the significance of FTC as well as survey material are to be found in [6, 7], whilst the book by [8] provides a useful theoretical framework.

All physical systems are non-linear and can be subject to system faults (e.g. in sensors, actuators or in internal process components). This motivates research into the field of robust FTC methods based on the use of nonlinear model dynamics and robust control and estimation. An approach to control of non-linear systems that is becoming popular in the literature with the availability of efficient LMI solutions [13] is the use of T-S fuzzy and linear parameter varying modeling and control [9-11].

The T-S approach to multiple-model control is always characterized by a trade-off between the complexity of gaining suitably stabilizing control action and the degree of accuracy of representing the nonlinear system via T-S fuzzy inference modelling, i.e. in terms of the number of local models or "fuzzy rules" [12].

During the last 5 years several research papers have described the use of T-S modeling and control within the framework of FTC involving actuator and sensor faults [13-16].

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An excellent way to use the T-S framework in AFTC is to develop a model-reference strategy for robust tolerance to actuator faults as discussed by [14] for discrete time systems. The authors in [16] have used the approach of [14] directly for continuous-time systems. In their work they use a T-S (non-linear) reference model which is chosen to precisely replicate the fuzzy model of the real closed-loop plant. Clearly, the works in [14, 16] hide the main challenge involved in model-reference strategies which is the need to deal with the unmatched uncertainty in the tracking dynamics. Recently, in [17] the nonlinear model reference approach proposed in [14, 16] is extended to handle uncertain T-S fuzzy continuous system with unmeasurable premise variables. It is more logical and more practical to use a linear reference model with T-S estimation and control.

Whilst the current paper is also based on the use of T-S within a model-reference FTC system, the research enhances the strategy proposed in [18, 19], by the considering the effects of time-varying actuator faults using observer-based estimation and tracking control.

The example used is the nonlinear inverted pendulum system described via fuzzy-inference (T-S) modelling with the vertical angle used as the measurable premise variable. The idea is to use an approach of estimation and fault compensation, by augmenting a proportional observer with proportional plus integral feedback aiming to achieve *two* requirements: (i) to provide estimation of system states and fault signals simultaneously, and (ii) to cover the situation in which the fault is of fast variation with an assumed bounded fault first time derivative. The control law is designed to minimize the tracking error during normal operation (the fault-free case). The tracking system tolerates actuator fault effects, including varying friction forces. The actuator faults are compensated using an additive fault observer-based estimation signal. Finally, the design approach is formulated as an LMI optimization problem that can be solved using MATLAB software.

The paper is organized as follows: Section II summarizes the main T-S fuzzy inference modeling concepts. Section III describes the proposed observer based fault-tolerant tracking control technique. Section IV gives a tutorial example of a non-linear inverted pendulum with friction to illustrate the proposed strategy. The paper is concluded in Section V.

## II. TAKAGI-SUGENO FUZZY MODEL

Consider the general form of a nonlinear system without any exogenous effects, to illustrate the basic idea of system representing in T-S fuzzy model form:

$$\left.\begin{aligned} \dot{x} &= f\big(x(t), u(t)\big) \\ y &= g\big(x(t)\big) \end{aligned}\right\} \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $u(t) \in \mathcal{R}^m$ is the input vector and $y(t) \in \mathcal{R}^l$ is the output vector.

As mentioned, the basic idea of this approach is to describe the nonlinear system locally around different operating points using linear models and then the overall system can be obtained by a fuzzy blending of the appropriate local systems. Hence, the nonlinear system is then approximated by the following equation:

$$\left.\begin{aligned} \dot{x} &= \sum_{i=1}^{r} h_i\left(p(t)\right)\{A_i x(t) + B_i u(t)\} \\ y &= C\, x(t) \end{aligned}\right\} \tag{2}$$

where $A_i \in \mathcal{R}^{n*n}, B_i \in \mathcal{R}^{n*m},\ i = 1,..,r$ and $C \in \mathcal{R}^{l*n}$ are the system matrices, $r$ is the number of fuzzy rules and the term $h_i\left(p(t)\right)$ is the weighting function that depends on the premise variable $p(t)$ that is assumed to be measured. The weighting function must satisfy the following properties for all time $t$.

