

Time-varying formation control for unmanned aerial vehicles with switching interaction topologies

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Abstract—Time-varying formation control problems for unmanned aerial vehicle (UAV) swarm systems with switching interaction topologies are studied using a consensus based approach. Necessary and sufficient conditions for UAV swarm systems with switching interaction topologies to achieve predefined time-varying formations are proposed. An approach to design the formation protocol is given. A quadrotor formation platform including four quadrotors is introduced. Theoretical results are validated by outdoor time-varying formation experiments.

Index Terms—Time-varying formation, unmanned aerial vehicle, swarm system, switching interaction topology

I. INTRODUCTION

Formation control of unmanned aerial vehicle (UAV) swarm systems has broad potential applications in civilian and military fields such as surveillance [1], drag reduction [2] and telecommunication relay [3], etc. In the past decade, many results on UAV formation have been proposed based on leader-follower, virtual structure and behavior approaches (see [4]–[8] and references therein).

In recent years, consensus problems of swarm systems have been studied extensively and many results have been obtained [9]–[13]. With the development of consensus theory, more and more researchers realized that formation problems can be solved using consensus approaches. Ren [14] applied a consensus protocol to deal with the formation problems of second-order swarm systems and showed that leader-follower, virtual structure and behavior based formation approaches can be regarded as special cases of consensus based approaches. For UAV swarm systems to achieve time-invariant formations, Abdessameud and Tayebi [15] presented a consensus based controller with time delays. A consensus approach and an output feedback linearization method are used together to discuss the formation problems of UAV swarm systems in [16]. The approaches in [15], [16] are validated by numerical simulations. Indoor time-invariant formation flight experiments for quadrotor swarm systems can be found in [17]. Basing on consensus approaches, Yu et al. [18] presented outdoor time-varying formation experiments. However, the interaction topologies

in [17], [18] are assumed to be fixed. In practical applications, the interaction topologies of UAV swarm systems may be switching due to that the communication channel may fail and new channels may be created during flight. To the best of our knowledge, time-varying formation problems for UAV swarm systems with switching interaction topologies are still open.

In this paper, time-varying formation problems for UAV swarm systems with switching interaction topologies are investigated and formation experiments on the quadrotor platform are performed. Firstly, a time-varying formation protocol with switching interaction is given. Then based on consensus theory, necessary and sufficient conditions for UAV swarm systems to achieve time-varying formation are presented. In [15]–[17], the formation is time-invariant or partially time-varying. In [18], the interaction topology is fixed. Moreover, an approach to determine the gain matrices in the protocol is given by solving an algebraic Riccati equation. Finally, a quadrotor formation platform is introduced to validate the theoretical results. Both simulations and outdoor experiments for the quadrotor swarm system to achieve time-varying formations under switching interaction topologies are presented.

The rest of this paper is organized as follows. In Section 2, basic concepts and useful results on graph theory are summarized and the problem formulation is shown. Main theoretical results are proposed in Section 3. In Section 4, a quadrotor formation platform is introduced and experimental applications are given. Section 5 concludes the whole work.

Throughout this paper, for simplicity of notation, 0 will denote zero matrices of appropriate size with zero vectors and zero number as special cases. Let 1_N be a column vector of size N with 1 as its elements, and I_n represents an identity matrix with dimension n . Let \otimes denote Kronecker product.

II. PRELIMINARIES AND PROBLEM DESCRIPTION

In this section, firstly, basic concepts and results on graph theory are introduced, and then, we present a detailed description of the problem.

A. Basic concepts and results in graph theory

An undirected graph G can be described by $G = \{Q, E, W\}$, where $Q = \{q_1, q_2, \dots, q_N\}$ is the node set, $E \subseteq \{(q_i, q_j) : q_i, q_j \in Q\}$ is the edge set, and $W = [w_{ij}] \in \mathbb{R}^{N \times N}$ is a symmetric adjacency matrix with nonnegative elements w_{ij} . An edge of G is denoted by $q_{ij} = (q_i, q_j)$. The adjacency elements associated with the edges of G are positive, i.e., $w_{ji} > 0 \Leftrightarrow q_{ij} \in E$. Additionally, $w_{ii} = 0$ for all $i \in \{1, 2, \dots, N\}$. Let $N_i = \{q_j \in Q : (q_j, q_i) \in E\}$ denote the set of neighbors of node q_i . Define the in-degree of node q_i as $\deg_{in}(q_i) = \sum_{j=1}^N w_{ij}$. The degree matrix of G is denoted by $D = \text{diag}\{\deg_{in}(q_i), i = 1, 2, \dots, N\}$. The Laplacian matrix of G is defined as $L = D - W$. An undirected graph is said to be connected if there is a path from each node to any one of the other nodes. More details on graph theory can be found in [19]. The following lemma is useful in analyzing formation problems of UAV swarm systems.

