

# Adaptive Fuzzy Backstepping Control for Trajectory Tracking of Unmanned Aerial Quadrotor

Fouad Yacef<sup>1</sup>, Omar Bouhali<sup>2</sup> and Mustapha Hamerlain<sup>1</sup>

**Abstract**—This paper presents an adaptive fuzzy control strategy to solve the problem of trajectory tracking for quadrotor unmanned aerial vehicle in the presence of model parameters uncertainties and external disturbances. A fuzzy system is employed to approximate directly a model based control law developed using backstepping techniques. The adaptive laws for tuning the adjustable parameters of the fuzzy system are derived based on the Lyapunov theorem. The stability analysis of the designed adaptive fuzzy backstepping controller (AFBC) is shown by the Lyapunov theory. The proposed controller yields asymptotic tracking, robustness in the presence of external disturbances affecting the six degrees of freedom, and parameters uncertainties. It is proved that all signals in the closed-loop system are semiglobally uniformly ultimately bounded, and the tracking error converge to a small neighborhood of the origin. Numerical simulation results are provided to illustrate the good tracking performances of the proposed adaptive control approach.

## I. INTRODUCTION

Quadrotor unmanned aerial vehicle (UAV) consists of two pairs of counter-rotating rotors and propellers, is an attractive VTOL unmanned aircraft for both civilian and military applications. Due to wide range of application of rotary wings unmanned aerial vehicles (UAV), it's become an interesting field that has motivated the control community. Because of the high manoeuvrability and simple mechanical structure, the quadrotor UAV becomes a more interesting platform than other micro UAVs during the last few years. These vehicles are under-actuated mechanical systems, which complicates the control design stage. Techniques developed for fully actuated robots can't be directly applied to the under-actuated nonlinear mechanical systems [1].

Quadrotor helicopters have been an increasingly benchmark research platform. In designing a controller for these aircraft, there are several important considerations. These kinds of UAVs have a high nonlinear and time-varying behavior and they are constantly affected by aerodynamic disturbances. In addition, UAVs are usually models subject to un-modelled dynamics and parametric uncertainties. Therefore an advanced control strategy is required to achieve good performance in autonomous flight or at least to help the piloting of the vehicle, with high manoeuvrability and robustness with respect to external disturbances [2]. There are numerous sources of uncertainty in the system actuator

degradation, external disturbances, and potentially uncertain time delays in processing or communication [3]. Adaptive fuzzy control is an attractive candidate for this type of aircraft because of its ability to generate high performance tracking in the presence of parametric uncertainties.

Various studies have been made to control the quadrotor helicopter and many strategies have been developed to overcome the trajectory tracking problem for the quadrotor system. A nonlinear full model was proposed in [4], the authors presents the helicopter dynamics based on Newton-Euler formalism. The aerodynamic forces and moments acting on this model were considered. Where a simplified model was presented in [5] considering small variations of Euler angles. The trajectory tracking problem was handled using Backstepping and sliding mode control techniques. In [6] quadrotor trajectory tracking was achieved by combining backstepping and sliding-mode techniques with inner/outer loop, but the time scale separation assumption needs large inner-loop gain to guarantee closed-loop stability. In [7] the author give relationship between the attitude and linear acceleration, and use a new command-filtered backstepping technique to stabilize the attitude and a linear tracking differentiators to eliminate the classical inner/outer-loop structure. the quadrotor dynamics was divided into three interconnected subsystem [8] [9], then full state backstepping technique is used to track the desired trajectory.

Several control technique for quadrotor trajectory tracking don't take into account external disturbances, unmodelled dynamics and parameter uncertainties on the whole model, in [6] [10] the proposed controller can't reject external disturbances. In [1] disturbances are only taken into account in the rotational motion control. To overcome this drawback the authors in [11] proposed the use of sliding mode observer for quadrotor velocities observation and estimation of external disturbances, with trajectory tracking controller based on backstepping techniques which was robust to the unknown aerodynamic friction integrated in the dynamic modeling, where in [12] a neural network was used to estimate aerodynamic forces and moments using backstepping approach and Lagrange form dynamics. A robust backstepping controller based on the idea of practical stability instead of Lyapunov stability to deal with non-linearity, measurement disturbances and noise, sensor biases was proposed [13].

Backstepping control technique provides an intuitive solution since it solves underactuation problem iteratively through salving the actuated subsystem so as to provide a virtual controller in order to stabilize the underactuation subsystem [14]. Unfortunately, this systematic approach requires

<sup>1</sup>Fouad Yacef and Mustapha Hamerlain is with Productique and Robotics Laboratory, Centre for Development of Advanced Technologies (CDTA), 16000 Algiers, Algeria, yaceffouad@hotmail.fr, mhamerlain@cdta.dz

<sup>2</sup>Omar Bouhali with Mechatronic Laboratory, Faculty of Science and Technology, Jijel University, 18000 Jijel, Algeria, bouhali.omar@univ-jijel.dz

full knowledge of system dynamics; which stands for an extremely difficult assumption to meet in practice for UAVs. To undertake this problem and extend the applicability of the backstepping control technique, extension of backstepping with other schemes has been proposed, such as adaptive-based control [15] [16], function approximation techniques [17] [18], and other methodologies. These schemes exploit the backstepping method to introduce additional stability properties in order to deal with uncertainty and unmodelled dynamics, however resulting in an entangled algorithm that adds more complexity to the already involved backstepping algorithm.

In this paper, we are interested in the challenge of resorting on backstepping without adding significant complexity to deal with uncertainties and using a direct adaptive fuzzy control scheme for this purpose, [19] [20]. It is proposed a robust backstepping controller, based on the universal approximation theorem [21], that allows us to compensate some unknown nonlinear backstepping model based control law and handle smooth and bounded uncertainties and external disturbances. Simulations are presented for the simplified quadrotor model in Newton-Euler coordinates subject to aerodynamic disturbances such as wind gust and aerodynamic forces. As suggested in [22] quadrotor dynamics is written in a state space form suited for the proposed approach and full details of our proposed approach are presented and discussed.

This paper is organized as follows. In section 2, the dynamic model of the quadrotor is presented. Then, in section 3, the proposed controller, and stability of closed loop system is illustrated. In section 4, some simulation results are carried out to show the effectiveness of the proposed controller. Finally, some conclusions are written in section 5.

## II. QUADROTOR MODELING

The quadrotor, shown in Fig 1, has four rotors to generate the propeller forces  $F_1, F_2, F_3$  and  $F_4$ . Its configuration simplifies the displacement and increases the lift force. On varying the rotor speeds altogether with the same quantity, the lift forces will change, affecting in this case the altitude of the vehicle. The two pairs of rotors (1, 3) and (2, 4) turn in opposite directions in order to balance the moments and produce yaw motion as needed. Yaw angle is obtained by speeding up or slowing down the clockwise motors depending on the desired angle direction. The motion direction according to the horizontal plan depends on the sense of yaw angle and tilt angles (pitch and roll), whether they are positives or negatives.

