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Modeling and Identification of Electric Propulsion System for Multirotor Unmanned Aerial Vehicle Design

Grzegorz Szafranski, Roman Czyba and Marian Błachuta

Abstract— **The propulsion system plays a key role in each unmanned aerial vehicle. Considering different multirotor platforms we may observe that mostly the dynamics and performance of the particular vehicle is strictly depended on the drive unit. In this paper we focus on the problem of modeling and identification of the propulsion system consisting of an electronic speed controller, electric brushless direct current motor and propeller. We propose a multiple-input and multiple-output nonlinear model of the propulsion unit based on the block oriented modeling with the static nonlinearities at the input and linear dynamical part at the output. Such model enhanced with the aerodynamics theory of a propeller should be suitable and detailed enough to provide the simulation, control algorithms prototyping and solution verification under the circumstances of designing the vertical take-off and landing unmanned platform.**

I. INTRODUCTION

The propulsion system is the main element of each unmanned platform. Especially for multirotors e.g. quadrotor is very important. In general, the rotary-wing aircrafts do not have the lifting surface. The thrust force is generated by rotors and allows to achieve vertical climb and horizontal flight. Nowadays most of the vehicles use an electric motor without brushes (Brusless Direct Current Motor) directly connected with the propeller. The commutation process of the brushless DC motor is possible with the help of the electronic circuit. A six transistors inverter is needed [1]. Brushless motors offer to be more efficient than the brushed ones because of high torque per weight, per watt unit etc. [2].

In all configurations of mini unmanned rotary platforms increasing or decreasing simultaneously the velocity of propellers allow to achieve a vertical motion. However, depending on the exact construction, the propeller velocities variations resulting in horizontal plane motion due to changes in roll and pitch. It is undisputed that the propulsion system has an effect on unmanned vehicle platform dynamics. In most cases for modeling and control algorithm design the multirotor platforms are considered as a rigid body dynamics with the external forces and moments applied, proportional to the power of the propeller velocity [3-6], in some work even the motor dynamics is neglected [9]. Nevertheless the rotors give the ability to change the orientation of the vehicle only. Therefore the propulsion system must be investigated carefully. In recent years many research studies considered more advanced modeling, paying attention to the aerodynamics of the propeller as well [7-10]. Different aspects of the propeller dynamics has been also examined e.g. blade flapping [8,10]. However, the main force that affects the body dynamics of the multirotor platform is a thrust force and torque originated from drive. Although the thrust generated by the propulsion unit is a function of an advanced ratio [12], which implies measuring the propulsion unit under the dynamic conditions, the static thrust is much easier to estimate and has a meaning for hover, which is a frequent state of flight for rotary-wing vehicles. On the basis of the static thrust measurements some approximation of the real thrust value can be achieved by using e.g. the momentum or blade element theory [11-13], which is also mentioned in [7,10] and will be presented in the next sections taking into consideration the vertical movement of the flying platform.

Other aspect related to the propulsion system is a flight endurance which is also a very common issue. As stated in the beginning most of the rotary platforms do not have lifting surface. Almost all energy stored in the batteries is used for developing the thrust force. In order to examine the flight range it is necessary to dispose of the propulsion model that is able to estimate the current consumption. Models of the propulsion system mentioned in the previous paragraph result in one dimensional dynamics without considering the nonstationary impact of the propulsion unit to the whole model of the multirotor. A shortage in the more developed models of propulsion system especially ones that include time varying dependencies brings the concept of multidimensional model of a drive unit for electric unmanned aerial vehicles.

The considerations in this paper are dealing with the process of modeling and identifying the electric propulsion system for the sake of the total dynamics of the vertical take-off and landing platform. For the purpose of the nonlinear identification of multirotor drive unit the Hammerstein model has been employed. Under the circumstances mentioned before the multiple-input, multiple-output (MIMO) model of drive unit has been proposed. The dependencies between the model variables are indicated. The presented findings includes both conceptual and practical meanings for the identification and modeling of the multirotor propulsion system.

