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Quadrotors Data Fusion using a Particle Filter*

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Abstract— **This paper presents a data fusion algorithm by means of a particle filter (PF) for estimate the position of a quadrotor equipped with multiple sensors. A global positioning system (GPS) is considered for position measurement in a loosely coupled scheme, velocity can be obtained from an optic flow sensor, whilst an inertial measurement unit (IMU) can provide orientation and angular rate measurements. Simulations were carried out, where real noise from a not expensive GPS is added to the simulated position to test the proposed algorithm.**

I. INTRODUCTION

Unmanned Aerial Vehicles (UAVs) have gained great relevance in the last years thanks to their huge potential in many civilian and military applications such as exploration, surveillance, search and rescue, in a cheap way and without risking human lives in dangerous situations. Particularly, four rotors rotorcrafts, also known as quadrotors, have received special interest since their rotor's configuration produces cancellation of the reactive torques, considerably simplifying their analysis and control. Also, they are suitable for vertical take off and landing, as well as hovering, making them a good choice for maneuvering in small spaces or perform high precision tasks.

Several stabilization and trajectory tracking control laws have been proposed for this kind of systems and validated through simulations and experiments in indoor applications, relying on a good measurement of the position and velocity. To cite some examples, [1] proposes an attitude stabilization control strategy for hover flight using nested saturations, while in [2] a sliding mode approach is used to accomplish position control of a quadrotor. In [3] a trajectory tracking control by means of a discrete time feedback linearization control scheme is proposed.

However, big effort is still required to accomplish autonomous flight for outdoor applications, due to presence of external perturbations, especially the wind, and the lack of a good measurement for the position and velocity. Not expensive Global Positioning Systems (GPS) sensors can provide an estimation of the position and velocity, however, the errors, of 2m at best, and their low measurement rate of about 5Hz, are not suitable for precise applications and can interfere with the system stability, even more, GPS can easily lost its signal leaving the system without a position measurement. Another alternative widely studied are the optical flow sensors which use computer vision algorithms for estimating the motion velocity of a system, but they are noisy and sensibles for lighting changes.

Data fusion algorithms are an interesting solution to this problem, they take information from multiple sensors, especially GPS, cameras and IMU, to improve the estimation of the position and velocity. The Kalman filter an its variations are the mos popular approach, for example, in [5] an observer-control scheme for quadrotors using the Extended Kalman Filter (EKF) is proposed and tested in real time indoor experiments, using data from an optic flow algorithm and an IMU. In [6], a Robust Adaptive Kalman Filter is proposed for estimation of UAV dynamics in the presence of sensor faults, see also [7], [8], [9]. However, this techniques assume that the process and measurement noise are Gaussian, which is not the case of a GPS in a loosely coupled scheme.

The great advances in micro controllers processing capacity allow to explore other alternatives. The Particle Filter (PF) presents an interesting option to this problems, at the cost of computational expense, because it is not restricted to Gaussian noise. Some interesting works about particle filtering can be found in the literature, like in [11] where a 3-D model based visual tracking approach in a particle filtering framework, while in [10] is presented a vision based algorithm using a particle filter tracking for estimating the position of a quadrotor. Both of them relay on information obtained by computer vision algorithms, while this work focus on GPS data.

This paper is aimed to improve the inaccurate and noisy measurements obtained by not expensive GPS sensors, in order to accomplish high precision tasks in autonomous outdoor flight. The use of a PF is explored, through simulations, for data fusion and state estimation, where real GPS data was collected and the obtained error was added to the simulated position.

The paper proceeds as follows: In section II the dynamic model of the quadrotor is given. In section III, the PF for the quadrotor state estimation is proposed. In section IV the performance of the PF is tested in simulation. Section V gives the conclusions and future work.

