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Robust Range-only SLAM for Aerial Vehicles

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Abstract— **This paper presents a robust method to map the position of a set of radio range sensors while at the same time an aerial vehicle is being localized with respect that map employing only range measurements even in the presence of noisy measurements. The method makes use of a pre-filtering algorithm to detect and remove measurements outliers and extends the original approach presented in [1] to model and estimate the measurement model of each radio sensor in order to correct the computed range of each node. The method is validated in simulation involving an aerial vehicle and radio-based range sensors.**

I. INTRODUCTION

Sensor networks are widely used in most inspection and monitoring applications involving aerial vehicles. These low-cost sensors are used to provide measurements of different nature and provides some processing capabilities, but a common problem when using these wireless sensor networks is the need to localize each sensor of the network. Wireless Sensor Networks (WSN) are typically localized using range measurements between the aerial vehicle and each beacon or even between static beacons. The main problem with this kind of measurements is that they are very noisy due to multi-path effects and other radio-related noises, the second main problem with this kind of measurements is the lack of bearing information which makes the initialization of the beacon position a challenging problem due to its multi-modal nature.

Many proposed solutions for range-only mapping are based on probabilistic filters which are able to model the uncertainty of the observation and action model as well as the imprecisions of sensors [2]. Thus, for example, [3] proposes a probabilistic approach based on a particle filter to get the initial estimation of each beacon position aided by an aerial vehicle with another radio range sensor onboard. Once the particle filter of each beacon has converged into a single Gaussian solution, the estimation is switched to an Extended Kalman Filter (EKF). The results of the method are accurate but there is a huge amount of information which is lost by the EKF due to the delayed initialization with the particle filter. Later, to deal with the delayed initialization, the authors proposed a multiple hypotheses solution [4] where the initial 2D location uncertainty of beacons is modelled with a Gaussian Mixture Model integrated in a single EKF since the first range measurement received. Then, new range measurements are are used to update these hypotheses and to reduce the number of them to decrease the computational resources required by these multi-hypotheses solutions. The method uses the polar parametrization proposed in [5] to model the bearing information. The results of this method are accurate and, in contrast to previous work, it allows to initialize the EKF with a single range measurement.

In the domain of range-only SLAM (RO-SLAM), different SLAM frameworks have been proposed and compared [6], [7], being the Fast-SLAM approach considered the most accurate and efficient solution. This Fast-SLAM framework is used in [8], [9], where a particle filter for both localization and mapping problem is used. Two different optimization techniques for the particle filter used to locate the different beacons are applied. Another Fast-SLAM solution is proposed in [10], [11] where the particle filter of the mapping problem is switched into an EKF once each particle filter converges into a Gaussian distribution reducing the computational burden of previous approaches. For 3D RO-SLAM, [12] proposes an undelayed solution which models the multi-modality of the range measurement model with a Gaussian Mixture Model so that each beacon is integrated in an independent EKF. This solution presents accurate results but does not allow the integration of inter-beacon measurements in such a way that cross correlations between beacons can be taken into account. Thus, a previous work of the authors of this paper [1] proposes an undelayed 3D RO-SLAM method based on a centralized EKF-SLAM framework which allows the integration of inter-sensor measurements considering the cross correlation between them. The main problem of this solution is that, since it is based on an EKF, the solution is very sensible to the presence of measurement outliers so, in this work, a robust filter is proposed to avoid the divergence of the EKF filter. On the other hand, with radio-based sensors, it is common to found some linear errors related with the propagation model of each node which is affected by the environment where these are placed. This paper propose an extension of their previous work to estimate the range measurement model parameters which corrects this linear errors, allowing then a better localization of an UAV with the results of the mapping process.

In the field of robust range-only localization an mapping [13] proposed a robust outlier rejection method to improve the performance of its EKF-SLAM based on a spectral graph partitioning. The main problem of this solution is that, as in [14], it requires a minimum network connectivity constraint to make it work properly which is not always

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available in real applications. The solution proposed in [14] is based on what they call *robust quadrilaterals* to avoid the ambiguities related with the multi-modal nature of range measurements. In [15] the authors propose another solution based on gating techniques, such as chi-square filter.