The paper is organized as follows. First, the theoretical background has been given for phenomenological modeling of a propulsion system. Also the aerodynamics in vertical motion has been considered. The second part includes the discussion of a hardware which has been used for the identification experiments. The next section presents an approach for identifying the propulsion system. First of all the nonlinear method and model is chosen for describing the whole components of the driving unit then the exact form of the model together with signal dependencies is explained.

G. Szafranski, , R. Czyba, M. Blachuta are with the Silesian University of Technology, Akadmicka 16, 44-100 Gliwice, Poland (corresponding author to provide phone: +48322372143; fax: +48322371165; e-mail: grzegorz.szafranski@polsl.pl).

Finally, the results of the identification procedure and the conclusions have been briefly discussed.

II. ROTOR SYSTEM MODELING

In this section some theoretical background about modelling the propulsion system is presented. It has been divided into three parts. First the mathematical model of a brushless DC motor is explained. Secondly some aspects of the propulsion aerodynamics are presented. Finally the participation of the propulsion system in the whole multirotor body dynamics is clarified.

A. Motor and propeller modelling.

Although the brushless DC motor is a kind of a synchronous motor with the internal feedback for the optimal electronic switching circuit, it can be modelled as a conventional DC motor [1]. If such assumption is being made one will receive the model in terms of quality.

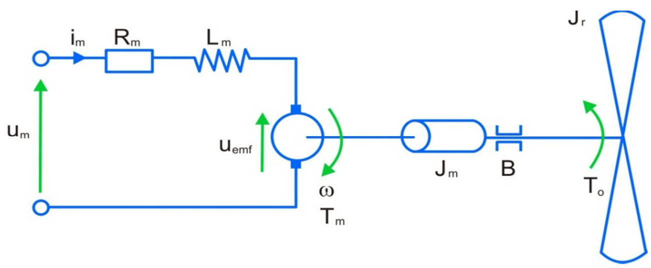

Figure 1. The propulsion system mathematical model.

Considering the above picture the equations for a motor dynamics can be written. From basic physics principles the electrical equation of the system has the following form:

$$u_m - i_m R_m - L_m \frac{di_m}{dt} - e_m = 0, \quad e_m = k_{emf}\Omega \qquad (1)$$

where: u_m - voltage applied to the motor, i_m - armature current, R_m - motor winding resistance, L_m – inductance, e_m - back electromotive force, k_{emf} - motor electrical constant, Ω – shaft velocity.

Motor torque T_m is proportional to the armature current i_m. The load has an opposing torque T_o. The mechanical equation can be expressed as follows:

$$J\dot{\Omega} - T_m - T_o - B\Omega = 0, \quad T_m = k_m i_m \qquad (2)$$

where: J - rotor and propeller inertia, B - viscous friction coefficient, k_m - torque constant.

The motor shaft inertia is relatively small. In many cases it can be neglected. However, the inertia of the propeller is significantly large and is not easy to determine. Some geometrical simplifications can be made in order to estimate the plausible value of the inertia. For example the rod and the cuboid solids may be used (see Fig. 2).

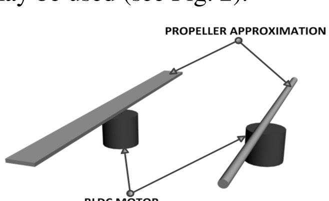

Figure 2. The geometric approximation of the propulsion system.

One of the main parameter of the propeller is stiffness which prevents fluttering, a kind of a propulsion vibration. Such vibrations can easily be transferred to the multiplatform frame and cause extra disturbances in measuring system. Amongst the other forces and torques acting on the rotor that we can distinguish as follows: bending which also depends on the propeller rigidity, centrifugal force – the effect of the rotating blade mass at high speeds. In the following considerations these forces are omitted, only main force such as thrust is discussed. The torque can be evaluated by means of the current model dynamics.