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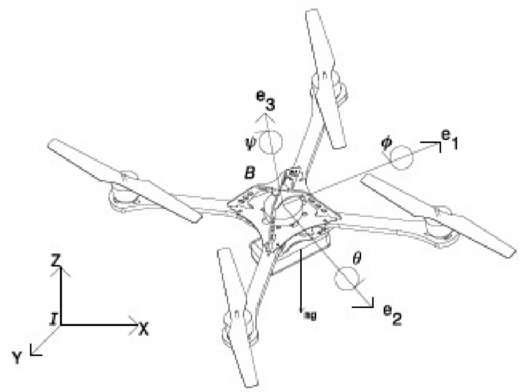

Fig. 1. Quadrotor in an inertial reference frame.

II. QUADROTOR DYNAMIC MODEL

The quadrotor can be represented as a rigid body in space with mass m and inertia matrix J, subject to gravitational and aerodynamic forces. Let us consider an inertial coordinate frame $I = \{X\ Y\ Z$, fixed to the ground and a body fixed coordinate frame, $B = \{e_1, e_2, e_3\}$, see Fig. 1. Consider the vectors

$$\xi = [x\ y\ z]^T \tag{1}$$

$$\Phi = [\phi\ \ \theta\ \ \psi]^T \tag{2}$$

which stand for the position of the center of gravity, with respect to the inertial frame I, and the Euler angles roll, pitch and yaw, respectively. The motion equations are given by the Newton-Euler equations in the inertial frame I [12]

$$m\ddot{\xi} = TRe_3 - mge_3 \tag{3}$$

$$J\dot{\Omega} = -\Omega_x J\Omega + \Gamma \tag{4}$$

where $T \in \Re^+$ is the total thrust of the motors, g is the gravity constant and $\Gamma \in \Re^3$ is the input torque defined in the body fixed frame B. $R \in SO(3) : B \to I$ is the rotational matrix from the body frame to the inertial frame.

$$\Omega = [p\ q\ r]^T \tag{5}$$

represents the angular velocity in the body frame B. Ω_x stands for the skew symmetric matrix such that $\Omega_x \upsilon = \Omega \mathrm{x} \upsilon$ is the vector cross product. The kinematic relation between the generalized velocities $\dot{\Phi} = (\dot{\phi}, \dot{\theta}, \dot{\psi})$ and the angular velocity Ω is given by [13]

$$\Omega = Q\dot{\Phi} \tag{6}$$

with

$$Q = \begin{bmatrix} 1 & 0 & -s\theta \\ 0 & c\phi & c\theta s\phi \\ 0 & -s\phi & c\theta c\phi \end{bmatrix} \tag{7}$$

III. PARTICLE FILTER

Consider the state vector $\chi \in \Re^{12}$

$$\chi = [\xi\ \ \dot{\xi}\ \ \Phi\ \ \dot{\Phi}]^T \tag{8}$$

Then from (3), (4), with a sampling time T_s small enough, the discrete time evolution model is given by

$$\dot{\chi}_k = \begin{bmatrix} \dot{\xi}_{k-1} T_s + \xi_{k-1} \\ (\frac{1}{m} T_{k-1} R_{k-1} e_3 - g e_3) T_s + \dot{\xi}_{k-1} \\ \dot{\Phi}_{k-1} T_s + \Phi_{k-1} \\ \bar{\Gamma}_{k-1} T_s + \dot{\Phi}_{k-1} \end{bmatrix} + \omega_k \tag{9}$$

where the total thrust T and the control torque $\bar{\Gamma}$ are the system inputs. ω stands for the model noise with a probability density p_ω.

Consider also the measurement vector $Y = [\xi_{GPS}\ \ \dot{\xi}_{OF}\ \ \Phi_{IMU}\ \ \dot{\Phi}_{IMU}]^T$, it is the position measured by a GPS, the velocity measured by an optic flow sensor and the orientation and angular rate obtained by an IMU. Hence, the observation model is

$$Y_k = \chi_k + \nu_k \tag{10}$$

ν represents the measurement noise with a probability density p_ν.