$$\sum_{i=1}^{r} h_i\left(p(t)\right) = 1\,;\, 1 \geq h_i\left(p(t)\right) \geq 0;\ \mathrm{i} = 1, 2, \ldots, \mathrm{r} \tag{3}$$

## III. ACTUATOR FTC STRATEGY

The faulty system considered here is represented by the following smooth and differentiable non-linear differential equation:

$$\left.\begin{aligned} \dot{x}_f &= f\left(x_f(t), u_f(t), f_a(t)\right) \\ y &= g\left(x_f(t)\right) \end{aligned}\right\} \tag{4}$$

where $f_a(t) \in \mathcal{R}^m$ is the fault signal. Based on the brief introduction given in Section II the T-S fuzzy model of the system (4) is of the following form:

$$\left.\begin{aligned} \dot{x}_f &= \sum_{i=1}^{r} h_i(p)\{A_i x_f + B_i\left(u_f + f_a\right)\} \\ y &= C\, x_f \end{aligned}\right\} \tag{5}$$

Consider the following reference model which has the same order $n$ as an individual local linear model of the T-S fuzzy system:

$$\dot{x}_d = A_d x_d + B_d d \tag{6}$$

where $x_d \in \mathcal{R}^n$ is the desired trajectory for $x_f$ for all $t \geq 0$, $d \in \mathcal{R}^d$ is the bounded reference input, $A_d \in \mathcal{R}^{n*n}$ and $B_d \in \mathcal{R}^{n*d}$ are a stable and controllable state space pair.

The control objective here is to force the nonlinear plant state in both faulty and healthy cases to follow the states of the reference model. To achieve this objective the following fuzzy controller is proposed:

$$u_f = \sum_{i=1}^{r} h_i(p)\{K_i\left(\hat{x}_f - x_d\right) + K_{i2} x_d + K_{i3} d - \hat{f}_a\} \tag{7}$$

where $\hat{x}_f$ is the estimate of the faulty state.

Subtracting (6) from (5) and substituting for $u_f$ from (7) the dynamics of the tracking error ($e_t = x_f - x_d$) are given by:

$$\begin{aligned} \dot{e}_t = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(p) h_j(p) \{&\left(A_i + B_i K_j\right) e_t - B_i K_j e_x \\ &+ \left(B_i K_{j2} + A_i - A_d\right) x_d + (B_i K_{j3} \\ &- B_d) d + B_i e_f\} \end{aligned} \tag{8}$$

where $e_x$ and $e_f$ are the state and fault estimation errors. It can clearly be seen from (7) that the performance of the proposed control strategy is highly related to the accuracy of simultaneous estimation of both the system state and the fault signal. It is also of interest to consider cases fast fault scenarios for which it can be assumed that the first time derivative of each fault signal is bounded. Therefore, we use a fuzzy observer-based *fast fault estimator* where the observer is designed to estimate the system state and fault signal. The observer proposed originally for linear systems by [20] is illustrated below:

Assuming that the time derivative of the fault ($\dot{f}$) is bounded, then the following fuzzy observer is proposed to simultaneously estimate the system states and actuator fault signal:

$$\left.\begin{aligned} \dot{\hat{x}}_f &= \sum_{i=1}^{r} h_i(p)\{A_i \hat{x}_f + B_i(u_f + \hat{f}_a) + L_i\left(y - C\hat{x}_f\right)\} \\ \dot{\hat{f}}_a(t) &= \sum_{i=1}^{r} h_i(p) F_i C(\dot{e}_x + e_x) \end{aligned}\right\} \tag{9}$$

