2014 International Conference on Unmanned Aircraft Systems (ICUAS)
May 27-30, 2014. Orlando, FL, USA

Self Healing Control Method Against Unmanned Helicopter Actuator Stuck Faults

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Abstract— **A self healing (SH) control framework is proposed in this paper against actuator stuck faults. For unmanned helicopters (UHs), the framework is composed by an active fault-tolerant control (FTC) system and reference redesign. The FTC system is based on linear-quadratic regulator (LQR) and pseudo inverse technique, which can compensate stuck faults of the post-fault system with output analytical redundancy (OAR) feature. The post-fault system may not achieve the original reference because remaining actuators' margin will degrade after stuck-fault compensation. A new reference is necessary for the post-fault system, which can be achieved by reference redesign method based on solving an optimal problem. At last the proposed SH framework is illustrated with a linear unmanned helicopter model, which includes rotor-speed control input and swashplate configuration.**

I. INTRODUCTION

During the past few decades, unmanned helicopters (UHs) have attracted more and more attention. Compared to unmanned fixed-wing aircraft, UH has stronger coupling and less hardware redundancy of actuators due to its structural features. In order to ensure the reliability and safety of UHs, fault diagnosis and identification (FDI) and fault-tolerant control (FTC) is necessary. A number of related methods have been proposed [1] against actuator faults. But only few papers focus on stuck problem. In this paper, we will focus on actuator stuck faults of UHs which are also known as lock-in-place faults.

[2], [3] investigated swashplate reconfiguration with rotor-speed control method against actuator stuck faults. In this method, when one of the three actuators used for main rotor control is stuck, the attitude of UH can be controlled by the remaining actuators and the altitude can be controlled by rotor speed. Generally speaking, in order to simplify the control, rotor-speed controller is independent of flight controller. Some FTC methods only based on flight controller are also investigated. In [4], FTC architecture is investigated against actuator faults, which is based on active fault estimation using adaptive unscented Kalman filter and feedback linearization. In [5], tail rotor failure is considered through fuzzy logic. In [6], adaptive failure compensation for coaxial rotor helicopter under propeller failure was proposed.

In addition to special methods for UHs, some general methods against actuator stuck faults have been proposed. [7] proposed precompensator and proportional-integral (PI) controller based FTC method against stuck faults. [8] presented adaptive control methods for actuator stuck faults. [9] proposed adaptive control methods with new adaptive laws based on iterative learning observer. However, these methods do not take into account the constraints of actuators. In other words, in order to compensate actuator faults, especially actuator stuck faults, actuator margin will degrade. The post-fault system may not achieve the original reference. Under these circumstances, reference redesign is proposed in this paper to solve the problem.

Reference redesign have been presented against actuator stuck faults with actuator constraints for other control plants. [10] developed reference inputs generation methods for this problem. In the on-line method, an optimization problem was built for minimizing the energy of control inputs and the error between the real and the desired outputs. [11] proposed a novel method which can modify the steady-state reference on-line by a model predictive control strategy. The problem is solved by an optimization problem at last. The main weak point of these methods is that the new reference have to be computed real-time. In other words, the steady-state reference cannot be obtained before the system operates so that the final states of the system cannot be known. [12] presented a command input management approach to modify the steady-state reference of the post-fault system according to the steady-state open-loop gain under no fault condition. But the choice of new reference are based on experience so that it is not guaranteed to be optimal.

Therefore, a new reference redesign approach against actuator stuck faults is investigated in this manuscript which can provide a new optimal steady-state reference based on information of faults, post-fault system and input constraints. At the same time, the controller is redesigned to keep system stability and tracking performance. The main contribution of this paper is to propose self healing (SH) framework

*This work was supported by National Key Technology Research and Development Program of China under Grant: 2013BAK03B01; The National Natural Science Foundation of China under Grant: 61273025; The Chinese Academy of Sciences Visiting Professorship for Senior International Scientists under Grant 2012T1G0007.