Two frames are used to study the system motion: a frame integral with the earth  $E^a(O^a, e_1^a, e_2^a, e_3^a)$  which is supposed to be inertial, and a body-fixed frame  $E^b(O^b, e_1^b, e_2^b, e_3^b)$  where  $O^b$  is fixed to the center of mass of the quadrotor. The absolute position of the quadrotor is described by  $\xi = [x, y, z]^T$  and its attitude by the Euler angles  $\eta = [\phi, \theta, \psi]^T$ , used corresponding to aeronautical convention. The attitude angles are respectively called Yaw angle ( $\psi$  rotation around z-axis), Pitch angle ( $\theta$  rotation around y-axis) and Roll angle

( $\phi$  rotation around x-axis). Let  $V = [u, v, w]^T \in E^b$  denote the linear velocity and  $\Omega = [p, q, r]^T \in E^b$  denote the angular velocity of the airframe expressed in the body-fixed-frame. The relation between the velocities vectors ( $V, \Omega$ ) and ( $\xi, \eta$ ) is:

$$\begin{cases} \dot{\xi} = R(\eta)V \\ \dot{\Omega} = N(\eta)\dot{\eta} \end{cases} \quad (1)$$

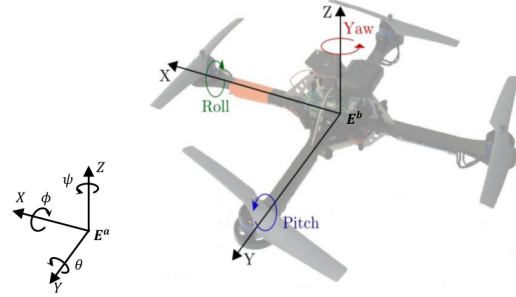


Fig. 1. Quadrotor scheme

Where  $R(\eta)$  and  $N(\eta)$  are respectively the transformation velocity matrix and the rotation velocity matrix between  $E^a$  and  $E^b$ , such as:

$$R(\eta) = \begin{bmatrix} C_\theta C_\psi & C_\theta S_\psi & -S_\theta \\ S_\phi S_\theta C_\psi - C_\phi S_\psi & S_\phi S_\theta S_\psi + C_\phi S_\psi & S_\phi C_\theta \\ C_\phi S_\theta C_\psi + S_\phi S_\psi & C_\phi S_\theta S_\psi - S_\phi C_\psi & C_\phi C_\theta \end{bmatrix}$$

and

$$N(\eta) = \begin{bmatrix} -S_\theta & 0 & 1 \\ C_\theta S_\phi & C_\phi & 0 \\ C_\theta C_\phi & -S_\phi & 0 \end{bmatrix}$$

The derivation of (1) with respect to time gives

$$\begin{cases} \ddot{\xi} = R(V + \dot{\Omega} \times V) \\ \dot{\Omega} = N\dot{\eta} + \left(\frac{\partial N}{\partial \phi}\dot{\phi} + \frac{\partial N}{\partial \theta}\dot{\theta}\right)\dot{\eta} \end{cases} \quad (2)$$

Using the Newton's laws in the reference frame  $E^b$ , about the quadrotor helicopter subjected to forces  $\sum F_{ext}$  and moments  $\sum T_{ext}$  applied to the center of mass, one obtains the dynamic equation motions:

$$\begin{bmatrix} mI_{3 \times 3} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \dot{V} \\ \dot{\Omega} \end{bmatrix} + \begin{bmatrix} \Omega \times mV \\ \Omega \times I\Omega \end{bmatrix} = \begin{bmatrix} \sum F_{ext} \\ \sum T_{ext} \end{bmatrix} \quad (3)$$

where  $m$  and  $I = diag[I_x, I_y, I_z]$  are respectively the mass and the total inertia matrix of helicopter,  $I_{3 \times 3}$  is the identity matrix,  $\sum F_{ext}$  and  $\sum T_{ext}$  includes the external forces/torques developed in the center of mass of the quadrotor according to the direction of the reference frame  $E^b$ , such as:

$$\begin{cases} \sum F_{ext} = F + F_{aero} + F_{grav} \\ \sum T_{ext} = T + T_{aero} + T_{gyro} \end{cases} \quad (4)$$

where  $F$  and  $T$  are the forces and torques produced by the propeller system, respectively.  $F_{grav}$  is the gravity effect force with  $G = [0, 0, g]^T$  the gravity vector,  $F_{aero}$  and  $T_{aero}$  are the aerodynamic forces and moments acting on the quadrotor, respectively, and  $T_{gyro}$  define the gyroscopic effects resulting from the propeller rotations. These variables are defined as

$$F = \begin{bmatrix} 0 \\ 0 \\ b \sum_{i=1}^4 \omega_i^2 \end{bmatrix}, \quad T = \begin{bmatrix} lb(\omega_3^2 - \omega_1^2) \\ lb(\omega_4^2 - \omega_2^2) \\ d \sum_{i=1}^4 (-1)^{i+1} \omega_i^2 \end{bmatrix}$$

$$T_{aero} = -[A_p, A_q, A_r]^T, \quad F_{aero} = -[A_u, A_v, A_w]^T$$

$$F_{grav} = -mR(\eta)^T G, \quad T_{gyro} = - \sum_{i=1}^4 \Omega \times I_r (-1)^{i+1} \omega_i$$

where  $l$  is the distance from the center of mass to the rotor shaft,  $d$  is the drag factor,  $b$  is the thrust factor and  $I_r$  is the rotor inertia. Using (3) and (4) the equation of the dynamics of rotation of the quadrotor, expressed in the reference frame  $E^a$ , will be:

$$\begin{cases} \ddot{\xi} = \frac{1}{m} R \sum F_{ext} \\ \ddot{\eta} = (IN)^{-1} [\sum T_{ext} - I(\frac{\partial N}{\partial \phi} \dot{\phi} + \frac{\partial N}{\partial \theta} \dot{\theta}) \dot{\eta} \\ \quad - (N \dot{\eta}) \times (IN \dot{\eta})] \end{cases} \quad (5)$$

The aerodynamic functions  $A_i$  are highly nonlinear and dependent on numerous physical variables such as the angle between airspeed and the body-fixed frame and geometric form of the helicopter. Generally, they are approximated from the non-dimensional coefficients  $C_i$  as  $A_i = \frac{1}{2} \rho C_i W^2$  where  $\rho$  is the air density, and  $W = \Omega - \Omega_{air}$  the velocity of the aircraft with respect to the air [28].