B. Rotor aerodynamics considerations

The aerodynamics of the propeller cannot be omitted while we consider the detailed model of the propulsion unit because it interferes with the rigid body dynamics of the multirotor vehicle. The principle of the lift process generation is explained by one of the basic approach which is a momentum theory [11-13]. While using this kind of theory we can skip the knowledge about the geometric shape of the propeller but we have to reckon with more inaccurate results. There are also other limitations that must be taken into account. However, with some modifications it can be very useful and satisfactory.

Multirotor vehicles very often operate in a variety of flight regimes. While performing e.g. a precise observation, some tracking tasks or taking-off and landing from the small areas the horizontal speed is not so demanding (small pitch and roll angles). Thus, we are about to consider the vertical movement, which includes hover, climb and descent – the most common maneuvers performed by the multirotors platforms.

The brushless DC motor produces torque which drives the propeller with the same angular velocity as a motor shaft because of the direct mechanical connection between those two. In fact the propeller torque is generated by the aerodynamic drag affecting the rotor when it is spinning. When propeller and motor torque are equal the propeller is rotating at the constant velocity. Having the ideal motor, the power in steady state is as follows:

$$P_m = u_m i_m = T_m \cdot u_m / k_m \qquad (3)$$

In hover, the flow is axisymmetric and the flow through the rotor is either upward or downward. For the ideal hover condition the power is the thrust force multiply the induced velocity v_i, which is for that stage of flight equal to the hover velocity stream v_h. If the wind-free hover conditions are fulfilled the ideal propeller power is expressed in the following formula:

$$P_{hi} = T_i \cdot v_{hi} \qquad (4)$$

If we denote that ρ is the air density, A is the disk active area, the induced velocity in hover is [11]:

$$v_{hi} = \sqrt{T_i / 2\rho A} \qquad (5)$$

In this case the static thrust can be rewritten if we accept that the torque is proportional to the thrust $T_m = T_i \cdot k_q$:

$$T_{istat} = \frac{2\rho A k_q^2 \cdot u_m^2}{k_m^2} \tag{6}$$

Although the momentum theory is idealized, assumes no losses, it can easily serve as an estimation of a minimal amount of power that is needed for hovering. We can also assume the power losses ratio which brings better calculations certainty.

For the climbing rotor the relationship is as follows:

$$T = 2\rho A(V_c + V_i)V_i \tag{7}$$

where: V_c – vertical rotor velocity in the up direction.

In the climb the induced velocity V_i is the terms of the v_h has the solution:

$$\frac{V_i}{V_h} = -\left(\frac{V_c}{2V_h}\right) + \sqrt{\left(\frac{V_c}{2V_h}\right)^2 + 1}, \quad V_c > 0 \tag{8}$$

This is normal working state of the rotor with hover being lower limit. Note that if the climbing speed increases the induced velocity decreases.

For the axial descent the above model cannot be used, and it is required to consider two following cases.

$$\frac{V_i}{V_h} = -\left(\frac{V_c}{2V_h}\right) - \sqrt{\left(\frac{V_c}{2V_h}\right)^2 - 1}, \quad V_c \le 2V_h \tag{9}$$

First the rotor is extracting power from the airstream and this operating condition is known as the windmill brake state, because the rotor decreases or brakes the velocity of the flow.

Second case, it is the region between hover and windmill state. When $-2V_h \le V_c \le 0$ momentum theory is invalid because the flow can take on two possible directions and a well-defined slipstream ceases to exist. Unfortunately descending flight accentuates interactions of the tip vortices with other blades and so the flow becomes rather unsteady and turbulent, and experimental measurements of rotor thrust and power are difficult to make. In this case the curve of V_i is not analytically predictable. The experimental estimates can be used to find the best approximation for induced velocity at any rate of descent. Many authors, including Young and Johnson, suggest a linear approximation given in [11,12].