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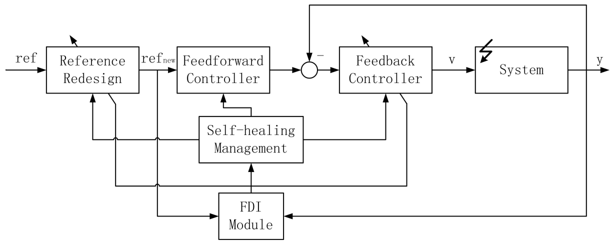

Fig. 1. The structure of self healing framework

composed by the above two parts.

The structure of SH framework is shown in Fig. 1 which includes self-healing management, reference redesign, controller redesign and FDI. The self-healing management module is used to detect whether the original reference can be achieved asymptotically or not. If it is impossible, a new optimal reference will be calculated by reference redesign module. The feedback controller is used to keep system stability while the feedforward one keeps tracking performance. The function of FDI module is to detect, isolate and identify the stuck faults. In the following discussion, the investigation of FDI methods is not included. Stuck-fault magnitude is assumed to be provided by FDI module no delay. At last, the proposed framework is illustrated with a linear UH model which includes rotor-speed control input and swashplate configuration.

The paper is organized as follows. Problem statement is dedicated in Section II. Section III is devoted to the development of the proposed method. In section IV, an linear unmanned helicopter model is investigated and a simulation based on the model is considered to illustrate the proposed method. Section V ends the paper.

II. PROBLEM STATEMENT

Consider a Linear Invariant-System (LTI) system which is always stabilizable using state feedback

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x \in R^n$ is the system state vector, $u \in R^p$ is the system control input vector and $y \in R^m$ is the system output vector. $A \in R^{n\times n}$, $B \in R^{n\times p}$ and $C \in R^{m\times p}$ are constant matrices. Furthermore, $B = [b_1\, b_2\, ...\, b_p]$, where b_i represents the ith column of matrix B.

The actuator stuck faults can be modeled as

$$u(t) = \Gamma v(t) + (I - \Gamma)\bar{u} \tag{2}$$

where $v(t)$ represents the output of controller and $\bar{u}$ is a constant vector. $\Gamma = diag(\gamma_1,\ \gamma_2, ...,\ \gamma_p)$ and $\gamma_i = 1\ or\ 0$ with $\gamma_i = 1$ represents that the ith actuator is fault-free and $\gamma_i = 0$ represents that the ith actuator is lock-in-place because the actuator cannot respond the control signal totally.

Hence, the control input vector can be divided into two parts $u(t) = [u_0^T(t)\ \bar{u}_f^T]^T$ in stuck-fault case, where $u_0(t) \in R^{p_0}$ is the normal control input vector, $\bar{u}_f \in R^{p_f}$ is the stuck control input vector which is constant and $p_0 + p_f = p$. The control matrix B can be decomposed in $[B_0\ B_f]$ with $B_0 \in R^{n\times p_0}$ and $B_f \in R^{n\times p_f}$. Then system (1) in stuck-fault case can be described as

$$\begin{cases} \dot{x}(t) = Ax(t) + B_0u_0(t) + B_f\bar{u}_f \\ y(t) = Cx(t) \end{cases} \tag{3}$$

Compared with the normal system defined by (1), the post-fault system has different input matrix and an additional constant item. In order to achieve the desired or degraded performance, one way is to use the inputs of remaining normal actuators to compensate the constant item.

If matrix B can be decomposed in $[B_1\ \ B_2]$ and $Rank(B) = Rank(B_1) = q < p$ and the system (A, B) is controllable, it will be obvious that the system (A, B_1) is also controllable because deleting linearly dependent columns of B will not affect the controllability of the system [13]. Thence, after analyzing the former research achievements, two definitions for different kinds of redundancy are summarized as following according to the structure of matrix B with the control objective which is to guarantee the state controllability.