Lemma 1 [10]: Let $L \in \mathbb{R}^{N \times N}$ be the Laplacian matrix of an undirected graph G , then

- (i) L has at least one zero eigenvalue, and $\mathbf{1}_N$ is the associated eigenvector; that is, $L\mathbf{1}_N = 0$;
- (ii) If G is connected, then 0 is a simple eigenvalue of L , and all the other eigenvalues are positive.

B. Problem description

Consider a UAV swarm system with N UAVs. The interaction topology of the UAV swarm system can be described by the undirected graph G . For $i, j \in \{1, 2, \dots, N\}$, a UAV i can be denoted by a node s_i in G and the interaction from UAV i to UAV j can be denoted by e_{ij} . It is assumed that G is connected.

For each of these UAVs, since the translational dynamics have a much larger time constant than the attitude dynamics, the formation control can be implemented with an outer/inner-loop structure [20]. The current paper mainly considers the formation control in the outer-loop. In this case, on the formation control level, the UAV can be regarded as a point-mass system, and the dynamics of each UAV can be described by the following double integrator [6], [16].

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = u_i(t), \end{cases} \quad (1)$$

where $i = 1, 2, \dots, N$. In this equation set, $x_i(t) \in \mathbb{R}^n$ and $v_i(t) \in \mathbb{R}^n$ denote the position and velocity vectors of the i th UAV respectively, while $u_i(t) \in \mathbb{R}^n$ is for the control input of UAV i . In the following, if not otherwise specified, it is assumed that $n = 1$ for simplicity of description. However, it should be pointed out that similar analysis can also be done for the higher dimensional case by using Kronecker product, so all the results hereafter remain valid for $n > 1$.

Let $\xi_i(t) = [x_i(t), v_i(t)]^T$, $B_1 = [1, 0]^T$, and $B_2 = [0, 1]^T$, then UAV swarm system (1) can be rewritten as

$$\dot{\xi}_i(t) = B_1 B_2^T \xi_i(t) + B_2 u_i(t) \quad (2)$$

Let $h_i(t) = [h_{ix}(t), h_{iv}(t)]^T$ ($i = 1, 2, \dots, N$) be piecewise continuously differentiable vectors and $h(t) = [h_1^T(t), h_2^T(t), \dots, h_N^T(t)]^T \in \mathbb{R}^{2N}$. A time-varying formation is specified by a vector $h(t)$.

Definition 1: UAV swarm system (2) is said to achieve the time-varying formation $h(t)$ if for any given bounded initial states, there exists a vector-valued function $r(t) \in \mathbb{R}^2$ such that

$$\lim_{t \rightarrow \infty} (\xi_i(t) - h_i(t) - r(t)) = 0 \quad (i = 1, 2, \dots, N),$$

where $r(t)$ is called a *formation reference function*.

Definition 2: UAV swarm system (2) is said to achieve consensus if for any given bounded initial states, there exists a vector-valued function $c(t) \in \mathbb{R}^2$ such that

$$\lim_{t \rightarrow \infty} (\xi_i(t) - c(t)) = 0 \quad (i = 1, 2, \dots, N),$$

where $c(t)$ is called a *consensus function*.

Remark 1: From Definitions 1 and 2, it can be seen that if $h(t) \equiv 0$, UAV swarm system (2) achieves consensus if it achieves formation. In this case, the time-varying formation reference function is equivalent to the consensus function. Therefore, consensus problem is just a special case of formation problem for UAV swarm system described by (2).

For the i th UAV, consider the following time-varying formation protocol expression

$$u_i(t) = K_1 (\xi_i(t) - h_i(t)) + K_2 \sum_{j \in N_i(t)} w_{ij} ((\xi_j(t) - h_j(t)) - (\xi_i(t) - h_i(t))) + \dot{h}_i(t), \quad (3)$$

where $K_1 = [k_{11}, k_{12}]$ and $K_2 = [k_{21}, k_{22}]$ are constant gain matrices, and $N_i(t)$ the time-varying neighbor sets.