The main contributions of this paper are the use of a two-step pre-filtering method which improve the robustness of the algorithm presented in a previous work of the authors [1] by rejecting outliers with a simple and efficient algorithm. This pre-filtering method is based on some heuristics which depends on on the current state of the UAV position and the previous range measurements received. The second contribution of this paper is an extension of [1] to model the range measurements model, the method estimates the model parameters including this parameters in the EKF estimation. The methods are validated in simulation.

The paper is organized as follows. Section II summarizes the previous work of authors in the domain of range-only SLAM. Section III details the proposed method to filter range measurements outliers, and explains how to include the estimation of the range measurements model parameters in the EKF state vector. The method is then validated in simulation in Sect. IV. Finally the conclusions in Sect. V close the paper.

II. UNDELAYED 3D RO-SLAM

The method presented in this paper extends the RO-SLAM method introduced by authors in [1] making it more robust against range measurement outliers and improving the measurement correction by considering a different model parametrization for each beacon. This method uses a centralized EKF to estimate not only the 3D position of the UAV, but also the 3D position of each static beacon. The paper will show how a refined mapping of the sensor node positions together with a tuned radio propagation model can benefit the UAV localization, making it more robust and accurate.

The state vector $\mathbf{x}$ of the centralized EKF is composed by the UAV position $\mathbf{x}_r = [x_r, y_r, z_r]^T$ and the position of m beacons (features of the map $\mathbf{f}_i$):

$$\mathbf{x} = [\mathbf{x}_r^t, \mathbf{f}_1^t, \mathbf{f}_2^t, \ldots, \mathbf{f}_m^t]^T \tag{1}$$

For the localization of the aerial vehicle, any kinematic and dynamic model of the vehicle can be used. Then to correct the position predicted, it is possible to use some range sensors whose positions are known (anchors) as a mean to trilaterate the 3D position of the aerial robot when beacons are not initialized or have not converged yet. The positions of the anchors must be placed in a way that the vehicle can trilaterate its position as it moves around the entire scenario. The equation applied to update the vehicle position in the EKF with range measurements from an anchor node is

$$r_i = \sqrt{(x_{a_i} - x_r)^2 + (y_{a_i} - y_r)^2 + (z_{a_i} - z_r)^2} \tag{2}$$

where $[x_{a_i}, y_{a_i}, z_{a_i}]$ is the known 3D position of the i-th anchor which generated the range measurement r_i.

As it was mentioned above, the mapping solution proposed in this paper uses a multiple-hypotheses strategy which integrates two Gaussian Mixture Models (GMM) in the EKF to model the bearing uncertainty since the very first measurement. The state vector of each beacon $\mathbf{f}_i$ is represented using a reduced spherical parametrization (see Fig. 1) which is composed by the position of the aerial vehicle $\mathbf{x}_i = [x_i, y_i, z_i]^T$ from which the first range measurement was received, the range measurement received $\rho_i = r_i$ (the radius of the spherical parametrization), n_θ samples of possible azimuth angles θ_i and n_ϕ samples of possible elevation angles ϕ_i. Hence, the state vector of a beacon during the initialization stage is

$$\mathbf{f}_i = [\mathbf{x}_i^t, \rho_i, \theta_{i1}, \theta_{i2}, \ldots, \theta_{in_\theta}, \phi_{i1}, \phi_{i2}, \ldots, \phi_{in_\phi}]^T \tag{3}$$

It should be noticed how this spherical parametrization uses only $4 + n_\theta + n_\phi$ parameters to represent the $n_\theta n_\phi$ hypotheses instead of $4 + n_\theta n_\phi$ as in other solutions [12]. Within this parametrization, each sample θ_{ij} is the mean value of a Gaussian mode in a GMM used to model the azimuth parameter θ_i. The weights of each mode in the GMM are initialized uniformly according to the number of samples/modes employed. In the same manner, the standard deviation $\sigma_{\theta_{ij}}$ of each sample is initialized uniformly and depends on the number of hypotheses $h = n_\theta n_\phi$ used to model the real uniform spherical distribution. The same procedure is used to initialize elevation samples of the ϕ_i GMM. The number of hypotheses h are initialized automatically according to the first measurement received and the optimal density of hypotheses as described in [1]. Finally, the covariance of the spherical parametrization $\mathbf{x}_i$ is initialized with the current state of the covariance matrix of the UAV at the moment of the initialization. In this parametrization the deviation σ_{ρ_i} represents the thickness of the initial uniform sphere distribution represented in Fig.1. Then, once a beacon has been initialized, next range measurements are used to update beacon state and to prune the samples of both GMMs according to a prune strategy described in [1]. But, in this paper, a pre-filtering algorithm of the range measurements received is proposed bellow to avoid the filter divergence.

