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Rolling-horizon trajectory planning for multiple UAVs based on pseudospectral collocation

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Abstract— **This paper proposes a method based on a Legendre pseudospectral collocation to generate trajectories in scenarios with multiple aerial vehicles in order to avoid collisions. The method uses a rolling horizon policy in which collision-free trajectories are planned up to a given time horizon, thus considering a much smaller problem space. Then, the system is applied iteratively. Using this approach, the multi-UAV trajectory generation problem can be solved exactly to yield an optimal solution to the sub-problem given by the considered time horizon. The proposed method allows for both changes of speed and heading for each aerial vehicle to guarantee that the safety distance between them is always maintained. Many simulations have been performed to compute the better values for the relevant parameters of the method (i.e. look-ahead time and the number of collocations points). The number of collocation points affects significantly the computation time, so a tradeoff between feasibility and optimality should be reached. The computational load and scalability of the method are also studied in randomly generated scenarios to test its application in real time. Experiments to test the validity of the approach have been also carried out in the multivehicle aerial testbed of the Center for Advanced Aerospace Technologies (Seville, Spain).**

I. INTRODUCTION

Coordination and collision avoidance is a critically important aspect in applications with multiple Unmanned Aerial Vehicles (UAVs) to successfully perform a coordinated mission [1] [2] [3]. Trajectory planning methods with low computational load play an important role to ensure the safety in dynamic and uncertain environments. In this case, an efficient re-planning is needed to address the changes in the environment.

Cooperation and coordination of many mobile entities such as aerial vehicles are being performed in the EC-SAFEMOBIL FP7 European Project (http://www.ec-safemobil-project.eu/). This project is developing perception and control methods and technologies in order to reach levels of reliability and safety to facilitate UAV deployment in a broad range of applications. Safe trajectory planning techniques for multiple autonomous vehicles are very important in this context to avoid collisions between the vehicles, and between them and the environment.

UAV planning algorithms and collision avoidance methods have been studied extensively. A detailed survey on the former is presented in [4] and [5] reviews papers on the latter.

Planning methods can include non-linear programming (NLP) [6], integer programming [7], Rapidly-exploring Random Trees (RRT) [8], particle swarm optimization [9], evolutionary computation methods [10], ant colony optimization methods [11], among many others. These methods are not good candidates for trajectory planning in dynamic environment and a small time horizon.

The stochastic method based on the Monte Carlo method presented in [12] solves conflicts in air traffic control but the computation time is also high. A CDR method based on a mixed-integer linear program (MILP) optimizes the total flight time by modifying velocity or heading [7].

Pseudospectral collocation methods have been used in the last years. Among them highlights the direct collocation methods and the pseudospectral methods. The former computes the solution considering a fixed degree polynomial state approximation in each segment and the convergence is achieved by increasing the number of segments [13]. The latter uses a single segment and convergence is achieved by increasing the degree of the polynomial. Both methods chose the collocation points based on accurate quadrature rules and the basic functions are typically Chebyshev or Lagrange polynomials. The more commonly used pseudospectral methods are the Gauss pseudospectral method (GPM) [14], the Radau pseudospectral method [15] (RPM), and the Lobatto pseudospectral method [16] (LPM).

An hp-adaptive pseudospectral method has been proposed in [17]. This method can increase the number of segments and the degree of the polynomial within a segment to achieve an error less than the tolerance error allowed. However, each segment adds collocation points in each iteration, so the number of collocation points could be too large and that provokes larger computation times.

Pseudospectral and Direct Collocation methods have been applied to compute aircraft trajectories [18] [19] [20] but their computation times are too large to plan trajectories in dynamic and uncertain environments. Moreover, most of published works consider trajectory generation for a standalone vehicle [18] [19].

This paper addresses the problem of collision avoidance with multiple UAVs to ensure the safety and reliability of the mission in dynamic environments. The information of the environment, the rest of vehicles, is limited for a look-ahead time. Moreover, the dynamics of the vehicles

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is considered to compute more realistic trajectories. The proposed method is based on pseudospectral collocation techniques and is iterative. Look-ahead times are determined by a rolling horizon approximation in order to quickly compute solution trajectories to the sub-problem considered. Two maneuvers are allowed to solve the detected collisions: change of speed and heading. Its main characteristic is the low computational load which makes that it can be applied in dynamic environments. Moreover, the method presents a good scalability. A study is presented to analyze the look-ahead time and the number of collocations points which quicker compute the solution and ensure a safe one.