Remark 2: One sees that, only the states of neighbors are required to achieve formation. In protocol (3), K_1 can be chosen to assign the motion modes [21] of the time-varying formation reference, while K_2 can be designed to drive all the UAVs to achieve the desired formation.

In the following, the switching interaction topologies case will be considered. Let I denote all the possible interaction topologies with an index set $S \subset \mathbb{N}$, where \mathbb{N} represents the set of natural numbers. Let $\delta(t) : [0, +\infty) \rightarrow S$ be a switching signal whose value is the index of the topology at the time of t , and $L_{\delta(t)}$ stands for the corresponding Laplacian matrix. Throughout this paper, it is assumed that admissible switching signals have a dwell time $T_d > 0$, and all the interaction topologies in I are connected.

Let $\xi(t) = [\xi_1^T(t), \xi_2^T(t), \dots, \xi_N^T(t)]^T$, $h_x(t) = [h_{1x}(t), h_{2x}(t), \dots, h_{Nx}(t)]^T$ and $h_v(t) = [h_{1v}(t), h_{2v}(t), \dots, h_{Nv}(t)]^T$. Under protocol (3), UAV swarm system (2) can be expressed in a compact form as follows

$$\begin{aligned} \dot{\xi}(t) = & \left(I_N \otimes (B_2 K_1 + B_1 B_2^T) - L_{\delta(t)} \otimes (B_2 K_2) \right) \xi(t) \\ & - (I_N \otimes (B_2 K_1) - L_{\delta(t)} \otimes (B_2 K_2)) h(t) \\ & + (I_N \otimes B_2) \dot{h}_v(t). \end{aligned} \quad (4)$$

The present paper mainly investigates the following three problems for the UAV swarm system (4) with switching

interaction topologies: (i) under what conditions can the time-varying formation $h(t)$ be achieved; (ii) how to design protocol (3) to achieve the predefined time-varying formation $h(t)$; and (iii) how to validate the theoretical results on a practical quadrotor formation platform.

III. CONTROLLER DESIGN

In this section, firstly, necessary and sufficient conditions for UAV swarm system (4) with switching interaction topologies to achieve the time-varying formation $h(t)$ are presented using a consensus based approach. Then an approach to determine the gain matrices in the protocol is given.

Let $\tilde{\xi}_i(t) = \xi_i(t) - h_i(t)$ ($i = 1, 2, \dots, N$) and $\tilde{\xi}(t) = [\tilde{\xi}_1^T(t), \tilde{\xi}_2^T(t), \dots, \tilde{\xi}_N^T(t)]^T$. Then swarm system (4) can be rewritten as follows

$$\begin{aligned} \dot{\tilde{\xi}}(t) = & \left(I_N \otimes (B_2 K_1 + B_1 B_2^T) - L_{\delta(t)} \otimes (B_2 K_2) \right) \tilde{\xi}(t) \\ & + (I_N \otimes B_1) (h_v(t) - \dot{h}_x(t)). \end{aligned} \quad (5)$$

Based on Definitions 1 and 2, the following lemma holds directly.

Lemma 2: UAV Swarm system (4) with switching interaction topologies achieves time-varying formation $h(t)$ if and only if swarm system (5) achieves consensus.

Let $U = [\bar{u}_1, \bar{u}_2, \dots, \bar{u}_N]$ be an orthogonal constant matrix with $\bar{u}_1 = \mathbf{1}_N / \sqrt{N}$, then one gets $U^T L_{\delta(t)} U = \text{diag}\{0, \tilde{U}^T L_{\delta(t)} \tilde{U}\}$, where $\tilde{U} = [\bar{u}_2, \bar{u}_3, \dots, \bar{u}_N]$. Let $\zeta(t) = (\bar{u}_1^T \otimes I) \tilde{\xi}(t)$ and $\varsigma(t) = (\tilde{U}^T \otimes I) \tilde{\xi}(t)$, then swarm system (5) can be transformed into

$$\begin{aligned} \dot{\zeta}(t) = & (B_2 K_1 + B_1 B_2^T) \zeta(t) \\ & + \frac{1}{\sqrt{N}} (\mathbf{1}_N^T \otimes B_1) (h_v(t) - \dot{h}_x(t)), \end{aligned} \quad (6)$$

$$\begin{aligned} \dot{\varsigma}(t) = & \left(I_{N-1} \otimes (B_2 K_1 + B_1 B_2^T) - (\tilde{U}^T L_{\delta(t)} \tilde{U}) \otimes B_2 K_2 \right) \varsigma(t) \\ & + (\tilde{U}^T \otimes B_1) (h_v(t) - \dot{h}_x(t)). \end{aligned} \quad (7)$$