The update scheme proposed is also optimized so that it only use $n_\theta + n_\phi$ equations, instead of the $n_\theta n_\phi$ equations used in other works. This optimization is based on the independence of azimuth and elevation samples. Hence, the $n_\theta + n_\phi$ equations employed are as follows:

$$r_i = \sqrt{(x_{f_i} - x_r)^2 + (y_{f_i} - y_r)^2 + (z_{f_i} - z_r)^2} \tag{4}$$

where x_{f_i}, y_{f_i} and z_{f_i} stand for

$$\begin{aligned} x_{f_i} &= \quad x_i + \rho_i cos(\theta_i) cos(\phi_i) \\ y_{f_i} &= \quad y_i + \rho_i sin(\theta_i) cos(\phi_i) \\ z_{f_i} &= \qquad\quad z_i + \rho_i sin(\phi_i) \end{aligned} \tag{5}$$

Thus, to update one θ_{ij} sample the method proposes to substitute the value ϕ_i of (4) by the expected elevation angle

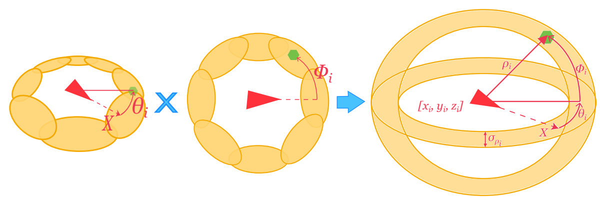


Fig. 1. Spherical parametrization of beacons. The yellow area represents the probability distribution where the feature can be located once a first range measurement is received. The annulus at left represents the GMM used to model the theta samples (azimuth angles), the annulus at center represent the GMM used to model the phi samples (elevation angles) and finally the sphere at right represents the real uniform and spherical distribution modelled with the combination of both GMM. The green polygon represents the real position of the range-sensor (beacon). The center of the sphere is composed by the position where the robot was located when the first range measurement was received.

$\overline{\phi_i}$ of the ϕ_i GMM and propagating its equation properly through the Jacobian $\mathbf{H}$. Analogously, the expected azimuth value $\overline{\theta_i}$ is used to update each elevation sample ϕ_{ij}. More details are given in [1].

Finally, to update the weights of both GMMs the following equations are used:

$$\omega_{\theta_{ij_\theta}} = \omega_{\theta_{ij_\theta}} max(p(r_i|\mathbf{x}_r^t, \mathbf{x}_i^t, \rho_i, \theta_{ij_\theta}, \phi_{ij_\phi})|j_\phi = 1..n_\phi) \tag{6}$$

$$\omega_{\phi_{ij_\phi}} = \omega_{\phi_{ij_\phi}} max(p(r_i|\mathbf{x}_r^t, \mathbf{x}_i^t, \rho_i, \theta_{ij_\theta}, \phi_{ij_\phi})|j_\theta = 1..n_\theta) \tag{7}$$

III. ROBUST RO-SLAM

As previously introduced, it is well known that outliers and distorted measurements significantly affects range-only simultaneous and mapping approaches [16], making them less applicable in real application scenarios. This paper proposes improving the multi-hypothesis approach presented in [1] with specially adapted new measurement filtering algorithms in order to elevate the approach to a new level in terms of accuracy and robustness.

Thus, next lines will propose estimating the propagation model associated to each radio beacon in order to account for the installation particularities and sensor characteristics of each radio beacon considered into the system. It will be assumed that the range measurements are biased and scaled by unknown parameters given by the environment, the multi-hypotheses approach will be updated with new information in order to refine not only the position of the range sensors but also their radio propagation characteristic.

In addition a method for outlier rejection will be presented here. This is very important when dealing with Gaussian Filters as measurements outlier can eventually lead to the filter divergence.

A. Measurement Model

It is very common that range sensors based on radio produce biased and, at some point, scaled measurements. This effect is the combination of the sensors environment (position, structure, ...) and the inaccuracies of the ranging mechanism. This is why both, bias and scale factor, are different for every range sensor in the system. Thus, an accurate range-only localization will not only depend on the anchors position but also on the characterization of their measurement models.