### III. CONTROLLER DESIGN

The quadrotor helicopter is an under-actuated system, since the dynamic model (5) of the quadrotor has six outputs  $(x, y, z, \phi, \theta, \psi)$ , and only four independent inputs. Therefore we are not able to control all of the states at the same time [8]. A possible combination of controlled outputs can be  $(x, y, z, \psi)$  in order to track the desired positions, more to an arbitrary heading and stabilize the other two angles, which introduces stable zero dynamics into the system [29]. A good controller should be able to reach a desired position and a desired yaw angle while guaranteeing stability of the pitch and roll angles.

$$\dot{X} = f(X, U) \quad (6)$$

with the following state vector  $X = [x_{11}, x_{21}, x_{12}, \dots, x_{26}]^T \in \mathbb{R}^{12}$  and input vector  $U = [u_1, u_2, u_3, u_4]^T$

$$\begin{aligned} x_{11} &= \phi, & x_{22} &= \dot{\theta}, & x_{14} &= z, & x_{25} &= \dot{x} \\ x_{21} &= \dot{\phi}, & x_{13} &= \psi, & x_{24} &= \dot{z}, & x_{16} &= y \\ x_{12} &= \theta, & x_{23} &= \dot{\psi}, & x_{15} &= x, & x_{26} &= \dot{y} \end{aligned}$$

$$\begin{aligned} U_1 &= F_3 - F_1 \\ U_2 &= F_4 - F_2 \\ U_3 &= F_1 - F_2 + F_3 - F_4 \\ U_4 &= F_1 + F_2 + F_3 + F_4 \\ \omega &= \omega_1 - \omega_2 + \omega_3 - \omega_4 \end{aligned}$$

where  $F_i = b\omega_i^2$  ( $i = 1, \dots, 4$ ) is the force generated by the  $i$ th rotor. The transformation matrix between the rate of change of the orientation angles  $\dot{\eta}$  and the body angular velocities  $\Omega$  can be considered as an identity matrix if the perturbations from hover are small [5]. Then,  $\dot{\eta}$  can be taken approximately as  $\Omega$  this is ( $\dot{\eta} \approx \Omega$ ), therefore

$$f(X) = \begin{pmatrix} x_{11} \\ x_{22}x_{23}a_1 + x_{22}a_2\omega - A_{\dot{\phi}} + b_1U_1 \\ x_{22} \\ x_{21}x_{23}a_3 + x_{21}a_4\omega - A_{\dot{\theta}} + b_2U_2 \\ x_{23} \\ x_{21}x_{22}a_5 - A_{\dot{\psi}} + b_3U_3 \\ x_{24} \\ -A_z - g + \frac{C_{x_{11}}C_{x_{12}}}{m}U_4 \\ x_{25} \\ -A_x + \frac{U_4}{m}(C_{x_{11}}S_{x_{12}}C_{x_{13}} + S_{x_{11}}S_{x_{13}}) \\ x_{26} \\ -A_y + \frac{U_4}{m}(C_{x_{11}}S_{x_{12}}S_{x_{13}} - S_{x_{11}}C_{x_{13}}) \end{pmatrix}$$

where  $A_F = \frac{1}{m} R F_{aero} = -[A_x, A_y, A_z]^T$ ,  $A_T = I^{-1} T_{aero} = -[A_{\dot{\phi}}, A_{\dot{\theta}}, A_{\dot{\psi}}]^T$  are two complex nonlinear functions resulting aerodynamic forces and moments,  $c = \frac{d}{b} > 0$  is a positive constant.  $C(\cdot)$  and  $S(\cdot)$  represent  $\cos(\cdot)$  and  $\sin(\cdot)$  and

$$\begin{aligned} a_1 &= \frac{I_y - I_z}{I_x}, & a_4 &= \frac{-I_r}{I_y}, & b_2 &= \frac{l}{I_y} \\ a_2 &= \frac{I_r}{I_x}, & a_5 &= \frac{I_x - I_y}{I_z}, & b_3 &= \frac{c}{I_z} \\ a_3 &= \frac{I_z - I_x}{I_y}, & b_1 &= \frac{l}{I_x} \end{aligned}$$

#### A. Problem statement

The main objective of the control step is to design an output-feedback controller and parameter adaptive laws such that all involved signals in the closed loop system remain bounded, and the tracking  $e_{1i} = x_{1i,d} - x_{1i}$ , ( $i = 1, \dots, 6$ ) are as small as desired. Which mean that the signals  $(x(t), y(t), z(t), \psi(t))$  tracks asymptotically the desired trajectory  $(x_d(t), y_d(t), z_d(t), \psi_d(t))$ .

The state space model (6) can be rearranged as follow:

$$\begin{cases} \dot{x}_{1i} = x_{2i} \\ \dot{x}_{2i} = f_i(x) + g_i(x)u_i(t) \\ y_i = x_{1i} \end{cases} \quad (7)$$

where  $f_i(x)$  and  $g_i(x)$ , ( $i = 1, \dots, 6$ ) of the six subsystem are nonlinear smooth functions,  $U = [u_1, \dots, u_6]^T \in \mathbb{R}^6$  is the control input vector,  $y = [y_1, \dots, y_6]^T \in \mathbb{R}^6$  is the output vector.

$$\begin{aligned} f_1(x) &= x_{22}x_{23}a_1 + x_{22}a_2\omega - A_{\dot{\phi}}, & g_1(x) &= b_1 \\ f_2(x) &= x_{21}x_{23}a_3 + x_{21}a_4\omega - A_{\dot{\theta}}, & g_2(x) &= b_2 \\ f_3(x) &= x_{21}x_{22}a_5 - A_{\dot{\psi}}, & g_3(x) &= b_3 \end{aligned}$$

$$\begin{aligned} f_4(x) &= -A_z - g, \quad g_4(x) = \frac{C_{x_{11}}C_{x_{12}}}{m} \\ f_5(x) &= -A_x, \quad g_5(x) = 1 \\ f_6(x) &= -A_y, \quad g_6(x) = 1 \end{aligned}$$

The terms  $u_5$  and  $u_6$  represent the relation between pitch and  $x$  motion; roll and  $y$  motion respectively. This term will be used as inputs control to overcome under-actuated problem.

$$\begin{cases} u_5 = \frac{u_4}{m}(C_{x_{11}}S_{x_{12}}C_{x_{13}} + S_{x_{11}}S_{x_{13}}) \\ u_6 = \frac{u_4}{m}(C_{x_{11}}S_{x_{12}}S_{x_{13}} - S_{x_{11}}C_{x_{13}}) \end{cases} \quad (8)$$

Let us denote:

$$\ddot{y} = \begin{bmatrix} \ddot{y}_1 \\ \vdots \\ \ddot{y}_6 \end{bmatrix}, \quad F(x) = \begin{bmatrix} f_1(x) \\ \vdots \\ f_6(x) \end{bmatrix}$$

$$G(x) = \begin{bmatrix} g_1(x) & 0 & \cdots & 0 \\ 0 & g_2(x) & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \vdots & 0 & g_6(x) \end{bmatrix}$$

All over this paper the following assumptions are considered. **A1:** The signals  $\xi$ ,  $\eta$ ,  $\dot{\xi}$  and  $\dot{\eta}$  can be measured or estimated by on-board sensors.

**A2:** The velocities  $\dot{\xi}$ ,  $\dot{\eta}$  and the forces  $F_i$  ( $i = 1, \dots, 4$ ) are bounded.

**A3:** The yaw, pitch and roll angles are limited to  $(-\pi < \psi < \pi)$ ,  $(-\frac{\pi}{2} < \theta < \frac{\pi}{2})$  and  $(-\frac{\pi}{2} < \phi < \frac{\pi}{2})$ .

**A4:** The matrix  $G(x)$  is positive definite and bounded as  $0 < \underline{g}I_{6 \times 6} < G(x) < \bar{g}I_{6 \times 6}$ , where  $I_{6 \times 6}$  is the identity matrix,  $\underline{g}$  and  $\bar{g}$  are some positive constants.