The particle filter (PF) or sequential Monte Carlo technique is a kind of recursive Bayesian filter for state estimation [14]. It consists in generating N_p particles according to the a priori probability, with an importance weight W defined as

$$W_k = \frac{p(Y_{1:k}|\chi_{0:k})p(\chi_{0:k})}{q(\chi_{0:k}|Y_{1:k})} \tag{11}$$

where $p(Y|\chi)$ is the probability density function of Y given χ, and $q(\chi_k|Y_k)$ is the so-called proposal distribution.

At each iteration, the outputs are measured by the sensors and the weights can be updated recursively, using the bootstrap method, by the following expression

$$\begin{gathered} W_k^i = W_{k-1}^i p(Y_k|\chi_k^i) = W_{k-1}^i p_\nu(Y_k - \chi_k^i) \\ i = 1, ..., N_p \end{gathered} \tag{12}$$

for a normal probability distribution in the measurement, with covariance matrix Q

$$W_k^i = W_{k-1}^i e^{(-\frac{1}{2}(Y_k - \chi_k^i)Q^{-1}(Y_k - \chi_k^i))} \tag{13}$$

normalizing

$$\bar{W}_k^i = \frac{W_k^i}{\Sigma_{i=1}^{N_p} W_k^i} \tag{14}$$

Then, the set of particles evolves following the evolution model (9), with ω_k^i randomly generated according to p_ω.

In order to avoid the weight degeneracy problem, a resampling step is required. In order to do so, the effective sample size N_{eff} is employed as a measure of the number of active particles

$$N_{eff} = \frac{1}{\Sigma_{i=1}^{N_p}(W_k^i)^2} \tag{15}$$

it can be noticed that this value is maximum ($N_{eff} = N_p$) when all the particles have the same weight, and minimum ($N_{eff} = 1$) when only one weight is different from zero. If the effective sample size is lower than a certain threshold $N_{eff} < N_{th}$, the resampling step is performed by means of the Kitagawa method [15].
The estimated stated is given by the particles barycenter, it is

$$\bar{\chi}_k = \Sigma_{i=1}^{N_p} W_k^i \chi_k^i \tag{16}$$

Since GPS sample frequency is slow (5Hz) with respect to the others sensors, and in order to deal with the GPS loss signal, the measured position is updated in the following form

$$\xi_{GPSk} = \xi_{GPSk-n} + nT_s\dot{\xi}_k \tag{17}$$

where n is the number of sample times T_s passed since the last valid GPS data.

IV. SIMULATIONS

Simulations were carried out to test the proposed estimation algorithm for a quadrotor with an inaccurate position measurement, which is the case for a not expensive GPS. GPS errors were obtained from real experiments in an urban environment surrounded by buildings, and added to the simulated actual position, taking also into account the GPS measurement rate (5Hz) and including signal losses of 20 seconds every 100 seconds. The described GPS errors can be depicted in Fig. 2, where the zero values every 100 seconds represent GPS signal losses. Optical flow sensors can provide a noisy velocity measurement, hence some white noise was added to the velocity signal. The noisy measured velocity is shown in Fig. 3.

The other measurements, from the IMU and the visual flow sensors, are supposed to work at a frequency of 100Hz, which is an usual value for IMUs. The PF updates at the same sampling rate of 100Hz, while the slower GPS measurements, at 5Hz, are updated according to 17.

The performance of the particle filter for position estimation while the UAV evolves in a spiral trajectory can be seen in Fig. 4, using $N_p = 100$ particles and a resampling threshold of $N_{th} = 10$. It can be noticed that the PF performs an excellent position estimation despite the poor position measurement, this is clearer from Fig. 5 where the estimation error is displayed, which can be compared against the measurement error in Fig. 2. It is clear that the position