C. *Linkage between the propulsion system and multirotor frame.*

Having the propulsion system theoretically characterized the interactions with the rigid body dynamics of multirotor platform can be introduced. Forces and torques generated by a drive unit act on the flying platform frame and their impact is depended on the geometrical distribution of the propulsion systems. From the mechanical point of view the multirotor vehicle is a multi-body system which consists of the *N+1* rigid bodies, where *N* is a number of the actuators. Let us consider the *i*-th propulsion system. The $F_e : \{O_w; , X_w, Y_w, Z_w\}$ is an Earth inertial frame and

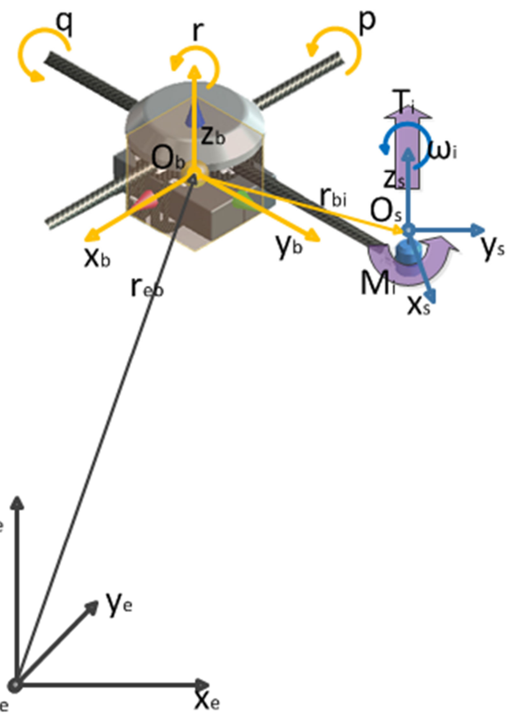

Figure 3. I-th rotor geometric configuration with the Earth, body and motor frames.

$F_b : \{O_b; , X_b, Y_b, Z_b\}$ is a frame attached to the multirotor body in the COG. For the *i*-th actuator the $F_s : \{O_s; , X_s, Y_s, Z_s\}$ frame is established (see Fig. 3).

The rotation matrices can be specified for orientations between each frame. Let us denote R_b^e as an orientation of body frame in the earth one. Whereas R_s^b is an orientation of the motor drive unit with respect to the body frame.

Using the notation of the elemental rotation matrices in the following form: for the rotation around x-axis about φ angle $R_x(\varphi)$, around y-axis $R_y(\varphi)$ and around z-axis $R_z(\varphi)$ we can denote that motors orientation in the body frame is as follows (for the uniform distribution of the actuator):

$$R_{si}^b = R_z\left(2(i-1)\pi/N\right), \; i = 1..N \tag{10}$$

Assuming that the distance from the centre of propeller frame to the body centre is given as a $\mathbf{r}_{bs} \in \mathbb{R}^3$ the propeller origin in the body can be noted as:

$$O_{si}^b = R_z\left(2(i-1)\pi/N\right) \cdot \mathbf{r}_{bs}, \; i = 1..N \tag{11}$$

The body coordinates in the Earth frame are given by the vector $\mathbf{r}_{eb} \in \mathbb{R}^3$ and orientation matrix R_b^e. Using the above equations the configuration of the multirotor is determined.

Next we will provide the Newton-Euler approach to obtain the dynamics of the multirotor vehicles. Taking into account the external forces and torques including the aerodynamics we will receive a complete model description.

Let us denote the body angular velocities as $\Omega_b \in \mathbb{R}^3$ and angular velocity of the propeller as $\omega_i \in R$ around the z-axis direction in propeller frame. The formula describing the angular velocities can be written:

$$\Omega_{si} = R_b^s \Omega_b + \begin{bmatrix} 0 & 0 & \omega_i \end{bmatrix}^T \tag{12}$$