**A5:** The desired trajectory and its time derivatives are smooth and bounded.

The Assumption 1 is a sufficient condition ensuring that the matrix  $G(x)$  is always regular and, therefore, system (7) is feedback linearizable by a static state feedback. According to the assumption3, so we have  $g_4(x) > 0$  and as consequent the assumption 4 is verified for system (7).

Using backstepping approach [6], with the assumption ( $A_F = A_T = 0$ ) we can obtained an output-feedback controller that ensures the boundedness of all signals in the closed-loop system and guarantees output tracking of the desired trajectory. As the subsystems (7) are second order systems we have two step for backstepping algorithm.

**Step 1.** Defining the tracking error as

$$e_{1i} = x_{1i,d} - x_{1i}, (i = 1, \dots, 6) \quad (9)$$

Now, let us consider the Lyapunov function  $V_i$ , and its derivative  $\dot{V}_i$  as follow:

$$\begin{cases} V_i = \frac{1}{2}e_{1i}^2 \\ \dot{V}_i = e_{1i}\dot{e}_{1i} = e_{1i}(\dot{x}_{1i,d} - \dot{x}_{2i}) \end{cases} \quad (10)$$

We choose  $\varphi_i = x_{2i}, (i = 1, \dots, 6)$  as our control input to have an exponential convergence of  $\dot{V}_i$

$$\varphi_i = \dot{x}_{1i,d} + \alpha_{1i}e_{1i} \quad (11)$$

where  $\alpha_{1i}$  is a positive constants, in this case we have exponential convergence of the error  $e_{1i}$  to zero  $\dot{V}_i = -\alpha_{1i}e_{1i}^2$ .

**Step 2.** The tracking error for second state variable  $x_{2i}$  can be defined as

$$e_{2i} = \varphi_i - x_{2i} = \dot{x}_{1i,d} + \alpha_{1i}e_{1i} - x_{2i} \quad (12)$$

From (7), (10) and (12) we obtained:

$$\begin{cases} \dot{e}_{1i} = e_{2i} - \alpha_{1i}e_{1i} \\ \dot{e}_{2i} = \alpha_{1i}(e_{2i} - \alpha_{1i}e_{1i}) + \ddot{x}_{1i,d} - f_i(x) - g_i(x)u_i(t) \end{cases} \quad (13)$$

Now the augmented Lyapunov function will be

$$\begin{cases} V_i = \frac{1}{2}e_{1i}^2 + \frac{1}{2g_i(x)}e_{2i}^2 \\ \dot{V}_i = e_{1i}\dot{e}_{1i} + \frac{1}{g_i(x)}e_{2i}\dot{e}_{2i} - \frac{\dot{g}_i(x)}{2g_i^2(x)}e_{2i}^2 \end{cases} \quad (14)$$

Using (13) the time derivative of Lyapunov function is

$$\begin{aligned} \dot{V}_i &= e_{1i}(e_{2i} - \alpha_{1i}e_{1i}) + \frac{e_{2i}}{g_i(x)}[\alpha_{1i}(e_{2i} - \alpha_{1i}e_{1i}) \\ &\quad + \ddot{x}_{1i,d} - f_i(x) - g_i(x)u_i(t) - \frac{\dot{g}_i(x)}{2g_i(x)}e_{2i}] \\ &= -\alpha_{1i}e_{1i}^2 + e_{2i}[\frac{e_{1i} + \alpha_{1i}(e_{2i} - \alpha_{1i}e_{1i}) + \ddot{x}_{1i,d} - f_i(x)}{g_i(x)} \\ &\quad - \frac{\dot{g}_i(x)}{2g_i^2(x)}e_{2i} - u_i(t)] \end{aligned}$$

Then, we have

$$\dot{V}_i = -\alpha_{1i}e_{1i}^2 + e_{2i} [u_{eq,i}^*(t) - u_i(t)] \quad (15)$$

where  $u_{eq,i}^*(t)$  is the ideal control law that insure the stability of the closed loop system.

$$\begin{aligned} u_{eq,i}^*(t) &= \frac{1}{g_i(x)}[e_{1i} + \alpha_{1i}(e_{2i} - \alpha_{1i}e_{1i}) + \ddot{x}_{1i,d} \\ &\quad - f_i(x) - \frac{\dot{g}_i(x)}{2g_i(x)}e_{2i}] \end{aligned} \quad (16)$$

Then if  $u_i(t) = u_{eq,i}^*(t) + \alpha_{2i}e_{2i}$  that  $\dot{V}_i = -\alpha_{1i}e_{1i}^2 - \alpha_{2i}e_{2i}^2 \leq 0$ , so  $e_{1i}, e_{2i} \rightarrow 0$  as  $t \rightarrow \infty$ ; with  $\alpha_{2i}$  positive constant. According to the above analysis, the ideal control law (16) is easily obtained if the nonlinear functions  $f_i(x)$  and  $g_i(x)$  are known. However, in our case, these nonlinear functions are not exactly known, so the above design method cannot be implemented. In this case, we propose the use of an adaptive fuzzy scheme to approximate directly the control law  $u_{eq,i}^*(t)$ .

### B. Adaptive fuzzy backstepping control design

In this section a fuzzy logic system (FLS) is used to approximate a model based control law, developed using backstepping approach. Then, a well-defined adaptive fuzzy backstepping controller is developed in order to meet control objectives.

A fuzzy logic system (FLS) consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine, and the defuzzifier. The knowledge base is composed of a

collection of fuzzy IF-THEN rules in the following form [19]:

$$R^i : \mathbf{IF} \ x_1 \text{ is } F_1^i \text{ and } \dots \ x_n \text{ is } F_n^i, \ \mathbf{THEN} \ y \text{ is } G^i,$$

where  $i = 1, 2, \dots, N$ ,  $x = [x_1, \dots, x_n]^T \in R^n$  and  $y \in R$  are input vector and output, respectively.  $F_j^i (j = 1, \dots, n)$  and  $G^i$  are fuzzy set associated with the fuzzy membership functions  $\mu_{F_j^i}(x_j)$  and  $\mu_{G^i}(y)$ , respectively.  $N$  is the number of rules.