where $\hat{x}_f \in \mathcal{R}^n$ is the estimate of the state vector $x_f$, $L_i \in \mathcal{R}^{n\times l}$ *and* $F_i \in \mathcal{R}^{m\times l}$ are the observer gains to be design, $e_x$ is the state estimation error defined as:

$$e_x = x_f - \hat{x}_f \tag{10}$$

The state estimation error dynamics are given by:

$$\dot{e}_x = \sum_{i=1}^{r} h_i(p)\{(A_i - L_i C) e_x + B_i e_f\} \tag{11}$$

where $e_f$ is the fault estimation error defined as follows:

$$e_f = f_a - \hat{f}_a \tag{12}$$

From Eqs. (9) & (11) the fault estimation error dynamics can be written as:

$$\begin{aligned} \dot{e}_f = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(p) h_j(p)\{&\dot{f}_a \\ &- F_i C\left(A_j - L_j C + I\right) e_x - F_i C B_j e_f\} \end{aligned} \tag{13}$$

In Eq. (13) the first time derivative of the fault is considered as a bounded exogenous input signal that affects the estimation dynamics. Using Eqs. (6), (8), (11) & (13), the augmented system takes the form:

$$\dot{\tilde{x}}_a(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(p) h_j(p)\{\tilde{A}_{ij}\tilde{x}_a + \tilde{N}_{ij}\tilde{z}\} \tag{14}$$

where:

$$\tilde{A}_{ij} = \begin{bmatrix} A_i + B_i K_j & -B_i K_j & B_i \\ 0 & A_i - L_i C & B_i \\ 0 & -F_i C\left(A_j - L_j C + I\right) & -F_i C B_j \end{bmatrix};\ \tilde{x}_a = \begin{bmatrix} e_t \\ e_x \\ e_f \end{bmatrix}$$

$$\tilde{z} = \begin{bmatrix} x_d \\ d \\ \dot{f} \end{bmatrix};\ \ \tilde{N}_{ij} = \begin{bmatrix} B_i K_{j2} + A_i - A_d & B_i K_{j3} - B_d & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix}$$

The objective here is to compute the gains $L_i, F_i, and\ K_i$ such that the input $\tilde{z}$ in (14) is attenuated below the desired level $\bar{\gamma}$, to ensure robust tracking performance.

**Remark 1:** The estimator of Eq. (9) is proposed to overcome the limitation inherited in the use of the output signal for control. Also, the estimated fault signal can be used to compensate the effect of the actual fault.

Clearly, a stability proof is required when considering the closed-loop behaviour of the overall augmented system. This stability requirement becomes more complicated for multiple-model systems since the complexity increases as the number of local models is increased. Therefore, to obviate a "trial and error" design methodology the proposed LMI design formulation is derived so that the design gains are obtained through a one-step solution to the set of LMIs.

***Theorem1***: *for t>0 and $h_i(p)h_j(p) \neq 0$, The closed-loop fuzzy system in (14) is asymptotically stable and the $H_\infty$ performance is guaranteed with an attenuation level $\bar{\gamma}$, Provided that the signal $(\tilde{z})$ is bounded, and $rank(CB_i) = m$, if there exist symmetric positive definite matrices $P_1, P_2$, and matrices $H_i, Y_j, F_i$, and a scalar $\mu$ satisfying the following LMI constraints ((15), (16), and (17)):*

$$Minimise\ (\bar{\gamma} + \tau)\ such\ that$$

$$P_1 > 0, \qquad P_2 > 0 \tag{15}$$

$$\begin{bmatrix} \tau I & B_i^T P_2 - F_i C \\ * & \tau I \end{bmatrix} > 0 \tag{16}$$