The following lemma presents a necessary and sufficient condition for UAV swarm system (4) to achieve formation $h(t)$.

Lemma 3: UAV Swarm system (4) with switching interaction topologies achieves the time-varying formation $h(t)$ if and only if $\lim_{t \rightarrow \infty} \varsigma(t) = 0$.

Proof: Let $e_1 \in \mathbb{R}^N$ be a vector with 1 as its first component and 0 elsewhere. Let

$$\tilde{\xi}_C(t) = \frac{1}{\sqrt{N}} \mathbf{1}_N \otimes \zeta(t), \quad (8)$$

$$\tilde{\xi}_{\bar{C}}(t) = \tilde{\xi}(t) - \tilde{\xi}_C(t). \quad (9)$$

Since $[\xi^T(t), 0]^T = e_1 \otimes \xi(t)$, one has

$$\tilde{\xi}_C(t) = (U \otimes I_2) [\xi^T(t), 0]^T. \quad (10)$$

Note that $[\zeta^T(t), \varsigma^T(t)] = (U^T \otimes I_2) \tilde{\xi}(t)$, by (8)-(10), it can be shown that

$$\tilde{\xi}_{\bar{C}}(t) = (U \otimes I_2) [0, \varsigma^T(t)]^T. \quad (11)$$

Due to the fact that $U^T \otimes I_2$ is nonsingular, one can see that $\tilde{\xi}_C(t)$ and $\tilde{\xi}_{\bar{C}}(t)$ are linearly independent by (10) and (11). Therefore, from (8) and (9), it holds that the subsystems with states $\tilde{\xi}_C(t)$ and $\tilde{\xi}_{\bar{C}}(t)$ describe the consensus dynamics and disagreement dynamics of swarm system (5) respectively. Furthermore, from Lemma 2, it follows that UAV swarm system (4) achieves the time-varying formation $h(t)$ if and only if $\lim_{t \rightarrow \infty} \tilde{\xi}_{\bar{C}}(t) = 0$; or that is to say, $\lim_{t \rightarrow \infty} \varsigma(t) = 0$. This completes the proof.

Let $\lambda_{\delta(t)}^i$ ($i = 1, 2, \dots, N$) be the eigenvalues of the Laplacian $L_{\delta(t)}$. Without loss of generality, it is assumed that $\lambda_{\delta(t)}^1 \leq \lambda_{\delta(t)}^2 \leq \dots \leq \lambda_{\delta(t)}^N$. Furthermore, from Lemma 1, one sees that $\lambda_{\delta(t)}^1 = 0$ with an associated eigenvector $\bar{u}_1 = \mathbf{1} / \sqrt{N}$. Let $\lambda_{\min} = \min\{\lambda_m^i (\forall m \in S; i = 2, 3, \dots, N)\}$.

Theorem 1: If for $\forall i \in \{1, 2, \dots, N\}$

$$\lim_{t \rightarrow \infty} \left((h_{iv}(t) - h_{jv}(t)) - (\dot{h}_{ix}(t) - \dot{h}_{jx}(t)) \right) = 0, \quad j \in N_i(t), \quad (12)$$

UAV swarm system (2) achieves time-varying formation $h(t)$ by protocol (3) with $K_2 = (2\lambda_{\min} R)^{-1} B_2^T P$ where P is the positive definite solution to the following algebraic Riccati equation

$$PA + A^T P - PB_2 R^{-1} B_2^T P + I = 0, \quad (13)$$

where $R > 0$ and $A = B_2 K_1 + B_1 B_2^T$.