Definition 1 (Hardware Redundancy): Actuators $u_i(t)$ and $u_j(t), i \neq j$ of the system defined in (1) are said to be mutual Hardware Redundancy (HR) if 1) the system is controllable; 2) $b_i = kb_j, k \neq 0$. The system is said to be HR if all of its actuators are HR. [14] proposed solutions for this condition with hot standby and cold standby technology.

Definition 2 (Strong Analytical Redundancy): Actuators $u_{i_1}, u_{i_2}, ..., u_{i_q}, i_k \neq i_j (k \neq j), 2 < q \leq p$ are said to be mutual Strong Analytical Redundancy (SAR) if 1) the system is controllable; 2) $\alpha_1 b_{i_1} + \alpha_2 b_{i_2} + ... + \alpha_q b_{i_q} = 0,\ \alpha_i \neq 0$. The system is said to be SAR if all of its actuators are SAR.

Remark 2: If a system is SAR, it is easily to achieve $B_f = B_0Q$, where Q is a constant matrix determined by B_0 and B_f. In this case, the output of the stuck actuators $\bar{u}_f$ can be compensated by the remaining normal actuators u_0 through $B_0u_0(t) + B_f\bar{u}_f = 0$. Several FTC methods have been proposed against this condition [9], [8]. On the other hand, if constraints of control inputs are also considered, at the same time, as $u_{min} \leq u(t) \leq u_{max}$ where u_{min} and u_{max} are constant vectors and represent the minimum and maximum values of control input respectively, the stuck faults in (2) may not be compensated.

On the other hand, if $Rank(B) = p$ and one actuator is stuck, the constant input produced by the actuator cannot be compensated totally by remaining actuators and the controllability of the system may not be guaranteed. In this case, output analytical redundancy is proposed which is to guarantee functional controllability of the system. A system is said to be functional controllability if $Rank([CB\ \ CAB\ \ \cdots\ \ CA^{n-1}B]) = m$. It can also be achieved if there exist at least one nonzero $m \times m$ minor in the system transfer function matrix $G(s) = C(sI - A)^{-1}B$ [7].

Definition 3 (Output Analytical Redundancy) [7]: The system is said to be Output Analytical Redundancy (OAR) if 1) the system is always stabilizable using the state feedback;

2) $p > m$; 3) $Rank(B) > m$ and there exist more than one nonsingular $m \times m$ square submatrices in the system transfer function matrix $G(s)$.

Remark 3: the condition (3) can also be described as there exist at least $m+1$ columns and m linearly independent columns. In this case, $G(s)$ can be decomposed in $[G_1(s)\ G_2(s)]$ with $G_1(s) \in R^{m\times m}$, $Rank(G_1(s)) = m$ and $G_2(s) \in R^{m\times(p-m)}$. Hence, $G_1(s)P_o = G_2(s)$, where P_o is a constant matrix.

For the following discussion, the system described by (1) is assumed to be OAR. [7] proposed a FTC method for an OAR system and the output of failure actuator is compensated by remaining actuators successfully. Compared to the paper, the novel point of this paper is to take into account actuator constraints.

In other words, the system is always stabilizable using state feedback. On the other hand, the relationship between control inputs and system outputs in fault-free and steady-state case can be described such as:

$$y(\infty) = H(\infty)u(\infty) \tag{4}$$

where $H(\infty)$ is open-loop gain in steady-state case. Compared with fault-free case, the remaining actuators also need compensate the constant inputs produced by stuck actuators. Therefore, the margin of remaining actuators will degrade. Taking into account actuator constraints, the post-fault system may not achieve the original reference. Hence, new reference are required.

In consideration of actuator stuck faults and actuator constraints, both FTC and reference redesign are needed to obtain satisfactory performance. An SH management method is proposed for selecting correct strategy.