$$\begin{bmatrix}
\Psi_{11} & -B_iY_j & B_iK_{j2} + A_i - A_d & B_i & B_iK_{j3} - B_d & 0 & 0 & 0 & 0 & 0 & 0 & X_1 \\
* & -2\mu X_1 & 0 & 0 & 0 & 0 & \mu I & 0 & 0 & 0 & 0 & 0 \\
* & * & -2\mu I & 0 & 0 & 0 & 0 & \mu I & 0 & 0 & 0 & 0 \\
* & * & * & -2\mu I & 0 & 0 & 0 & 0 & \mu I & 0 & 0 & 0 \\
* & * & * & * & -2\mu I & 0 & 0 & 0 & 0 & \mu I & 0 & 0 \\
* & * & * & * & * & -2\mu I & 0 & 0 & 0 & 0 & \mu I & 0 \\
* & * & * & * & * & * & \Psi_{77} & \Psi_{78} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & \Psi_{88} & 0 & 0 & I & 0 \\
* & * & * & * & * & * & * & * & -\bar{\gamma} I & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\bar{\gamma} I & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & -\bar{\gamma} I & 0 \\
X_1 & * & * & * & * & * & * & * & * & * & * & -w_1^{-1}
\end{bmatrix} < 0 \tag{17}$$

$where$:

$K_j = Y_j X_1^{-1},\ L_i = P_2^{-1} H_i,\ \gamma = \sqrt{\bar{\gamma}},\ X_1 = P_1^{-1}, \Psi_{11} = A_i X_1 + (A_i X_1)^T + B_i Y_j + (B_i Y_j)^T$

$\Psi_{77} = P_2 A_i + (P_2 A_i)^T - H_i C - (H_i C)^T, \Psi_{88} = -\left(B_i^T P_2 B_j + \left(B_i^T P_2 B_j\right)^T\right) + w_3, \Psi_{78} = (B_i^T H_j C - B_i^T P_2 A_j)^T.$

***Proof:*** From Theorem 1 the tracking performance objective can be presented mathematically as follows [18, 19, 21]:

$$\int_0^\infty \tilde{x}_a^T \bar{W}\, \tilde{x}_a\, dt - \gamma^2 \int_0^\infty \tilde{z}^T \tilde{z} \le 0 \tag{18}$$

Where $\bar{W} = \begin{bmatrix} w_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & w_3 \end{bmatrix}$

Consider the following candidate Lyapunov function for the augmented system (14):

$\upsilon(\tilde{x}_a) = \tilde{x}_a^T\, \bar{P}\, \tilde{x}_a$ , $where\ \bar{P} > 0$

To achieve the performance required by (18) and the required closed-loop stability of (14) the following inequality must hold [22]:

$$\dot{\upsilon}(\tilde{x}_a) + \tilde{x}_a^T \bar{W}\, \tilde{x}_a - \gamma^2 \tilde{z}^T \tilde{z} < 0 \tag{19}$$

where $\dot{\upsilon}(\tilde{x}_a)$ is the time derivative of the candidate Lyapunov function. Using Eq.(14), this becomes:

$$\dot{\upsilon}(\tilde{x}_a) = \sum_{i=1}^r \sum_{j=1}^r h_i h_j \{\tilde{x}_a^T (\tilde{A}_{ij}^T \bar{P} + \bar{P} \tilde{A}_{ij}) \tilde{x}_a + \tilde{x}_a^T \bar{P} \tilde{N}_{ij} \tilde{z} + \tilde{z}^T \tilde{N}_{ij}^T \bar{P} \tilde{x}_a\} \tag{20}$$

The inequality (19) (represented in matrix form) after substituting $\dot{\upsilon}(\tilde{x}_a)$ from Eq. (20) becomes:

$$\sum_{i=1}^r \sum_{j=1}^r h_i h_j \left\{ \begin{bmatrix} \tilde{x}_a \\ \tilde{z} \end{bmatrix}^T \begin{bmatrix} \tilde{A}_{ij}^T \bar{P} + \bar{P} \tilde{A}_{ij} + \bar{W} & \bar{P} \tilde{N}_{ij} \\ \tilde{N}_{ij}^T \bar{P} & -\gamma^2 I \end{bmatrix} \begin{bmatrix} \tilde{x}_a \\ \tilde{z} \end{bmatrix} \right\} < 0 \tag{21}$$