Proof: If for $\forall i \in \{1, 2, \dots, N\}$, (12) holds, it is easy to know that

$$\lim_{t \rightarrow \infty} (L_{\delta(t)} \otimes B_1) (h_v(t) - \dot{h}_x(t)) = 0. \quad (14)$$

Substituting $L_{\delta(t)} = U \text{diag}\{0, \tilde{U}^T L_{\delta(t)} \tilde{U}\} U^{-1}$ into (14), then pre-multiplying both sides of (14) by $U^{-1} \otimes I_2$ leads to

$$\lim_{t \rightarrow \infty} \left((\tilde{U}^T L_{\delta(t)} \tilde{U} \tilde{U}^T \otimes B_1) (h_v(t) - \dot{h}_x(t)) \right) = 0. \quad (15)$$

Since the interaction topology is connected, from Lemma 1 and the structure of U , one sees that $\tilde{U}^T L_{\delta(t)} \tilde{U}$ is nonsingular. Pre-multiplying both sides of (15) by $(\tilde{U}^T L_{\delta(t)} \tilde{U})^{-1} \otimes I$, one has

$$\lim_{t \rightarrow \infty} (\tilde{U}^T \otimes B_1) (h_v(t) - \dot{h}_x(t)) = 0. \quad (16)$$

Consider the stability of the following system

$$\dot{\theta}(t) = \left(I_{N-1} \otimes A - (\tilde{U}^T L_{\delta(t)} \tilde{U}) \otimes B_2 K_2 \right) \theta(t). \quad (17)$$

Choose the following Lyapunov function candidate

$$V(t) = \theta^T(t) (I_{N-1} \otimes P) \theta(t). \quad (18)$$

Taking the derivative of $V(t)$ with respect to t along the solution to system (17), one has

$$\dot{V}(t) = \theta^T(t) \left(I_{N-1} \otimes \Xi_1 - (\tilde{U}^T L_{\delta(t)} \tilde{U}) \otimes \Xi_2 \right) \theta(t), \quad (19)$$

where $\Xi_1 = A^T P + PA$ and $\Xi_2 = (K_2^T B_2^T P + PB_2 K_2)$.

Since $U^T L_{\delta(t)} U = \text{diag}\{0, \tilde{U}^T L_{\delta(t)} \tilde{U}\}$ and U is orthogonal, one can suppose that the eigenvalues of $\tilde{U}^T L_{\delta(t)} \tilde{U}$

are $\lambda_{\delta(t)}^2, \lambda_{\delta(t)}^3, \dots, \lambda_{\delta(t)}^N$. Note that $\tilde{U}^T L_{\delta(t)} \tilde{U}$ is symmetric. Therefore, there exists an orthogonal matrix $\hat{U}_{\delta(t)}$ such that

$$\hat{U}_{\delta(t)}^T \tilde{U}^T L_{\delta(t)} \tilde{U} \hat{U}_{\delta(t)} = \text{diag} \left\{ \lambda_{\delta(t)}^2, \lambda_{\delta(t)}^3, \dots, \lambda_{\delta(t)}^N \right\}. \quad (20)$$

Let

$$\begin{aligned} \bar{\theta}_{\delta(t)}(t) &= \left(\hat{U}_{\delta(t)}^T \otimes I_2 \right) \theta(t) \\ &= \left[\left(\bar{\theta}_{\delta(t)}^2(t) \right)^T, \left(\bar{\theta}_{\delta(t)}^3(t) \right)^T, \dots, \left(\bar{\theta}_{\delta(t)}^N(t) \right)^T \right]^T. \end{aligned}$$

Then by (19) and (20), it can be obtained that

$$\dot{V}(t) = \sum_{i=2}^N \left(\bar{\theta}_{\delta(t)}^i(t) \right)^T \left(\Xi_1 - \lambda_{\delta(t)}^i \Xi_2 \right) \bar{\theta}_{\delta(t)}^i(t). \quad (21)$$

Substituting $K_2 = (2\lambda_{\min} R)^{-1} B_2^T P$ into (21), from (13), one has

$$\dot{V}(t) = \sum_{i=2}^N \left(\bar{\theta}_{\delta(t)}^i(t) \right)^T \left(-I + \left(1 - \frac{\lambda_{\delta(t)}^i}{\lambda_{\min}} \right) P B_2 B_2^T P \right) \bar{\theta}_{\delta(t)}^i(t).$$