By using the singleton fuzzifier, product inference engine, and center average defuzzification [18], the output of the fuzzy logic system can be expressed as follow:

$$y(x | \theta) = \frac{\sum_{i=1}^N \bar{y}_i \prod_{j=1}^n \mu_{F_j^i}(x_j)}{\sum_{i=1}^N [\prod_{j=1}^n \mu_{F_j^i}(x_j)]} \quad (17)$$

where  $\bar{y}_i = \max_{y \in R} \mu_{G^i}(y)$ . Let

$$w_i(x) = \frac{\prod_{j=1}^n \mu_{F_j^i}(x_j)}{\sum_{i=1}^N [\prod_{j=1}^n \mu_{F_j^i}(x_j)]} \quad (18)$$

Denoting  $w(x) = [w_1(x), w_2(x), \dots, w_N(x)]^T$  as the vector of fuzzy basis functions, and  $\Theta^T = [\bar{y}_1, \bar{y}_2, \dots, \bar{y}_N]$  the vector of consequent parameters. Then, the fuzzy logic systems can be rewritten as follows:

$$y(x | \Theta) = w^T(x)\Theta \quad (19)$$

*Theorem 1 ([18]):* For any given real continuous function  $f(x)$  on the compact set  $\Omega_f \subset R^n$  and arbitrary  $\delta > 0$ , there exists an FLS such that

$$\sup_{x \in \Omega_f} |f(x) - w^T(x)\Theta| \leq \delta$$

The control input system that ensure closed loop system stability can be determined as

$$u_i(t) = u_{a,i} + u_{r,i} + u_{pd,i} \quad , \quad (i = 1, \dots, 6) \quad (20)$$

where  $u_{a,i}$  is the fuzzy adaptive control term (22) which is designed to approximate the ideal control law  $u_{eq,i}^*$  (16),  $u_{r,i}$  is a bounded robust term employed to compensate the fuzzy approximation error and the external disturbances, and

$$u_{pd,i} = \alpha_{2i} e_{2i} \quad (21)$$

is the proportional derivative term.

By theorem 1 and the proof given by [21], FLS are universal approximators, i.e., they can approximate any smooth function on a compact space. Than, the term  $u_{a,i}$  can be approximated as

$$u_{a,i} = w_i^T(x)\Theta_i \quad (22)$$

where  $w_i^T(x)$  is fuzzy basis vectors fixed by the designer,  $\Theta_i$  is the corresponding adjustable parameters vector of each fuzzy system.

According to [19] [20], the optimal parameter vector  $\Theta_i^*$  are defined as

$$\Theta_i^* = \arg \min_{\Theta_i \in \Omega_{\Theta}} \left\{ \sup_{x \in \Omega_x} |u_{eq,i} - u_{a,i}(x | \Theta_i)| \right\} \quad (23)$$

where  $\Omega_x$  and  $\Omega_{\Theta}$  are a compact set for  $x$  and  $\Theta_i$  respectively. Also, the minimum fuzzy approximation errors  $\delta_i$  is defined as

$$\delta_i = u_{eq,i} - u_{a,i}(x | \Theta_i^*) \quad (24)$$

In this case the ideal control law  $u_{eq,i}^*$  can be rewritten as

$$\begin{aligned} u_{eq,i}^* &= u_{a,i}(x | \Theta_i^*) + \delta_i(x) \\ &= w_i^T(x)\Theta_i^* + \delta_i(x) \end{aligned} \quad (25)$$

**A6:** There exist known constants  $\bar{\delta}_i > 0$ , such that  $|\delta_i| \leq \bar{\delta}_i$ . Define

$$\tilde{\Theta}_i = \Theta_i^* - \Theta_i \quad (26)$$

as the parameters vector estimation error.

### C. Stability Analysis

Consider the following Lyapunov function candidate:

$$V = \frac{1}{2} \sum_{i=1}^6 e_{1i}^2 + \frac{1}{g_i(x)} e_{2i}^2 + \frac{1}{\gamma_i} \tilde{\Theta}_i^T \tilde{\Theta}_i + \frac{1}{\eta_i} \tilde{\delta}_i^T \tilde{\delta}_i \quad (27)$$

where  $\gamma_i > 0$  and  $\eta_i > 0$  are positive design parameters, respectively.  $\tilde{\delta}_i = \delta_i^* - \hat{\delta}_i$ , and  $\hat{\delta}_i$  is the estimate of  $\delta_i^*$ .

Differentiating  $V$  and using (16) yields

$$\begin{aligned} \dot{V} &= \sum_{i=1}^6 -\alpha_{1i} e_{1i}^2 + e_{2i} [u_{eq,i}^*(t) - u_i(t)] \\ &\quad + \frac{1}{\gamma_i} \tilde{\Theta}_i^T \dot{\tilde{\Theta}}_i + \frac{1}{\eta_i} \tilde{\delta}_i^T \dot{\tilde{\delta}}_i \end{aligned} \quad (28)$$

Substitute (20) and (25) in (28) obtained

$$\begin{aligned} \dot{V} &= \sum_{i=1}^6 -\alpha_{1i} e_{1i}^2 + e_{2i} [w_i^T(x)\Theta_i^* + \delta_i(x) \\ &\quad - w_i^T(x)\Theta_i - u_{r,i} - \alpha_{2i} e_{2i}] \\ &\quad + \frac{1}{\gamma_i} \tilde{\Theta}_i^T \dot{\tilde{\Theta}}_i + \frac{1}{\eta_i} \tilde{\delta}_i^T \dot{\tilde{\delta}}_i \end{aligned} \quad (29)$$

The optimal parameters vector  $\theta_i^*$ , and  $\delta_i^*$  are constants, and then derivative of estimation error will be  $\dot{\tilde{\Theta}}_i = -\dot{\Theta}_i$ , and  $\dot{\tilde{\delta}}_i = -\dot{\hat{\delta}}_i$ . One have

$$\begin{aligned} \dot{V} &= \sum_{i=1}^6 -\alpha_{1i} e_{1i}^2 - \alpha_{2i} e_{2i}^2 + e_{2i} w_i^T(x) \tilde{\Theta}_i \\ &\quad + e_{2i} [\delta_i(x) - u_{r,i}] - \frac{1}{\gamma_i} \tilde{\Theta}_i^T \dot{\tilde{\Theta}}_i - \frac{1}{\eta_i} \tilde{\delta}_i^T \dot{\tilde{\delta}}_i \end{aligned} \quad (30)$$

$$\begin{aligned} \dot{V} &\leq \sum_{i=1}^6 -\alpha_{1i} e_{1i}^2 - \alpha_{2i} e_{2i}^2 + \frac{1}{\gamma_i} \tilde{\Theta}_i^T [\gamma_i e_{2i} w_i(x) - \dot{\tilde{\Theta}}_i] \\ &\quad + |e_{2i}| \delta_i^* - e_{2i} u_{r,i} + \frac{1}{\eta_i} \tilde{\delta}_i^T [\eta_i e_{2i} \tanh(\frac{e_{2i}}{\epsilon_i}) - \dot{\tilde{\delta}}_i] \\ &\quad - \delta_i^* e_{2i} \tanh(\frac{e_{2i}}{\epsilon_i}) + \hat{\delta}_i e_{2i} \tanh(\frac{e_{2i}}{\epsilon_i}) \end{aligned} \quad (31)$$

where  $\epsilon_i > 0$  is a design constant.

Choosing the robust terms  $u_{r,i}$ , parameter adaptive laws  $\dot{\tilde{\Theta}}_i$  and  $\dot{\tilde{\delta}}_i$  as follow:

$$u_{r,i} = \hat{\delta}_i \tanh(\frac{e_{2i}}{\epsilon_i}) \quad (32)$$

$$\dot{\tilde{\delta}}_i = \eta_i e_{2i} \tanh(\frac{e_{2i}}{\epsilon_i}) \quad (33)$$

$$\dot{\Theta}_i = \gamma_i e_{2i} w_i(x) \quad (34)$$

Note that the following inequality hold for any  $\zeta > 0$ :

$$|e_{2i}| - e_{2i} \tanh\left(\frac{e_{2i}}{\epsilon_i}\right) \leq \zeta \epsilon_i = \varsigma \quad (35)$$

where  $\zeta$  is a constant that satisfies  $\zeta = e^{-(\zeta+1)}$ , i.e.  $\zeta = 0.2785$ .