Thus, this paper proposes estimating the measurement model bias b_i and scale factor s_i for each sensor node i into the system. This way, the feature descriptor of (3) will include these two parameters, obtaining the following new parameterization of sensor node in the filter:

$$\mathbf{f}_i = [\mathbf{x}_i^t, \rho_i, \theta_{i1}, \theta_{i2}, \ldots, \theta_{in_\theta}, \phi_{i1}, \phi_{i2}, \ldots, \phi_{in_\phi}, s_i, b_i]^T \tag{8}$$

Notice how it is included only two new parameters and they are not affected at all by the multi-hypotheses model associated to the sensor position, leading to a fast and clean convergence of the estimation even in the presence of many hypotheses. In addition, the total overhead added to the filter is two parameters per sensor, obtaining a eight state parameterization when the feature converges to a single hypotheses.

The correction equation has also been modified according to the new range measurement model. The following correction equation is proposed:

$$r_i = s_i\sqrt{(x_{f_i} - x_r)^2 + (y_{f_i} - y_r)^2 + (z_{f_i} - z_r)^2} + b_i \tag{9}$$

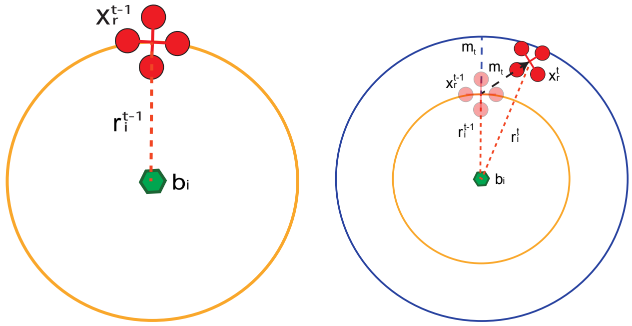
where x_{f_i}, y_{f_i} and z_{f_i} are defined in (5). Hence, the associated Jacobian $\mathbf{H}'$ of the new correction equation 9 is also extended with respect the Jacobian $\mathbf{H}$ employed in [1] as $\mathbf{H}' = [s_i\mathbf{H} \quad \mathbf{h} \quad 1]$ where $\mathbf{h}$ is the estimated measurement vector proposed in [1].

In general, the scale factor will be initialized to 1 while the bias will be initialized to 0, but with some initial noise in both initializations to let the filter adjust the real values of the parameters. Remember that the objective of this parameterization is to account for the small deviations of the range measurement model in commercial sensors, so the scaled factor and bias are expected to be around 1 and 0 respectively.

B. Outlier rejection filter

The outlier filter proposed in this paper is composed by two simple heuristics: the first heuristic filters those range measurements which are not consistent with the motion suffered by the aerial vehicle since the last range measurement received, and the second step is based on a median filter of the range measurements received in the last l meters tracked by the aerial vehicle.

The first step is executed when a range measurement is received, this pre-filtering process takes the position of the aerial vehicle at the time the new range measurement r_i^t is received so that the current position of the aerial vehicle $\mathbf{x}_r^t$ is compared with the position of the vehicle $\mathbf{x}_r^{t-1}$ when the last range measurement was received r_i^{t-1} (see 2(a)) using the euclidean distance. Thus, the euclidean distance



(a) (b)

Fig. 2. First step of the outlier rejection filter. Figure (a) represents the instant of the first range measurement reception whereas (b) represent the instant where a new range measurement is received. The black arrow represents the real direction of the vehicle and the blue discontinuous line represents the radial trajectory with respect the beacon position. In this case the distance r_i^t is lower than $r_i^{t-1} + m_t + \sigma_{r_i^t}$ so the range measurement is valid.

represents the position increment m_t of the vehicle between both range measurements as depicted in 2(b), hence, the new range measurement must be lower than $r_i^{t-1} + m_t + \sigma_{r_i^t}$ and higher than $r_i^{t-1} - m_t - \sigma_{r_i^t}$, i.e. the position increment m_t represents the maximum distance increment allowed ($\sigma_{r_i^t}$ represents the uncertainty of range measurement r_i^t). This maximum and minimum allowed distance increment of the range measurement corresponds to the cases where the aerial vehicle moves in the radial direction with respect the beacon hypotheses so, if the new range measurement is outside this range, the range measurement is considered an outlier and then it is not included in the correction stage to avoid filter divergences.