Finally, pseudospectral techniques only compute the solution valid in the collocation points. That is, the minimum separation among vehicles is maintained in these points. Therefore, an evaluation considering a model of vehicle should be carried out in order to ensure that the minimum separation is not violated during the solution trajectory to the sub-problem considered in each instant.

The paper is organized into seven sections. Section II describes the problem formulation addressed. The proposed method is explained in Section III. Simulations and experiments performed are showed in Section IV and V, respectively. Finally, the conclusions are detailed in Section VI.

II. PATH PLANNING FOR MULTIPLE UAVS

The problem of collision avoidance of multiple UAVs to perform the coordinated missions proposed by the EC-SAFEMOBIL project is considered in this paper. The project considers a cooperative tracking and surveillance scenario. This scenario considers a situation where a region is to be observed, and multiple targets within that region tracked, in the presence of obstacles. The scenario incorporates the need for cooperative collision avoidance techniques to perform the coordinated missions, assuring that the aerial vehicles do not collide with each other.

The proposed method to avoid collisions allows changes of the speed profile and the heading of the aerial vehicles involved in the conflict by considering the dynamics of each vehicle.

The trajectory of each UAV is given by an initial waypoint and a final waypoint. Each waypoint is defined by: 2D coordinates (x,y), speed from that waypoint (v), and the Estimated Time of Arrival (ETA) to the waypoint, t. It is assumed that all UAV trajectories are known in the time interval given by the look-ahead time. We consider that the UAVs maintain the safety separation if they are separated by a minimum distance, D.

The inputs of the method are the following:

- Time interval to define the sub-problem of the trajectory planning of each UAV given by the look-ahead time considered
- Model of each UAV

The objective is to find collision-free trajectories that minimize the probability of having a collision.

Several definitions are needed to perform the study presented in Section IV:

- **Number of nodes per segment**: it is also called collocation points. This number defines the degree of the polynomial of interpolation used in each segment.
- **Look-ahead time**: it is the time in which each UAV knows the information on the rest of UAVs.

Look-ahead time and the number of collocation points should be analyzed. The best values of both parameters should ensure a safe solution in each computation and with low computational load to iteratively perform trajectory re-planning in dynamic environment.

III. LEGENDRE PSEUDOSPECTRAL METHOD

The pseudospectral method numerically solves optimal control problems. The basic approach is to transform the optimal control problem into a sequence of nonlinear constrained optimization problems by discretizing the state and control variables. It computes a set of collocation points that provides an accurate approximation to the solution of the optimal control problem. The proposed method is based on a Legendre pseudospectral method known as DIDO [21]. Its novelty is how an iterative implementation is carried out in order to obtain an efficient trajectory re-planning.

The optimal control problem is considered in Bolza form. The following cost function should be minimized:

$$J = \phi(x(-1), t_0, x(+1), t_f) + \frac{t_f - t_0}{2} \int_{-1}^{1} \mathcal{L}(x(\tau), u(\tau), \tau) d\tau \tag{1}$$

subject to the dynamic constraints:

$$\frac{dx}{d\tau} = \frac{t_f - t_0}{2} f(x(\tau), u(\tau), \tau) \tag{2}$$

Boundary conditions are considered:

$$\phi(x(-1,), t_0, x(+1), t_f) = 0 \tag{3}$$

and the inequality path constraints:

$$C(x(\tau), u(\tau), \tau, t_0, t_f) \leq 0 \tag{4}$$

where $x(\tau)$ is the state, $u(\tau)$ is the control, and τ is time. The variable $\tau \epsilon [-1, 1]$ and $t \epsilon [t_0, t_f]$ are related as

$$t = \frac{t_f - t_0}{2} \tau + \frac{t_f - t_0}{2} \tag{5}$$

The proposed method is iterative such that it computes an optimal solution to a sub-problem in every iteration, that is, each path is generated piece by piece. The look-ahead time, T_{la}, is determined by the knowledge of the environment in every iteration and the maximum speed of the vehicle that is the worst case for trajectory re-planning. Figure 1 shows an example. Two UAVs fly from A to B and from C to D, respectively. Dashed lines represent each initial trajectory but both UAVs change their trajectory (blue curves lines). Black arrows indicate the look-ahead time that each

UAV will consider to compute the trajectory of the current sub-problem. Note that UAVs always consider its heading pointing to the goal waypoint.