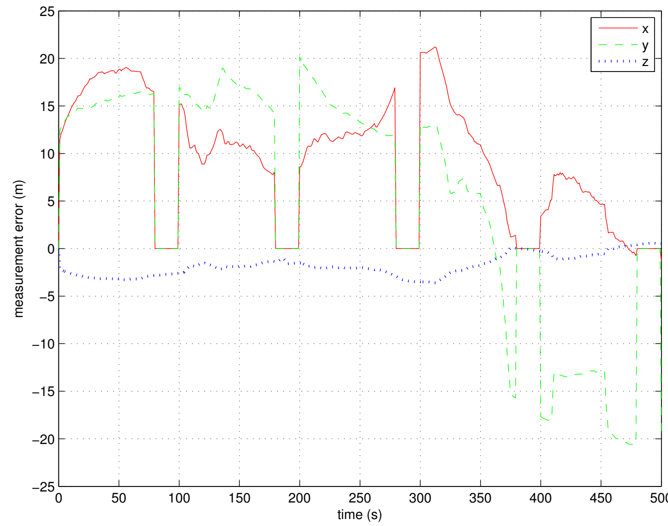

Fig. 2. GPS measurement error.

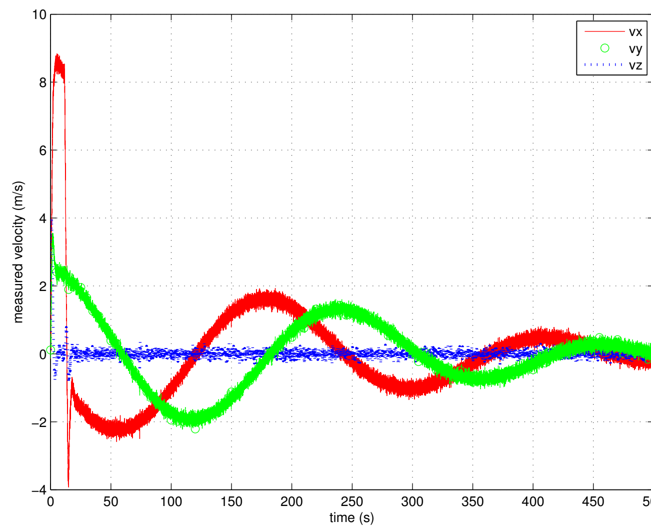

Fig. 3. Measured velocity.

estimation greatly improved the measured one and perfectly handled the signal losses and from measurement errors of until 20 meters are obtained error of less than one meter, making it suitable for precision tasks. In order to provide a comparison against other estimation techniques, the popular EKF was also tested under the same conditions and the obtained estimation errors are presented at Fig. 6, where it can be seen that the errors are much higher than the obtained with the PF, this can be explained due to the fact that the EKF assumes that measurement noise is Gaussian, which is not the case for a GPS in a loosely coupled scheme (see Fig. 2). Finally, Figs. 7 and 8 contain the XY plane view and the three dimensional one with the real and estimated positions.

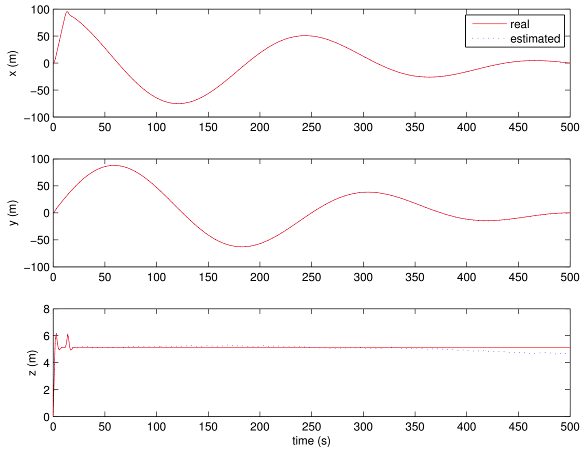

Fig. 4. Position.

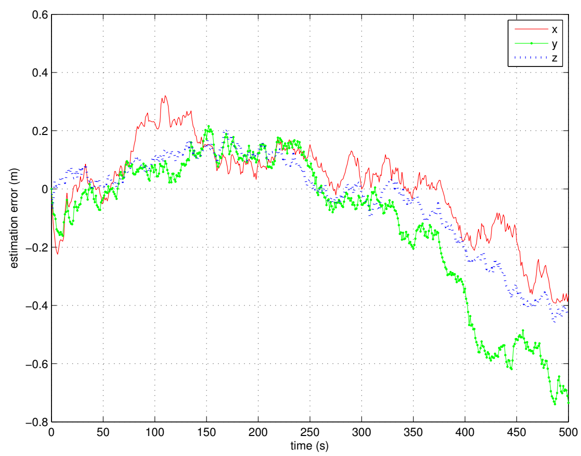

Fig. 5. PF estimation error.