Using a time differentiation and Euler equations of motion we receive:

$$M_{si} = I_{si}\dot{\Omega}_{si} + \Omega_{si} \times I_{si}\Omega_{si} - M_i \tag{13}$$

where $I_{si} \in \mathbb{R}^{3x3}$ is the inertia matrix of the propeller (see above) and M_i is an torque applied to the *i*-th propeller. Finally writing the equation for the multirotor platform we will obtain:

$$M_b = I_b\dot{\Omega}_b + \Omega_b \times I_b\Omega_b + \sum_{i=1}^{N} R_{si}^b M_{si} \tag{14}$$

in which $I_b \in \mathbb{R}^{3x3}$ is the inertia matrix of the multirotor platform, $M_b \in \mathbb{R}$ is the external torques component. Denoting the thrust force for *i*-th actuator as T_i the external torques which causes change in orientation can be rewritten in the next form:

$$M_b = \sum_{i=1}^{N} \left(O_{si}^b \times R_{si}^b T_i \right) \tag{15}$$

Next for the body frame on the basis of the Newton law the equation for the translational motion can be denoted:

$$\sum_{i=1}^{N} R_{si}^b T_{si} = m\dot{V}_b + \Omega_b \times mV_b + R_e^b F_e \tag{16}$$

where: m is a mass of multirotor $V_b \in \mathbb{R}^3$ is a body velocity vector, $F_e = \begin{bmatrix} 0 & 0 & -g \end{bmatrix}$- is a gravitational force in the Earth frame.

From this moment to design the control algorithms the above equations must be transformed to the Earth frame which is explained in [3].

The considerations presented above aim at the effects that propulsion system takes on the rigid body of the vertical take-off and landing platform.

After analysing the basic theoretical model of a propulsion system together with its aerodynamics in vertical movement, some remarks can be pointed out:

Remark 1. The relative order of the propulsion system can be denoted as one, maximally two.

Remark 2. Taking into account different phases of vertical flight (climb, descent and hover) the nonlinearities make difficulties in the synthesis of the control system.

Remark 3. The real thrust can be determined using the static thrust in some stages of multirotor flight.

Remark 4. Due to the problems with the parameters determination of the phenomenological model, an identification techniques are recommended.

Remark 5. Using the hover induced velocity the real thrust can be estimated for the ideal conditions by means of the momentum theory.

III. HARDWARE DESCRIPTION

To perform the necessary tests such as selection of a proper motor and propeller pair for the specific platform, possibility to enable the identification procedure of the selected drive unit, a suitable test-bed has been designed. The mechanics consists of the lever with the pivot point and transmission ratio 2:1. The impact produced by the propulsion unit is transferred to the strain gauge (sensing element) that gives the voltage signal proportional to the thrust force. Motor current and rotational velocity of the propeller are also measured. All the data are gathered via the DAQ card and passed to the PC computer. The block diagram of the laboratory setup is depicted in Fig. 4.

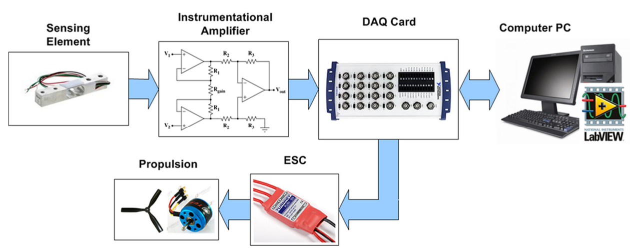

Figure 4. Data acquisition and control block diagram.

After multiple experiments with different motors and propellers in regard to the hover condition of the multirotor platform some choices have been made. The motor has been selected as a drive for the real multirotor, type quadrotor construction weighting approximately 2 kilograms. The exact motor parameters have been given in the Table I. The propeller has been matched with the following parameters: 12 inches in diameter, and 3.8 in pitch. The electric propulsion system beside the propeller, brushless DC motor, consists also of an electronic speed controller (ESC) (Fig. 5). The speed controller is off-the-shelf product. The frequency rate is up to 450 Hz which is nine times faster than standard controllers for the radio control planes.