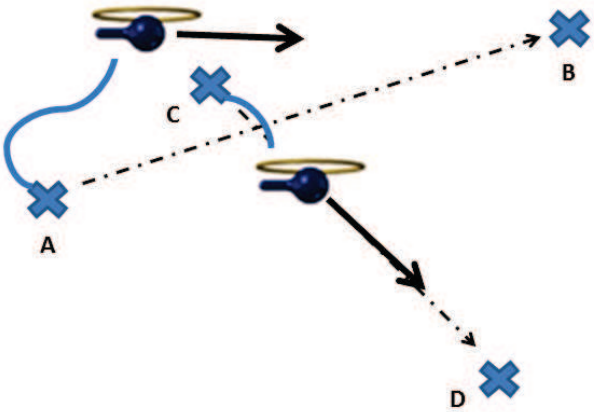

Fig. 1. Flight scheme of 2 UAVs

Number of collocations points considered in each iteration also influences on the proposed method. This number affects the quality of solution and the computation time. The quality means that minimum separation distance should be met by all UAVs. If constraints are met in every piece of the path, it would ensure the minimum separation distance in the whole trajectory. Moreover, a low computational time is required.

Finally, a frequency of computation should be determined in order to ensure that each UAV does not fly the whole trajectory computed in the previous iteration during the corresponding time between two iterations, T_f. This time should be larger that the computation time of the UAV trajectories, T_c:

$$T_f > T_c \tag{6}$$

Figure 2 illustrates the performance of the proposed method. Five collocation points have been considered (blue points) and a look-ahead time is considered from the knowledge of the environment in every iteration and the maximum speed of the vehicle. First, the trajectory of each UAV is computed within look-ahead time. An iteration is computed every T_f, so the computation time, T_c in the second iteration should fulfill:

$$T_{la} - T_f > T_c \tag{7}$$

A. Implementation

A model of aerial vehicle by considering a simplified dynamics is used in this paper. The altitude is assumed to be constant. The state vector is defined by $(x_i, y_i, \Phi_i, \Theta_i, t_i)$ where x_i, y_i the 2D position of the aerial vehicle, Φ_i, Θ_i are the pitch and roll angles and t_i the time of arrival in each collocation point. The control inputs are the pitch and roll torques u_{Φ_i}, u_{Θ_i}.

The model considered is:

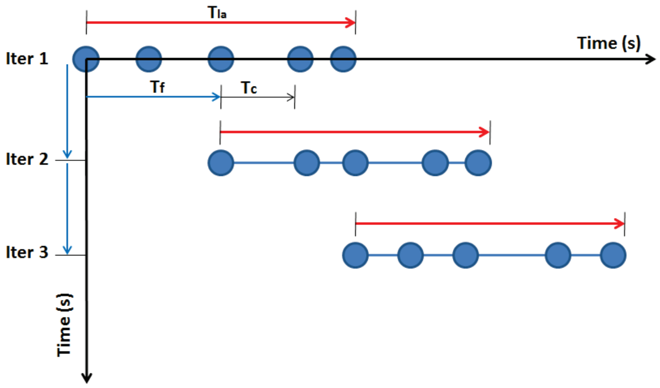

Fig. 2. Description of the proposed method.

$$\dot{x}_i = \frac{T}{m} \cdot sin(\Phi_i) \tag{8}$$

$$\dot{y}_i = \frac{T}{m} \cdot sin(\Theta_i) \tag{9}$$

$$\ddot{\Theta}_i = \frac{u_{\Theta_i}}{I_y} \tag{10}$$

$$\ddot{\Phi}_i = \frac{u_{\Phi_i}}{I_x} \tag{11}$$

where T is the thrust needed to maintain constant altitude, m is the mass, Θ_i is the roll angle, Φ_i is the pitch angle, I_x and I_y are the moments of inertia with respect to the axes x and y, respectively.

The multi-UAV system is defined by concatenating the state of all the UAVs. Therefore, the state vector and control vector are defined as follows:

$$\begin{aligned} X = [&x_1, y_1, \Phi_1, \dot{\Phi}_1, \Theta_1, \dot{\Theta}_1, t_1, ..., \\ &x_n, y_n, \Phi_n, \dot{\Phi}_n, \Theta_n, \dot{\Theta}_n, t_n] \end{aligned} \tag{12}$$

$$U = [u_{\Phi_1}, u_{\Theta_1}, u_{\Phi_2}, u_{\Theta_2} ...., u_{\Phi_n}, u_{\Theta_n}] \tag{13}$$

where n is the number of UAVs.