Substitute (32)-(35) into (31) results in

$$\dot{V} \leq \sum_{i=1}^6 -\alpha_{1i} e_{1i}^2 - \alpha_{2i} e_{2i}^2 + \delta_i^* \varsigma \quad (36)$$

Define  $c = \min\{2\alpha_{1i}, 2\alpha_{2i}\}$ , then (36) becomes

$$\dot{V} \leq -cV + \rho \quad (37)$$

where  $\rho = \sum_{i=1}^6 \delta_i^* \varsigma$ .

Integrating (37) over  $[0, t]$ , obtain

$$V(t) \leq V(0)e^{-ct} + \mu \quad (38)$$

where  $\mu = \rho/c$ .

From (37) and using the same proof as in [19] [20] it can be shown that the signals  $x_{1i}$ ,  $x_{2i}$ ,  $\dot{\Theta}_i$ ,  $\dot{\delta}_i$ , and  $u_i(t)$  are semiglobally uniformly ultimately bounded. Moreover, the tracking errors  $e_{1i} = x_{1i,d} - x_{1i}$  can be made as small as desired by adjusting the design parameters. Especially, from (38) we have

$$\lim_{t \rightarrow \infty} e_{1i}^2 \leq 2\mu$$

#### D. Control scheme

As quadrotor is an under-actuated system, we need tow virtual control inputs ( $u_5, u_6$ ), in addition to four control inputs of the quadrotor ( $u_1, u_2, u_3, u_4$ ) for the control of this system. The control scheme advocated for the overall system is then logically divided in an attitude controller, an altitude controller and position controller as schematized in Fig. 2 [30].

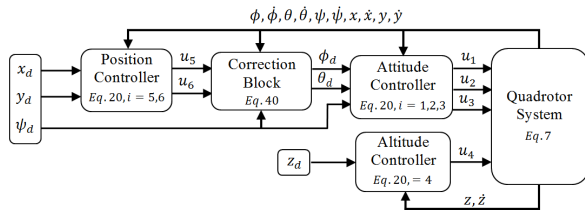


Fig. 2. Control scheme

In hovering flight control law  $u_4$  can be written

$$u_4 = \frac{mg}{C_\phi C_\theta} \quad (39)$$

Substituting (39) in (8), one have

$$\begin{cases} \theta_d = \text{atan}\left(\frac{u_5 \cos \psi + u_6 \sin \psi}{g}\right) \\ \phi_d = \text{atan}\left(\frac{u_5 \sin \psi - u_6 \cos \psi}{g} \cos \theta_d\right) \end{cases} \quad (40)$$

Which gives the relation between virtual control inputs ( $u_5, u_6$ ) and desired angles ( $\phi_d, \theta_d$ )

The proposed adaptive fuzzy controller AFBC is schematized in Fig. 3. The effectiveness of this controller can be verified by the following numerical simulation results.

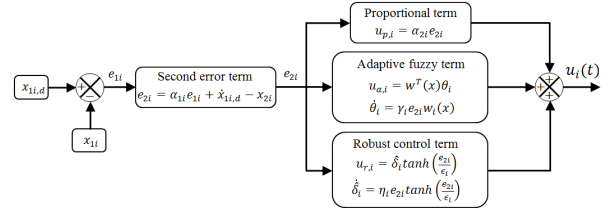


Fig. 3. Adaptive fuzzy controller for  $i$ -th sub-system

#### IV. NUMERICAL SIMULATION

In order to validate the designed AFBC control strategy, test simulations are made in order to check the performance attained for the trajectory tracking problem. besides, simulations comparing the proposed controller with the classical backstepping model-based control approach have been performed, taking in to account external disturbances (aerodynamics moments and forces) and parameter uncertainties (inertia parameters and thrust factor). This classical backstepping model-based strategy has been chosen for the comparison analysis because it is able to present suited performances, and also to show the capability of the proposed strategy to give similar and better performances.

The uncertainties can be appear in inertia parameters  $I_x, I_y$ , and  $I_z$ , and in thrust parameter  $b$ . We made a variation of 75% for inertia parameters, and variation of 40% for thrust factor, between 20 and 40 second. Assuming that our quadrotor needs, under ideal conditions, a thrust value of about  $U_4 \approx 4.76N$  to perform hovering flight, the following wind gusts are considered as external disturbances on the aerodynamic forces and moments:  $A_x = 1.2N$  at  $t = 7.5s$ ;  $A_{\dot{\phi}} = 1.2Nm$  at  $t = 15s$ ; and  $A_y = 1.2N$  at  $t = 22.5s$ ;  $A_{\dot{\theta}} = 1.2Nm$  at  $t = 30s$ ; and  $A_z = 1.2N$  at  $t = 37.5s$ ;  $A_{\dot{\psi}} = 1.2Nm$  at  $t = 45s$ .

Quadrotor parameters model using in simulation, are  $m = 0.486kg$ ,  $l = 0.25m$ ,  $I_x = I_y = 0.0038kgm^2$ ,  $I_z = 0.0076kgm^2$ ,  $I_r = 3.357 \cdot 10^{-5}kgm^2$ ,  $b = 2.984 \cdot 10^{-5}$ ,  $d = 3.232 \cdot 10^{-7}$  and  $g = 9.81m/s^2$ . Where the AFBC controller parameters are chosen as  $\alpha_{11} = 1, \alpha_{12} = 1, \alpha_{13} = 1, \alpha_{14} = 3, \alpha_{15} = 1, \alpha_{16} = 1, \alpha_{21} = 2, \alpha_{22} = 2, \alpha_{23} = 2, \alpha_{24} = 6, \alpha_{25} = 2, \alpha_{26} = 2$ . The gains of adaptive laws are chosen as  $\gamma_i = 1.2, i = (1, 2, 3, 5, 6)$ ,  $\gamma_4 = 2.8$ ,  $\eta_i = 0.3, i = (1, 2, 3, 5, 6)$ ,  $\eta_4 = 0.9$ ,  $\epsilon_i = 0.001, i = (1, \dots, 6)$ . The sampling time is given by  $\Delta t = 0.001s$ , initial positions and Euler angles are  $x(0) = 0, y(0) = 0, z(0) = 0.5, \phi(0) = 0, \theta(0) = 0, \psi(0) = 0.35rad$  respectively, linear and angular velocities are chosen equal to zeros. The input variables of the fuzzy system (22) are chosen as  $x = [x_{1i}, \dot{x}_{1i}]^T$  for angles system ( $i = 1, 2, 3$ ), and  $e = [e_{1i}, \dot{e}_{1i}]^T$  for

position system ( $i = 4, 5, 6$ ). For each variable  $x$  and  $e$ , three Gaussian membership functions are defined as

$$\begin{aligned} \mu_{F_{x_i}^1} &= \exp\left(-\frac{1}{2} \left(\frac{x_i+0.26}{0.1}\right)^2\right), & \mu_{F_{e_i}^1} &= \exp\left(-\frac{1}{2} \left(\frac{e_i+10}{4}\right)^2\right) \\ \mu_{F_{x_i}^2} &= \exp\left(-\frac{1}{2} \left(\frac{x_i}{0.1}\right)^2\right), & \mu_{F_{e_i}^2} &= \exp\left(-\frac{1}{2} \left(\frac{e_i}{4}\right)^2\right) \\ \mu_{F_{x_i}^3} &= \exp\left(-\frac{1}{2} \left(\frac{x_i-0.26}{0.1}\right)^2\right), & \mu_{F_{e_i}^3} &= \exp\left(-\frac{1}{2} \left(\frac{e_i-10}{4}\right)^2\right) \end{aligned}$$