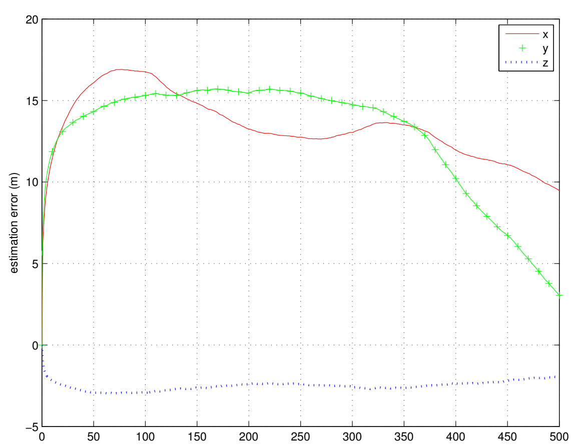

Fig. 6. EKF estimation error.

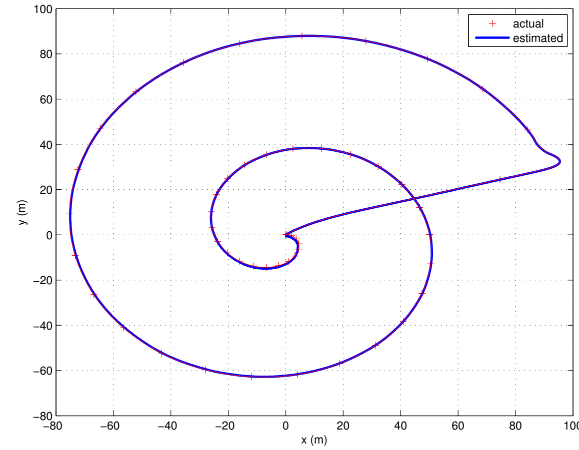

Fig. 7. XY plane.

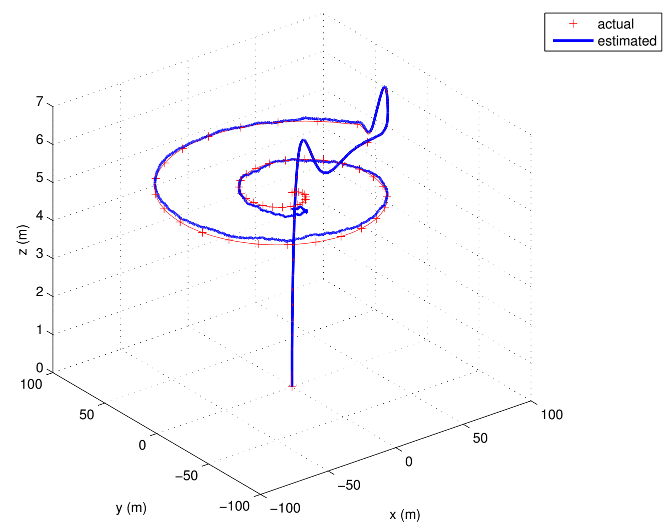

Fig. 8. 3d.

V. CONCLUSIONS AND FUTURE WORK

Simulations have shown promising results of the proposed data fusion strategy, considerably reducing the measurement error while handling the GPS signal losses.

The PF proved to be more adequate in this case than the EKF, since the measurement noise of a GPS in a loosely coupled scheme is not Gaussian.

Future work includes to implement the proposed strategy embedded on a quadrotor equipped with a GPS sensor, for real time experiments. This can be done as long as a good measurement of the velocity is available, some optical flow sensors are already found in the market and are to be tested for this purpose.

It is also desired to use the estimated data to close the loop in a control strategy to accomplish trajectory tracking for outdoor applications.

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