The second part of the outlier filter takes advantage on the fact that the robot does not move very quickly and that single beacon is measured by the UAV several times per second to filter noisy range measurements. This part of the algorithm makes a low-pass filtering procedure by taking the last range measurements $\mathbf{r}_l$ received in the last l meters travelled by the aerial vehicle then it is computed the median value $\hat{r}_i^t$ of the set $\mathbf{r}_l$ and, finally, the returned range measurement is the mean value of the range measurements which are around the median range value $\hat{r}_i^t$ (the interval around the median value is selected empirically as a percentage of the current number of range measurements in $\mathbf{r}_l$). The complete pre-filtering method is summarized in Algorithm 1.

Algorithm 1: Outlier prefiltering

Data: r_i^t, $\sigma_{r_i^t}$ and $\mathbf{x}_r^t$
Result: Filtered range measurement $\bar{r}_i^t$
begin
// First method step
$m_t \longleftarrow EuclideanDistance(\mathbf{x}_r^{t-1}, \mathbf{x}_r^t)$;
if $r_i^t > r_i^{t-1} + m_t + \sigma_{r_i^t}$ **or** $r_i^t < r_i^{t-1} - m_t - \sigma_{r_i^t}$ **then**
Discard r_i^t //Is an outlier
// Second method step
Remove range measurements in $\mathbf{r}_l$ taken further than l meters with respect $\mathbf{x}_r^t$;
Add $r_i^t \longrightarrow \mathbf{r}_l$;
$\mathbf{r'}_l \longleftarrow Order(\mathbf{r}_l)$;
$\hat{r}_i^t \longleftarrow Median(\mathbf{r'}_l)$;
$\mathbf{r''}_l \longleftarrow ValuesAround(\hat{r}_i^t, \mathbf{r'}_l)$;
$\bar{r}_i^t \longleftarrow Mean(\mathbf{r''}_l)$;

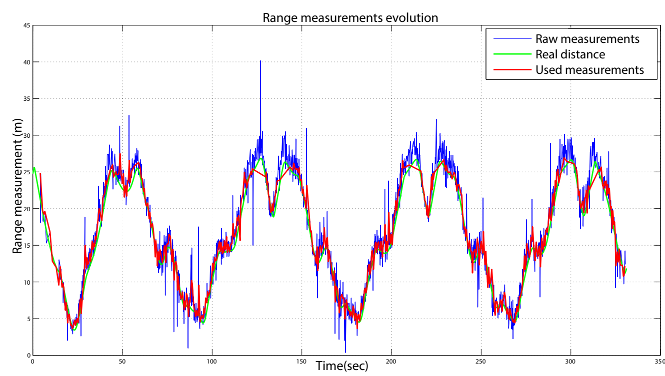


Fig. 3. Evolution of the range measurements received in the UAV from one of the nodes during the whole simulation. It can be seen the evolution of the actual distance between beacon and UAV (green solid line), the noisy measurement generated by the simulator (blue solid line) and the results of the outlier filter (red solid line)

IV. RESULTS IN SIMULATION

The RO-SLAM previously introduced with the outlier filter and range measurement model estimation has been implemented in C++ integrated into the Robot Operating System (ROS). A set of simulations have been developed in order to carefully evaluate the benefits of the method.

Thus, the simulation setup is composed by a set of 12 radio-based nodes which measurement noise characterization has been identified and included into the simulation (Gaussian Noise with $\sigma = 1m$). In addition, the UAV was piloted following smooth trajectories though the sensors area acquiring range information to each of the nodes in the setup. All this information, UAV trajectory and sensor node pose, is used as ground-truth for the sensor node mapping and vehicle localization.

A. Outlier filtering

The first experiments consisted consisted on determining whether the outlier rejection filter is working properly or not. Figure 3 shows the results of the outlier filter over the noisy range measurement between the UAV and one of the beacons. It can be seen how the filter removes all outliers and also produces a low-pass filtering over the range data reducing the effect of the noise.