The desired trajectory used is a circle of 8 meter high evolving in the Cartesian space defined by

$$\begin{aligned} x_d &= 1 - \sin\left(\frac{\pi t}{30} + \frac{\pi}{2}\right), & y_d &= -2 \sin\left(\frac{\pi t}{30} + \pi\right) \\ z_d &= 5 - 3 \sin\left(\frac{\pi t}{30} + \frac{\pi}{2}\right), & \psi_d &= 0.43 \sin\left(\frac{\pi t}{30} + \pi\right) \end{aligned}$$

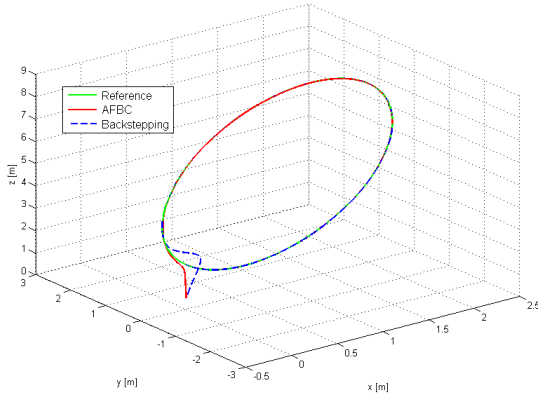


Fig. 4. Trajectory following

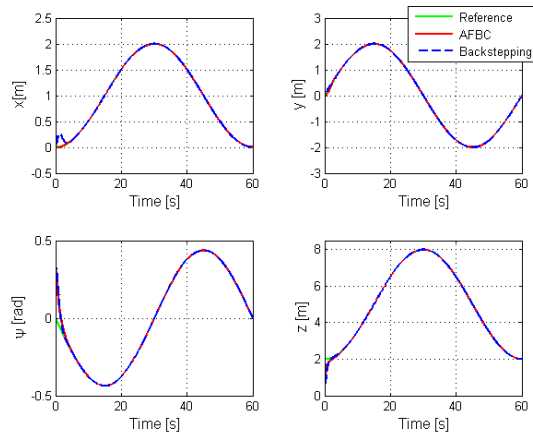


Fig. 5. Quadrotor outputs

Simulation results presented in Figs. 4-8 show the trajectory following of this desired trajectory for both control strategy. The way in which the quadrotor outputs follows the desired trajectory is presented in Fig. 4 (3D space) and Fig.5, where roll and pitch angles are presented in Fig. 6. Besides, some marks have been included in this last figure, indicating the external disturbances that have been applied.

Fig. 6 show how the desired roll and pitch angles varies in its attainment of an appropriate performance in the translation motion i.e  $x$  and  $y$  motion. On the other hand, Fig.

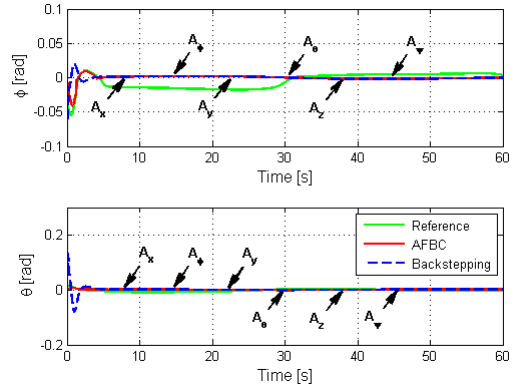


Fig. 6. Roll and pitch angles

7 present outputs errors, It can be seen that null error is achieved in all outputs for the AFBC controller even if parameters uncertainties and external disturbances are considered in the vehicle, which is not the case for backstepping controller ( $y$  position). As presented in Fig. 8, the adaptive fuzzy backstepping control strategy generates smooth input control signals, underlining the quality and the feasibility of the proposed control approach. The proposed AFBC proved better tracking performances than the backstepping controller, but input signals are less smooth.

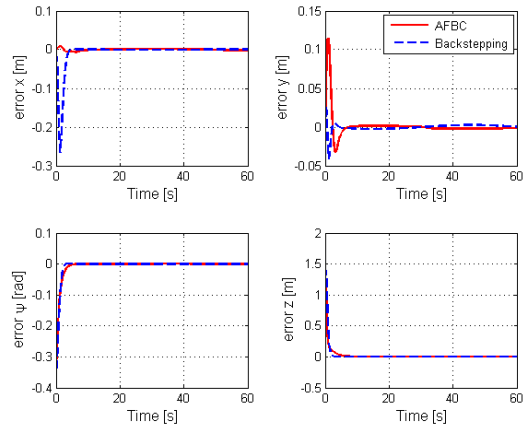


Fig. 7. Quadrotor outputs error

From these figures It can be seen how starting from an initial position far from the desired one, the proposed control strategy is able to make the vehicle follow the desired trajectory despite the presence of external disturbances and parameters uncertainties.

## V. CONCLUSION

In this paper, an adaptive fuzzy backstepping control technique has been developed for quadrotor trajectory tracking problem. FLS is used to approximate the unknown model-based control law. Using adaptive backstepping technique, an adaptive fuzzy output-feedback backstepping control approach is proposed. The proposed control approach can

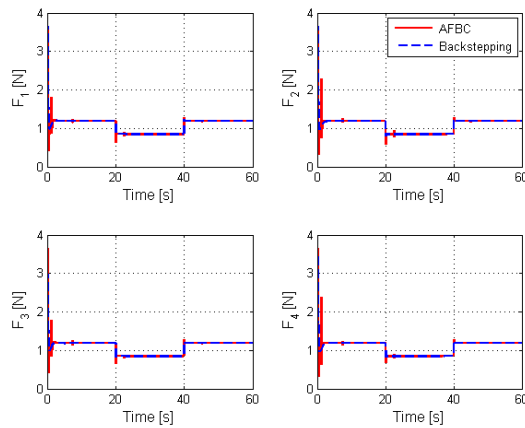


Fig. 8. Quadrotor inputs

overcome the problem of explosion of complexity inherent in the backstepping algorithm, and don't required the dynamic model of quadrotor for controller design stage. It is proved that all signals in the closed-loop control system are semiglobally uniformly ultimately bounded, and the tracking error converge to a small neighborhood of the origin. This controller provide good trajectory tracking performances, and robustness despite the presence of parameters uncertainties and external disturbances.