The solution should satisfy constraints taking into account the physical limitations of each UAV inputs:

$$u_{\theta min} \leqslant u_{\theta_i} \leqslant u_{\theta max} \tag{14}$$

$$u_{\Phi min} \leqslant u_{\Phi_i} \leqslant u_{\Phi max} \tag{15}$$

And the separation between UAV_i and UAV_j should meet:

$$distance(UAV_i, UAV_j) \geq D \tag{16}$$

where D is the safety distance.

IV. SIMULATIONS

Many simulations have been carried out in different scenarios randomly generated with several UAVs to test the proposed method. The problems have been solved by using the pseudospectral optimal control software DIDO [21]. The algorithms have been run in a PC with a CPU Intel Core i7-3770 @ 3.4 Ghz and 16 GB of RAM. The operating system used in the simulations was Windows 7 OS and the code has been implemented in Matlab.

First study considers one hundred random scenarios with five UAVs to analyze the best values of: look-ahead time, T_{la}, and number of collocation points, N_c. The size of the scenarios is $9x9m^2$. The minimum separation distance among UAVs is $1.15m$. The minimum and maximum torques considered are $-0.02N$ and $0.02N$. The criterion considered is to minimize the changes of the inputs. The maximum T_{la} considered is $3s$. A study should be done to set the T_{la} and the number of collocation points to ensure a safe solution and with a low computation time.

The considered values of each parameter in the study are: $T_{la} = [1.0, 1.5, 2.0, 2.5, 3.0]s$ and $N_c = [3, 4, 5, 6, 7, 8]$ points. Therefore, thirty possible combinations are explored in each scenario. The computation mean time obtained and its standard deviation for each combination which ensures a safe solution in all the scenarios is shown in Table I. Only eight combinations met it. In order to meet equation (6), T_f should be greater than $1.107s$ (the larger value of $T_c+\sigma_{Tc}$ in Table I), so $T_f = 1.25s$ is considered. First, second, fourth and sixth cases only meet equation (7). Among these options, a minimum computation time is chosen to carry out the next studies. Therefore, $T_{la} = 2.5s$ and $N_c = 4$ are chosen.

TABLE I

COMBINATIONS THAT ENSURE A SAFE SOLUTION AMONG THE THIRTY ONES CONSIDERED.

T_{la}(s)	N_c	T_c (s)	σ_{Tc} (s)
2.5	4	0.808	0.207
2.5	3	0.819	0.212
1.0	4	0.815	0.258
3.0	3	0.824	0.304
1.5	3	0.837	0.197
2.5	6	0.847	0.219
1.0	7	0.851	0.207
2.0	6	0.873	0.234

The next study analyzes the scalability and shows how the computation time depends on the the number of UAVs. Figure 3 and 4 show the two scenarios considered with up to eight UAVs.

Table II and III show the computation mean time and standard deviation. The proposed method adapts well considering up to six UAVs in scenario S1 and considering up to seven UAVs in scenario S2 because equation (6) is met.

Next, a simulation is presented to show the behavior of the proposed method, that is, how it computes a solution every T_f. Figure 5 shows the scenario considered with four UAVs.

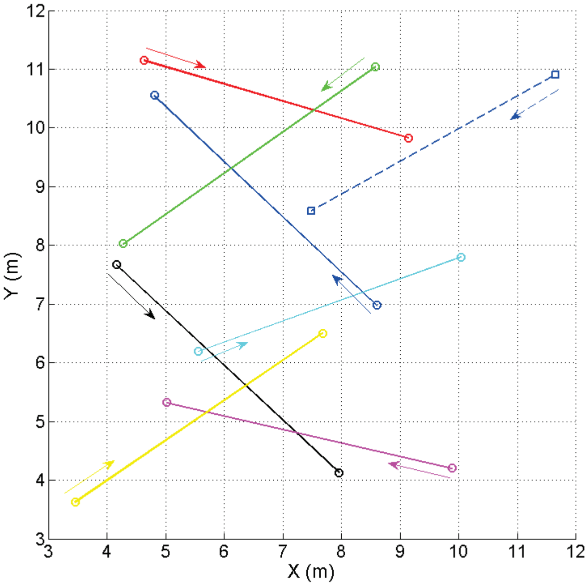

Fig. 3. Scenario S1 considered in the simulations with up to eight UAVs: UAV1 in blue, UAV2 in red, UAV3 in black, UAV4 in green, UAV5 in pink, UAV6 in clear blue, UAV7 in yellow and UAV8 in dashed blue.