In order to evaluate the impact of the outlier filter over the overall performance of the approach, the mapping results with and without considering outlier rejection has been analyzed. In both experiments the measurement model estimation was disabled to help recognizing the filter benefits. Figure 4 shows the 3D absolute error of the estimated position for one of the beacons with and without outlier

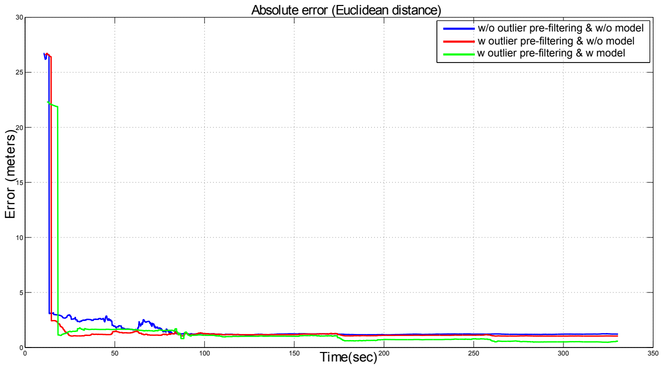


Fig. 4. 3D absolute error mapping the position of beacon number 4 at different conditions.

TABLE I

MAPPING ERRORS FOR BEACON NUMBER 4

	Final Abs. Error (m)
w/o outlier pre-filtering & w/o model	1.221
w outlier pre-filtering & w/o model	1.042
w outlier pre-filtering & w model	0.5749

rejection (red an blue lines respectively). The errors are similar in both cases but 15% smaller with filtering, see Table I. In addition, the absence of outliers increase the reliability of the EKF estimation.

Similar results are obtained for the rest of nodes of the experiments but they are not shown in the paper for space restrictions. In general, the localization error of beacons is improved and the EKF increase in immunity to wrong measurements.

B. SLAM results

Once the advantages of the outlier filter are presented, this section will show how the mapping of the sensor nodes are improved by including the estimation of the measurement model. The range information generated by the simulator have been distorted for some nodes, using a scale factor and a a small bias to corrupt the range information. The objective of this section is then analyze if the inclusion of the measurement model really improves the beacon mapping.

Thus, Fig. 4 provides a comparative analysis of the three method used for range-only mapping: without model and without outlier filtering, without model but with outlier filtering and, with model and outlier filtering. It can be seen how the 3D mapping error of the beacon decreases as we include the filtering and later the measurement model, resulting in something less than 1m in error reduction, see Table I.

The evolution of the hypotheses weight for beacon number 4 is presented in Fig. 5. It can be seen how almost all hypotheses are discarded after 25 seconds and at 50 seconds the filter converges to a single solution.

Finally, Fig. 6 shows the estimated scale factor and offset for beacon 4. In this case, the real measurement had a small bias of 0.1m and a scale factor of approximately 1. It can be seen how the estimations are consistent in mean and standard deviation, having the actual solution into the 3σ interval of the estimation.

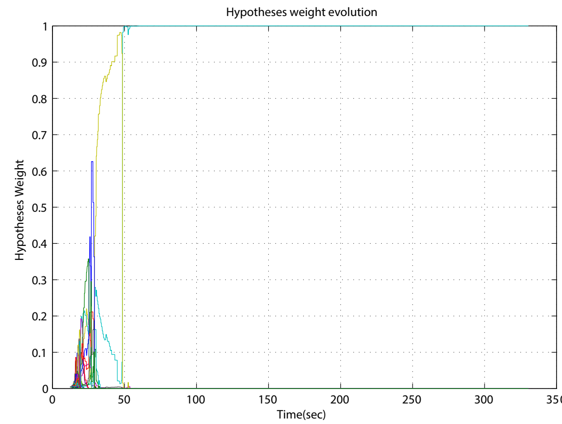


Fig. 5. Evolution of the weights for different hypotheses of beacon 4.

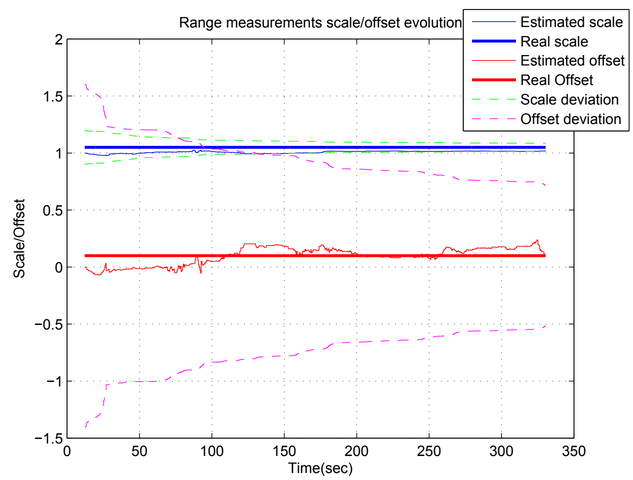


Fig. 6. Estimated scal factor and bias for beacon 4. It can be seen how the estimations are always very close the ground-truth (straight lines) and how the actual value is into the 3σ interval of the estimation.