III. MAIN RESULTS

A. Fault-tolerant control method

The post-fault controller $u_0(t)$ which provides control signals to the remaining actuators can be described as

$$u_0(t) = K_0x(t) + K_{r0}ref + K_f \tag{5}$$

where K_0 is state feedback control matrix to keep system stable, K_{r0} is feedforward control matrix to keep system tracking the reference offset-free, K_f is a constant vector to compensate the inputs of stuck actuators [15] and ref is the original reference. Then the closed-loop system are achieved such that

$$\begin{cases} \dot{x}(t) = (A + B_0K_0)x(t) + B_0K_{r0}ref + B_0K_f + B_f\bar{u}_f \\ y(t) = Cx(t) \end{cases} \tag{6}$$

where $B_0K_f + B_f\bar{u}_f$ is an additional item compared with fault-free case. According to adjusting the value of K_f, the influence of this item to the system should be as small as possible.

The feedback matrix K_0 is designed by linear-quadratic regulator (LQR) method to guarantee the closed-loop system $(A + B_0K_0)$ stable and the feedforward matrix K_{r0} based on the pseudo inverse matrix of $-[C(A + BK)^{-1}B]$.

The steady-state output of the closed-loop system with actuator stuck faults can be described by

$$\begin{aligned} y(\infty) = &-C(A + B_0K_0)^{-1}B_0K_{r0}ref \\ &- C(A + B_0K_0)^{-1}(B_0K_f + B_f\bar{u}_f) \end{aligned} \tag{7}$$

Because system (1) is OAR and state feedback have no effect on functional controllability, so the closed-loop system is also OAR. Assume the number of stuck actuators is not more than $p - m$. So there at least exists one nonsingular $m \times m$ square submatrices in the system transfer function matrix $H(s)$. Then $H(s)$ of the post-fault system can be decomposed in $[H_0\ H_f]$ in steady case with $H_0 \in R^{m\times m}$, $H_f \in R^{m\times(p-m)}$ and $H_0P_c = H_f$, where P_c is a constant matrix. Then the second item of (7) can be rewritten as $H_0K_f + H_f\bar{u}_f = H_0K_f + H_0P_c\bar{u}_f$. In order to guarantee the item is zero, K_f can be calculated by

$$K_f = -[C(A + B_0K_0)^{-1}B_0]^{+}C(A + B_0K_0)^{-1}B_f\bar{u}_f \tag{8}$$

Compared to the proposed method, [7] compensate actuator stuck faults by a dynamic precompensator to transform the original system to a new one with higher order but satisfying SAR.

Because of the existing of K_f and actuator constraints, the actuator output may not satisfy the original reference so that reference redesign is required.

B. Reference redesign method for steady case

The target of reference redesign problem is to find a new reachable reference to keep system safety. In other words, the steady-output $y(\infty)$ should satisfy the following condition:

$$y(\infty) \in \Omega_y \tag{9}$$

where Ω_y is an output admissible set defined by:

$$\Omega_y = \{G_f(\infty)u_0(\infty) \mid u_0(\infty) \in C_u\} \tag{10}$$

where $G_f(t)$ is the open-loop transfer function of the post-fault system, $u_0(t)$ is the control input of remaining normal actuators and C_u is the constraint set of inputs.

The outputs of post-fault system are expected to be as close as possible to the original reference ref. Hence, the optimal $y(\infty)$ should be

$$y^*(\infty) = arg\min \|y(\infty) - ref\| \in \Omega_y \tag{11}$$

where $\|\cdot\|$ is a norm used to define the distance between the output and the target. If $y^*(\infty)$ is equivalent to the original reference, it shows that the original reference is reachable for the post-fault system.

In order to achieve a new suitable reference for the post-fault system, two main methods have been proposed. The first one [12] proposed that the new reference can be achieved through $ref_{new} = y_f = G_\infty Wu_\infty$, where u_∞ is the steady-state closed-loop control signal without fault and $G_\infty = \lim_{s\to 0} C(Is - A)^{-1}B$ is the steady-state open-loop gain of the fault-free system. W is a diagonal weight matrix where larger valuers are assigned to healthy actuators. The new steady reference is obtained by adjusting the values of W.