Considering that $1 - \lambda_{\delta(t)}^i \lambda_{\min}^{-1} < 0$, by the assumption of the dwell time $T_d > 0$, one gets that $\dot{V}(t) \equiv 0$ if and only if $\bar{\theta}_{\delta(t)}^i(t) \equiv 0$ ($i = 2, 3, \dots, N$), which means that $\bar{\theta}_{\delta(t)}(t) \equiv 0$. Note that $\bar{\theta}_{\delta(t)}(t) = \left(\hat{U}_{\delta(t)}^T \otimes I \right) \theta(t)$, $\dot{V}(t) \equiv 0$ if and only if $\theta(t) \equiv 0$. Therefore, system (17) is asymptotically stable. From (7) and (16), one knows that $\lim_{t \rightarrow \infty} \zeta(t) = 0$. From Lemma 3, one gets that UAV swarm system (4) under protocol (3) achieves the time-varying formation $h(t)$. This completes the proof.

IV. SIMULATIONS AND EXPERIMENTS

In this section, firstly, a quadrotor formation platform is introduced. Then both simulation and experiment are shown to validate the theoretical results.

As shown in Fig. 1, the formation platform consists of four quadrotors with flight control system (FCS) and a ground control station (GCS). Each quadrotor has a tip-to-tip wingspan of 55 cm, a height of 26 cm, a weight of about 1.4 Kg, a battery life of 12 minutes and a maximum take-off weight of 1.8 Kg. The FCS is based on a TMS320F28335 Digital Signal Processor (DSP). The acceleration and attitude of the quadrotor are estimated by three 1-axis gyroscopes, 3-axis accelerometer and 3-axis magnetometer. A GPS is utilized to measure the position and velocity of each agent at a rate of 10 Hz. A barometer and an ultrasonic range finder work together to measure the height. A micro SD card is mounted on board for recording real time data. The wireless communications among quadrotors are realized by Zigbee modules. Through the Zigbee network, control commands can also be sent to specified quadrotors, and the states of all quadrotors can be monitored by the GCS. For safety reasons, the remote control (RC) is reserved. However, when quadrotors are achieving formation, neither the RC nor GCS will send commands to quadrotors.

For simplicity, assume that there exist four interaction topologies in set I (as shown in Fig. 2), where $\delta(t) =$



Fig. 1. Quadrotor formation platform.

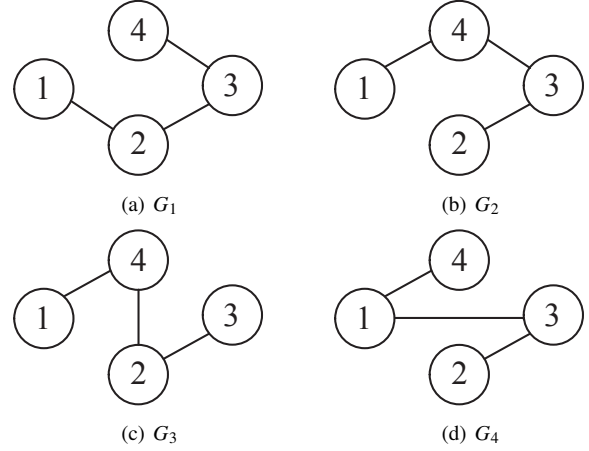


Fig. 2. Interaction topologies.

$\{1, 2, 3, 4\}$ and $G_{\delta(t)}$ represents the corresponding topology at the time of t . The interaction topologies have 0 – 1 weights and are randomly chosen from I with interval $T_d = 10$ s. The formation control for four quadrotors are performed in the XY plane; that is $n = 2$. In this case, $\xi_i(t)$ and $u_i(t)$ can be rewritten as $\xi_i(t) = [x_{iX}, v_{iX}, x_{iY}, v_{iY}]^T$ and $u_i(t) = [u_{iX}, u_{iY}]^T$ ($i = 1, 2, 3, 4$). The height and the yaw angle of each quadrotor are specified to be constants. The attitude controller runs at 500 Hz while the formation controller runs at 5 Hz. Choose the following time-varying formation

$$h_i(t) = \begin{bmatrix} r \cos(\omega t + (i-1)\pi/2) \\ -\omega r \sin(\omega t + (i-1)\pi/2) \\ r \sin(\omega t + (i-1)\pi/2) \\ \omega r \cos(\omega t + (i-1)\pi/2) \end{bmatrix} \quad (i = 1, 2, 3, 4),$$