#### REFERENCES

[1] G. V. Raffo, M. G. Ortega, and F. R. Rubio, "Backstepping/Nonlinear  $H_\infty$  Control for Path Tracking of a Quadrotor Unmanned Aerial Vehicle", in *American Control Conference (ACC)*, Washington, USA, 2008, pp. 3356-3361.

[2] G. V. Raffo, M. G. Ortega, and F. R. Rubio, "An integral predictive/nonlinear  $H_\infty$  control structure for a quadrotor helicopter", *Automatica*, vol. 46, pp. 29-39, 2010.

[3] Z. T. Dydek, A. M. Annaswamy, and E. Lavretsky, "Adaptive Control of Quadrotor UAVs: A Design Trade Study With Flight Evaluations", *Control Systems Technology*, IEEE Transactions on, vol. 21, pp. 1400-1406, 2013.

[4] T. Madani and A. Benallegue, "Backstepping control with exact 2-sliding mode estimation for a quadrotor unmanned aerial vehicle", in *Intelligent Robots and Systems, 2007. IROS 2007. IEEE/RSJ International Conference on*, 2007, pp. 141-146.

[5] S. Bouabdallah and R. Siegwart, "Full control of a quadrotor," in *IEEE/RSJ International Conference on Intelligent Robots and Systems*, San Diego, CA, USA, 2007, pp. 153-158.

[6] S. Bouabdallah and R. Siegwart, "Backstepping and Sliding-mode Techniques Applied to an Indoor Micro Quadrotor", in *Robotics and Automation, 2005. ICRA 2005. Proceedings of the 2005 IEEE International Conference on*, 2005, pp. 2247-2252.

[7] Z. Zuo, "Trajectory tracking control design with command-filtered compensation for a quadrotor", *Control Theory & Applications, IET*, vol. 4, pp. 2343-2355, 2010.

[8] T. Madani and A. Benallegue, "Backstepping Control for a Quadrotor Helicopter", in *IEEE/RSJ International Conference on Intelligent Robots and Systems*, Beijing, China, 2006, pp. 3255-3260.

[9] T. Madani and A. Benallegue, "Control of a Quadrotor Mini-Helicopter via Full State Backstepping Technique", in *45th IEEE Conference on Decision and Control*, San Diego, CA, USA, 2006, pp. 1515-1520.

[10] P. Castillo, R. Lozano, and A. Dzul, "Stabilization of a mini rotorcraft with four rotors", *Control Systems, IEEE*, vol. 25, pp. 45-55, 2005.

[11] T. Madani and A. Benallegue, "Sliding Mode Observer and Backstepping Control for a Quadrotor Unmanned Aerial Vehicles", in *American Control Conference, 2007. ACC '07, 2007*, pp. 5887-5892.

[12] A. Das, K. Subbarao, and F. Lewis, "Backstepping Approach for Controlling a Quadrotor Using Lagrange Form Dynamics", *Journal of Intelligent & Robotic Systems*, vol. 56, pp. 127-151, 2009.

[13] P. Lorenzo and M. Alessandra, "Simulation and Robust Backstepping Control of a Quadrotor Aircraft", in *AIAA Modeling and Simulation Technologies Conference and Exhibit*, ed: American Institute of Aeronautics and Astronautics, 2008.

[14] H. Ramirez-Rodriguez, V. Parra-Vega, A. Sanchez-Orta, and O. Garcia-Salazar, "Robust Backstepping Control Based on Integral Sliding Modes for Tracking of Quadrotors", *Journal of Intelligent & Robotic Systems*, vol. 73, pp. 51-66, 2014.

[15] H. Mu, X. Bin, D. Chen, Y. Kaiyan, and F. Yu, "Adaptive tracking control of underactuated quadrotor unmanned aerial vehicles via backstepping", in *American Control Conference (ACC)*, Baltimore, Maryland, USA, 2010, pp. 2076-2081.

[16] M. Krstic, L. Kanellakopoulos, and P. V. Kokotovic, *Nonlinear and Adaptive Control Design*. New York: Wiley-Interscience, 1995.

[17] T. Madani and A. Benallegue, "Adaptive Control via Backstepping Technique and Neural Networks of a Quadrotor Helicopter", in *Proceedings of the 17th World Congress The International Federation of Automatic Control*, Seoul, Korea, 2008, pp. 6513-6518.

[18] L. X. Wang, *Adaptive Fuzzy Systems and Control: Design and Stability Analysis*: Prentice-Hall, Englewood Cliffs, NJ, 1994.

[19] T. Shao-Cheng, L. Yong-Ming, F. Gang, and L. Tie-Shan, "Observer-Based Adaptive Fuzzy Backstepping Dynamic Surface Control for a Class of MIMO Nonlinear Systems", *Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on*, vol. 41, pp. 1124-1135, 2011.

[20] Z. Qi, S. Peng, L. Jinjun, and X. Shengyuan, "Adaptive Output-Feedback Fuzzy Tracking Control for a Class of Nonlinear Systems", *Fuzzy Systems, IEEE Transactions on*, vol. 19, pp. 972-982, 2011.

[21] L. Hyeongcheol and M. Tomizuka, "Robust adaptive control using a universal approximator for SISO nonlinear systems", *Fuzzy Systems, IEEE Transactions on*, vol. 8, pp. 95-106, 2000.

[22] L. Luque-Vega, B. Castillo-Toledo, and A. G. Loukianov, "Robust block second order sliding mode control for a quadrotor", *Journal of the Franklin Institute*, vol. 349, pp. 719-739, 2012.

[23] P. Castillo, R. Lozano, and A. Dzul, *Modelling and control of mini-flying machines*: Springer-Verlag New York Inc, 2005.

[24] J. Escareno, S. Salazar-Cruz, and R. Lozano, "Embedded control of a four-rotor UAV", in *American Control Conference, (ACC)*, Minnesota, USA, 2006, pp. 3936-3941.

[25] F. Kendoul, D. Lara, I. Fantoni, and R. Lozano, "Nonlinear control for systems with bounded inputs: Real-time embedded control applied to UAVs", in *Decision and Control, 2006 45th IEEE Conference on*, 2006, pp. 5888-5893.

[26] G. Hoffmann, H. Huang, S. Waslander, and C. Tomlin, "Quadrotor helicopter flight dynamics and control: Theory and experiment", in *Proc. of the AIAA Guidance, Navigation, and Control Conference*, 2007.

[27] P. Pounds, R. Mahony, and P. Corke, "Modelling and control of a quadrotor robot", in *Proceedings of the Australasian Conference on Robotics and Automation*, 2006.

[28] A. Gessow and G. C. Myers, *Aerodynamics of the Helicopter*. Ungar: New York, 1967.

[29] E. Altug, J. P. Ostrowski, and C. J. Taylor, "Quadrotor control using dual camera visual feedback", in *IEEE International Conference on Robotics and Automation ICRA*, Taipei, Taiwan, 2003, pp. 4294-4299.

[30] F. Yacef, O. Bouhali, H. Khebbache, and F. Boudjema, "Takagi-Sugeno Model for Quadrotor Modelling and Control Using Nonlinear State Feedback Controller," *International Journal of Control Theory and Computer Modelling (IJCTCM)* vol. 2, pp. 9-24, 2012.