The other one [10] achieves the new reference by solving an optimization problem $J = \min_{y_f, u_f}(\|y_d - y_f\|^2 + \|u_0\|^2)$, where y_f is the desired output, y_d is the desired output before actuator stuck and u_0 is control input of post-fault system.

Considering the fault system described by (3), the goal is to find an optimal new reference to make the system satisfies equation (9).

In steady case, the output should be equivalent to the reference $y(\infty) = ref = Cx(\infty)$, so the controller can be described by

$$u_0(\infty) = K_0 x(\infty) + K_{r0} C x(\infty) + K_f \quad (12)$$

Then the state equation of the closed-loop system in steady case can be described by

$$(A+B_0K_0)x(\infty)+B_0[K_{r0}Cx(\infty)+K_f]+B_f\bar{u}_f = 0 \quad (13)$$

At the same time, the new reference are expected to be as close as possible to the original reference $ref_d = ref$. Then the target is to make the following function minimum.

$$J = \min_{x(\infty)} \|Cx(\infty) - ref\|_W \quad (14)$$

and the constraints of (14) are

$$\begin{aligned} (A + B_0K_0)x(\infty) + B_0[K_{r0}Cx(\infty) + K_f] + B_f\bar{u}_f = 0 \\ u_{min} \le K_0x(\infty) + K_{r0}Cx(\infty) + K_f \le u_{max} \end{aligned} \quad (15)$$

where W is constant weight matrix, $\|x\|_W = x^T W x$, and u_{max}, u_{min} are actuator constraints.

In order to guarantee steady-state performance and satisfy actuator constraints, feedforward matrix K_{r0} is also solved by the optimal problem. In other words, the variables to be optimized are $x(\infty)$ and K_{r0}. Under this circumstance, a linear optimal problem becomes a bilinear problem.

To simplify the calculation, it is possible to make

$$S = K_{r0}Cx(\infty) \quad (16)$$

Then the above optimal problem can be considered as

$$J = \min_{x(\infty),R}(1-\varepsilon)\|Cx(\infty) - r\|_{W_1} + \varepsilon\|S\|_{W_2} \quad (17)$$

Subject to:

$$(A + B_0K_0)x(\infty) + S + B_0K_f + B_f\bar{u}_f = 0 \quad (18)$$

$$u_{min} \le K_0x(\infty) + S + K_f \le u_{max} \quad (19)$$

where ε is the weight to adjust the proportion of items in the function.

C. Self healing management

Based on information produced by FDI module, self healing management module decides whether the system is in fault-free or fault case. For fault case, the first step is to redesign a new feedback controller with remaining control inputs and then through solve the optimal problem (17)-(19), the steady-state $x(\infty)$ and S can be achieved. Additionally, the new reference ref_{new} and feedforward matrix K_{r0} can be calculate by

$$ref_{new} = Cx(\infty) \quad (20)$$

$$K_{rnew} = S[Cx(\infty)]^+ \quad (21)$$

where $(\cdot)^+$ is pseudo inverse. If $\|ref_{new} - ref\| > threshold$, ref_{new} will be the new reference of the post-fault system and K_{rnew} will be the new feedforward matrix. Otherwise, just FTC is needed and reference redesign is not required.

IV. UNMANNED HELICOPTER SIMULATION

In this section, a linear UH model includes swashplate configuration and rotor speed control input is proposed based on a simplified full-state nonlinear UH model proposed by He Y. [16]. The UH is shown in Fig. 2(a). Then the SH framework is illustrated with the linear UH model.

A. Linear unmanned helicopter model

Generally speaking, in order to simplify the control of UH, rotor-speed controller is independent of flight controller. Therefor, most of UH models do not include rotor-speed control input. But it is necessary to add rotor-speed control input into UH models so that the redundancy of UH actuators increases and more choices can be selected for FTC strategies.