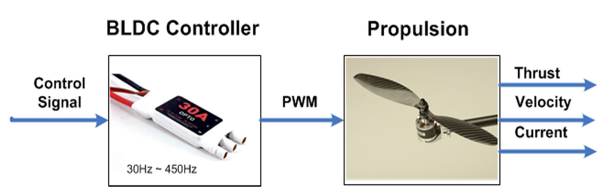

Figure 5. The propulsion unit: controller, propeller and BLDC motor.

Using the laboratory setup for the thrust measurement equipped with the current and velocity sensors it is possible to collect data for the variety of static characteristics, transient analysis and identification experiments.

TABLE I. MOTOR PARAMETERS

Type	IBM2814-8
Nominal Voltage in Cells	2-4S
KV – Electromotive Force Constant	1000 RPM/min/V
Power	432W
Length	36
Diameter	35
Shaft diameter	5
Weight	105

IV. PROPULSION IDENTIFICATION

To explain the dynamic properties of the propulsion system the model is needed. The common approach is very often based on the phenomenological modeling as stated in one of the previous section. However, such model always consists of some simplifications which can have an impact on neglecting some other dynamics, crucial for the control algorithms design procedure. Some problems may also occurred while evaluating the parameters of the mathematical model. Another way to acquire the adequate description of the dynamical system is to perform the identification experiment. For that purpose the proper model needs to be selected.

Phenomenological model of the electric propulsion system indicates that the system is nonlinear. Mainly because of the propeller load and aerodynamics, thus to obtain a suitable model for the simulation purposes the class of nonlinear block-oriented model has been chosen. When a nonlinear block precedes and follows a linear dynamic system the considered model is a Hammerstein-Wiener system[14,15].

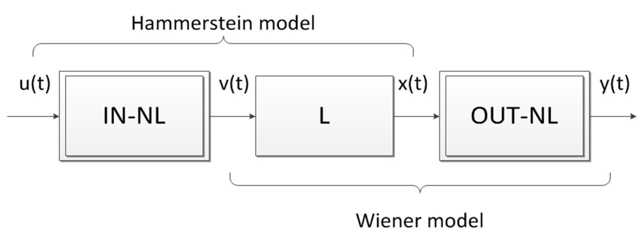

Figure 6. Nonlinear Hammerstein-Wiener Model.

While preparing the identification experiment one must consider the model structure. In this case some premises can be made on the basis of the mathematical model. After that the parameters will be determined. Taking into considerations the aerodynamics of the rotor the first approximation of the nonlinear input block can be assumed as a 2nd degree polynomial. The output nonlinearity can be omitted. So in the first step we assume that the model structure consists of the static non-linearity followed by a linear dynamic system which can be adopted as an output error model that is describing the deterministic and stochastic parts of a system separately. The presented structure is a Hammerstein model. Only a specific class of nonlinear systems can be identified in that manner because the dynamical part is considered as a Linear Time Invariant system.

However, in fact the electric propulsion unit is time-varying system because of the power supply source. During the flight the energy from the lithium-polymer battery is being reduced. The different regime of flight consumes different amount of current that affects the performance of the multirotor vehicle. Also the endurance is dependable of the maneuver intensity.

All above mentioned remarks lead to the proposal of the following electric propulsion model with two inputs and two outputs (Fig. 7).

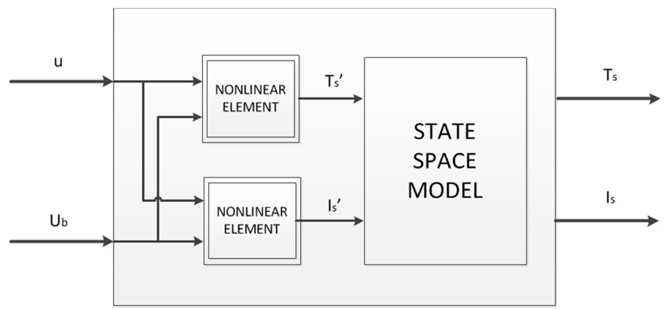

Figure 7. Propulsion model propositin.