where $r = 10$ m and $\omega = 0.1$ rad/s. If $h(t)$ is achieved, then both the positions and velocities of the four quadrotors locate at the vertexes of a rotating regular square respectively in the plane XY , Choose $K_1 = I_2 \otimes [-1 \ -0.8]$ and the initial states of four quadrotors as $\xi_1(0) = [9.84 \ -0.11 \ 0.19 \ 0.07]^T$, $\xi_2(0) = [-0.41 \ 0.04 \ 10.51 \ 0.22]^T$, $\xi_3(0) = [-10.47 \ 0.08 \ 0.48 \ 0.02]^T$, $\xi_4(0) = [-0.93 \ -0.08 \ -9.11 \ -0.25]^T$. Using

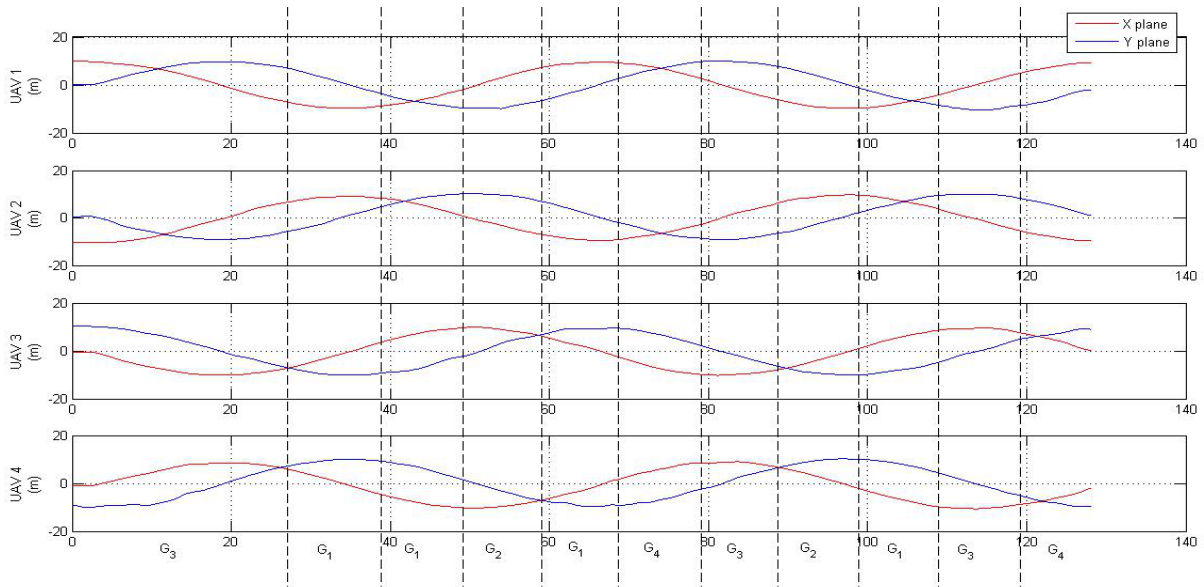
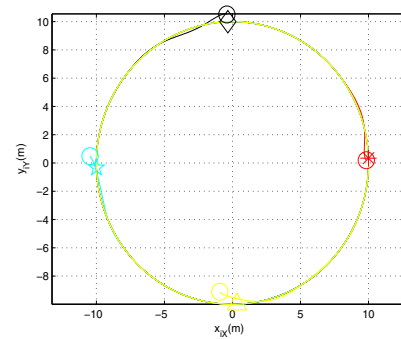


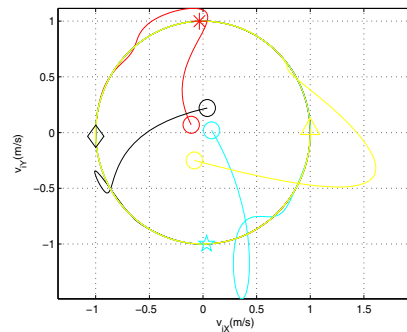
Fig. 3. Trajectories of $\xi_i(t)$ ($i = 1, 2, 3, 4$) with respect to time along the X axis and the Y axis respectively in experiment.

the approach in Theorem 1, choose $R=3$ and one can obtain a K_2 to ensure the UAV swarm system achieves the desired formation as $K_2 = I_2 \otimes [0.26 \ 0.57]$. It should be pointed out that the positions and velocities of neighbors, which are estimated by GPS, are transmitted via the Zigbee network to construct the protocol (3).