In this case, the control inputs of the UH model are $\theta_{a_s}, \theta_{b_s}, \theta_M, \theta_T$ and Ω, where θ_{a_s} and θ_{b_s} represent main rotor blade cyclic pitch, θ_M and θ_T represent collective pitch of main and tail rotors and Ω represents rotor speed. Generally speaking, the five or the former four variables are elected as control inputs for most UH models. Among them, the former three variables are used for main rotor controlling through swashplate. It means that the three variables are not the real outputs of actuators and they are control inputs in name only. In other words, the actuator faults can not be modeled or identified by them directly.

For this reason, swashplate configuration which provides the relationship between nominal control inputs and real actuator outputs should be added into UH models. The swashplate is shown in Fig. 2(b). and the relationship can be described by

$$\begin{pmatrix} \theta_M \\ \theta_{a_s} \\ \theta_{b_s} \end{pmatrix} = R_s \begin{pmatrix} \theta_{M1} \\ \theta_{M2} \\ \theta_{M3} \end{pmatrix} \quad (22)$$

where θ_{M1}, θ_{M2} and θ_{M3} are the outputs of actuators and R_s is a 3×3 constant matrix depending on the connection between actuators and swashplate.

According to the above non-linear model, a linear UH model can be achieved through linearization method. The linear model can be described as

$$\dot{x} = Ax + Bu \quad (23)$$

$$x = [u\ v\ w\ \varphi\ \theta\ \psi\ p\ q\ r\ a_{1s}\ b_{1s}]^T \quad (24)$$

$$u = [\theta_{M1}\ \theta_{M2}\ \theta_{M3}\ \theta_T\ \Omega]^T \quad (25)$$

(a) Unmanned helicopter (b) Swashplate

Fig. 2. Unmanned helicopter and swashplate

where $\theta_{M\cdot}$ is the actuator outputs of main rotor; θ_T is the actuator output of tail rotor and Ω is the control input of rotor-speed.

B. Simulation results

The output vector of the UH model is $y = [u\ v\ w\ \psi]^T$. The system matrix $A = [A_1\ A_2]$ and control matrix B are shown as follows:

$$A_1 = \begin{pmatrix} 0 & -0.0126 & -0.0003 & 0 & -9.7985 & 0 \\ 0.0126 & 0 & -0.0002 & 9.7816 & -0.0100 & 0 \\ 0.0003 & 0.0002 & 0 & 0.5758 & 0.1702 & 0 \\ 0 & 0 & 0 & 0 & -0.0126 & 0 \\ 0 & 0 & 0 & 0.0126 & 0 & 0 \\ 0 & 0 & 0 & -0.0004 & 0.0002 & 0 \\ 0 & -3.2812 & 0 & 0 & 0 & 0 \\ 0.1971 & 0 & 0.0699 & 0 & 0 & 0 \\ 0 & 0.0363 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (26)$$

$$A_2 = \begin{pmatrix} 0 & -0.3611 & 0.1183 & 0 & 0 \\ 0.3611 & 0 & -0.1626 & 0 & 0 \\ -0.1183 & 0.1626 & 0 & 0 & 0 \\ 1 & 0.0010 & -0.0174 & 0 & 0 \\ 0 & 0.9983 & 0.0588 & 0 & 0 \\ 0 & -0.0588 & 0.9984 & 0 & 0 \\ 0 & 0.0040 & -0.0001 & 0 & -238.2097 \\ -0.0102 & 0 & -0.0002 & 157.6842 & 0.0162 \\ -0.0002 & 0.0001 & 0 & 0 & -1.7699 \\ 0 & -1 & 0 & -15.2439 & 9.7200 \\ -1 & 0 & 0 & 10.704 & -15.2439 \end{pmatrix} \quad (27)$$

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -62.9212 & -62.9212 & -62.9212 & 0 & -0.1486 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 22.3608 & 22.3608 & 22.3608 & -35.6075 & -0.0053 \\ -16.4446 & -16.4446 & -16.4446 & 6.6662 & -0.0344 \\ 65.4286 & 65.4286 & 65.4286 & 267.5699 & 0.4889 \\ 5.0813 & 5.0813 & -10.1626 & 0 & 0 \\ 8.8011 & -8.8011 & 0 & 0 & 0 \end{pmatrix} \quad (28)$$

For fault-free case, $Rank(B) = 5$ and the feedback matrix K is calculated by LQR method with matrix $Q = diag(1\ 0.5\ 1.5\ 10\ 10\ 1\ 0.1\ 0.1\ 0.1\ 10\ 10)$ and matrix $R = diag(100\ 100\ 100\ 100\ 0.1)$.