Inequality (21) satisfied if condition (22) hold:

$$\begin{bmatrix} \tilde{A}_{ij}^T \bar{P} + \bar{P} \tilde{A}_{ij} + \bar{W} & \bar{P} \tilde{N}_{ij} \\ \tilde{N}_{ij}^T \bar{P} & -\gamma^2 I \end{bmatrix} < 0 \tag{22}$$

To be consistent with (14) $\bar{P}$ is structured as follows:

$$\bar{P} = \begin{bmatrix} P_1 & 0 & 0 \\ 0 & P_2 & 0 \\ 0 & 0 & I \end{bmatrix} > 0 \tag{23}$$

Then after simple manipulation and using the following equality:

$$F_i C = B_i^T P_2 \tag{24}$$

Hence, the inequality (22) can be re-formulated as:

$$\Pi_{ij} = \begin{bmatrix} \Omega_{11} & -P_1B_iK_j & P_1B_i & P_1\left(B_iK_{j2}+A_i-A_d\right) & P_1(B_iK_{j3}-B_d) & 0 \\ * & \Omega_{22} & (B_i^TH_jC-B_i^TP_2A_j)^T & 0 & 0 & 0 \\ * & * & \Omega_{33} & 0 & 0 & I \\ * & * & * & -\gamma^2I & 0 & 0 \\ * & * & * & * & -\gamma^2I & 0 \\ * & * & * & * & * & -\gamma^2I \end{bmatrix} < 0 \quad (25)$$

where: $\Omega_{11} = P_1A_i + (P_1A_i)^T + P_1B_iK_j + (P_1B_iK_j)^T + w_1$, $\Omega_{22} = P_2A_i + (P_2A_i)^T - H_iC - (H_iC)^T$,

$\Omega_{33} = -\left(B_i^TP_2B_j + (B_i^TP_2B_j)^T\right) + w_3$

Inequality (25) contains several nonlinear terms and the next step is to formulate this as an LMI. The single step design formulation of the LMI in (25) is proposed to avoid the complexity of separate design steps characterised by repeated iteration to achieve the gains required to satisfy performance requirements. Hence, after partitioning the matrix inequality shown in (25) $\Pi_{ij}$ becomes:

$$\Pi_{ij} = \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} \quad (26)$$

$\Pi_{12}$ is the upper right block of (25) and $\Pi_{22}$ is the lower right block of (25).

***Lemma 1*.** (Congruence) Consider two matrices $P$ and $Q$, if $P$ is positive definite and if $Q$ is a full column rank matrix, then the matrix $Q * P * Q^T$ is positive definite.

Let $Q = \begin{bmatrix} P_1^{-1} & 0 \\ 0 & X \end{bmatrix}$ , and $X = \begin{bmatrix} P_1^{-1} & 0 \\ 0 & I \end{bmatrix}$

Then $Q * \Pi_{ij} * Q^T < 0$ is also true and it can be re-written as:

$$\begin{bmatrix} P_1^{-1}\Pi_{11}P_1^{-1} & P_1^{-1}\Pi_{12}X \\ X\Pi_{12}P_1^{-1} & X\Pi_{22}X \end{bmatrix} < 0 \quad (27)$$

Inequality (27) implies that $\Pi_{22} < 0$ so that the following inequality holds true:

$$(X + \mu\Pi_{22}^{-1})^T\Pi_{22}(X + \mu\Pi_{22}^{-1}) \leq 0 \Leftrightarrow X\Pi_{22}X \leq -2\mu X - \mu^2\Pi_{22}^{-1} \quad (28)$$

where $\mu$ is a scalar.