Fig. 3 shows the trajectories of the four quadrotors relative to the center of the predefined rotating regular square in the plane XY within 128.6 s, where the dotted lines illustrate the switching time of the four topologies, and the specific topology in each time period is given at the bottom of the Fig. 3. One can see that the changes of the position of the four quadrotors are continuous and smooth, so it indicates that the switching interaction topologies do not influence the stability of the formation. Figs. 4 and 5 show the trajectories of positions and velocities of the four quadrotors obtained by simulation and experiment respectively, where the initial states of the four quadrotors are marked by circles and the final states are denoted by asterisks, pentagrams, diamonds, and triangles respectively. Fig. 6 depicts the snapshots of positions and velocities of the four quadrotors in the experiment from 63 s to 70 s. From Figs. 3-6, one sees that both in simulation and experiment examples, the positions of the four quadrotors locate at a circle with radius of 10 m, and the four quadrotors fly smoothly along the circumference with 90-degree phase difference between neighbors, so the quadrotor swarm system achieves the predefined time-varying formation under switching interaction topologies. The video of the flight experiment can be found at http://v.youku.com/v_show/id_XNjY3OTE4OTI0.html or <https://www.youtube.com/watch?v=9cVyDAvDi3M>.

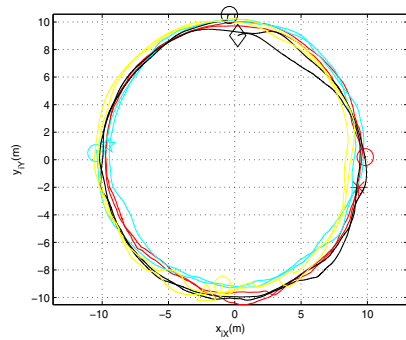


(a) Positions

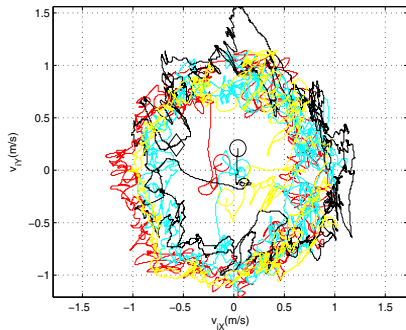


(b) Velocities

Fig. 4. Trajectories of $\xi_i(t)$ ($i = 1, 2, 3, 4$) in simulation.



(a) Positions



(b) Velocities

Fig. 5. Trajectories of $\xi_i(t)$ ($i = 1, 2, 3, 4$) in experiment.

V. CONCLUSIONS

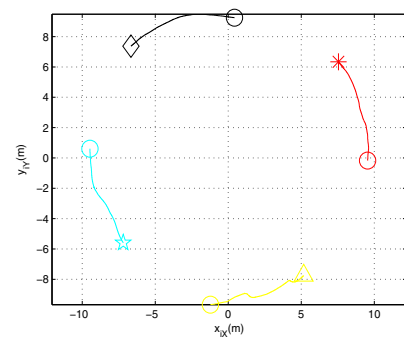
Time-varying formation control problems for unmanned aerial vehicle swarm systems with switching interaction topologies were investigated using a consensus based approach. Necessary and sufficient conditions for UAV swarm systems to achieve predefined time-varying formations were presented. An approach to determine the gain matrices in the formation protocol was proposed. Theoretical results are validated by outdoor time-varying formation experiments on a quadrotor formation platform with four quadrotors.

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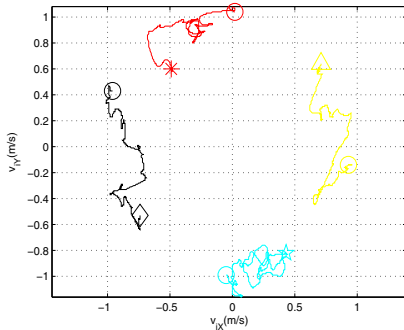
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(a) Positions



(b) Velocities

Fig. 6. Snapshots of $\xi_i(t)$ ($i = 1, 2, 3, 4$) in the experiment within [63 s, 70 s].

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