The reference is $ref = [4\ 4\ 0\ 0]^T$ and actuator constraints are $u_{max} = [0.15\ 0.15\ 0.15\ 0.15\ 94.20]^T$ and $u_{min} = -u_{max}$. At the same time, measurement noise is also considered.

In order to illustrate the proposed framework, assume a stuck fault occurs at instant 20s. The first actuator is stuck at 0.05 position such as $u_1 = \bar{u}_f = 0.05$ so that $B_0 = [b_2\ b_3\ b_4\ b_5]$, $Rank(B_0) = 4$ and $Rank([CB\ \ CAB\ \cdots\ CA^{n-1}B]) = 4$. The parameters of the optimal problem are such as $W_1 = W_2 = diag(1\ 1\ 1\ 1)$ and $\varepsilon = 0.001$. The simulation result without SH is shown in Fig. 3 where the real line represents the output and the dash line represents the reference. The control inputs is shown in Fig. 4 where the dash line represents the constraints of actuators. As illustrated in Fig. 3, the system outputs out of order after stuck fault occurring.

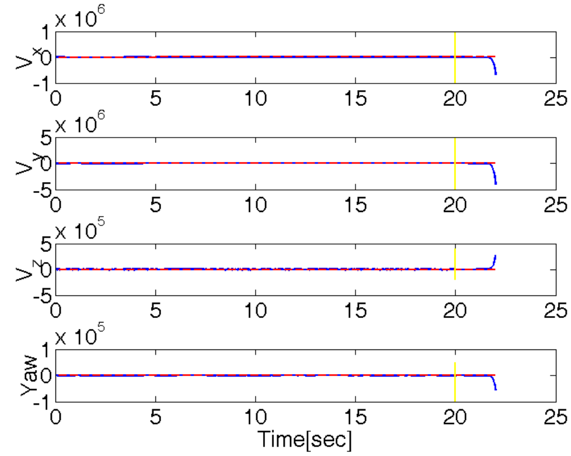

Fig. 3. System outputs without SH

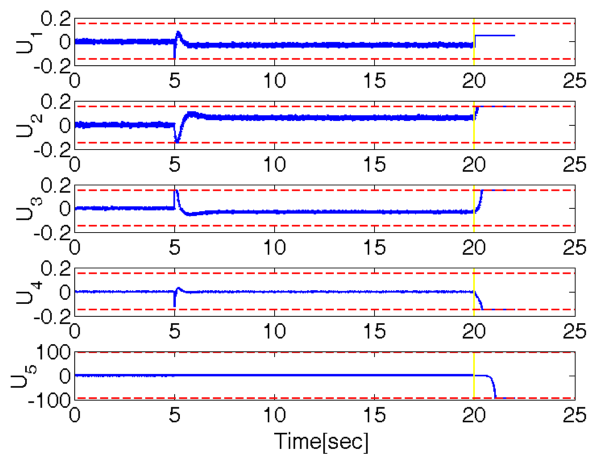

Fig. 4. Control inputs without SH

FTC controller can be designed according to (5) and K_f is achieved by (8) such as:

$$K_f = [0.0351\ 0.0506\ 0.0772\ -60.8372]^T; \quad (29)$$

The system outputs with FTC controller is shown in Fig. 5 where the dotted line is the original reference, the chain line is the output without reference redesign and control inputs is shown in Fig 6. As illustrated in the figures, the system outputs is stable at last but there exist non-zero offset. It means the FTC controller can not track the original reference with taking into account actuator constraints.