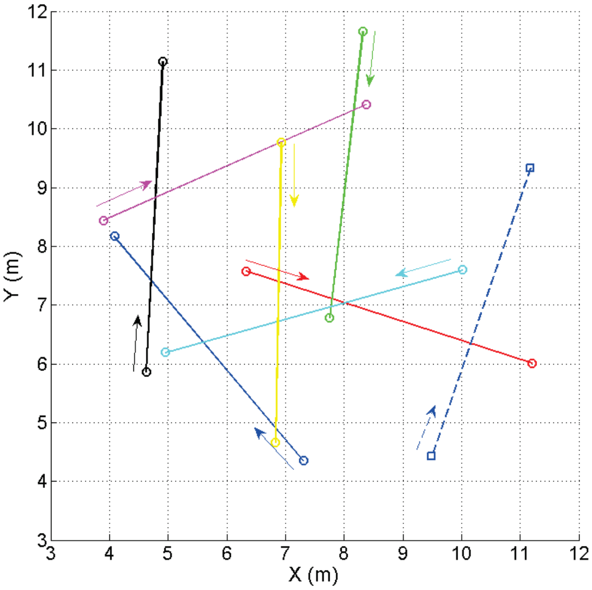

Fig. 4. Scenario S2 considered in the simulations with up to eight UAVs: UAV1 in blue, UAV2 in red, UAV3 in black, UAV4 in green, UAV5 in pink, UAV6 in clear blue, UAV7 in yellow and UAV8 in dashed blue.

Figures 6, 7, 8 and 9 shows four instants which correspond to $t = 1.25, 3.75, 5.00, 6.25s$. The sub-problems solved are presented in every iteration. Note that each UAV trajectory converges to its goal waypoint.

Figure 10 depicts the speed profile computed. Note that the initial and final speed of each UAV are equal. Finally, the separation between UAVs is presented in Figure 11. The trajectories are safe.

TABLE II

MEAN COMPUTING TIME WHEN THE NUMBER OF UAVS INCREASES BY CONSIDERING THE SCENARIO S1.

Number of UAVs	UAVs	Time (s)	σ_t (s)
3	1-3	0.365	0.338
4	1-4	0.455	0.250
5	1-5	0.599	0.162
6	1-6	0.686	0.174
7	1-7	1.279	0.514
8	1-8	1.438	0.341

TABLE III

MEAN COMPUTING TIME WHEN THE NUMBER OF UAVS INCREASES BY CONSIDERING THE SCENARIO S2.

Number of UAVs	UAVs	Time (s)	σ_t (s)
3	1-3	0.308	0.150
4	1-4	0.374	0.204
5	1-5	0.480	0.269
6	1-6	0.689	0.279
7	1-7	0.961	0.294
8	1-8	1.297	0.472

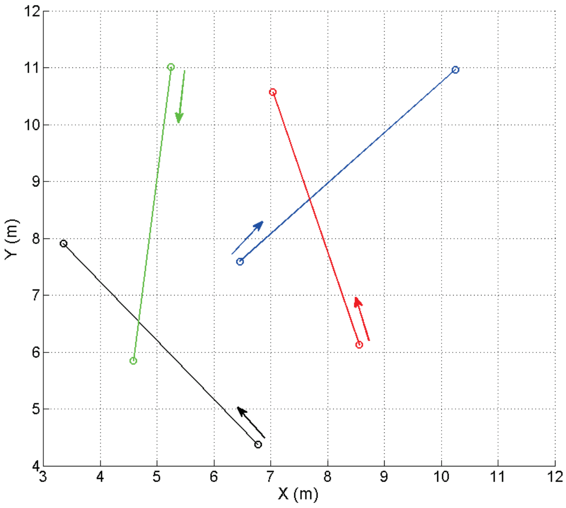

Fig. 5. Scenario with four UAVs: UAV1 in blue, UAV2 in red and UAV3 in black and UAV4 in green.