By substituting (28) into (27) and using the Schur complement, then (27) holds if the following inequality holds:

$$\begin{bmatrix} P_1^{-1}\Pi_{11}P_1^{-1} & P_1^{-1}\Pi_{12}X & 0 \\ X\Pi_{12}P_1^{-1} & -2\mu X & \mu I \\ 0 & \mu I & \Pi_{22} \end{bmatrix} < 0 \quad (29)$$

After substitution for $\Pi_{11}, \Pi_{12}, \Pi_{12}, \Pi_{22}$ from (26) and by simple manipulation, the LMI in (17) is obtained. The equality constraints given in (24) add conservatism to the design problem which may be reduced through an approximation via a minimization problem. Using the strategy proposed by [20, 23]:

$$Minimise\ \tau$$
$$\begin{bmatrix} \tau I & B_i^TP_2 - F_iC \\ * & \tau I \end{bmatrix} > 0 \quad (30)$$

This completes the proof.

## IV. SIMULATION RESULTS

The control of systems that involve friction in moving mechanical components presents interesting challenges [24]. Recently, some interesting publications [25, 26] suggest that friction can be compensated by considering it to act as a type of actuator fault in the system. The system is made fault-tolerant by fault estimation and compensation as an active FTC problem.

Here a tutorial example is presented using a non-linear simulation of an inverted pendulum and cart together with simulated Stribeck friction force [26].

The nonlinear inverted pendulum and cart system model is given as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \\ \dfrac{g\,sin(x_1) - mlax_2^2\,sin(2x_1)/2}{4l/3 - mla(cos(x_1))^2} \\ \dfrac{-mag\,sin(2x_1)\ /2 + alx_2^2\,sin(x_1)4m/3}{4/3 - mla(cos(x_1))^2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dfrac{-a\,cos(x_1)}{4l/3 - mla(cos(x_1))^2} \\ \dfrac{4a/3}{4/3 - mla(cos(x_1))^2} \end{bmatrix}(u - f_c) \quad (32)$$

where:

$x_1$: Pendulum angular position , $x_2$: Cart position (m),
$x_3$: Pendulum angular velocity , $x_4$: Cart velocity (m s[-1]).
*m*: Pendulum mass (2kg), *2l*: Pendulum length (1m),*M:* Cart mass (8kg), $a = \frac{1}{m+M}$

The T-S model of the nonlinear system (32) is obtained by linearizing the system around *two* operating points at which: $x_1 = 0\ and\ x_1 = \pm\pi/4$ radians, respectively. Details of the fuzzy model of this system can be found in [27].

By using $\mu = 10$, $w_1 = 10^{-3} * \begin{bmatrix} 37 & 0 & 0 & 0 \\ 0 & 11 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, and $w_3 = 10$ and solving the optimization design problem in Theorem 1, the minimum attenuation value $\gamma = 2.2361$, with:

$k_1 = [801.6 \quad 154.1 \quad 231.0 \quad 162.6]$

$$k_1 = [950.3 \quad 181.4 \quad 282.3 \quad 194.4]$$

$$k_{21} = [290.8 \quad 19.6 \quad 46.4 \quad 11.8]$$

$$k_{22} = [376.1 \quad 29.4 \quad 74.4 \quad 17.8]$$

$$k_{31} = [-3.8 \quad -15.7] \qquad k_{32} = [-5.9 \quad -23.5]$$

$$F_1 = [34.8 \quad -177.0 \quad 991.3]$$

$$F_2 = [5.5 \quad -166.3 \quad 914.2]$$

$$L_1 = \begin{bmatrix} 65.2 & 5.5 & 0.3 \\ -0.2 & 0.9 & 0.8 \\ 791.0 & 69.8 & 14.5 \\ -2.9 & -0.1 & 1.0 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 58.2 & 5.2 & 0.06 \\ -0.1 & 0.9 & 0.8 \\ 698.7 & 64.7 & 11.3 \\ -1.3 & -0.04 & 0.7 \end{bmatrix}$$

Fig.1 shows the simulation signals illustrating the performance of the proposed strategy applied to the system affected by the bearing friction (causing uncertainty). The observer provides good estimation of the friction force the presented simulation signals shows how the proposed control strategy can also passively tolerate the effect of friction on the tracking performance.