The MIMO model has the following inputs. Control signal which is a duty cycle of the PWM signal running at the ESC frequency and voltage of the battery level which can be easily measured. The first output is a thrust force which is mainly dependent on the control signal and also on the voltage roughly indicating the amount of charge in the battery. Second output is the current consumed by the motor loaded with the propeller which is depended upon the previous two inputs.

To establish the model parameters a series of identification experiments has been conducted. Using the laboratory setup the static thrust and current have been recorded as a function of control signal and power supply voltage. For the thrust output the dynamic responses have been normalized. They are depicted in Fig. 8. After that the nonlinear input polynomial functions have been recalculated. The result is presented in the next figure (Fig. 9).

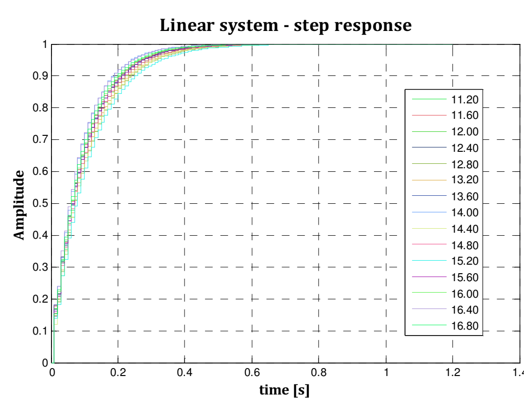

Figure 8. Step response of the propulsion system being the function of the battery voltage.

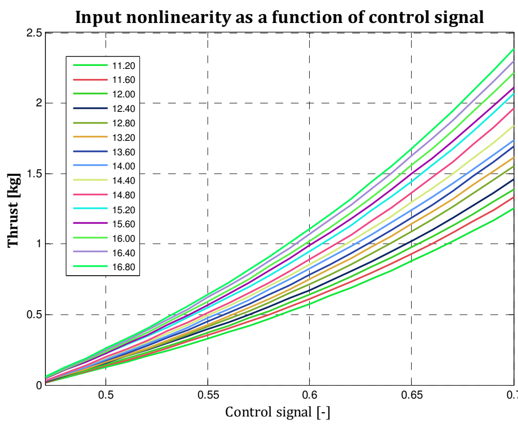

Figure 9. Approximation of the thrust force based on experimental data.

The same steps were undertaken in order to determine the dynamic response for the motor current. Despite the fact that the dynamics is faster than the previous one and in some cases it could be omitted, we have considered its participation in a propulsion model. Mainly due to the torque which is proportional to the motor current.

The model input dependencies have been formulated as a two dimensional functions with a dependent thrust force and current values. Functions have been fitted using the following formulas:

$$T_s'(u,U_b) = p_{00} + p_{10}\cdot u + p_{01}\cdot U_b + + p_{20}\cdot u^2 + p_{11}\cdot uU_b + p_{30}\cdot u^3 + p_{21}\cdot u^2U_b \quad (17)$$

$$I_s'(u,U_b) = q_{00} + q_{10}\cdot u + q_{01}\cdot U_b + q_{20}\cdot x^2 + q_{11}\cdot uU_b \quad (18)$$

where $\boldsymbol{p}$ and $\boldsymbol{q}$ are a polynomial coefficient matrices. The R-squared factor is higher than the 0.98 in both cases.

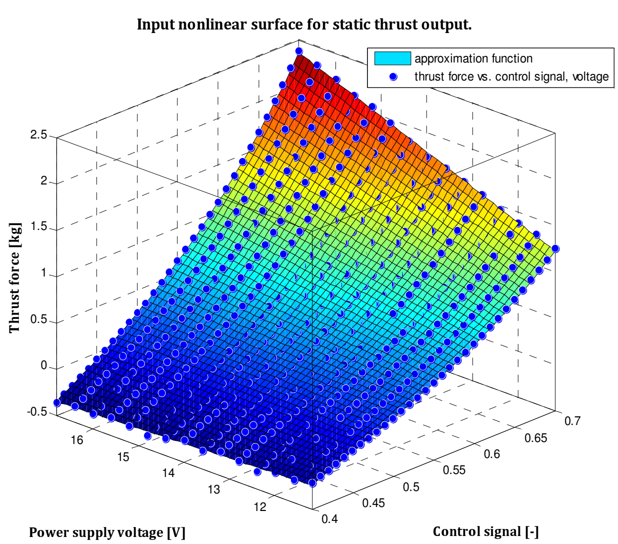

Figure 10. Thrust function approximation.