V. EXPERIMENTS

Experiments have been carried out in the indoor multi-UAV testbed of the CATEC with four Hummingbird quadrotors (see Figure 12) with 200gr payload and up to 20 minutes flight autonomy. The testbed has an indoor localization system based on 20 VICON cameras. This system is able to provide, in real time, the position and attitude of each UAV with centimeter accuracy. The minimum separation is $0.8m$ in the experiments. In this case, the optimization criteria minimizes the heading changes.

An experiment is shown to demonstrate the performance

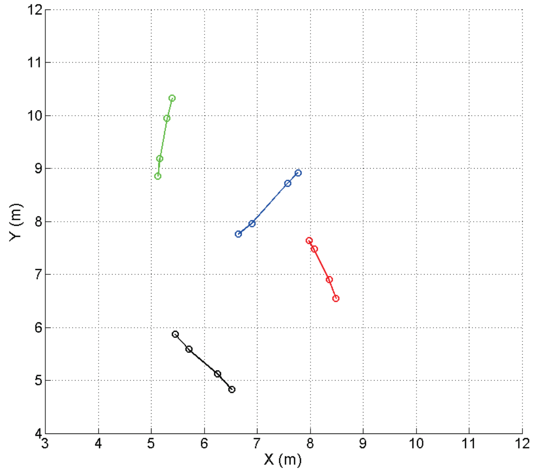

Fig. 6. Trajectory computed by the method at the instant $t = 1.25s$.

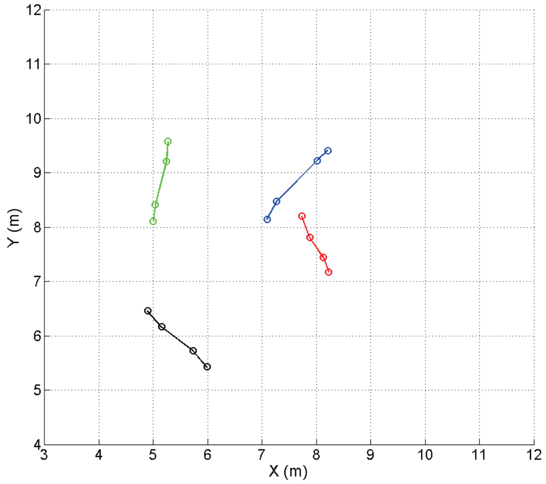

Fig. 7. Trajectory computed by the method at the instant $t = 3.75s$.

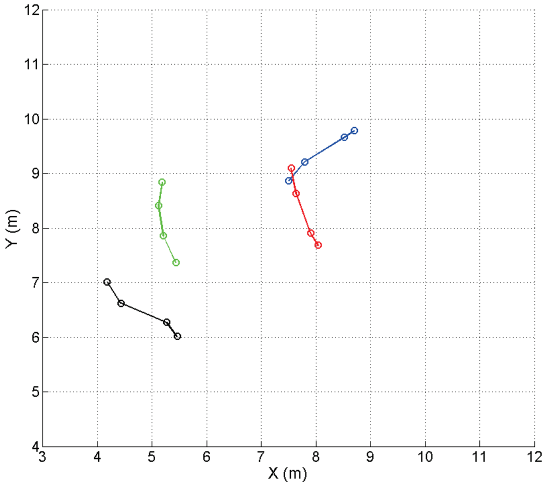

Fig. 8. Trajectory computed by the method at the instant $t = 5.0s$.

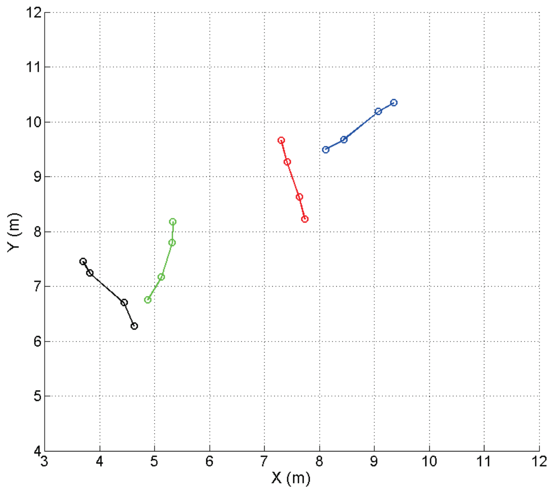

Fig. 9. Trajectory computed by the method at the instant $t = 6.25s$.