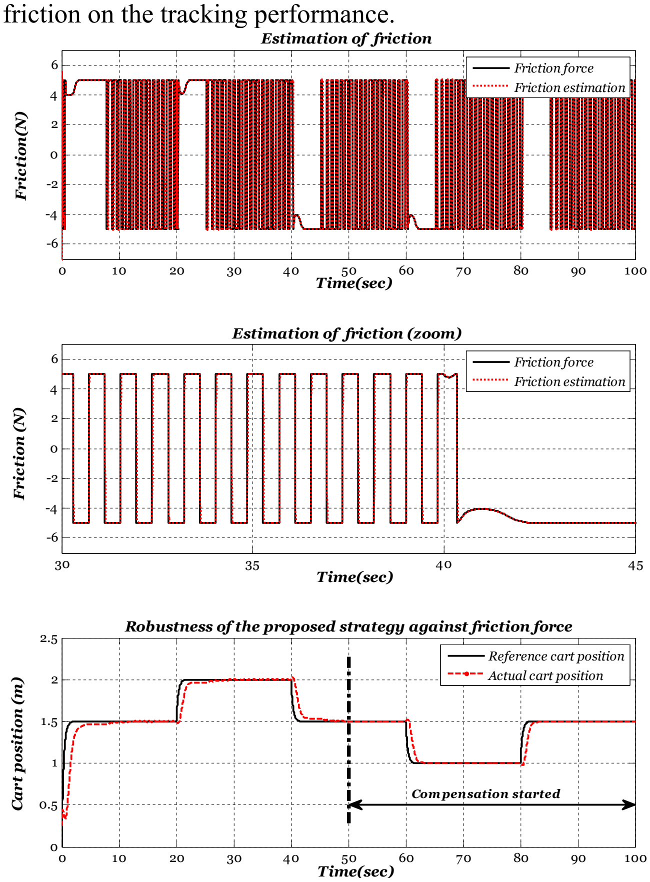



Figure. 1 Results for friction tolerant control.

Fig.2 simulation signals when friction and additive fault signal are simultaneously affect the system. The results are also show how the proposed control strategy can passively tolerate the effects of friction and additive fault without compensation with performance degradation. However, the use of fault compensation recovers the fault-free traction performance.