In this case, reference redesign is required. According to solve the optimal problem (17)-(19),the new reference $ref_{new}$and new feedforward matrix $K_{rnew} = S[Cx(\infty)]^+$ is obtained

$$ref_{new} = [4.0028\ 3.7060\ -0.0052\ 0.0006]^T \quad (30)$$

$$K_{rnew} = \begin{pmatrix} 0.042492 & 0.039342 & -0.000055 & 0.000006 \\ 0.083019 & 0.076863 & -0.000107 & 0.000013 \\ -0.050765 & -0.047001 & 0.000065 & -0.000008 \\ -3.616373 & -3.348229 & 0.004660 & -0.000548 \end{pmatrix} \quad (31)$$

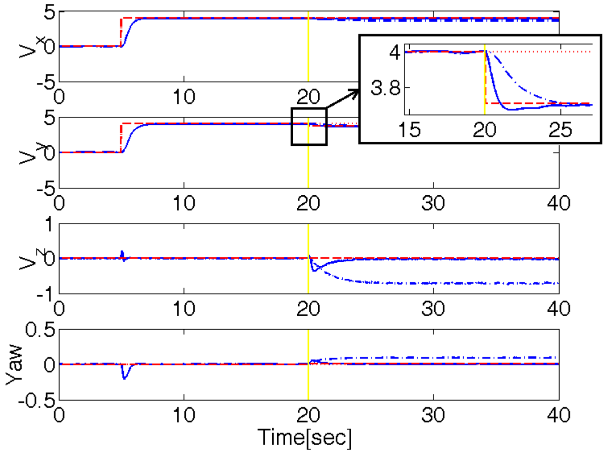

Fig. 5. System outputs with SH (reference redesign), ··: original reference; −·: output with FTC; −−: new reference −: output with SH.

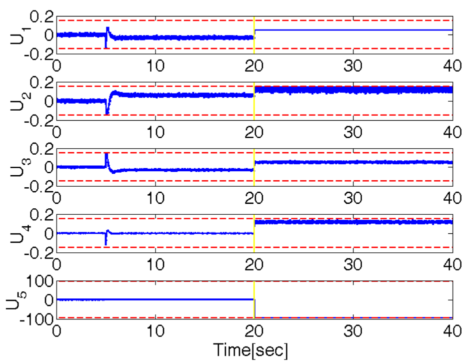

Fig. 6. Control inputs without reference redesign

As illustrated in Fig. 5, where the dash line is the new reference and the real line is the output, the system output can track the new reference offset-free and the error between the original and the new reference is less than the error without reference redesign. A comparison between the control inputs with/without reference redesign is shown in Fig. 7 (without noise),they almost have the same value in steady case but the dynamic process are different.

V. CONCLUSIONS

Self healing framework is proposed in this manuscript which can guarantee the system to track the safety or desired reference as close as possible with actuator stuck faults and actuator constraints. As parts of the framework, a new actuator-stuck compensating method based on output analytical redundancy and reference redesign method for achieving new steady-state reference have been investigated. At the same time, a UH model which includes both rotor-speed control input and swashplate configuration is proposed. Compared with normal UH models, the proposed one has more actuator redundancy and direct relationship with real actuator outputs. Finally, the simulation results based on the UH model illustrate the effectiveness of the proposed self

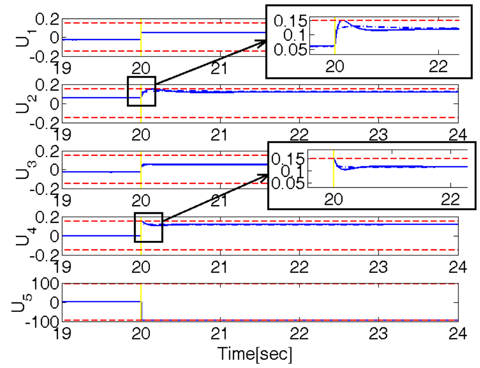

Fig. 7. Control inputs with SH (reference redesign), −·: control inputs without reference redesign −: control inputs with reference redesign.

healing framework.

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