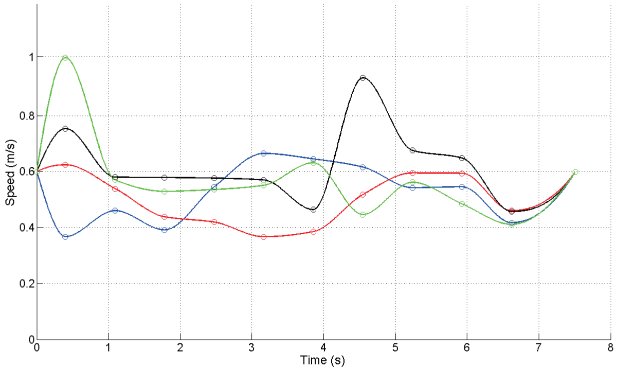

Fig. 10. Speed profile of each UAV during the whole flight considering four UAVs in scenario S1: UAV1 in blue, UAV2 in red and UAV3 in black and UAV4 in green.

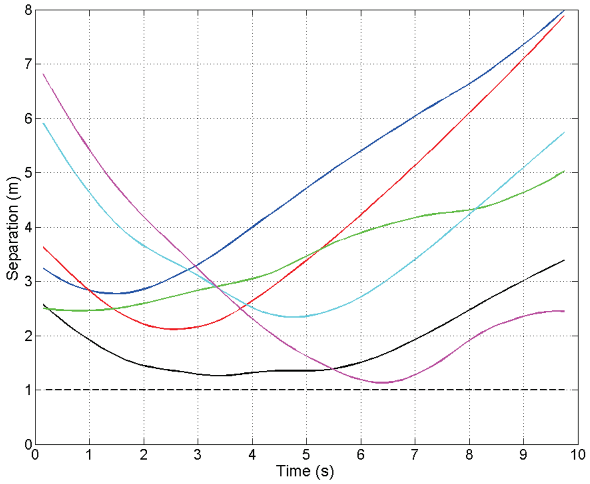

Fig. 11. Separation between UAVs during the whole flight considering four UAVs in scenario S1: UAV1-UAV2 in black, UAV1-UAV3 in blue, UAV1-UAV4 in red,UAV2-UAV3 in green, UAV2-UAV4 in clear blue, UAV3-UAV4 in pink and the minimum separation in dashed black line.

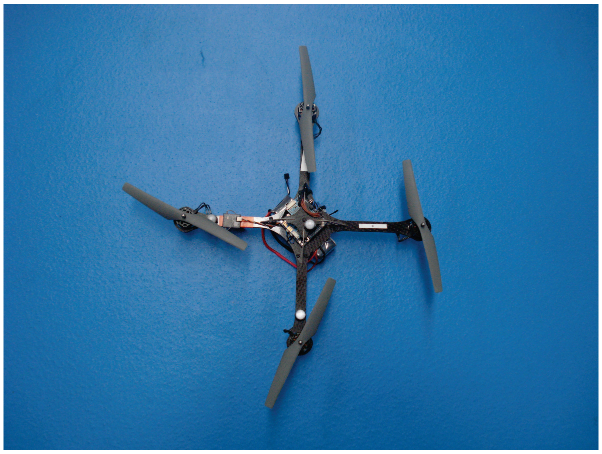

Fig. 12. Quad-rotor used in the experiments.

of the proposed method. Figure 13 shows the collocation points computed and Figure 14 the speed profile of each UAV.

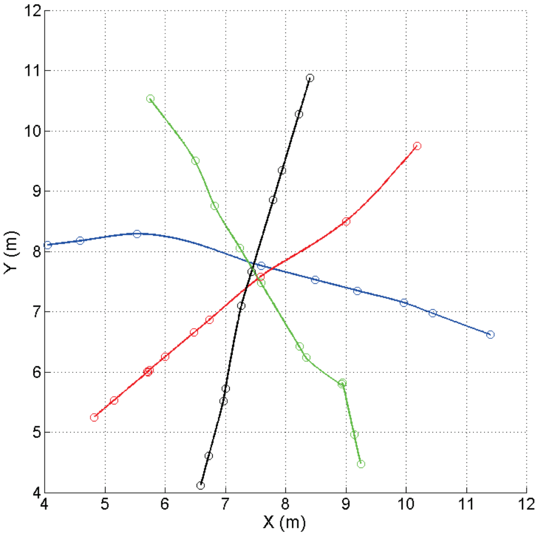

Fig. 13. Collocation points computed in the experiment: UAV1 in blue, UAV2 in red and UAV3 in black and UAV4 in green.