The last part of this section aims to show the localization results obtained during the experiments described above while simultaneously the wireless sensor network is being mapped together with range measurement models. The results are shown in Fig. 7. It can be seen how the estimation follows the ground-truth with small errors most of the time. Also can be noticed how the estimation is always into the 3σ interval of the estimation and how estimated deviations are well computed except for Z axis, this is because the UAV poorly trilaterates in Z.

V. CONCLUSIONS

This paper has presented a robust method to simultaneously map the position of a set of radio range sensors and localize an UAV with only range measurements even in the presence of noisy measurements. The method makes use of a pre-filtering algorithm to detect and remove range measurements outliers and extends the original approach to model and estimate the measurement model of each radio

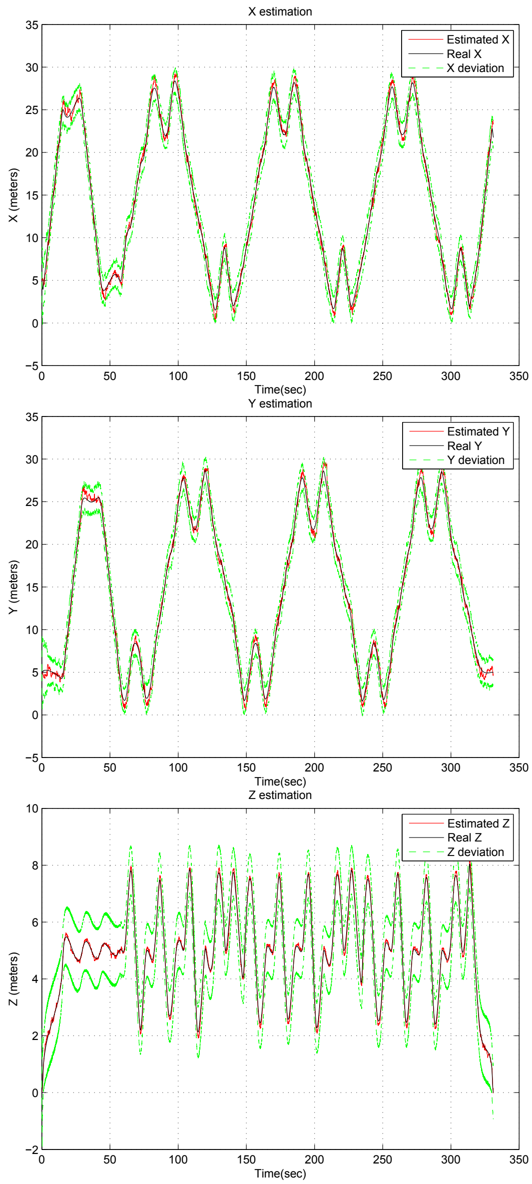


Fig. 7. Estimated trajectory based on range-only localization and refined nodes mapping.

sensor in order to correct the computed range of each node. Different simulation results have been performed to validate the method, the results show how the use of the pre-filtering method rejects several spurious range measurements which decreased the estimation error of the mapping process in more or less a 15%. On the other hand, the extension of the previous authors work to model the propagation model of range measurements have demonstrated an improvement not only in the mapping results (near 1 meter of error reduction) but also for the localization of the UAV.

The simple propagation model presented in this paper works with accurate results on environments where the noise conditions of the nodes are not affected with new obstacles or other dynamic elements, future work will consider environments where the noise characteristic of sensors is dynamic according to the current position of the UAV, this makes necessary to consider the inclusion of a prediction stage in these model parameters which allows the evolution of the deviation of this parameters. On the other hand, offline learning solutions will also be considered so that the vehicle can construct during an exploration mission a lookup table which maps the current UAV position with the associated propagation model parameters to be used by each beacon of the environment. All these methods will be validated with real experiments on more realistic environments.

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