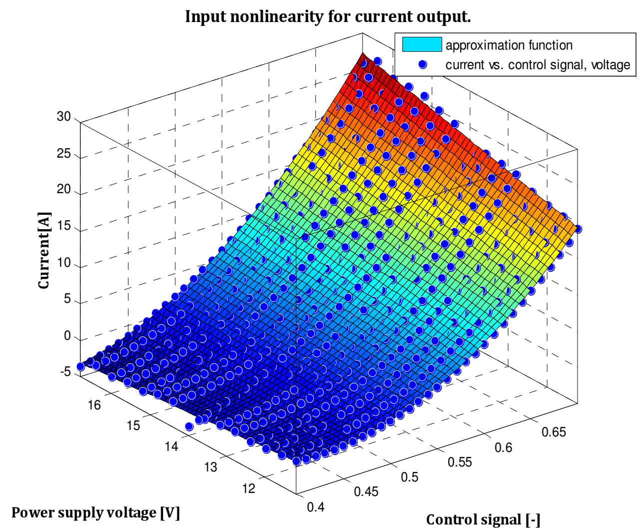

Figure 11. Current function approximation.

The dynamic part of a propulsion model has been given in the state space form below.

$$\begin{aligned}\dot{\mathbf{x}} &= \begin{bmatrix} -10.526 & 0 \\ 0 & -8.446 \end{bmatrix}\cdot\mathbf{x} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\cdot\mathbf{u} \\ \mathbf{y} &= \begin{bmatrix} 10.526 & 0 \\ 0 & -2.9675 \end{bmatrix}\cdot\mathbf{x} + \begin{bmatrix} 0 & 0 \\ 0 & 1.3514 \end{bmatrix}\cdot\mathbf{u}\end{aligned} \quad (19)$$

where: **x**- state vector, **u** – input vector – control signal and power supply voltage, **y** – output vector – static thrust and motor current. Basing on the above we can distinguish the dynamics with respect to the thrust and current.

V. CONCLUSIONS

In this paper an approach to the propulsion system modeling and identification is described. In the first step the propulsion dynamics is identified using the nonlinear Hammerstein model. After that the function of two inputs is used to approximate the given output. In this way the multiple-input and multiple-output model of the drive unit has been obtained. The peculiarity of this method is the fact that the model is strictly based on the experimental data. The signals correspond to the actual physical values. Such model can be easily used for the detailed multirotor vehicle simulations and control algorithms design in future.

There should be remarked that the model taken from the identification experiment represents the whole propulsion unit. Starting from the ESC controller to the thrust force and motor current consumption which causes the nonstationary system in fact. Here the approach allows to treat the nonlinear varying-time propulsion system as a linear time invariant one with the static nonlinearities at the input. Combining the results with the basic aerodynamics theory, the real thrust force can be calculated on the basis of the static thrust values.

The results can be used for the range prediction in order to assure enough power for the return. The model allows to calculate the amount current drawn from the battery. In aspects of a regime flying the precise simulations can be done to give the idea about the optimal path planning.

In the future research the propeller torque value can be also estimated to enhanced the model of a propulsion system. As a result it will be strictly based only on the empirical data. Moreover the battery model will be incorporated into the whole vertical take-off and landing platform model in order to completely test the drive unit in simulation environment. Also the idea of the real thrust calculation will be verified using the sensors information from the real quadrotor flying robot.

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