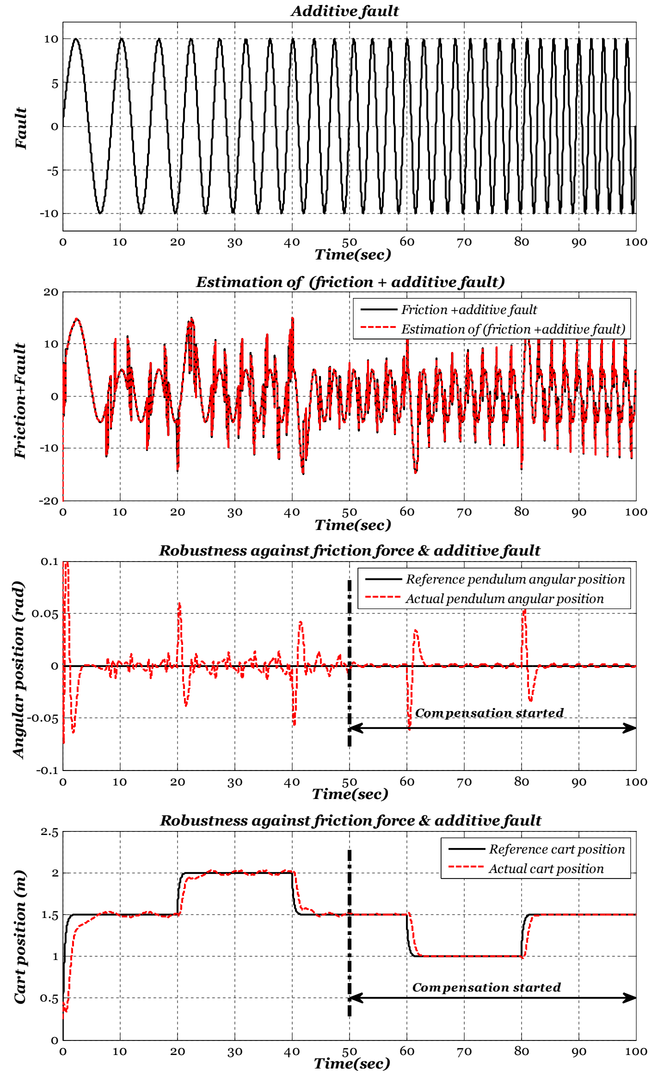
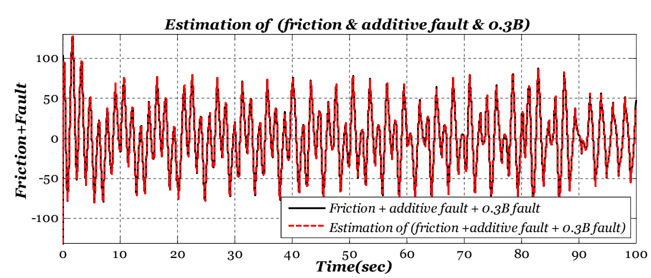



Figure. 2 Fault tolerant results for friction and additive fault.

Fig.3 simulation signals when friction, additive fault, and loss of effectiveness of the actuator represented by $(0.3 * B_i)$ are simultaneously affect the system. Although the system remains stable for this severe fault scenario, the tracking performance degraded by this fault and hence the estimation of this combined fault has been used to compensate the effect of the fault. Results show how the proposed control strategy can actively tolerate the effects of this combined fault scenario.



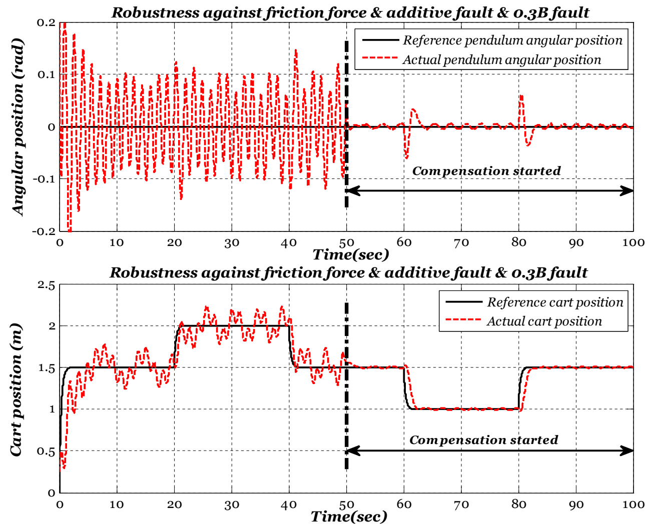



Figure. 3 FTC results for active friction, additive fault, and $0.3 * B_i$ fault.

## V.CONCLUSION

In this paper, a strategy for actuator FTC for nonlinear systems described via T–S fuzzy modelling is proposed. Robustness of the strategy is ensured through attenuation of the $\mathcal{L}_2$ norm of the exogenous disturbance inputs on the dynamics of the augmented system. It is assumed that the friction force is inherited as a normal function of the system. The proposed approach shows a very effective discrimination between the friction force and the actuator fault. Furthermore, based on the fact that the fault can affect the system in different scenarios the dependency of the estimation and compensation FTC approach on accurate estimation requires that the estimation strategy must also take into account the effect of the fault derivative on the estimation accuracy.

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