The real trajectories are represented in 15 and Figure 16 shows the separation among UAVs. Each UAV maintains the minimum separation distance.

VI. CONCLUSIONS

This paper addresses the problem of collision avoidance with multiple UAVs in coordinated missions by ensuring the safety and reliability of the mission in dynamic environments. An efficient re-planning is needed to address the changes in the environment. Moreover, the dynamics of the vehicles to compute more realistic trajectories.

A method based on a Legendre pseudospectral collocation to generate trajectories is proposed and is applied iteratively. It presents a novel way to use these techniques in dynamic

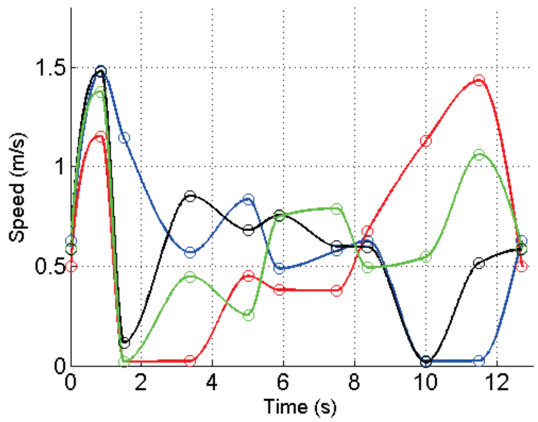

Fig. 14. Speed profile of each UAV in the experiment: UAV1 in blue, UAV2 in red and UAV3 in black and UAV4 in green.

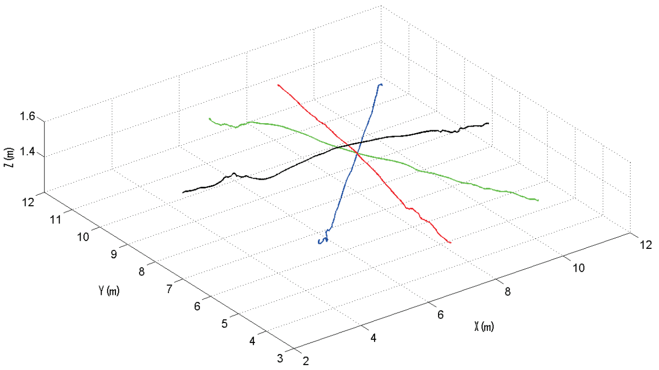

Fig. 15. UAV trajectories in the experiment in 3D: UAV1 in black, UAV2 in blue, UAV3 in red and UAV4 in green.

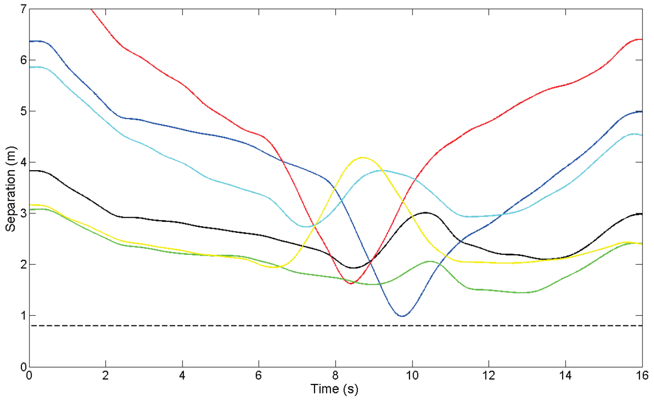

Fig. 16. Separation between UAVs in the experiment: UAV1-UAV2 in black, UAV1-UAV3 in blue, UAV1-UAV4 in red,UAV2-UAV3 in green, UAV2-UAV4 in clear blue, UAV3-UAV4 in pink and the minimum separation in dashed black line.

environment. The collision-free trajectories are planned in a sub-problem defined by the time horizon. The maneuvers allowed are both changes of speed and heading.

Look-ahead time and number of collocation points are the more relevant parameters of the method in the kind of applications considered. A tradeoff between feasibility and optimality should be reached. Thus, they have been analyzed and set up to ensure safe solutions and low computation times by considering random scenarios.

Experiments to verify the solution computed by the method have been carried out in the multivehicle aerial testbed of the Center for Advanced Aerospace Technologies (Seville, Spain).

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